PHYS*4150: Problem Set 1

Distributed: Friday January 11, 2019
Due: Friday January 25, 2019 at 10:30am

Problem 1 (15 pts): Drude model basics

a) The resistivity ρ is a proportionality constant which relates an applied electric field to the current density it induces:

$$\mathbf{E} = \rho \mathbf{j}. \tag{1}$$

Consider a current flowing through a simple metallic wire with a length of L and cross-sectional area A. Express the potential difference (V) between the ends of the wire, current (I) flowing through the wire, and resistance (R) of the wire using only these five parameters (E, ρ, j, L, and A). Use these to confirm Ohm's Law.

b) Use simple arguments to justify A&M eq. 1.4:

$$\mathbf{j} = -ne\mathbf{v} \tag{2}$$

where n is the carrier density of the metal. Use unit analysis to confirm this result.

c) Calculate the average volume per electron in metallic copper at room temperature. Use values of Z and n from A&M Table 1.1. Using the measured resistivity of copper shown in A&M Table 1.2 (use value for 273 K), calculate the mean relaxation time τ. Calculate the mean free path of the electrons between collisions. How does this compare to the atomic spacing of the metal?

Problem 2 (15 pts): Drude equation of motion

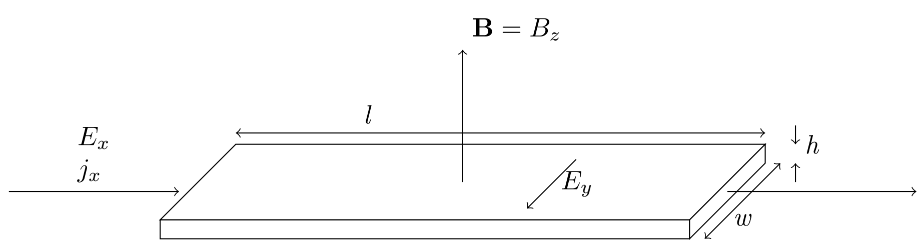
Starting from the expression for current density, $\mathbf{j} = -ne\mathbf{v}$, derive the Drude model equation of motion, A&M eq. 1.12:

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{f}(t) - \frac{\mathbf{p}}{\tau}. \tag{3}$$

Be explicit in your reasoning and explain each step.

Problem 3 (10 pts): Hall effect experiment

Consider the simple ***Hall effect*** apparatus displayed below:



Imagine you are performing this experiment in your lab. You have control over, and can vary, the applied magnetic field **B** and have access only to a voltmeter/ammeter and a ruler (a really good one) to make measurements. Describe in detail how you would obtain an experimental value for the *Hall coefficient* R_H:

$$R_H = \frac{E_y}{j_x B_z}. \tag{4}$$

From this measurement, what two things can you infer about the charge carriers in the metal? (*Hint: use A&M eq. 1.21 to guide you.*)

Problem 4 (10 pts): EM wave equation

Starting from Maxwell's equations, derive the electromagnetic wave equation for the Drude model, as expressed by A&M eq. 1.34:

$$-\nabla^2 \mathbf{E} = \frac{\omega^2}{c^2}\epsilon(\omega)\mathbf{E} \tag{5}$$

Ensure this is consistent with the result for the dielectric parameter shown in class and in A&M eq. 1.35. You may use *either* S.I. units or cgs units, but make sure you are consistent. In S.I. units, Maxwell's equations can be expressed as

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0 \tag{6a}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{6b}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{6c}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{6d}$$

where $\mathbf{j}$ is the current density. Please be explicit with each step of the derivation and note any assumptions that are made.

Problem 5 (20 pts): Joule Heating

Ashcroft & Mermin question 1.2

Problem 6 (20 pts): Classical Limit of Fermi-Dirac Statistics

Ashcroft & Mermin question 2.3