# Midterm Solu.

## Prob. 1

1. c) $n \approx 10^{28}\ m^{-3} \approx 10^{22}\ cm^{-3}$

$\Rightarrow n = N_A \frac{Z \rho}{A}$ $\Rightarrow$ $Z$ − # valance $e^-$
$A$ − atomic number
$\rho$ − density of metal $(g/m^3)$

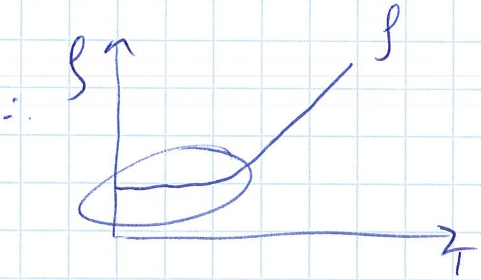
2.

| | $\varepsilon_F$ | $\mu$ |
|---|---|---|
| a) | ✓ | ✓ |
| b) | ~x | ✓ |
| c) | ✓ | x |
| d) | ~x | ✓ |

$\Rightarrow$ $\mu$ in general is energy to add $e^-$

$\varepsilon_F$ is energy of HO state @ $T=0$.

3. Drude: $\sigma = \frac{1}{\rho}$ = constant w/ temp.




4. $\omega > \omega_p$ $\Rightarrow$ $\omega_p$ = plasma freq.

$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}$

5. Drude $\Rightarrow$ classical values of $v_0$ & $C_v$.

Sommerfeld $\Rightarrow$ use QM for free particle to estimate $v_0$ & $C_v$.

6. FEM: $B = \frac{1}{K} = \frac{2}{3} n \varepsilon_F$

higher B = harder $\therefore$ higher $\varepsilon_F \Rightarrow$ harder.

7. SC $\rightarrow$ BCC $\rightarrow$ FCC

$a^3$ $\quad \frac{a^3}{2}$ $\quad a^3/4$

8. real lattice & reciprocal lattice are Fourier transforms of each other.

9. (100), (111), (121), (411)

highest density $\longrightarrow$ lowest.

10.
a) true
b) true
c) ~~true~~ $\rightarrow$ false.
d) true

Prob. 2

$$-\frac{\hbar^2}{2m}\psi''(x) = \varepsilon\,\psi(x)$$

$$\psi = \sqrt{\frac{1}{L}}\,e^{i\vec{k}\cdot\vec{x}} = \sqrt{\frac{1}{L}}\,e^{ik_x x}$$

a) $\psi(x) = \psi(x+L)$

$$\therefore\ \cancel{\sqrt{\frac{1}{L}}}\,e^{ikx} = \cancel{\sqrt{\frac{1}{L}}}\,e^{ik(x+L)}$$

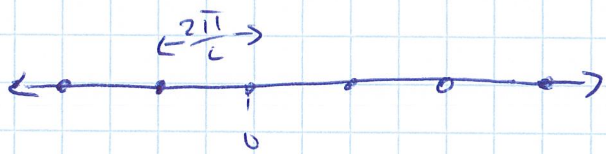
4

$$\therefore\ e^{ikL} = 1$$

$$\therefore\ kL = 2\pi n \qquad n \Rightarrow \text{integer.}$$

$$\boxed{k = \frac{2\pi}{L}\,n}$$

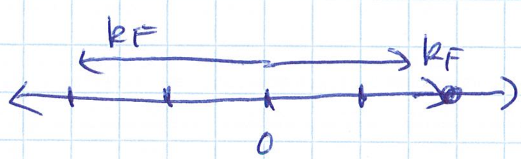
B)




k-space volume per $k \Rightarrow \Omega_k = \frac{2\pi}{L}$

$$\Delta k = \frac{1}{\Omega_k} = \frac{L}{2\pi} \Rightarrow \text{states per volume}$$

b) $\Delta k = \frac{L}{2\pi}$




3

Fill to $k = k_F \Rightarrow \Omega_k = 2k_F$

2 $\bar{e}$ per state

$$\#\text{ of } e^- = N = 2 \cdot \Delta k \cdot \Omega_k$$

$$= 2 \cdot \frac{L}{2\pi} \cdot 2k_F.$$

$$\therefore n = \frac{N}{L} = \frac{2}{\pi} k_F \Rightarrow k_F = \frac{\pi}{2} n$$

c) $$D = \frac{1}{L} \sum_{\text{states}} \delta(\varepsilon - \varepsilon_k)$$

Recall: $\sum_k F_k$

$\Rightarrow \left(\frac{L}{2\pi}\right)^d \int_k F_k \, dk.$

$$= \frac{2}{L} \sum_k \delta(\varepsilon - \varepsilon_k)$$

$$= 2 \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\varepsilon - \varepsilon_k) \, dk = \frac{2 \cdot 2}{2\pi} \int_0^{\infty} \delta(\varepsilon - \varepsilon_k) \, dk$$

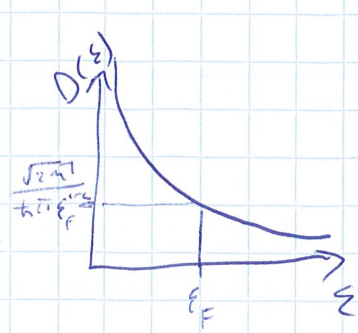
change of limits gives factor of 2.

$$dk = dk. \qquad \varepsilon = \frac{\hbar^2 k^2}{2m}$$

$$d\varepsilon = \frac{\cancel{2}\hbar^2 k}{\cancel{2}m} \, dk.$$

$$\frac{m}{\hbar^2 k} \, d\varepsilon = dk.$$

$$\Rightarrow \frac{m}{\sqrt{2m\varepsilon}\,\hbar} \, d\varepsilon = \sqrt{\frac{m}{2\varepsilon \hbar^2}} \, d\varepsilon.$$




$$D = \frac{2 \cdot 2}{2\pi} \int_0^{\infty} \delta(\varepsilon - \varepsilon_i) \sqrt{\frac{m}{2\hbar^2}} \frac{1}{\sqrt{\varepsilon}} \, d\varepsilon.$$

$$= \sqrt{\frac{2m}{\hbar^2 \pi^2}} \cdot \frac{1}{\varepsilon^{1/2}} = \frac{\sqrt{2m}}{\hbar \pi} \frac{1}{\varepsilon^{1/2}}$$

d) show $u = \frac{1}{3} n \varepsilon_F$

@ $\cancel{T} = 0 \Rightarrow f(\varepsilon) = 1 \quad , \varepsilon < \varepsilon_F$

$= 0 \quad , \varepsilon > \varepsilon_F .$

5

$u = \int D(\varepsilon)\, \varepsilon\, f(\varepsilon)\, d\varepsilon .$

$$u = \int_0^{\varepsilon_F} \frac{\sqrt{2m}}{\pi \hbar} \frac{1}{\sqrt{\varepsilon}} \varepsilon \, d\varepsilon .$$

$$= \frac{\sqrt{2m}}{\pi \hbar} \int_0^{\varepsilon_F} \varepsilon^{1/2} \, d\varepsilon$$

$$= \frac{\sqrt{2m}}{\pi \hbar} \frac{2}{3} \varepsilon_F^{3/2}$$

Note $n = \frac{2}{\pi} k_F \Rightarrow \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$

$$\therefore n = \frac{2}{\pi} \frac{\sqrt{2m}}{\hbar} \varepsilon_F^{1/2}$$

$$= \frac{1}{3} n \varepsilon_F$$

e) even though $D(0) = \infty$, the energy $= 0$.

$\therefore \quad 0 \cdot D(0) = 0$

$\rightarrow$ goes to $\infty$ as $\frac{1}{\varepsilon^{1/2}}$

2

i.e. even though $D(\varepsilon)$ is large as $\varepsilon \to 0$

$\varepsilon \to 0$ faster.

Prob. 3

2 a) $P = 3\bar{a}_1 - 2\bar{a}_2$

3 b) $\bar{a}_1 = b\hat{x}$

$\bar{a}_2 = \frac{b}{2}\hat{x} + \frac{c}{2}\hat{y} = \frac{1}{2}(b\hat{x} + c\hat{y})$

c) volume Area $\Rightarrow |\bar{a}_1 \times \bar{a}_2|$

3

$A = \frac{1}{2} b \cdot c$

2 d) 2 petagons. $\Rightarrow$ can see there are 2 distinct pentagon orientations.

e) a) yes. $\Rightarrow$ prim.

b) no $\rightarrow$ trans down/up does not move to same lattice pt.

5 + 1 + 1

justify

is prim cells

7

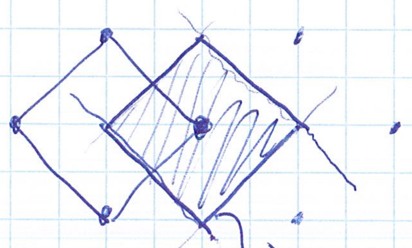
c) yes $\rightarrow$ prim.

d) no $\rightarrow$ trans L/R does not yield lattice pt.

e) yes $\rightarrow$ not prim (2 × prim.).

f)

3



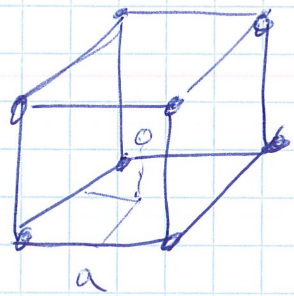
w-s cell.

# Problem 4

$Cs^+$ @ $(0\,0\,0)$ $\qquad$ $Cl^-$ @ $(\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})$

2 a) Coord. # = 8

b)




$a = 4.119$ Å

3

$Cs$ @ $(0\,0\,0)$

$Cl$ @ $(\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})$

$$r = \frac{1}{2}(\bar{a}_1 + \bar{a}_2 + \bar{a}_3)$$

$$|\bar{r}| = \sqrt{\frac{1}{4}(3a^2)} = \frac{\sqrt{3}}{2}a = 3.57 \text{ Å}.$$

c) $$S_K = \sum_j f_j \, e^{iK\cdot d_j}$$

5 $$= f_{Cs}\cdot e^0 + f_{Cl}\, e^{i\bar{K}\cdot(\frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{2}a_3)}$$

$$K = hb_1 + kb_2 + lb_3 \quad \& \quad b_i\cdot a_j = 2\pi\delta_{ij}$$

$$\therefore \; S_K = f_{Cs} + f_{Cl}\, e^{i\pi(h+k+l)}$$

if $h+k+l$ = even.

$$S_k = f_{cs} + f_{cl}\, e^{i 2\pi n} \qquad (n \text{ int}).$$
$$= f_{cs} + f_{cl}$$

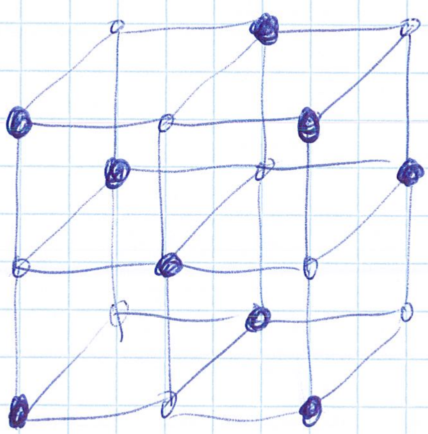
if $h+k+l$ = odd.

$$S_k = f_{cs} + f_{cl}\, e^{i(2n+1)\pi}$$
$$= f_{cs} - f_{cl}$$

$$\therefore \quad S_k = \begin{cases} f_{cs} + f_{cl} & h+k+l = \text{even} \\ f_{cs} - f_{cl} & h+k+l = \text{odd}. \end{cases}$$

scattering amp $= |S_k|^2$

d)



$\bullet \to f_{cs} + f_{cl}$

$\circ \to f_{cs} - f_{cl}$

3

2

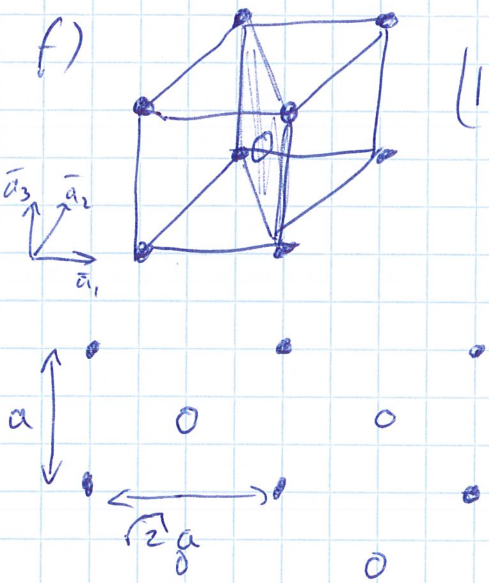
e) $\Omega_0 = a^3$

$$\Omega_k = \left(\frac{2\pi}{a}\right)^3 = \frac{8\pi^3}{a^3}$$

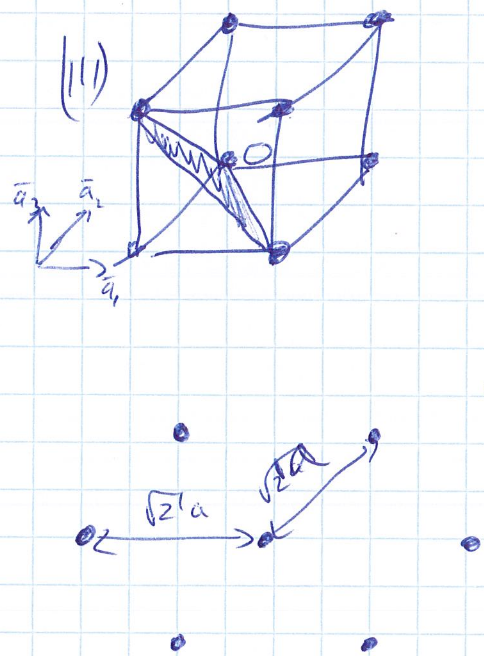
$a = 4.119$ Å

$\Omega_k =$ 3.55 Å$^{-3}$

f) (110)




(111)




simple cube
side: a.

all same
ions.

2

2

+1