

Interpreting regression coefficients when X or Y or both are log-transformed:

MBC 638

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level - level

$$\hat{y} = b_0 + b_1 \cdot x$$

Math (optional)

Example

Intercept: When $x=0$, $\hat{y}=b_0$.

Slope: $\frac{d\hat{y}}{dx} = b_1$
→ When $dx=1$,
 $d\hat{y} = b_1$

If X goes up by 1 unit of X,
Y is predicted to go up by b_1 units of Y.

Y = daily sales of beer (\$000)
X = temperature outside (°F)

$$\hat{y} = 3.5 + 2 \cdot x$$

Intercept: When temp = 0°F,
sales of beer are predicted
to be \$3,500

Slope: When temperature
increases by 1°F sales
are predicted to go up by
\$2,000.

log - level

$$\ln \hat{y} = b_0 + b_1 \cdot X$$

Math (optional)

$$\hat{y} = e^{b_0 + b_1 \cdot X}$$

Intercept: When $X=0$, $\hat{y} = e^{b_0}$

Slope: $\frac{d\hat{y}}{dX} = b_1 \cdot e^{b_0 + b_1 \cdot X} = b_1 \cdot \hat{y}$

$$\frac{d\hat{y}}{\hat{y}} = b_1 \cdot dX$$

Multiply both sides by 100:

$$\frac{d\hat{y}}{\hat{y}} \cdot 100 = b_1 \cdot dX \cdot 100 = b_1 \cdot 100$$

relative change in \hat{y}

$$\frac{d\hat{y}}{\hat{y}} \cdot 100 = b_1 \cdot 100$$

If X goes up by 1 unit of X ,
 \hat{y} is predicted to go up by $b_1 \cdot 100$ percent.

Example

Y = sales of beer (\$1000)

X = temp °F

$$\ln \hat{y} = 1.5 + 0.05X$$

Intercept: When temp
 = 0°F sales are
 predicted to be
 \$4,482. ($e^{1.5}$ thousand)

Slope: When temp.
 increases by 1°F
 sales are predicted
 to go up by 5%.

level - log

$$\hat{y} = b_0 + b_1 \cdot \ln X$$

Math (optional)

Intercept: When $X=1$, $\hat{y} = b_0$.

Slope: $\frac{d\hat{y}}{dx} = \frac{b_1}{x}$

$$d\hat{y} = b_1 \cdot \underbrace{\frac{dx}{x}}_{\text{relative change in } X}$$

Multiply and divide by 100:

$$d\hat{y} = b_1/100 \cdot \underbrace{\frac{dx}{x} \cdot 100}_{\text{percentage change in } X} = 1\%$$

If X goes up by 1%,
 \hat{y} is predicted to go up by $b_1/100$ units of \hat{y} .

Example

Y = sales of beer (\$'000)

X = temperature ($^{\circ}\text{F}$)

$$\hat{y} = 35 + 15 \cdot \ln X$$

Intercept: When temp = 1°F ,
sales are predicted to be
\$35,000.

Slope: When temperature
increases by 1%,
sales are predicted to go
up by \$150 (15/100 thousand).

log-log

$$\ln Y = b_0 + b_1 \cdot \ln X$$

Math (optional)

$$\hat{Y} = e^{b_0 + b_1 \cdot \ln X}$$

Intercept: When $X=1$, Y is predicted to be $\hat{Y} = e^{b_0}$.
($\ln 1 = 0$)

Slope: $\frac{d\hat{Y}}{dX} = \frac{b_1}{X} \cdot e^{b_0 + b_1 \cdot \ln X}$

$$\frac{d\hat{Y}}{\hat{Y}} = b_1 \cdot \frac{dX}{X}$$

relative change in Y

relative change in X

Multiply both sides by 100.

$$\frac{d\hat{Y}}{\hat{Y}} \cdot 100 = b_1 \cdot \frac{dX}{X} \cdot 100$$

% change in Y

% change in X
(1%)

If X goes up by 1%, Y is predicted to go up by b_1 %.

Example

Y = sales of beer (\$'000)

X = temp °F

$$\ln Y = 0.25 + 7 \cdot \ln X$$

Intercept: When temp = 1°F, sales are predicted to be \$1,280 ($e^{0.25}$ thou)

$$\$1.28_{\text{thou}} = \$1,280$$

Slope: When temperature increases by 1% sales are predicted to go up by 7%.