MBC 638
10/30/17

Lecture 10, slide 12

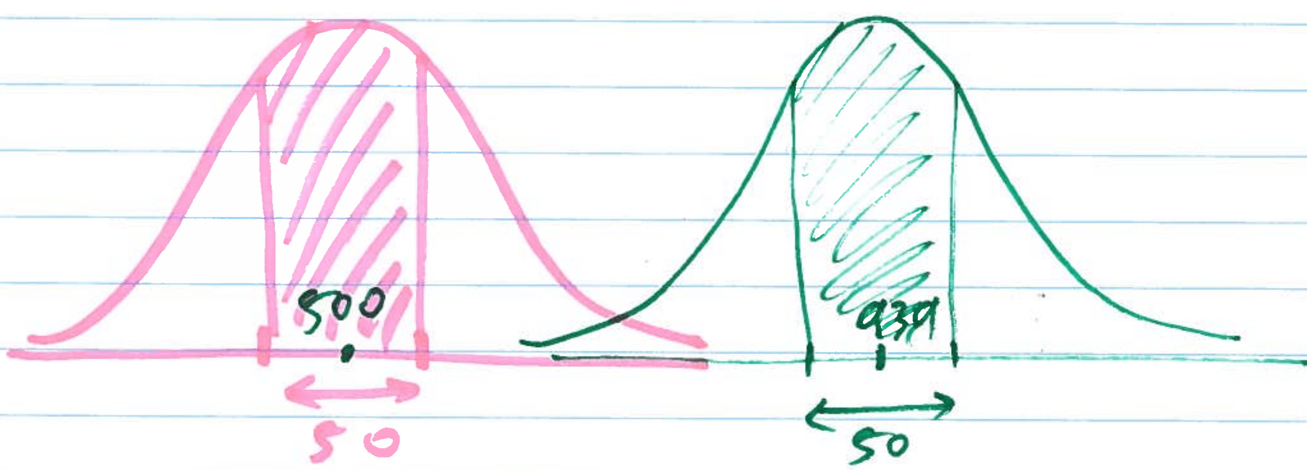
$\mu = 939$
$\sigma = 245$

a) $P(939-25 < \bar{x} < 939+25) = \boxed{0.5294}$

$\bar{x} \approx$ Normal $(n \geq 30)$
$n = 5$
$\mu_{\bar{x}} = \mu = 939$
$\sigma_{\bar{x}} = \sigma / \sqrt{n} = \frac{245}{\sqrt{50}} \ldots$

Lecture 10, slide 13

$\sigma = 245$
$P(\mu - 25 < \bar{x} < \mu + 25) = ??$



z-score transformation

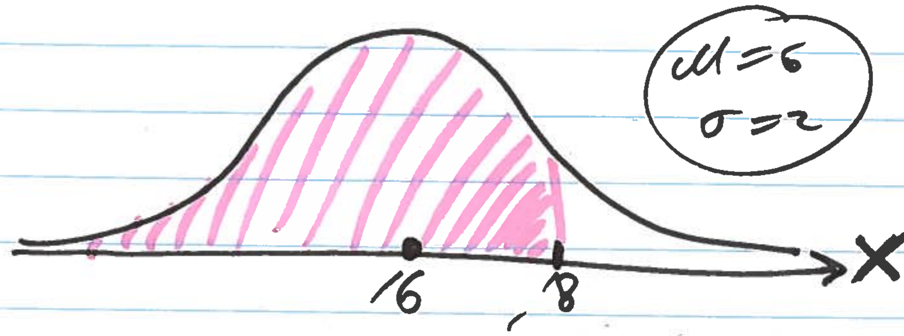
$$P\left(\frac{(\mu-25)-\mu}{\sigma/\sqrt{n}} < Z_{\bar{x}} < \frac{(\mu+25)-\mu}{\sigma/\sqrt{n}}\right)$$

$$P\left(-\frac{25}{245/\sqrt{50}} < Z < \frac{25}{245/\sqrt{50}}\right)$$

Example X is Normally distributed

$\mu = 6 \qquad \sigma = 2$

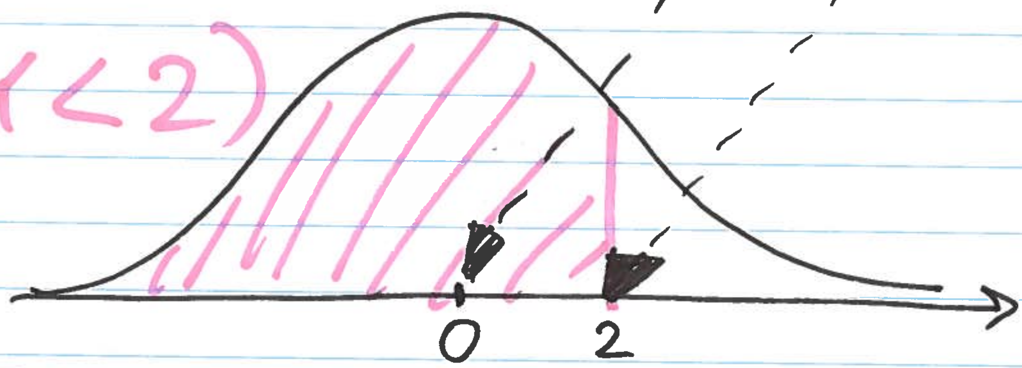
$\boxed{P(X < 8)}$



$Y = \underset{\text{r.v.}}{X} - \underset{\text{const.}}{\mu}$

$\mu_Y = 0$
$\sigma_Y = 2$

$P(Y < 2)$



$Z = \dfrac{Y}{\sigma}$ r.v. / const

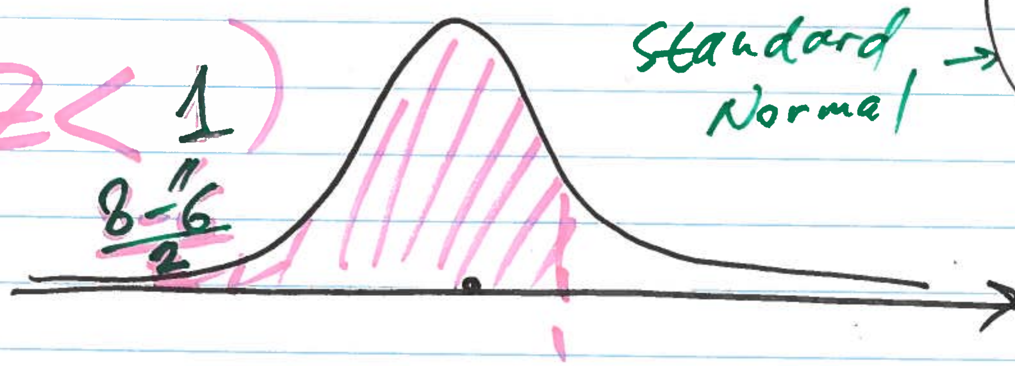
Standard Normal

$\mu_Z = 0$

$\sigma_Z = \frac{2}{\sigma} = 1$

$P(Z < 1)$

$1 = \frac{8-6}{2}$



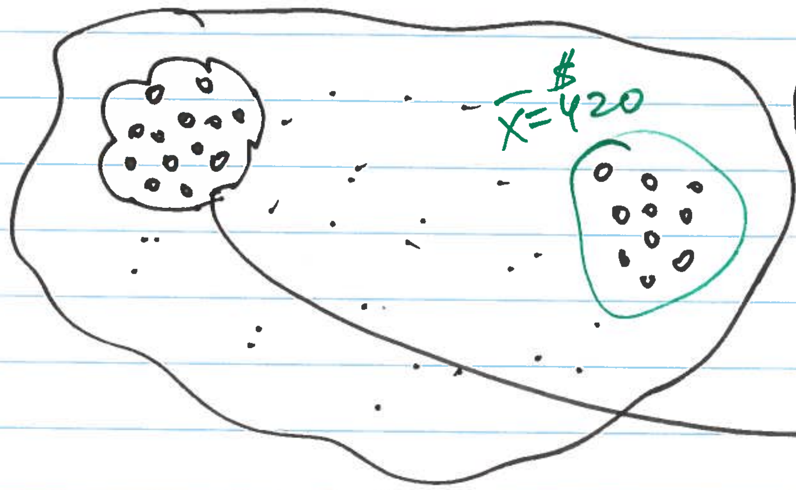
$Z = \dfrac{X - \mu}{\sigma}$

X		Z
$\mu = 6 \quad \sigma = 2$	$\longleftrightarrow$	$\mu = 0 \quad \sigma = 1$
$P(X < 8)$		$P\left(Z < \frac{8-6}{2}\right)$
		$P(Z < 1)$

CONFIDENCE INTERVAL

We want to estimate the average Halloween spending of U.S. households this year. → μ

X:



population μ = ???

σ

$\boxed{\bar{X}}$ = point estimate of μ

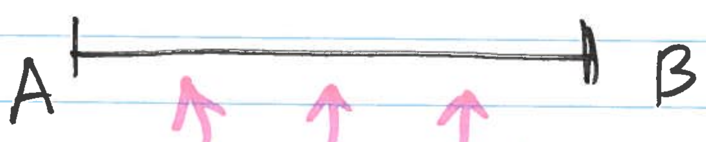
(\$350)

... but with an error (sampling error)

$\sigma_{\bar{x}} = \sigma / \sqrt{n}$

Instead:

Interval estimate of μ:



μ is here with high level of confidence (e.g. 95%).

$\boxed{Q1}$: What happens to the width of the interval if sample size (n) increases? Narrower.

$\boxed{Q2}$: ——— " ——— if σ increases? Wider.

$\boxed{Q3}$: ——— " ——— if confidence level increases?

5 - 5.5	10%
4.5 - 6	50%
3 - 10	~~100%~~

Derivation of 95% C.I. for μ

We need: $P(A < \mu < B) = 0.95$

We collect a sample of size n and observe $\bar{X}$.

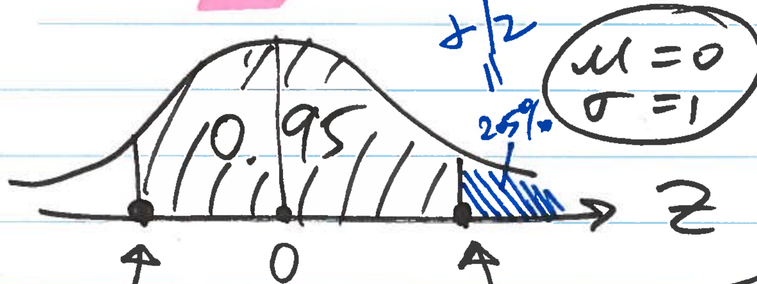
$$P(\text{left} < \bar{X} < \text{right}) = 0.95$$

$$P\left(\frac{\text{left}-\mu}{\sigma/\sqrt{n}} < \underbrace{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}_{Z} < \frac{\text{right}-\mu}{\sigma/\sqrt{n}}\right) = 0.95$$

$1-\alpha$

95% certain

5% error $= \alpha$



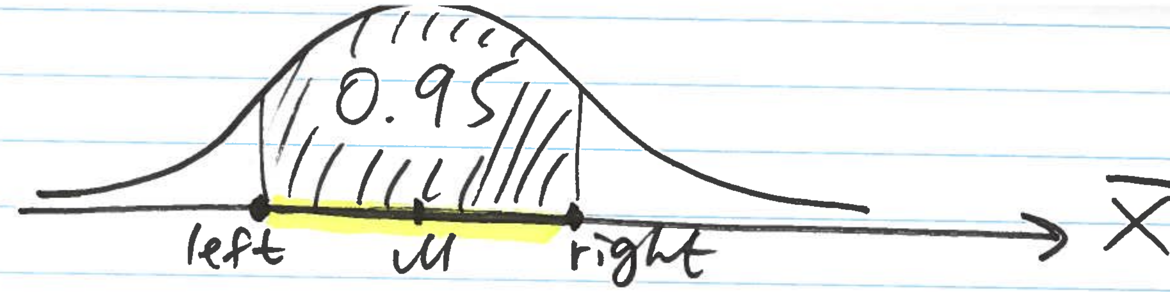
2.5th percentile $=$norm.inv(0.025,0,1) -1.9599

97.5th percentile $=$norm.inv(0.975,0,1) 1.9599

$$P\left(\underbrace{-1.96}_{-z_{2.5\%} = -z_{0.025} = -z_{\alpha/2}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \underbrace{1.96}_{z_{2.5\%} = z_{0.025} = z_{\alpha/2}}\right) = 0.95$$

$$P\left(-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 0.95$$

$$P\left(-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} < \bar{X}-\mu < z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

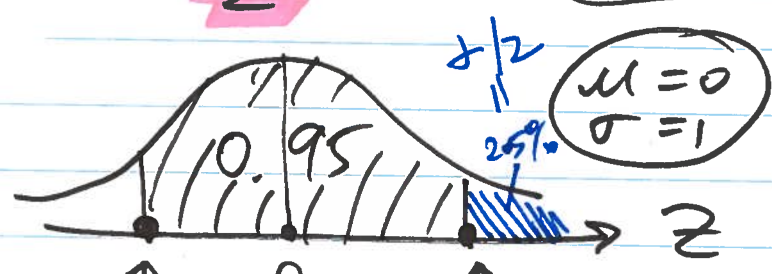


$$P(\text{left} < \bar{X} < \text{right}) = 0.95$$

$$P\Big(\underbrace{\frac{\text{left} - \mu}{\sigma/\sqrt{n}}} < \underbrace{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}_{Z} < \underbrace{\frac{\text{right} - \mu}{\sigma/\sqrt{n}}}\Big) = 0.95$$

$1-\alpha$

95% certain
5% error $= \alpha$



2.5th percentile
=norm.inv(0.025,0,1)
-1.9599

97.5th percentile
=norm.inv(0.975,0,1)
1.9599

$$P\Big(\underbrace{-1.96} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \underbrace{1.96}\Big) = 0.95$$

$-Z_{2.5\%} = -Z_{0.025} = -Z_{\alpha/2}$ $\qquad$ $Z_{2.5\%} = Z_{0.025} = Z_{\alpha/2}$

$$P\Big(-Z_{\alpha/2} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < Z_{\alpha/2}\Big) = 0.95$$

$$P\Big(-Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \bar{X} - \mu < Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\Big) = 0.95$$

$$P\Big(-\bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < -\mu < -\bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\Big) = 0.95$$

$\times(-1)$

$$\boxed{P\Big(\underbrace{\bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{A} < \mu < \underbrace{\bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{B}\Big) = 0.95}$$

$\times(-1)$

$(1-\alpha)\%$ C.I. for μ is $\boxed{\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$

Slide 9

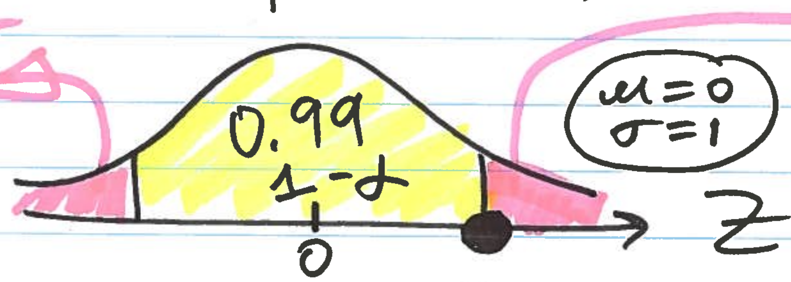
$n = 1000$
$\bar{x} = 69$
$\sigma = 5$

99% C.I. for μ : $\boxed{\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$

$$69 \pm \left(\boxed{2.5756} \cdot \frac{5}{\sqrt{1000}} \right)$$

$\frac{\alpha}{2} = 0.005$

$\alpha = 0.01$



$z_{\alpha/2}$ = 99.5th percentile of Standard Normal distr.
= norm.inv(0.995, 0, 1)
= norm.s.inv(0.995)
2.5756

69 ± 0.407

point estimate of μ — margin of error

$[68.59 \quad , \quad 69.41]$

We are 99% confident that McDonald's overall average satisfaction score (nationally) is between 68.59 and 69.41 points out of 100 possible.