MBC 638
10/11/2017

PROBABILITY

Independent events:

$$P(A|B) = P(A)$$

$$= \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A)\cdot\cancel{P(B)}}{\cancel{P(B)}}$$

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- H = high risk
  F = female

- $P(H) = 0.1$
  $P(F) = 0.49$

$\rightarrow P(H|F) = 0.08$

"given condition"

$P(F|H) = \dfrac{P(H \text{ and } F)}{P(H)}$

$= \dfrac{P(H \text{ and } F)}{P(F)}$

$P(H \text{ and } F) = P(H|F) \cdot P(F)$

$P(F|H) \cdot P(H)$

$= 0.08 \, (0.49)$

$= 0.0392$

$\cancel{P(H \text{ and } F) = P(H) \cdot P(F)}$

$0.1 \cdot 0.49 = 0.049$

only if independent

	H	H^c	
F	0.0392		0.49
F^c			0.51
	0.1	0.9	1

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D = default
M = miss a monthly payment

$$P(D) = 0.05 \qquad P(D^c) = 0.95$$

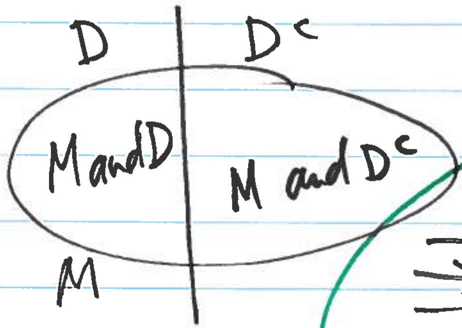
$$P(M \mid D^c) = 0.2 \qquad P(M^c \mid D^c) = 0.8$$

$$\frac{P(M \text{ and } D)}{P(D)} = P(M \mid D) = 1$$

a) $P(D \mid M) = ???$

$$= \frac{P(D \text{ and } M)}{P(M)}$$

$$= \frac{P(M \mid D) \cdot P(D)}{P(M)}$$



$$= \boxed{\frac{P(M \mid D) \cdot P(D)}{P(M \mid D) \cdot P(D) + P(M \mid D^c) \cdot P(D^c)}}$$

$$\Rightarrow P(M) = P(M \text{ and } D) + P(M \text{ and } D^c)$$
$$= P(M \mid D) \cdot P(D) + P(M \mid D^c) \cdot P(D^c)$$

$$= \frac{1 \cdot 0.05}{1 \cdot 0.05 + 0.2\,(0.95)}$$

$$= \boxed{0.21}$$

D = default

M = miss a monthly payment

$P(D) = 0.05$ $\qquad$ $P(D^c) = 0.95$

$P(M \mid D^c) = 0.2$

$P(M \mid D) = 1$

	D	D^c	
M	$P(M \text{ and } D) = P(M \mid D) \cdot P(D) = 1 \cdot 0.05 = 0.05$	$P(M \text{ and } D^c) = P(M \mid D^c) \cdot P(D^c) = 0.2(0.95) = 0.19$	$P(M)$ 0.24
M^c	$P(M^c \text{ and } D)$ 0	$P(M^c \text{ and } D^c)$ 0.76	$P(M^c)$ 0.76
	$P(D)$ 0.05	$P(D^c)$ 0.95	1

a) $P(D \mid M) = \frac{P(D \text{ and } M)}{P(M)} = \frac{0.05}{0.24} = \boxed{0.21}$

b) Yes, because $0.21 > 0.2$.

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F = fraudulent return

X = exceeds IRS standard for deductions due to contributions

$P(F \mid X) = 0.2$ $\quad P(X^c) = 0.92$

$P(F \mid X^c) = 0.02$

$P(X) = 0.08$

	F	F^c	
X	$P(F \text{ and } X) = P(F\mid X)\cdot P(X)$ $= 0.2(0.08)$ $= 0.016$	$P(F^c \text{ and } X)$ 0.064	$P(X)$ 0.08
X^c	$P(F \text{ and } X^c) = P(F\mid X^c)\cdot P(X^c)$ $= 0.02(0.92)$ $= 0.0184$	$P(F^c \text{ and } X^c)$ 0.9016	$P(X^c)$ 0.92
	$P(F) = 0.0344$	$P(F^c) = 0.9656$	1

a) $P(F) = \boxed{0.0344}$

b) $P(F^c \mid X^c) = \dfrac{0.9016}{0.92} = \boxed{0.98}$

OR: $P(F^c \mid X^c) = 1 - P(F \mid X^c)$
$= 1 - 0.02 = 0.98$

c*) Are events F and X independent?

Not independent

① $\underset{0.016}{P(F \text{ and } X)} \overset{???}{=} \underset{0.0344\,(0.08) = 0.0028}{P(F)\cdot P(X)}$

② $P(F \mid X) = 0.2$ but $P(F) = 0.0344$

③ $P(F \mid X) \neq P(F \mid X^c)$