CONFIDENCE INTERVAL FOR μ

MBC 638
11/1/2017

$$\boxed{\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$

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Data: Monthly op. expenses

Month	\$
1	5000
2	6500
3	⋮
⋮	⋮
36	5400

(=average(...))
$\bar{X} = \$5474$

$S = \$764$
(=stdev.s(...))

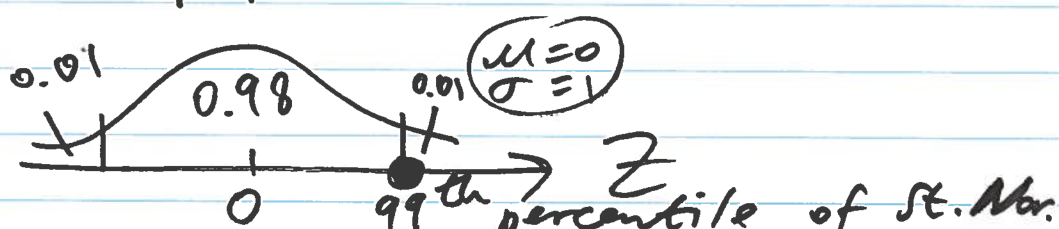
a) 98% confidence interval for μ

or: $t_{\alpha/2}$

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$$

$n = 36 \geq 30$

$$5474 \pm \boxed{2.326} \cdot \frac{764}{\sqrt{36}}$$



5474 ± 296.69

[5,177.31 to 5,770.69]

b) Margin of error $= 2.326 \cdot \frac{764}{\sqrt{n}} = \frac{296.69}{2}$

Solve for n:

$$\left(\frac{2.326 \cdot 764}{296.69/2}\right)^2 = 143.55$$

Round up.

$$\boxed{n = 144}$$

3 ⋮ 36   ⋮ 5400

$\bar{X} = \$5474$

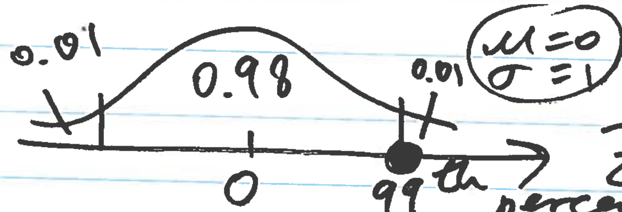
$S = \$764$ (=stdev.s(...))

a) 98% confidence interval for μ

$$\bar{X} \pm z_{\alpha/2} \cdot \frac{S}{\sqrt{n}} \qquad n = 36 \geq 30$$

or: $t_{\alpha/2}$

$$5474 \pm \boxed{2.326} \cdot \frac{764}{\sqrt{36}}$$



99th percentile of St. Nor.

$$5474 \pm 296.69$$

[5,177.31 to 5,770.69]

b) Margin of error $= 2.326 \cdot \frac{764}{\sqrt{n}} = \frac{296.69}{2}$

Solve for n:

$$\left(\frac{2.326 \cdot 764}{296.69/2}\right)^2 = 143.55$$

Round up.

$\boxed{n = 144}$

New Margin of error $= 2.326 \cdot \frac{764}{\sqrt{n}} \cdot \frac{1}{2}$

$= 2.326 \cdot \frac{764}{\sqrt{n}} \cdot \frac{1}{\sqrt{4}}$

$= 2.326 \cdot \frac{764}{\sqrt{4 \cdot n}}$

$4 \cdot n = 144$

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$n = 16$ < 30
$\bar{x} = \$5474$
$s = \$764$

98% C.I. for μ.

$$\bar{x} \pm \cancel{z_{\alpha/2}} \cdot \frac{s}{\sqrt{n}}$$

t

		parameters	
Normal	:	μ	σ
t	:	n	

$$5474 \pm \underbrace{t_{\alpha/2}}_{\substack{d.f. = n-1 = 15 \\ 2.60248}} \cdot \frac{764}{\sqrt{16}}$$

$$5474 \pm 497.07$$

98% C.I. for μ is: $[4976.93, 5971.07]$

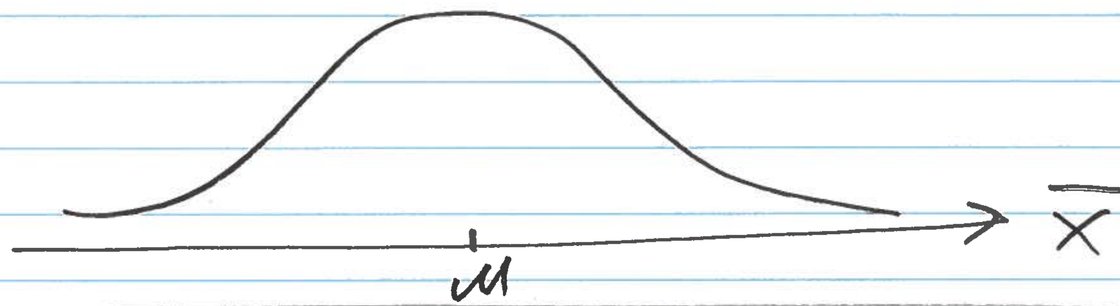
C.L.T.:

When $n \geq 30$ (large),

$\bar{x}$ is $\approx$ Normally distributed

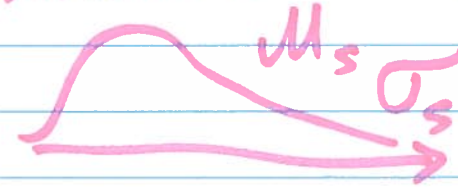
with $\mu_{\bar{x}} = \mu$

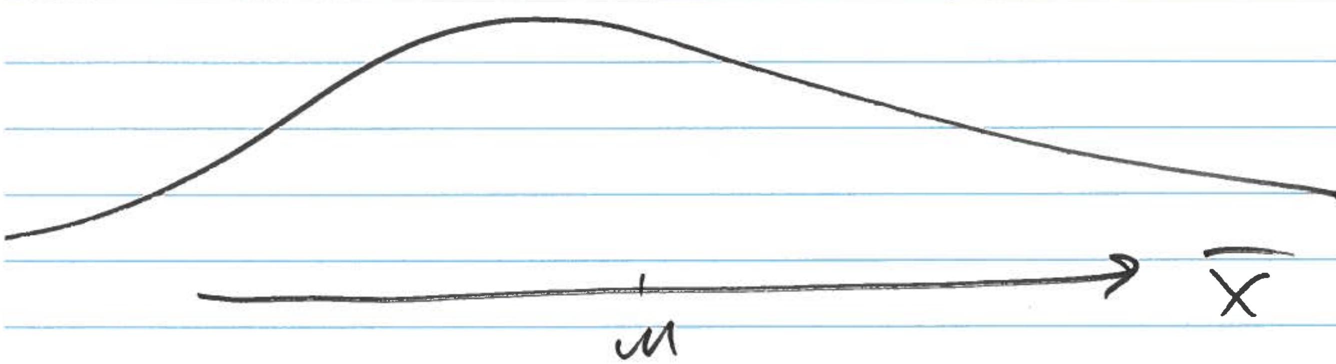
$\sigma_{\bar{x}} = \sigma / \sqrt{n}$ constant (parameter)



$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} = s / \sqrt{n}$ random variable





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10 yrs

$n = 56$ ≥ 30

$\bar{x} = 6.82$

$s = 0.64$

1 yr

$n = 40$ ≥ 30

$\bar{x} = 6.25$

$s = 0.75$

95% confidence

or: $z_{\alpha/2}$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$6.82 \pm 2.004 \cdot \frac{0.64}{\sqrt{56}}$$

$$6.82 \pm 0.171$$

$$[6.65, 6.99]$$

$1-\alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$6.25 \pm 2.023 \cdot \frac{0.75}{\sqrt{40}}$$

$$6.25 \pm 0.240$$

$$[6.01, 6.49]$$

We conclude that consultants with more experience provide better service (average overall rating is higher).

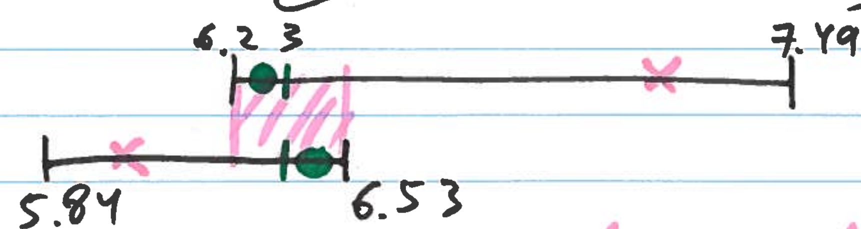
99.5% confidence.

☆ these numbers are made up!

$[6.23, 7.49]$ $[5.84, 6.53]$

10 yrs:

1 yr:



Because the 2 intervals overlap, it's inconclusive.

point estimate of μ

C.I. for μ: $\quad \bar{X} \pm z_{\alpha/2} \cdot \underbrace{\frac{\sigma}{\sqrt{n}}}_{\text{st. deviation of } \bar{X}}$

C.I. for $\mu_1 - \mu_2$:

$$\underbrace{(\bar{X}_1 - \bar{X}_2)}_{\text{point estimate of } \mu_1 - \mu_2} \pm z_{\alpha/2} \cdot \underbrace{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{\text{st. deviation of } \bar{X}_1 - \bar{X}_2}$$

s s

$t_{\alpha/2}$

d.f. $= n_1 + n_2 - 2$

Assume that our two samples are independent:

$$\text{variance}(\bar{x}_1 - \bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

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10 yrs

$n_1 = 56 \quad \bar{x}_1 = 6.82$

$s_1 = 0.64$

1 yr

$n_2 = 40 \quad \bar{x}_2 = 6.25$

$s_2 = 0.75$

$$(6.82 - 6.25) \pm 1.9855 \cdot \sqrt{\frac{0.64^2}{56} + \frac{0.75^2}{40}}$$

$= \text{t.inv}(0.975, 94)$

0.57 ± 0.29

$(0.28, 0.86)$ Yes!

99.5%: $\quad (-0.11, 0.95)$

Inconclusive

these numbers are made up!