# SCM 651 Fall 2017: Additional Notes for Session 2

## 1. Net Present Value (NPV)

**Definition:** Suppose you have several time periods of equal lengths, say years. You have a series of cash flows that occur now (period 0), at the end of the first period (period 1), at the end of the second period (period 2), and so on. If you pay money (for example, make an investment) in a period, cash flow is negative. If you receive money in a period (say, receive interest on a deposit) in a period, cash flow is positive. Denote the cash flows by $V_0$, $V_1$, $V_2$, ..., $V_K$. If the compound interest rate for the period is $r$, then the net present value of the series of cash flows is

$$\text{NPV} = \sum_{i=0}^{K} \frac{V_i}{(1+r)^i}$$

In the example in Excel, NPV is computed with assumption that you make an investment now ($V_0$ is negative), and receive a positive cash flow at the end of a year for a series of years. $r$ is the **annual** compound interest rate.

**Explanation:**

- The net present value is the sum of the present values of the cash flows.
- To understand present value, suppose a bank offers you an annual compound interest rate of 5%. If you invest \$100 now, it becomes \$105 after one year, $1.05 \times \$105$ after two years, and so on. So, after $i$ years, it becomes $\$100 \times (1.05)^i = 100 \times (1+r)^i$, where $r = .05$ is the annual compound interest rate. So, if you invest $\$100 \times (1+r)^{-i}$ now, it becomes \$100 after $i$ years.
- Thus, if you receive \$100 $i$ years from now, that is same as receiving $\$100 \times (1+r)^{-i}$ now.
- Therefore, the **present value** of \$100 $i$ years from now is $100 \times (1+r)^{-i}$.

**Important note on NPV from Excel:**

- When you use NPV in Excel, Excel assumes that the first cash flow occurs one year from now, **not now**.
- So, in the three examples given in the Excel files for sessions 1 and 2, the first cash flow, which is negative because it is an investment, is assumed to happen one year from now, not right now.
- If the first cash flow occurs now (not one year from now), you will get the correct NPV if you multiply the NPV from Excel by $(1 + r)$ (that is, by 1.07 in the examples given).

**IRR:** This is the value of $r$ that makes the net present value of a stream of cash flows zero. IRR is the solution of a polynomial equation and may not always be a real number (for example, if all cash flows are either positive or negative). Even when it is real, it may not be unique.

**Note on IRR from Excel:** The IRR remains same whether the first cash flow is now or one year from now. So, the IRR you get from Excel is fine.

## 2. XNPV

A series of cash flows may occur at unequal intervals, and interest may be compounded at other time intervals than years. In EXCEL, XNPV is computed with the assumption that interest is compounded daily. Let $\frac{r'}{365}$ denote the daily compound interest rate. If you invest \$100 now, after one day it becomes $100 \times (1 + \frac{r'}{365})$, after two days it becomes $100 \times (1 + \frac{r'}{365})^2$, and so on. Thus, the present value of a cash flow of \$100 $i$ days from now is $\$100 \times (1 + \frac{r'}{365})^{-i}$. This is the idea behind computing XNPV.

**Note 1.** $r'$ is **not** same as the annual compound interest $r$. To see that, suppose you deposited \$100, and interest is compounded every day at interest rate $\frac{r'}{365}$. Then, at the end of a year of 365 days, your deposit has become $100 \times (1 + \frac{r'}{365})^{365} = 100 \times (1 + r)$.
Therefore, $365 \ln(1 + \frac{r'}{365}) = \ln(1 + r)$

If you are given $r$, you can compute $r'$ from the above, if you wish to. Note that $r' < r$.

**Note 2.** More generally, suppose interest is compounded at $n$ equal intervals each year, each time at compound interest rate $\frac{r'}{n}$. Then, if you deposit \$100 now, it becomes $100 \times (1 + \frac{r'}{n})^n$ at the end of the year. As $n$ becomes large (say, 365), $(1 + \frac{r'}{n})^n \approx e^{r'}$. Thus, $e^{r'} \approx 1 + r$, where the $r$ is the compound interest rate you get if you only withdraw money at the end of a year. Again, $r' \approx \ln(1 + r) < r$.

**Note 3.** When you use XNPV in Excel, Excel assumes that the first date when a cash flow occurs is "now."

## 3. Customer Lifetime Value (CLV)

**Definition:**

- Suppose you have several time periods of equal lengths, say years.
- You have a customer, if she stays a client of your company, who generates a cash flow $V_0$ now (period 0), a cash flow $V_1$ in period 1, etc. So far, this is same as NPV.
- However, at the end of each period, there is probability $R$ the customer remains a client, and probability $(1-R)$ she leaves and never returns. Thus, $R$ is the probability of retention.
- The compound interest rate is $I$ for a period.

The net **expected present value** of the series of cash flows, also called **customer lifetime value** or CLV, is

$$\text{CLV} = \sum_{i=0}^{K} \frac{V_i R^i}{(1+I)^i} = \sum_{i=0}^{K} V_i \rho^i,$$

where $\rho = \dfrac{R}{1+I}$.

## 4. Correlation

Suppose you have $n$ observations on two variables $X$ and $Y$: $X_1, \ldots, X_n$, and $Y_1, \ldots, Y_n$. Let $\overline{X}$ and $\overline{Y}$ denote the sample means of $X$ and $Y$, that is, $\overline{X} = \dfrac{\sum X_i}{n}$, and $\overline{Y} = \dfrac{\sum Y_i}{n}$. Then, the correlation of $X$ and $Y$ from the sample, denoted by $r_{XY}$ or simply $r$, is given by:

$$r = \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\Sigma(X_i - \overline{X})^2} * \sqrt{\Sigma(Y_i - \overline{Y})^2}}$$

- We always have $-1 \leq r \leq +1$
- $r^2$ measures how close the plot of $X$ and $Y$ is to a straight line. If it is exactly a straight line, then $r^2 = 1$.
- $r > 0$ means if $X$ is higher (lower) than its average $\overline{X}$, $Y$ also tends to be higher (lower) than its average $\overline{Y}$.
- $r < 0$ means if $X$ is higher (lower) than its average $\overline{X}$, $Y$ also tends to be lower (higher) than its average $\overline{Y}$.
- $r = 0$ means that, for any value of $X$, $Y$ is equally likely to be higher or lower than its average.

## 5. Moving Average Model

In the time series example, we use a simple moving average model (more complex models use weights). Suppose you have a time series of observations $X_1, X_2, \ldots$. Then, the moving average for period $t$ is the average of the current and the previous $(n-1)$ values of $X$, that is,

$MA_t = \frac{1}{n}(X_1 + X_2 + \ldots + X_{n-1})$

The number $n$ has to be specified by the user.

**Example:** Suppose $n = 4$. Then,

$MA_t = \frac{1}{4}(X_t + X_{t-1} + X_{t-2} + X_{t-3})$

$MA_{t-1} = \frac{1}{4}(X_{t-1} + X_{t-2} + X_{t-3} + X_{t-4})$

$MA_t - MA_{t-1} = \frac{X_t}{4} - \frac{X_{t-4}}{4}$

To see how it helps, suppose

$X_t = at + B_t$

where $a$ is constant number, and $B_t$ is a seasonal component that takes the same value after every four periods. Thus, $B_t = B_{t-4} = B_{t-8}$ for any $t$.

Therefore,

$MA_t - MA_{t-1} = \frac{X_t}{4} - \frac{X_{t-4}}{4} = \frac{1}{4}(at + B_t) - \frac{1}{4}\{a(t-4) + B_{t-4}\} = \frac{1}{4}(at + B_t) - \frac{1}{4}\{a(t-4) + B_t\} = at$

Therefore, by looking at how the moving average changes, we eliminate any seasonal variation and only see the trend term $at$.

**Note:**

- $MA_t - MA_{t-1} = \frac{1}{n}(X_t - X_{t-n})$
- The first moving average is available for $t = n$.

| Period | Sales | Trend | Seasonality | Moving Average |
|---|---|---|---|---|
| 1 | 2 | 1 | 1 | ____ |
| 2 | 4 | 2 | 2 | ____ |
| 3 | 6 | 3 | 3 | ____ |
| 4 | 8 | 4 | 4 | 5 |
| 5 | 6 | 5 | 1 | 6 |
| 6 | 8 | 6 | 2 | 7 |
| 7 | 10 | 7 | 3 | 8 |
| 8 | 12 | 8 | 4 | 9 |
| 9 | 10 | 9 | 1 | 10 |
| 10 | 12 | 10 | 2 | 11 |
| 11 | 14 | 11 | 3 | 12 |
| 12 | 16 | 12 | 4 | 13 |
| 13 | 14 | 13 | 1 | 14 |
| 14 | 16 | 14 | 2 | 15 |
| 15 | 18 | 15 | 3 | 16 |
| 16 | 20 | 16 | 4 | 17 |

## 6. Exponential Model

This model assumes that at time $t$, the variable of interest (say $Y$) is given by $Y = Ae^{bt}$, where $A$ and $b$ are constant numbers.

Note that $\dfrac{dY}{dt} = bY$, that is, the rate of change of $Y$ is proportional to $Y$.

You can convert this to a linear form by taking logarithms of both sides:

$\ln Y = \ln A + bt$

$b$ can estimated either from the original equation, or using the regression of $\ln Y$ against $t$.

**Examples:**

- Population growth in a country.
- Amount of money under compound interest.
- The number of people who know of your product after you stop advertising. (In this case, $b < 0$.)

## 7. Power Curve

In the notes for sessions 1 and 2, the power curve is used to examine a phenomenon called the experience curve (also called the learning curve) effect.

**Experience Curve Effect:** This is the idea that every time cumulative production (number of units produced from the beginning) doubles, marginal cost (that is, the cost of producing one more unit) drops by a fixed percentage.

Formally, if $C_0$ is the marginal cost after a cumulative production of $Q_0$ units, then the marginal cost after a production of $Q$ units is given by:

$C(Q) = C_0(\frac{Q}{Q_0})^{-\alpha}$, where $\alpha > 0$ is constant.

Thus, for any $Q$, $\dfrac{C(2Q)}{C(Q)} = 2^{-\alpha}$, which is constant.

**Estimation:** You can estimate $\alpha$ in two ways.

- You can estimate $\alpha$ directly by using the power curve option in the scatter plot.
- You can first linearize the equation by taking logarithms:

  $\ln C = (\ln C_0 - \alpha \ln Q_0) - \alpha \ln Q$,

  that is,

  $\ln C = \text{Constant} \quad - \quad \alpha \ln Q$

  Then do the regression with $\ln C$ as the dependent variable against $\ln Q$ as the independent variable.

## 8. Regression Model

In regression models, we want to find how a quantitative dependent variable $Y$ depends on one or more independent variables $X$'s which may or may not be quantitative. For the present, we will only consider quantitative independent variables. Later on the course, we will discuss non-quantitative independent variables using dummy variables.

**Example 1. Two variable regression model:**

$Y = \beta_0 + \beta_1 X + \epsilon$

where

- $\beta_0$ and $\beta_1$ are model parameters that are same for all cases.
- $X$ is the independent variable.
- $\epsilon$ is a normally distributed random variable with mean zero and constant variance that is independent from case to case and is also uncorrelated with $X$. This represents the effect of other factors we did not include in the model.

**Example 1. Two variable regression model:**

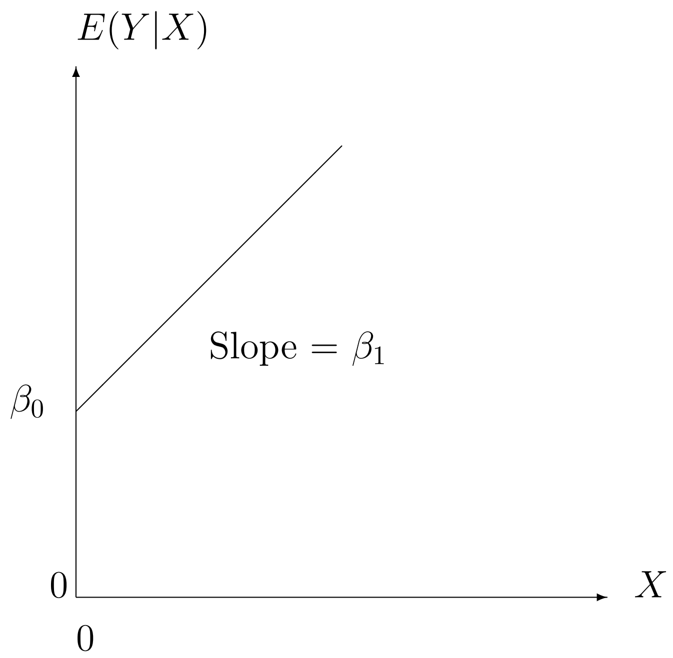
$Y = \beta_0 + \beta_1 X + \epsilon$

**Note:**

$E(Y|X) = \beta_0 + \beta_1 X$, that is, given $X$, on the average $Y$ is $(\beta_0 + \beta_1 X)$

The plot of $E(Y|X)$ against $X$ is a straight line with intercept $\beta_0$ and slope $\beta_1$.

If $X$ changes by a unit, on the average $Y$ changes by $\beta_1$ units.



**Example 2. Multiple Regression Model**

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m + \epsilon$

- $\beta$'s are same for all cases. These are the regression parameters. This model has $(m+1)$ parameters ($\beta_0$, $\beta_1$, …, $\beta_m$).
- $Y$ (dependent variable) is a quantitative variable.
- $X$'s are independent variables.
- $\epsilon$ is a normally distributed random variable with mean zero and constant variance that is independent from case to case and is also uncorrelated with $X$'s. This represents the effect of other factors we did not include in the model.
- If all the independent variables are quantitative, then $\beta_i$ represents how much $Y$ changes **on the average** if $X_i$ increases by a unit, keeping all the other $X$'s unchanged. It is also called the **marginal effect** of $X_i$ on $Y$.

## 9. Demand Curve and Profit Maximization

**Demand:** The demand for a product, $Q$ may depend on many variables, including price. We now consider demand as a function of price. In the following discussions:

- $P$ is the unit price
- $Q$ is the number of product units that will sell at price $P$
- Total cost $= F + (c * Q)$ where $F$ is the total fixed cost, and $c$ is the unit variable cost.
- Sales revenue (dollar sales) $= P * Q$
- Profit ($\Pi$) = Sales Revenue − Total Cost $= (P - c) * Q \;-\; F$

Normally (but not always), $Q$ is lower if $P$ is higher.

**Price Elasticity of Demand:** $\epsilon = \dfrac{P}{Q}\dfrac{\partial Q}{\partial P}$. This is approximately equal to the percent change in demand for a one percent change in price and is normally a negative number.

Price elasticity of demand can also be expressed as $\dfrac{\partial \ln Q}{\partial \ln P}$.

**Normal Demand:** $\epsilon < 0$, that is, a demand decreases as price increases.

**Inverse Demand:** $\epsilon > 0$, that is, demand is higher if price is higher. This happens when customers cannot assess quality of the product and uses price as an indicator of quality.

## 10. Relation between Elasticity and Profit

Assume normal demand, that is, $\epsilon < 0$. Note that the fixed cost $F$ is not affected by changes in price. Therefore,

$$\frac{\partial \Pi}{\partial P} = Q + (P-c)\frac{\partial Q}{\partial P} = Q[1 + (\frac{P-c}{Q})\frac{\partial Q}{\partial P}] = Q[1 + (\frac{P-c}{P})(\frac{P}{Q}\frac{\partial Q}{\partial P})]$$

$$= Q[1 + (\frac{P-c}{P})\epsilon] = Q[1 - (\frac{P-c}{P})E], \text{ where } E = |\epsilon|.$$

$PV \equiv \dfrac{P-c}{P}$ is called the **profit-volume** ratio.

Thus,

$$\frac{\partial \Pi}{\partial P} = Q[1 - PV * E] = Q * PV * [\frac{1}{PV} - E]$$

**Note:**

- $(P-c)$ is called the **unit gross margin**. Usually, $P > c$, that is, $P - c > 0$.
- $\dfrac{1}{PV} = \dfrac{P}{P-c}$ is price as a multiple of unit gross margin.
- As long as $P - c > 0$, $\dfrac{1}{PV} > 1$. It is large when $P$ is close to $c$.

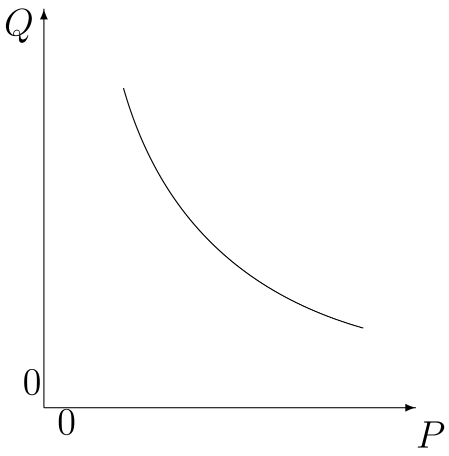
Since $Q > 0$ and usually $PV > 0$, we have the following results about how changes in price affect profit:

(1) If $\dfrac{1}{PV} > E$, $\Pi \uparrow$ if $P \uparrow$

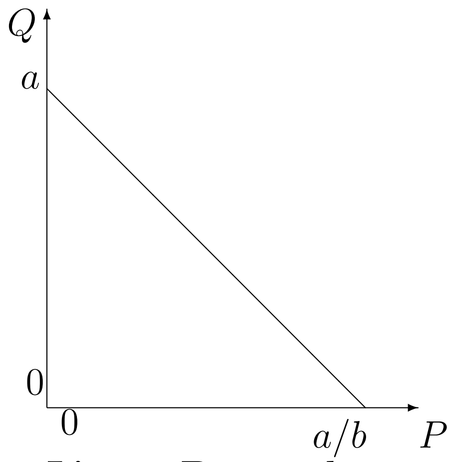
(2) If $\dfrac{1}{PV} < E$, $\Pi \uparrow$ if $P \downarrow$

(3) If $\dfrac{1}{PV} = E$ profit is maximized. This can only happen if $E > 1$, and the profit maximizing price is $P^* = c + \dfrac{c}{E-1}$

## 11. Two Common Demand Functions




**Constant Elasticity Demand**




**Linear Demand**

**1. Linear demand function:** $Q = a - bP$ if $P \leq \frac{a}{b}$, and $Q = 0$ if $P > \frac{a}{b}$.

- Profit is maximized if $P = P^* = \frac{a}{2b} + \frac{c}{2} = c + \frac{1}{2}(\frac{a}{b} - c)$
- $a$ and $b$ can be estimated by fitting a regression model with $Q$ as dependent variable, and $P$ as independent variable.

**2. Constant elasticity demand function:** $Q = AP^{-E}$, where $E > 0$ is the absolute value of the price elasticity of demand.

- If $E \leq 1$, an increase in price always increases profit.
- If $E > 1$, profit maximizing price is $P^* = \frac{E * c}{E - 1} = c + \frac{c}{E - 1}$.
- For this demand function, $\ln Q = \ln A - E \ln P$

  $E$ can be estimated by running a regression with $\ln Q$ as dependent variable, and $\ln P$ as independent variable.