PARTIAL REDUNDANCY ELIMINATION FOR GLOBAL VALUE NUMBERING

A Thesis

Submitted to the Faculty

of

Purdue University

by

Thomas John VanDrunen

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

August 2004

To my parents.

ACKNOWLEDGMENTS

Many individuals have helped me in this work in tangible and intangible ways. To begin, I would like to acknowledge my thesis committee. Thanks to my advisor, Tony Hosking, for supervising this work and giving me the freedom to pursue what interests me. To Jens Palsberg I have deep appreciation, who in many ways was a second advisor to me, working with me in this research and a side project and giving me priceless professional advice. Jan Vitek treated me as one of his own students in the sense of privileges, and only to a lesser extent in the sense of responsibilities. Thanks to Zhiyuan Li for also serving on my committee. Although not a committee member, I acknowledge Suresh Jagannathan as another supportive faculty member.

I had the privilege of working along side of many excellent labmates, including Carlos Gonzalez-Ochoa, David Whitlock, Adam Welc, Dennis Brylow (to whom special thanks are due as the lab Linux administrator), Krzysztof Palacz, Christian Grothoff, Gergana Markova, Chapman Flack, Ioana Patrascu, Mayur Naik, Krishna Nandivada, Bogdan Carbunar, Di Ma, and Deepak Bobbarjung.

I thank my parents for their support during this endeavor and the rest of my education. I also appreciate the encouragement from my siblings and their families: Becky, Tom, Dave, Katherine, and Jack. I am grateful for the many friendships I have had as a graduate student which made this effort much more bearable: Lisa Graves, Rose McChesney, Anastasia Beeson, Sergei Spirydovich, Jamie VanRandwyk, Zach Peachy, Jared Olivetti, Doug Lane, Katherine Pegors, Leila Clapp, David Vos, Anna Saputera, Ben Larson, and (the biggest encourager and encouragement during the writing of the dissertation) Megan Hughes.

Finally, may this work be a thank offering to Jesus Christ. "Except the LORD build the house, they labor in vain who build it." *SOLI DEO GLORIA.*

TABLE OF CONTENTS

Page

Page

LIST OF TABLES

LIST OF FIGURES

Figure Page

Figure Page

ABSTRACT

VanDrunen, Thomas John . Ph.D., Purdue University, August, 2004. Partial Redundancy Elimination for Global Value Numbering . Major Professor: Antony L Hosking.

Partial redundancy elimination (PRE) is a program transformation that removes operations that are redundant on some execution paths, but not all. Such a transformation requires the partially redundant operation to be hoisted to earlier program points where the operation's value was not previously available. Most well-known PRE techniques look at the lexical similarities between operations.

Global value numbering (GVN) is a program analysis that categorizes expressions in the program that compute the same static value. This information can be used to remove redundant computations. However, most widely-implemented GVN analyses and related transformations remove only computations that are fully redundant.

This dissertation presents new algorithms to remove partially redundant computations in a value-based view of the program. This makes a hybrid of PRE and GVN. The algorithms should be simple and practical enough to be implemented easily as optimization phases in a compiler. As far as possible, they should also to show true performance improvements on realistic benchmarks.

The three algorithms presented here are: ASSAPRE, a PRE algorithm for programs in a form useful for GVN; GVNPRE, the hybrid algorithm for value-based PRE; and LEPRE, an approximate PRE technique for object and array loads.

1 INTRODUCTION

"You look a little shy; let me introduce you to that leg of mutton," said the Red Queen. "Alice – Mutton; Mutton – Alice." The leg of mutton got up in the dish and made a little bow to Alice; and Alice returned the bow, not knowing whether to be frightened or amused.
"May I give you a slice?" she said, taking up the knife and fork, and looking from one Queen to the other.
"Certainly not," the Red Queen, very decidedly: "it isn't etiquette to cut any one you've been introduced to."
—Lewis Carroll, *Alice through the Looking Glass*

1.1 The thesis

1.1.1 General problem

A *programming language* is a system of expressing instructions for a real or theoretical machine which automates computation or some other task modeled by computation. Viewed this broadly, programming languages include things as diverse as the lambda calculus and the hole patterns in the punch cards of a Jacquard loom. The former is a minimalist language that can encode anything computable in the Church-Turing model of computation. The latter is of historic significance as the first use of instructions stored in some form of memory to control the operation of the machine.

Most modern computers interpret simple languages made up of binary-encoded instructions that move information in memory, send information to devices, and perform basic logical and arithmetic operations on the information. A non-trivial computer program requires millions of such instructions. A language like this is called a *machine language*.

As computing developed, it became evident that it was impractical for humans to compose programs in machine languages. This prompted the invention of *high-level languages* in which computational tasks could be described unambiguously but also for which humans could be trained easily and in which programs could be composed

quickly. Since computers understand only machine languages, programs written in a high-level language must be translated to a machine language.

In order to automate the process of translation from high-level languages to machine languages, programmers began composing computer programs whose purpose was reading in other programs and translating them from one language (the *source*) to another (the *target*). A computer program that translates computer programs is called a *compiler*. A compiler is divided into three parts: a front-end, which lexically analyzes and parses the source program; a middle, which performs analyses and transformations on the program for things like error checking or making improvements; and a back-end, which emits the target program. Of course, each compilation phase must represent the program in computer memory; a form of the program used internally by the compiler is called an *intermediate representation (IR).* Two desirable properties of an IR are *language independence* and *machine independence*; that is, programs in (nearly) any source high-level language can be represented by it, and it can represent a program bound for (nearly) any target machine language.

A fundamental goal of computer science across its sub-disciplines is making computational tasks faster. Naturally, a well-constructed compiler should produce target programs that are as efficient as possible. The process of transforming a program for better performance is called *optimization*. Optimization may happen at any phase of the compiler, but much research has focused on mid-compiler optimizations since they are widely applicable for languages and machine architectures.

A common form of inefficiency in a program is *redundancy*, when identical or equivalent tasks are performed twice. Consider the program fragment in Figure 1.1 written as a Java method: The computation `d + a` is redundant because it is equivalent to the previous computation `a + b`. It would be more efficient to eliminate that instruction. A more interesting case is the computation `d + e` at the end of the method. It is redundant if variable `x` is `false` and the second branch is taken, but it is not redundant otherwise. If a computation is redundant on at least one but not all traces of the program to that point, it is *partially redundant*. An example

```
static int f(int a, int b, boolean x){
  int c = 0;
  int d = 1;
  int e = 2;
  if (x) {
    c = a + b;
    d = b;
    e = d + a;
  }
  else {
    c = d + e;
  }
  return d + e;
}
```

Figure 1.1. Java program example

like this shows that a computation may need to be inserted in order to eliminate a redundancy. In this case, if we insert the instruction `t` = `d` + `e` after `e` = `d` + `a` and `t` = `c` after `c` = `d` + `e`, we could replace `return d + e` with `return t`. The net effect is that the computation `d` + `e` is *hoisted* to an earlier program point. This would not affect the running of the program if it branches for `x` being `false` (except that a move instruction is added), but it would result in one fewer addition operations if it branches for `x` being `true`. Note that this does not decrease the size of the program; in fact, it often increases, as in this case.

Transformations like this need analyses to determine program points where instructions should be inserted or can be eliminated. Elimination requires knowledge of what has been computed already, that is, what computations are *available*. Availability requires a forward analysis over the program. On the other hand, we must determine what computations will be computed later in the program to identify insertion points. We say that a computation is *anticipated* at a program point if it will be executed later in the program, and this requires analyzing the program backward. There are two major ways to consider equivalences among computations. If

two computations match exactly, including with respect to variable names, they are *lexically equivalent*, as in the case of the two computations of `d + e` in our example. Note, however, that even though `a + b` and `d + a` are not lexically equivalent, they still compute the same value on any run of the program. This is an example of a *value equivalence*. Analyses that consider value equivalences are stronger than those that restrict themselves to lexical equivalences.

Previous work on this problem has fallen typically into two categories. *Global value numbering* (GVN) analyzes programs to group expressions into classes, all of which have the same value in a static view of the program. *Partial redundancy elimination* (PRE) hoists computations to make partially redundant computations fully redundant and thus removable.

1.1.2 Thesis statement

This dissertation concerns the removal of value-equivalent computations. By presenting algorithms to this end, it seeks to demonstrate that

> Practical, complete, and effective analyses for availability and anticipation allow transformations to remove full and partial value-equivalent redundancies.

The algorithms presented here are **practical** in that they can be implemented simply, incorporate easily into the optimization sequence of existing compilers, have a reasonable cost, and represent an advancement over previous work in this area from a software engineering perspective. They are **complete** in that they subsume other algorithms with similar goals. They are **effective** in that they produce a performance gain on some benchmarks. Finally, they consider not only lexically equivalent but also value equivalent computations and remove not only full redundancies but also partial redundancies.

In particular, we present three algorithms. The first is a PRE algorithm for an IR property that is particularly useful for GVN. The second is a complete hybrid of

PRE and GVN. The third uses GVN to perform PRE on object and array references. An additional contribution of this dissertation is the framework we use to discuss programs and values for the purpose of optimizing analyses and transformations.

1.1.3 Outline

The remainder of this chapter explains preliminary matters relevant to all parts of the dissertation: definitions for the IR and other details of our framework (Section 1.2.1), the cost model and standards we use to make claims of our algorithms' worth (Section 1.2.2), an overview of our experimental infrastructure (Section 1.2.3), and a list of prior publications of the material presented here (Section 1.2.4). Chapter 2 traces the development of other approaches to this and similar problems. As implied earlier, there are two major strains in the evolution, motivating the hybrid approach which is the cornerstone of this dissertation.

Chapter 3 presents the **ASSAPRE** algorithm. Chow, Kennedy, et al introduced **SSAPRE** [1, 2], an algorithm for PRE on IRs with a property called *static single assignment* (SSA) form. We critique this algorithm in three areas. First, it is fundamentally weak in that it is completely lexical in its outlook and ignores redundancies involving secondary effects. Second, it actually makes requirements on the IR that are more strict than SSA, as it is usually understood, and if applied to a non-conforming program will perform erroneous transformations. Finally, it is difficult to understand and implement. ASSAPRE is an improvement in these areas. It reduces lexical restrictions on the expressions it considers, and it uses an anticipation analysis. Since SSA is useful for GVN, this algorithm illuminates how PRE, which is typically lexical, can be extended for use with GVN, thereby considering values-based equivalence.

Chapter 4 presents the **GVNPRE** algorithm. This is the chief contribution of the dissertation and is a novel hybrid of earlier GVN and PRE approaches. It uses a clean way of partitioning expressions into values that is an extension of a simple hash-

based GVN, uses a system of flow equations to calculate availability and anticipation and to determine insertion points, and removes full and partial redundancies.

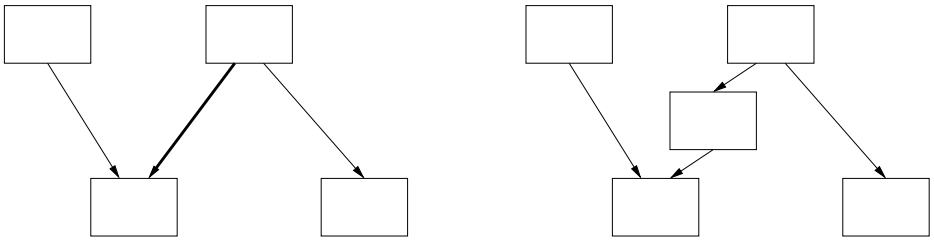
Chapter 5 presents the **LEPRE** algorithm. GVNPRE considers only scalar operations, not, for example, loads from objects and arrays. We show that it is impossible to extend GVNPRE to cover loads completely. Fink et al. introduced an algorithm for eliminating loads that are fully redundant [3]. Although not itself a GVN scheme, it relies on GVN as input to its analysis. However, that algorithm does not consider partial redundancies. As an approximate solution to the problem of extending GVNPRE for loads, we extend Fink et al's algorithm to use GVNPRE's analysis for doing PRE. We also sketch a different approach to cross-breeding GVN and PRE that would be more amenable to the removal of loads.

We compare each algorithm to related approaches, describe an implementation, and report on static and performance results. Chapter 6 concludes by considering future work in this area.

1.2 Preliminaries

1.2.1 IR definitions and assumptions

We assume input programs represented as *control flow graphs* (CFGs) over *basic blocks* [4]. A basic block is a code segment that has no unconditional jump or conditional branch statements except for possibly the last statement, and none of its statements, except for possibly the first, is a target of any jump or branch statement. A CFG is a graph representation of a procedure that has basic blocks for nodes and whose edges represent the possible execution paths determined by jump and branch statements. Blocks are identified by numbers; paths in the graph (including edges) are identified by a parenthesized list of block numbers. We define $\mathsf{succ}(b)$ to be the set of successors to basic block b in the CFG, and similarly $\mathsf{pred}(b)$ the set of predecessors. (When these sets contain only one element, we use this notation to stand for that element for convenience.) We also define $\mathsf{dom}(b)$ to be the dominator of b, the nearest



(a) Critical edge in bold (b) Critical edge replaced by landing pad

Figure 1.2. Critical edge removal

block that dominates b. This relationship can be modeled by a *dominator tree*, which we assume to be constructed before the performing of our algorithms [4]. The *dominance frontier* of a basic block is the set of blocks that are not dominated by that block but have a predecessor that is [5]. We assume that all *critical edges*—edges from blocks with more than one successor to blocks with more than one predecessor [6]—have been removed from the CFG by inserting an empty block between the two blocks connected by the critical edge, as illustrated in Figure 1.2. Formally, this property means that if $|\mathsf{succ}(b)| > 1$ then $\forall b' \in \mathsf{succ}(b), |\mathsf{pred}(b')| = 1$ and if $|\mathsf{pred}(b)| > 1$ then $\forall b' \in \mathsf{pred}(b), |\mathsf{succ}(b')| = 1$. This provides a landing pad for hoisting.

Figure 1.3(a) shows the sample program from Figure 1.1 as a CFG, and Figure 1.3(b) shows the optimized version. Note that we do not include branch instructions in the CFG illustrations.

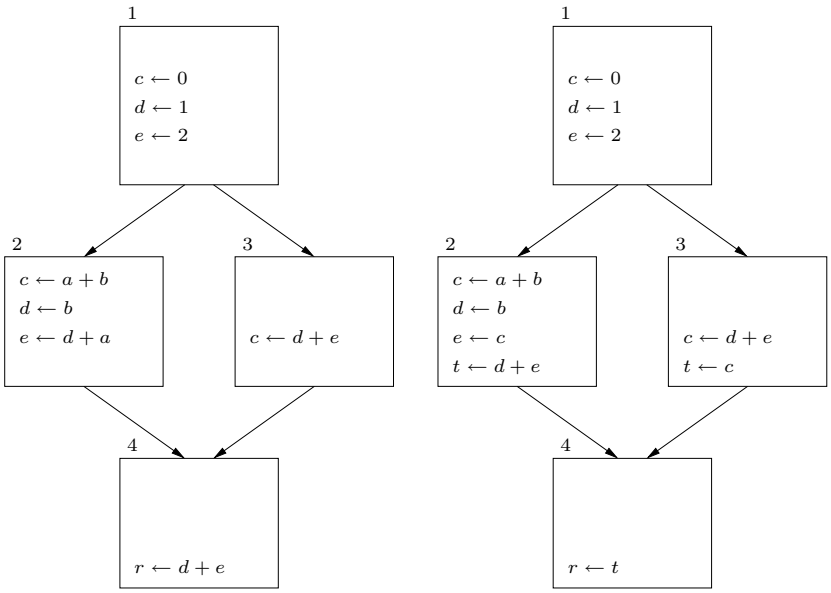
Static single assignment (SSA) form is an intermediate representation property such that each variable—whether representing a source-level variable chosen by the programmer or a temporary generated by the compiler—has exactly one definition point [5, 6]. Even though definitions in reentrant code may execute many times, statically each SSA variable is assigned exactly once. If several distinct assignments to a variable occur in the source program, the building of SSA form splits that variable into distinct versions, one for each definition. In basic blocks where execution paths merge (such as block 4 in our example) the consequent merging of variables

is represented by a *phi function*. Phi functions occur only in instructions at the beginning of a basic block (before all non-phi instructions) and have the same number of operands as the block has predecessors in the CFG, each operand corresponding to one of the predecessors. The result of a phi function is the value of the operand associated with the predecessor from which control has come. A phi function is an abstraction for moves among temporaries that would occur at the end of each predecessor. SSA form does not allow such explicit moves since all would define the same variable. SSA makes it easy to identify the live ranges of variable assignments and hence which lexically equivalent expressions are also semantically equivalent. On the other hand, SSA complicates hoisting since any operand defined by a merge of variable versions must have the earlier version back-substituted. Figure 1.3(c) shows the sample program in SSA form, where we name SSA variables using subscripts on the names of the source-level variables from which they come. In some examples, we ignore the relationship with pre-SSA variable names and consider all SSA variables to be independent temporaries.

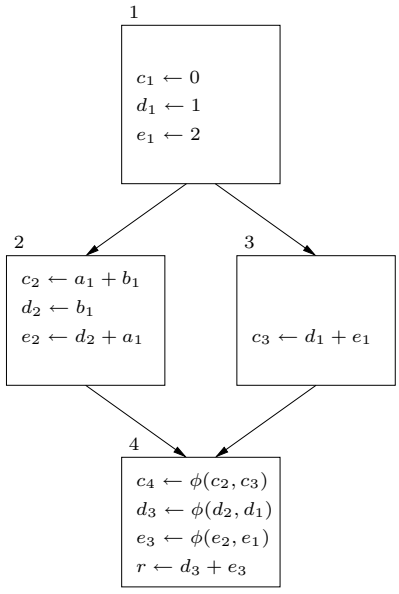
We now formally define the language for our examples (this excludes jumps and branches, which for our purposes are modeled by the graph itself):

k	::=	$t \mid t_1$ `op` t_2 ($\mid$ `gf` x t)	*Operations*
p	::=	$\phi(t^*)$	*Phis*
γ	::=	$k \mid p \mid \bullet$	*Right-hand terms*
i	::=	$t \leftarrow \gamma$ ($\mid$ `pf` x t_1 t_2)	*Instructions*
b	::=	i^*	*Basic blocks*

Note that basic blocks are considered vectors of instructions. The symbol $\bullet$ stands for any operation which we are not considering for optimization; it is considered to be a black box which produces a result. The meta-variable t ranges over (SSA) variables. Because of SSA form, no variable is reassigned, and a variable's scope is implicitly all program points dominated by its definition. For simplicity, we do not include constants in the grammar, though they appear in some examples; constants



(a) Original (b) Optimized



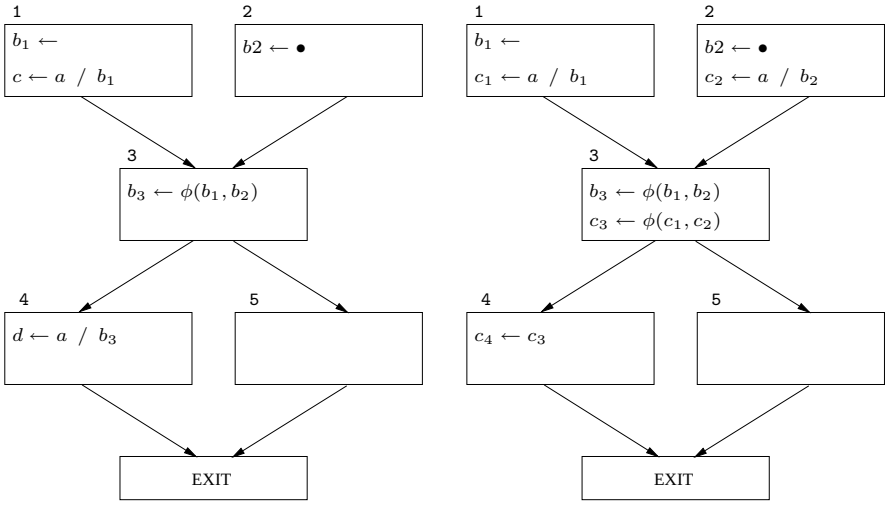
(c) In SSA form

Figure 1.3. Sample program

may be treated as globally-defined temporaries. We use the term *operation* instead of *expression* to leave room for a more specialized notion of expressions in Chapter 4. We let `op` range over operators, such as arithmetic or logical operators. `gf` x t is a getfield operation, that is, one that retrieves the value of field x of the object to which t points. x ranges over field names. `pf` x t_1 t_2 is a putfield operation, setting field x of the object pointed to by t_1 to hold the value of t_2. The productions involving `gf` and `pf` are bracketed because we use them only at certain points in this dissertation; otherwise getfields are replaced with $\bullet$ and putfields are ignored. For simplicity, the only types we recognize are integer and object, and we assume all programs type correctly, including that getfields and putfields use only legitimate fields.

1.2.2 Cost model and goals

The proof of an optimization's worth is real performance gain when applied to real-world benchmarks. That said, theoretical cost models are useful in evaluating optimizations abstractly, apart from a specific language or architecture. We consider each operation (t_1 `op` t_2 or `gf` x t) to cost one unit. Moves (such as $t_1 \leftarrow t_2$) are free. Our goal is to reduce the number of units on traces of the input program. This implies we may freely generate new temporaries and insert new moves in our effort to reduce other instructions. We consider this to be a realistic cost model because move instructions and extra temporaries largely can be cleaned up by good register allocation. Though it may seem naïve to give arithmetic operations and getfields equal cost when, for any realistic architecture, getfields are much more expensive (an important point in our experimental evaluation), we still feel justified in treating them as though equal because at no time in the algorithms presented would one be substituted for another; that is, one never needs to choose between an arithmetic operation and a getfield.

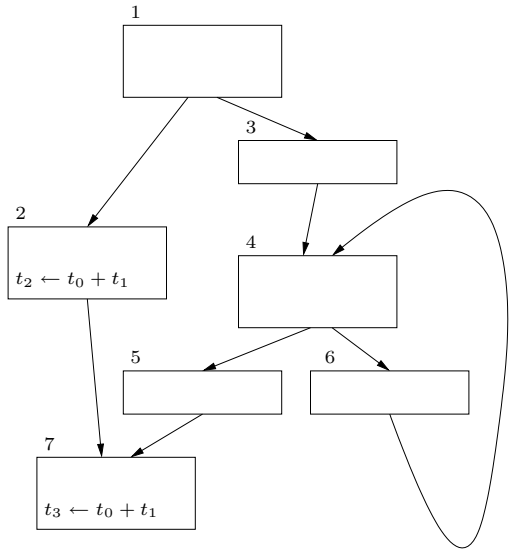


(a) Unoptimized (b) Incorrectly "optimized"

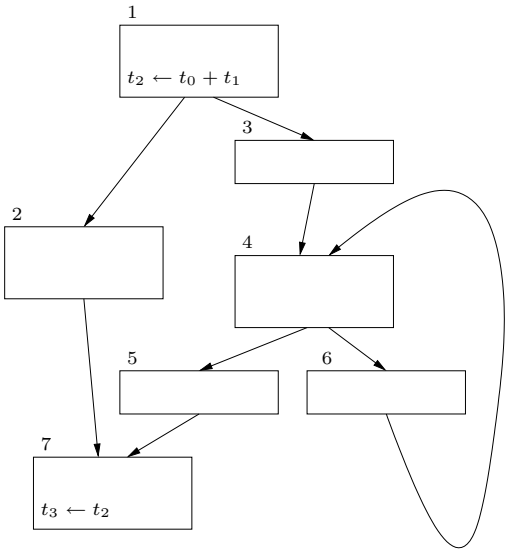
Figure 1.4. Illicit code motion

Our algorithms must be Hippocratic in that they should do no harm. Generally, a PRE hoist will improve one path while making no change on another. What is not acceptable is a transformation that improves one path while making another path worse. Consider the example in Figure 1.4(a). The operation $d \leftarrow a \ / \ b_3$ is partially redundant in block 4, since it already has been computed in path (1,3,4) but not in path (2,3,4). Hoisting the computation to block 2 as shown in 1.4(b) removes the block 4 computation and improves the path (1,3,4) without changing (2,3,4). However, this lengthens the path (2,3,5). Not only does this defy optimality, but if the hoisted instruction throws an exception (in our case, if b_2 is zero), then the transformation has changed the behavior of the program by introducing an exception where otherwise there was not one. This illustrates the need for anticipation analysis. A hoist should be performed only for an expression that is anticipated on all paths from that point to program exit.

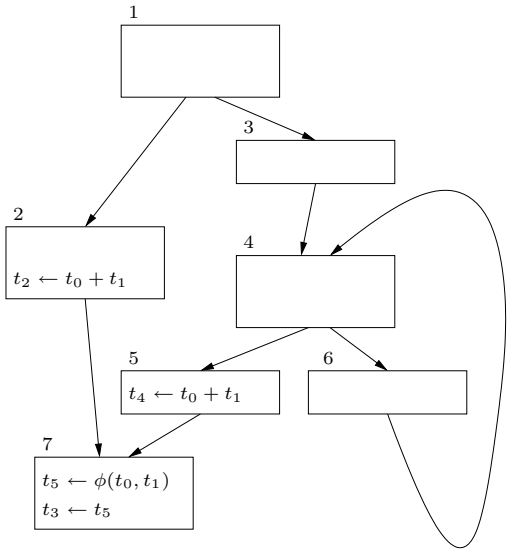
One potential casualty of hoisting instructions is that it lengthens the live ranges of variables, increasing register pressure and potentially harming performance. We believe that since most hoists are fairly local and many modern architectures have



(a) Original



(b) Early hoisting



(c) Late hoisting

Figure 1.5. Hoisting as late as possible

large register sets, this need not be a major concern. Nevertheless, to avoid undue register pressure, algorithms should make insertions as late as possible without missing any optimization opportunities. Consider the example in Figure 1.5(a), where $t_3 \leftarrow t_0 + t_1$ is redundant on path (1,2,7). It would be simple, safe, and, in the cost model presented so far, optimal to move $t_2 \leftarrow t_0 + t_1$ from block 2 to block 1 and reload from t_2 in block 7, as shown in 1.5(b). However, that forces another variable to be live through the loop of block 4. If block 4 happens to be register-heavy, this may cause a spill. Hoisting to block 5, as shown in 1.5(c), also fits our optimality criteria without the register pressure on the loop. Thus our algorithms should hoist instructions to program points that are as late as possible.

1.2.3 Experiments and implementation

All of our algorithms have been implemented in the Jikes RVM [7–9], a virtual machine that executes Java classfiles. We have implemented the algorithms described here as new compiler phases for the RVM's optimizing compiler. The optimizing compiler performs a series of code transformations on both SSA and non-SSA representations. It already has a GVN analysis based on Alpern et al [10], as well as loop-invariant code motion (LICM) and global common subexpression elimination (GCSE) phases, which rely on GVN. Fink et al's load elimination [3] is also available. The RVM's default optimization chain supplies optimizations having similar goals to ours, with which we can compare results and examine interactions. We use three sets of benchmarks in our experiments: eight from the SPECjvm98 suite [11], four from SciMark [12], twelve from the sequential benchmarks of the Java Grande Forum [13], and ten from the JOlden Benchmark Suite [14] Different versions of Jikes RVM, different architectures, and different subsets of the benchmarks have been used to evaluate the various algorithms depending on what was most readily available at each stage of this work.

1.2.4 Publications

The material in Chapter 3 is to appear in *Software—Practice & Experience* [15]. The material in Chapter 4 was presented at the Thirteenth International Conference on Compiler Construction in April 2004 [16], with more details given in an accompanying technical report at Purdue University [17].

2 RELATED WORK

Infandum, regina, iubes renovare dolorem.
(You command me, queen, to recall unspeakable pain.)

—Virgil, *The Aeneid*, II, 3.

These are the generations of redundancy elimination. One of the earliest recognizable efforts related to our problem arose in the work of Cocke, in which he identified our "aim to delete those computations which have been previously performed, [and] when possible to move computations from their original positions to less frequently executed regions," proposed a flow analysis to determine if expressions can be eliminated or moved, and explained a primitive form of critical edge splitting [18,19]. Kildall, infamous for losing a crucial contract with IBM to a young Bill Gates [20], outlined a generalized optimization technique which, parameterized by an "optimizing function," could be used for a variety of transformations on a program as a CFG [21,22]; in this work, he foresaw GVN by proposing an optimizing function for common subexpression elimination that partitions expressions into equivalence classes.

2.1 Partial redundancy elimination

PRE was invented by Morel and Renvoise [23, 24]. They identified partial redundancy as a generalization of loop invariance and defined a set of properties to determine appropriate transformations: *transparency*, true for an expression in a block which does not modify its operands; *availability*, true for an expression in a block which computes it and does not modify its operands after the final computation; *anticipation* (in situ, *anticipability*), true for an expression in a block that computes it and does not modify its operands before the first computation; and *insert*, true for an expression in a block where it is anticipated and also available in

at least one predecessor. Since these properties are computed per expression, the algorithm is inherently lexical. Since they involve both availability and anticipation, it is bidirectional. Drechsler and Stadel [25] and several others [26] modified the algorithm to better handle subexpressions. Dhamdhere adapted the algorithm to have more elimination opportunities by the splitting of critical edges [27] and to avoid code motion that does not produce real redundancy elimination [28].

The epoch for PRE is *Lazy Code Motion* (LCM), invented by Knoop, Rüthing, and Steffen [29, 30]. Its principle contribution is an algorithm that is provably optimal but takes register pressure into account by hoisting expressions no earlier than necessary. It uses four properties to determine hoisting and elimination of expressions: *downsafety*, equivalent to anticipation; *earliest*, true for an expression and a block if that expression is downsafe at the block but not on any path from the start to that block; *latest*, true for an expression and a block if the expression is optimally placed there but would not be optimally placed on any path from that block to the exit; and *isolated*, true for an expression and a block if that expression, placed at that block, would reach no other occurrence of that expression (that is, without reassignment of its operands) to program exit. Identifying these properties allows the typical bi-directional analysis to be broken down into uni-directional components (something foreseen by Dhamdhere et al [31]), which the researchers claimed to be more efficient. Drechsler and Stadel contributed to this work by showing how it could be used on full-sized basic blocks (as opposed to blocks of only one instruction as in the original presentation [29]) and by simplifying the flow equations [32]. As with original PRE, this approach is expression-based and thus lexical.

The world that came after LCM saw numerous minor PRE projects. Wolfe showed that the elimination of critical edges was all that is necessary to achieve unidirectionality as in LCM [33]; his PRE, however, did not attempt to minimize register pressure. Briggs and Cooper did a fascinating study on making PRE more effective [34]. They identified PRE's reliance on lexical equivalence as a major handicap; its inability to rearrange subexpressions also limits the redundant computations it can

recognize. They showed that by first propagating expressions forward to their uses (the opposite of what optimizations like common subexpression elimination and PRE do), sorting and reassociating the expressions, and renaming them using information gained from GVN, many more optimization opportunities are exposed for PRE. Note that this essentially uses GVN to enhance PRE, yet it is not a hybrid like the algorithm in Chapter 4 because it uses GVN in enabling transformations prior to PRE rather than forming a unified algorithm that performs both. Paleri et al claimed to have simplified LCM by incorporating the notion of *safety* (being either available or anticipated) into the definitions of partial availability and partial anticipation and adjusting the flow equations appropriately [35]. We noted earlier that PRE increases code size; LCM's own inventors developed a variant that takes code growth as well as register pressure into consideration in deciding among possible optimal transformations [36]. Hosking et al applied PRE to pointer dereferencing operations, which they termed "access paths" [37]. The most important development for the purposes of this dissertation is SSAPRE, designed by Chow, Kennedy et al to take programs in SSA form as input and to preserve SSA across the transformation [1, 2]; this algorithm will be discussed in detail in Chapter 3.

2.2 Alpern-Rosen-Wegman-Zadeck global value numbering

Identifying the origin of GVN is difficult, but it is clear at least that one GVN heritage was founded by two papers simultaneously published by Alpern, Rosen, Wegman, and Zadeck [6, 10]. One paper described how expressions can be partitioned into congruence classes globally, as opposed to merely in view of a single basic block or another restricted program fragment [10]. *Congruence* is a conservative approximation to *equivalence*, a property undecidable at compile-time, that a set of expressions all have the same value as each other at run-time. This algorithm initially makes the optimistic assumption that large classes of expressions are congruent and refines the partitioning by splitting congruence classes until reaching a fixed

point. The other paper did not outline a specific global value numbering analysis, but simply used SSA and an analysis of trivial moves to make a broader application of what otherwise would be a lexical scheme for eliminating redundancy, including partial [6]. It introduced a system of expression *ranks*, where a nested expression has a lower rank than a later expression in which its result is used, which the algorithm uses to cover *secondary effects*, opportunities which a transformation makes for more optimization. (This concept of ranks was also used by Briggs and Cooper, discussed above [34], and it removes the need to maintain the order of computations in a basic block, anticipating Click's "sea of nodes" IR [38].) The algorithm maintains a *local computation table* for each basic block containing the expressions that occur in the block. The *movable computation table* for each edge contains the expressions anticipated at the beginning of the block to which the edge leads. These are used to determine where to place hoists. After hoisting, the algorithm searches backwards from a computation to find an earlier occurrence and, if one is found, eliminates the redundant computation.

A side contribution of these papers is that they gave the first (to our knowledge) published description of SSA and an algorithm to build it. One of the papers demonstrated many advantages of SSA [6], and has inspired work on more efficient algorithms for SSA construction [5], the relationship between SSA and functional programming [39], and related IRs [3, 38, 40].

This ARWZ GVN launched a line of research in GVN, largely done at Rice University. Not all the algorithms produced by this heritage follow the ARWZ schemes—most, in fact, present themselves as alternatives. The unifying feature is the mind-set of what GVN should be: a method for grouping expressions by static value so that recomputations of available values can be eliminated. Click extended GVN to recognize algebraic identities (for example, $a + a$ has the same value as $a * 2$) and observed its interaction with constant copy propagation [41, 42]. He also devised an alternate, hashing GVN algorithm and combined it with a heuristic for pulling computations out of loops to approximate loop invariant code motion and compete

with PRE [43]. The clearest description of hash-based GVN comes from Briggs, Cooper, and Simpson [44, 45]. A hash table associates the name of an expression—a constant, a variable, or a computation—to a congruence class or value number. SSA guarantees that such an association is well defined (a variable always has the same value) and permits a global view. An assignment asserts that two expressions are in the same congruence class. If the hash table is built by walking over the dominator tree (so all code that dominates a point is visited before that point), then for any computation, we know that the operands have already been set in the hash table (except possibly for constants, whose congruence class is obvious). Since this approach is used in our GVN-PRE hybrid, we reserve the details for Chapter 4. Cooper and Simpson identified another approach to GVN, called *Strongly Connected Component* (SCC)- *Based Value Numbering* [45,46], which uses a hashtable but is flexible enough to allow for congruence classes to be refined and use the SSA graph (a representation of the relationship among SSA variables defined by the phis) instead of the CFG. In Chapter 7 of his dissertation, Simpson described a use of GVN to improve the LCM algorithm for PRE; the essence of this approach, which Simpson called Value-Driven Code Motion, is to capture more precisely under what conditions the recomputation of a subexpression kills the availability of an available expression [45]. In addition to the work of the Rice University group, Gargi extended ARWZ GVN to perform forward propagation and reassociation (as used by Briggs and Cooper [34]) and to consider back edges in the SSA graph to discover more congruences.

How does this understanding of GVN compare with PRE? Based on what we have surveyed so far, one might conclude that "PRE finds lexical congruences instead of value congruences" [43] but finds them even when only partially redundant, whereas GVN finds value congruences but can remove only full redundancies. This is the perspective given in Muchnick's treatment of redundancy elimination [47]. In Figure 2.1(a), $e \leftarrow c + b$ is partially redundant because of $d \leftarrow c + b$. By hoisting and preserving the result of the operation in temporary variable t, PRE produces the program in Figure 2.1(b). Because of the move $c \leftarrow a$ in Figure 2.1(a), the operations

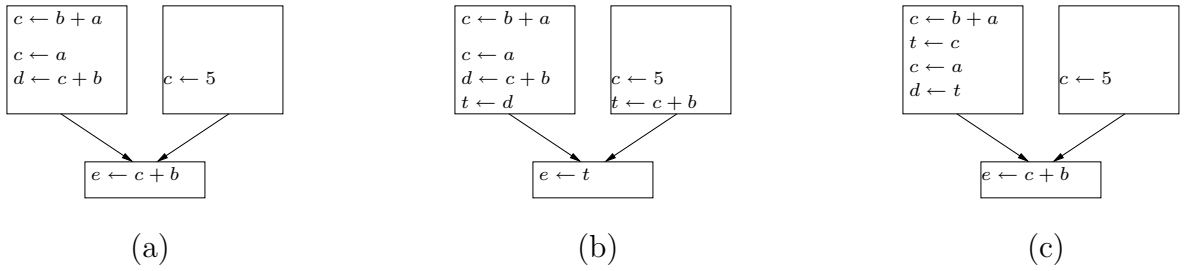


Figure 2.1. Basic example

$c \leftarrow b + a$ and $d \leftarrow c + b$ compute the same value. Accordingly, GVN preserves this value in a temporary and eliminates the re-computation, as in Figure 2.1(c). As this example shows, in this view, neither PRE nor GVN is strictly more powerful than the other. (Click asserts that "in practice, GVN finds a larger set of congruences than PRE" [43], but he seems unaware of LCM).

Are these limitations inherent to either PRE or GVN? Why can there not be a hybrid that covers both and more? This question has been a strong motivation for the research presented in this dissertation. There indeed has been work on hybrid approaches, but to set the stage we must first turn to another GVN tradition that grew up at the same time as ARWZ GVN, like cousins an ocean apart.

2.3 SKR global value numbering

One year before the ARWZ foundation was laid, Steffen presented an optimization [48, 49] which applied Kildall's vision of partitioning expressions into congruence classes to Morel and Renoise's PRE. This begot a line of value numbering approaches distinguished from ARWZ in several ways. While the ARWZ family of projects was usually implementation-driven, this branch, which we call SKR GVN, had a stronger burden for theoretical soundness and provable optimality, often framing the work in terms of abstract interpretation [50]. It did not rely on SSA. It saw no dichotomy between GVN and PRE. It was worked on almost exclusively by three researchers,

Steffen, Knoop, and Rüthing (the same heroes of LCM in PRE's golden age [29,30]), and while itself conscious of the ARWZ family, it was frustratingly overlooked by its counterparts. A scan of the references in Ph.D. theses from the Rice University group finds no citations of this work [42, 45]; even work as late as Gargi's cited only one paper, and then only for a discussion of ARWZ [51, 52].

The seminal paper notes that Morel and Renoise's PRE can move computations, but cannot recognize non-lexical equality, in both cases unlike Kildall [48]. It recasts Kildall's analysis as an abstract interpretation and proves it correct. Since SSA is not used, variables can be redefined and hence expressions can change value, and so the partitioning of expressions may be different at every program point. The paper considers the state of the partitioning at the beginning and end of every basic block (the *pre-* and *post-partitions* with respect to a block). The *Value Flow Graph* (VFG) has the congruence classes of pre- and post-partitions as nodes and edges that represent value equivalence among congruence classes from different partitions as affected by the assignments in a block or the join points in the CFG. A system of boolean equations to determine maximally connected subgraphs of the VFG reveals optimal computation points; from this code can be moved and the program optimized.

How does this compare with ARWZ and its seed? Steffen, with Knoop and Rüthing, claimed that the VFG was superior to SSA because SSA algorithms are optimal only for programs without loops and correct only for reducible CFGs [53]. However, it is worth noting that if SSA expressions are partitioned into global congruence classes such as in a simple hash-based GVN, a graph representing the relevant information of the VFG can be constructed merely by inspecting the phis. SKR also introduces more trivial redefinitions than ARWZ [54], and its researchers also conceded that its computational complexity, compared to ARWZ, was "probably one of the major obstacles opposing to its widespread usage in program optimization" [52]. The researchers maintained, however, that SKR was more complete, general, and amenable to theoretical reasoning.

What SKR lacked in practical acceptance, it compensated for by being a fruitful sanctuary for contemplating the deeper truths of redundancy elimination. The researchers' experience with both PRE and GVN allowed them to make a lucid description of the state of then-current research [55]. All redundancy elimination that involves hoisting hangs on the notion of *safety*, the property that a computation's value will be computed by every trace of the program passing that point. This property is approximated by decomposition into availability (called *upsafety* in SKR) and anticipation (*downsafety*); a computation is safe if it is either available or anticipated. In a lexical (or *syntactic*) view, this "if" is really "if and only if," that is, the approximation is precise. However, Knoop et al showed that in a value-based (in their terms, *semantic*) view, it is not. Consider the program in Figure 2.2 (taken from Knoop et al [55] in substance but put into our framework). Is either $a + b$ or $c_3 + b$ safe at the beginning of block 3? $a + b$ is available from block 2 but not block 1 and anticipated from block 4 but not block 5, and $c_3 + b$ is available from block 2 but not block 1 and anticipated from block 5 but not block 4; our approximation would say "no" to both. However, they indeed are safe because if control comes from block 2, they both will have been computed, and if control comes from block 1, they are the same value and thus definitely will be computed later, whether control takes block 4 or block 5. Knoop et al call algorithms that use the approximation *code motion*, whereas *code placement* refers to schemes, perhaps oracular, that find true safety. Knoop et al considered examples like this the "frontier" for algorithms attempting semantic code placement, and conjectured that, apart from algorithms that change the structure of the CFG [56], "there does not exist a satisfactory solution to the code placement problem" [55].

2.4 Bodík's VNGPRE

This was the fullness of research at the advent of Bodík. In two papers presented at about the same time, he and his co-authors answered the challenge of Knoop et

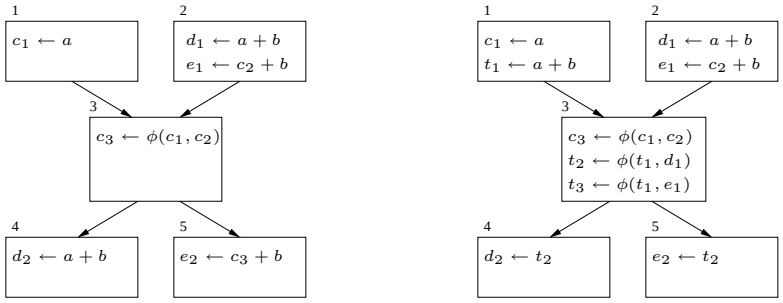


Figure 2.2. Knoop et al's frontier case

al [57, 58]. One of the papers used CFG-restructuring for a practical and complete PRE [58]. The other paper applied a series of analyses to a representation similar to the VFG and performed a PRE that tore through the SKR frontier [57]. In this framework, expressions are names of values. If the program is not in SSA, expressions represent different values at different program points (where a "program point" is recognized between each instruction, branch, and join). Bodík defined a structure called the *Value Name Graph* (VNG) whose nodes are expression / program-point pairs. The edges capture the flow of values from one name to another according to program control flow. Consider the program in Figure 2.3 with its VNG alongside. For each program point (between each instruction, branch, and join) there are three nodes, one for each of the expressions $a + b$, $c + b$, and $d + b$. Note that edges exist between nodes of the same expression unless there is a killing assignment to an operand. Such assignments produce edges between lexically different expressions. A path in such a graph is called a *value thread*. The edges (and therefore threads) are built by performing backward substitution—that is, starting at the end of the program and connecting threads based on the assignments at each instruction; see how the assignment $c \leftarrow a$ substitutes a for c to sew $c+b$ to $a+b$ at the appropriate point. The power of this back-substitution is determined by a parameterized *Value Transfer Function* (VTF), which depends on the language of expressions recognized (for

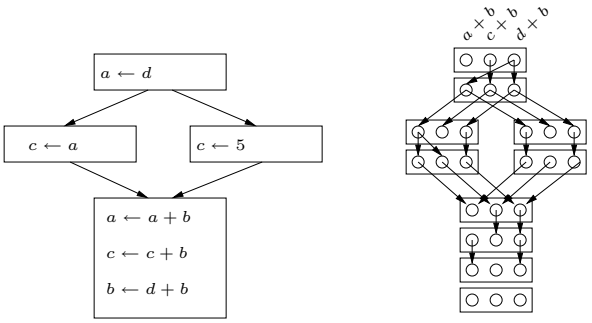


Figure 2.3. Example with a VNG

example, if only single-variable expressions are recognized, the VTF may consider only assignments, but if computational expressions are part of the VTF's language, it may make use of algebraic identities). Moving forward on the VNG, Bodík's algorithm performs what may be considered a path-sensitive GVN, which partitions expression/program-point pairs (that is, nodes of the VNG) into congruence classes, rather than just expressions (Bodík assumed the program is not in SSA). Familiar data flow equations are then used to compute properties like availability, anticipation, insert, earliest, etc. By making the definition of earliest very aggressive, the frontier is crossed. More information on the representation and algorithm can be found in Bodík's Ph.D. dissertation [59].

Bodík approached the problem with an ARWZ GVN mind set: "Commonly used techniques fail to optimize [an example with value-based partial redundancy] because... partial redundancy elimination [citing LCM [29] as the latest among other PREs] based on data flow problems that are solved over a space of lexical names can only detect value reuse between [lexically identical expressions]... Global value numbering [citing ARWZ [10]] is a method for determining which lexically different (static) names are equivalent among all program paths. This is ineffective [because two expressions in a partial redundancy] are equal only along [certain] path[s]" [57]. He concurred with Muchnick that "these techniques can be used independently to eliminate redundant computations, but none is strictly superior to the others" [57].

Nevertheless, note that this representation is essentially Knoop et al's VFG applied to instructions rather than basic blocks; Knoop et al also make use of a backwards substitution function δ, which is equivalent to the VTF [55]. This seems to have been more prophetic than influential, as Bodík says, "Knoop, Rüthing, and Steffen developed independently from us a representation called the Value Flow Graph" [57].

How do we evaluate Bodík's work? The power of his representation and algorithm is remarkable. Bodík himself conceded it produces significant register pressure [60]. Dhamdhere pointed out that its granularity to single instructions (as opposed to basic blocks) makes it very complex conceptually [61]. Although Bodík backed up his claims with an implementation [57,59], it is difficult to imagine a software engineer picking up one his papers and incorporating it into an optimizing compiler, given its complexity and memory demands. The research in this dissertation makes a complete, value-based PRE accessible to all compilers, that they may observe the goals Bodík made. We make comparisons to Bodík's till the close of the dissertation.

3 ANTICIPATION-BASED PARTIAL REDUNDANCY ELIMINATION

Sergeant Hunter: You know what I would do if I were you?
Pee-Wee: What?
Sergeant Hunter: I'd retrace my steps.

—*Pee-Wee's Big Adventure*

3.1 Introduction

In this chapter, we present a new algorithm for PRE in SSA. An earlier SSA-based algorithm, SSAPRE, by Chow, Kennedy, et al [1, 2], is weak in several ways. First, it makes stronger assumptions about the form of the program (particularly the lives ranges of variables) than are true for the traditional definition of SSA. Other SSA-based optimizations that are considered to preserve SSA may break these assumptions; if these other optimizations are performed before SSAPRE, the result may be incorrect. Moreover, the SSAPRE algorithm does not handle nested expressions (secondary effects) or address the frontier identified by SKR [55], as Bodík's work does [57]. Finally, from a software engineering perspective, the earlier PRE algorithm for SSA is difficult to understand and implement. In this chapter we will describe SSAPRE and illustrate its shortcomings. Next, we present our main contribution: a new algorithm for PRE that assumes and preserves SSA, called ASSAPRE. It is structurally similar to, though simpler than, SSAPRE. The key difference is that it discovers redundancy by searching backwards from later computations that can be eliminated to earlier computations, rather than searching forward from early computations to later. Our running example will demonstrate that our new algorithm addresses the concerns about SSAPRE. Finally, we present performance results for ASSAPRE.

3.2 SSAPRE

In this section, we explain the SSAPRE algorithm of Chow, Kennedy, et al. [1, 2], giving part of the motivation for the present work. First, we summarize the concepts and describe the algorithm's phases with an example. Second, we demonstrate weaknesses of the algorithm to show where improvements are necessary.

3.2.1 Summary

As mentioned before, one useful tool when arranging any set of optimizations is a suitable, common program representation. If all optimizations use and preserve the same IR properties, then the task of compiler construction is simplified, optimization phases are easily reordered or repeated, and expensive IR-rebuilding is avoided. LCM, the most widely recognized PRE algorithm at the time SSAPRE was presented, did not use SSA [29,30], and Briggs and Cooper [34] explicitly broke SSA before performing PRE. The principle motivation for SSAPRE is to do PRE while taking advantage of and maintaining SSA form.

We have already spoken of the distinction of lexical and value-based views of the program. SSAPRE is lexical, but important to its notion of lexical equivalence is the idea that variables that are different SSA versions of the same source-level variable are still considered lexically the same. SSAPRE associates expressions that are lexically equivalent when SSA versions are ignored. For example, $a_1 + b_3$ is lexically equivalent to $a_2 + b_7$. We take $a + b$ as the *canonical expression* and expressions like $a_1 + b_3$ as versions of that canonical expression; lexically equivalent expressions are assigned version numbers analogous to the version numbers of SSA variables. A *chi statement*[1] (or simply chi) merges versions of expressions at CFG merge points just as phis merge variables. The chis can be thought of as potential phis for a hypothetical temporary that may be used to save the result of the computation if

[1]In the original description of SSAPRE, Chow, Kennedy et al. [1, 2], chis are called Phis (distinguished from phis by the capitalization) and denoted in code by the capital Greek letter Φ.

an opportunity for performing PRE is found. Just as a phi stores its result in a new version of the variable, so chis are considered to represent a new version of an expression. The operands of a chi (which correspond to incoming edges just as phi operands do) are given the version of the expression that is available most recently on the path they represent. If the expression is never computed along that path, then there is no version available, and the chi operand is denoted by $\perp$. Chis and chi operands are also considered to be occurrences of the expression; expressions that appear in the code are differentiated from them by the term *real occurrences*.

SSAPRE has six phases. We will discuss these while observing the example in Figure 3.1. The unoptimized program is in Figure 3.1(a). The expression $b + c$ in block 4 is partially redundant because it is available along the right incoming edge.

1. **Chi Insertion** For any real occurrence of a canonical expression on which we wish to perform SSAPRE, insert chis at blocks on the dominance frontier of the block containing the real occurrence and at blocks dominated by that block that have a phi for a variable in the canonical expression. (These insertions are made if a chi for that canonical expression is not there already.) These chis represent places that a version of an expression reaches but where it may not be valid on all incoming edges and hence should be merged with the values from the other edges. In our example, a chi is inserted at the beginning of block 4 for canonical expression $b + c$. Compare with the placement of phis in Cytron et al's SSA-building algorithm [5].

2. **Rename** Assign version numbers to all real expressions, chis, and chi operands. This algorithm is similar to that for renaming SSA variables given in Cytron et al [5]. While traversing the dominator tree of the CFG in preorder, maintain a renaming stack for each canonical expression. The item at the top of the stack is the defining occurrence of the current version of the expression. For each block b in the traversal, inspect its chis, its real instructions, and its corresponding chi operands in the chis of its successors, assigning a version

number to each. If an occurrence is given a new version number, push it on the stack as the defining occurrence of the new version. When the processing of all the blocks that b dominates is finished, pop the defining occurrences that were added while processing $b$lock. Table 3.1 explains how to assign version numbers for various types of occurrences depending on the defining occurrence of the current version (this table is expanded from Table 1 of Chow, Kennedy et al. [2], using delayed renaming). Note that chi operands cannot be defining occurrences. Figure 3.1(b) shows our example after Rename. The occurrence in block 3 is given the version 1. Since block 4 is on its dominance frontier, a chi for this expression is placed there to merge the versions reaching that point, and the right chi operand is given version 1. Since the expression is unavailable from the left path, the corresponding chi operand is $\perp$. The chi itself is assigned version 2. Since that chi will be on top of the renaming stack when $b + c$ in block 4 is inspected and since the definitions of its variables dominate the chi, it is also assigned version 2.

3. **Downsafety** Compute downsafety with another dominator tree preorder traversal, maintaining a list of chis that have not been used on the current path. When program exit is reached, mark the chis that are still unused (or used only by operands to other chis that are not *downsafe*) as not *downsafe*. In our example, the chi is clearly *downsafe*.

4. **WillBeAvail** Compute *canBeAvail* for each chi by considering whether it is *downsafe* and, if it is not, whether its operands have versions that will be available at that point. Compute *later* by setting it to false for all chis with an operand that has a real use for its version or that is defined by a chi that is not *later*. Compute *willBeAvail* by setting it to true for all chis for which *canBeAvail* is true and *later* is false. Compute *insert* for each chi operand by setting it to true for any operand in a chi for which *willBeAvail* is true and either is $\perp$ or is defined by a chi for which *willBeAvail* is false. Since the chi

in Figure 3.1(b) is *downsafe*, it can be available. Since its operand from block 3has a version defined by a real occurrence, it cannot be postponed. Therefore, *willBeAvail* is true for the chi. [2]

5. **Finalize** Using a table to record what use, if any, is available for versions of canonical expressions, insert computations for chi operands for which *insert* is true (allocating new temporary variables to store their value), insert phis in place of chis for which *willBeAvail* is true, and mark real expressions for reloading that have their value available at that point in a temporary. In Figure 3.1(c), we have inserted a computation for its $\perp$ chi operand and a phi in the place of the chi to preserve the value in a new temporary. Strunk and White call the word *finalize* "a pompous, ambiguous verb" [63]. Whatever one thinks of the word itself, Chow, Kennedy, et al's choice of it as a name for this phase is as mysterious as the algorithm itself.

6. **Code Motion** Replace all real expressions marked for reloading with a move from the temporary available for its version. A move from the new temporary added in the previous step can then replace the real occurrence in block 4, as Figure 3.1(c) displays. This is another phase name that is not particularly descriptive, since the movement of code is actually the net effect of this combined with phase 5. A better name would be *elimination*.

3.2.2 Counterexamples

SSAPRE's concept of redundancy is based on source-level lexical equivalence. Thus it makes a stronger assumption about the code than SSA form: that the source-level variable from which an SSA variable comes is known and that there is a one-to-one correspondence at every program point. Dependence on this assumtion

[2] Whitlock quipped that with all the properties for chis, it is surprising they do not have a *shouldBeAvailNextThursday* property, and that the chis for which *willBeAvail* is false remind him of the women he asks out on dates [62].

Table 3.1
How to determine what version to assign to an occurrence of an expression

Definition	Occurrence being inspected		
	real	chi	chi operand
real	Assign the old version if all corresponding variables have the same SSA version; otherwise assign a new version and push the item on the stack.	Assign a new version and push the item on the stack.	Assign the old version if the defining occurrence's variables have SSA versions current at the point of the chi operand; otherwise assign $\perp$.
chi	Assign the old version if all the definitions of the variables dominate the defining chi; otherwise assign a new version and push the item on the stack.	Assign a new version and push the item on the stack	Assign the old version if the definitions of the current versions of all relevant variables dominate the defining chi; otherwise assign $\perp$.

Table 3.2
Chi and chi operand properties

Properties for chis	
downsafe	The value of the chi is used on all paths to program exit.
canBeAvail	None of the chi's operands are $\perp$ or the chi is downsafe (i.e., the value has been computed on all paths leading to this point or insertions for it can be made safely).
later	Insertions for the chi can be postponed because they will not be used until a later program point.
willBeAvail	The value of the chi will be available at this point after the transformation has been made; it can be available and cannot be postponed.
Property for chi operands	
insert	An insertion needs to be made so that this chi operand's value will be available.

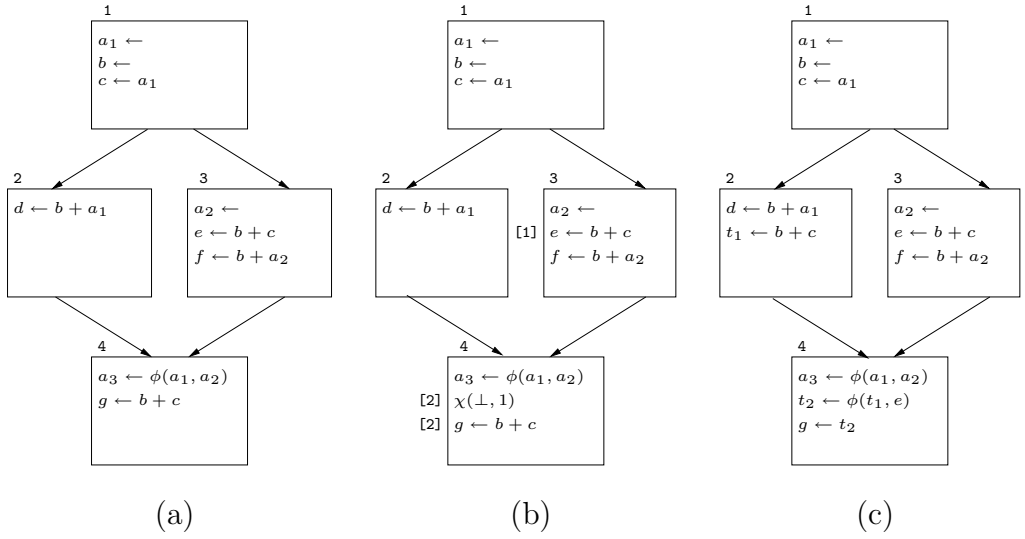


Figure 3.1. PRE example 1

weakens SSAPRE's ability to find all redundancies. In the example above, since the definition of c is a move from a_1, $b + c$ has the same value as $b + a_1$ in block 2. This means inserting $b+c$ in block 2 is not optimal; although it is not redundant lexically, it clearly computes the same value. Situations like this motivate research for making (general) PRE more effective, such as in Briggs and Cooper [34]. In our example, all we need is a simple constant copy propagation to replace all occurrences of c with a_1, as shown in Figure 3.2(a). Now the expression in block 4 is fully redundant even in the lexical sense. SSAPRE is fragile when used with such a transformation because it assumes that no more than one *version* of a variable may be simultaneously live. Copy propagation breaks this condition here, since both a_1 and a_2 are live at the exit of block 3, and, assuming a_3 is used later in the program, both a_1 and a_3 are live in block 4. (Appel points out this anomaly of SSA form in his exercise 19.11 [64].) In this case, $b + a_1$ is given the same version in blocks 2 and 3, while $b + a_2$ in block 3 is given its own version, since it does not match the current version $b + a_1$. See Figure 3.2(b).

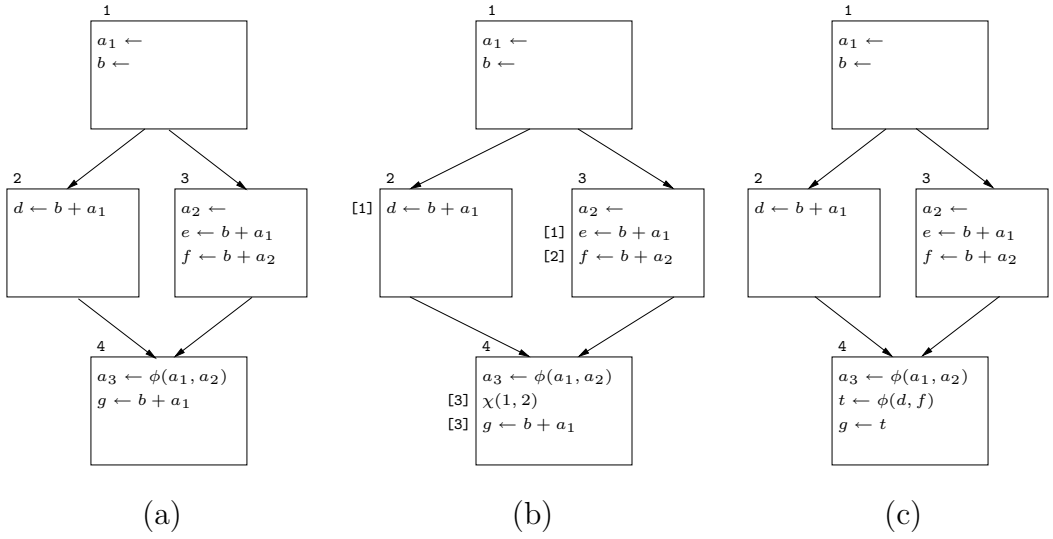


Figure 3.2. PRE example 1 after copy propagation

What version should be given to the right operand of the chi in block 4? Version 2 ($b + a_2$) is the current defining occurrence, and Table 3.1 suggests that its version should be used if its operands are the current versions of the variables. Since a_2 is the corresponding operand to the phi at block 4, it can be considered current, and we give the chi operand version 2. However, this produces incorrect code. In the optimized program in Figure 3.2(c), g has value $b + a_2$ on the right path, not $b+a_1$ as it should. Chow, Kennedy, et al. [1,2] give an alternative Rename algorithm called Delayed Renaming which might avoid producing wrong code, depending on how it is interpreted. In that algorithm, the "current" variable versions for a chi operand are deduced from a later real occurrence that has the same version as the chi to which the operand belongs, adjusted with respect to the phis at that block. In this case, the chi has the same version as $g \leftarrow b + a_1$. Since neither b nor a_1 are assigned by phis, they are the current versions for the chi operand, and are found to mismatch $b + a_2$; thus the chi operand should be $\perp$. However, this would still miss the redundancy between $e \leftarrow b + a_1$ and $g \leftarrow b + a_1$.

The entry in Table 3.1 for processing real occurrences when a chi is on the stack is also fragile. Figure 3.3 displays another program before, during, and after SSAPRE.

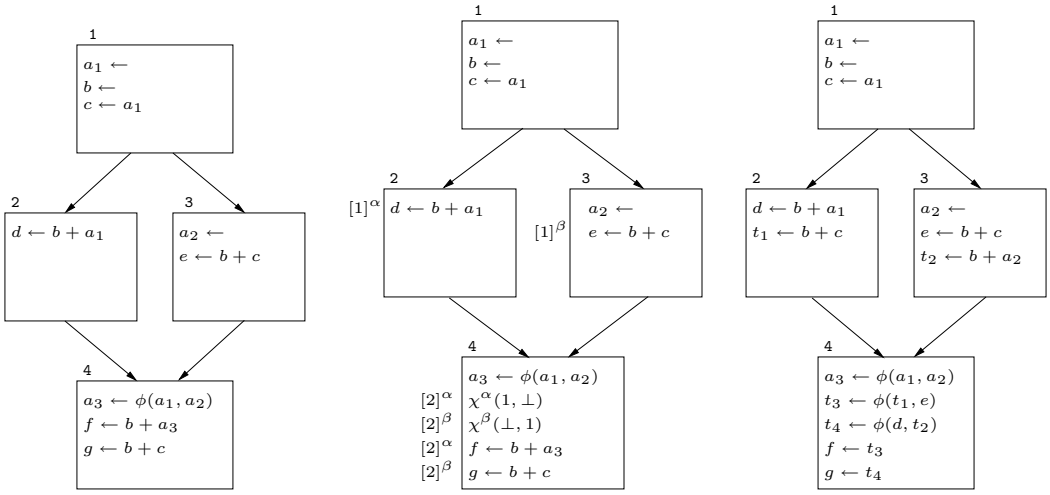


Figure 3.3. PRE example 2

Since two canonical expressions ($b+a$ and $b+c$) are being processed, we distinguish their chis and version labels with α and β, respectively. The same program, before SSAPRE but after constant copy propagation, is shown in Figure 3.4(a). There is now only one canonical expression. Chi Insertion and Rename produce the version in 3.4(b). The right chi operand is $\perp$ because a_1 in $b + a_1$ is not "current" on that edge, not being the corresponding operand to the phi. Such a chi represents the correct value for $f \leftarrow b + a_3$. However, $g \leftarrow b + a_1$ is also assigned the same version: the chi is the definition of the current version, and the definitions of all the operands of $g \leftarrow b + a_1$ dominate it, which are the conditions prescribed by Table 3.1. Consequently, the optimized version assigns an incorrect value to g in (c).

In both cases, the problem comes from the algorithm assuming that only one version of a variable can be live at a given time. Simple fixes are conceivable, but not without liabilities. Renaming variables to make SSAPRE's assumption valid will merely undo the optimizations intended to make PRE more effective. The rules in Table 3.1 could be made more strict; for example, we could associate versions of variables with each chi and make sure all versions match before assigning a real

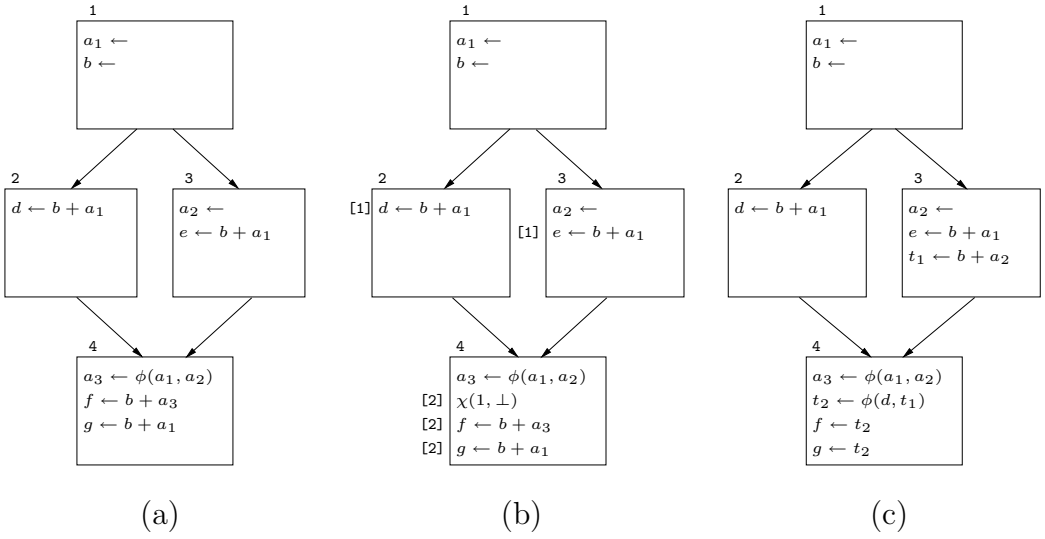


Figure 3.4. PRE example 2 after copy propagation

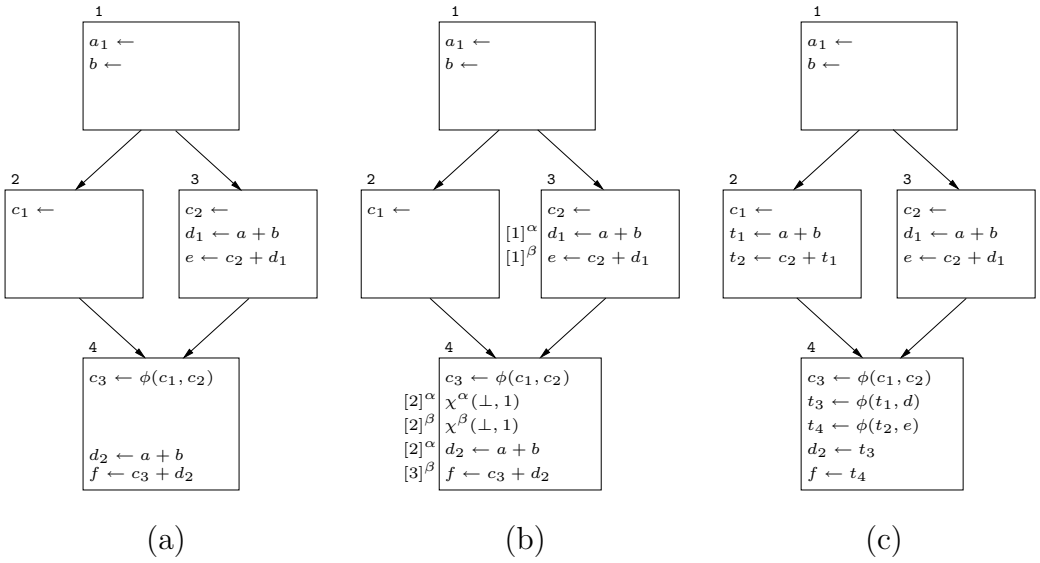


Figure 3.5. Nested redundancy

occurrence the same version as a chi. This would prevent the incorrect result in Figure 3.4, but since $b + a_1$ in the original is indeed partially redundant, ignoring it would not be optimal.

Another case when SSAPRE misses redundancy is when a redundant computation has subexpressions. Consider the program in Figure 3.5(a). The expression

$a + b$ in block 4 is clearly partially redundant; $c_3 + d_2$ is also partially redundant, since d_1 and d_2 have the same value on that path. Essentially, it is the complex expression $c + a + b$ that is redundant. Figure 3.5(b) shows the program after Chi Insertion and Rename. As before, version numbers and chis for the two different expressions are distinguished by Greek letters. $d_2 \leftarrow a + b$ is given the same version as the corresponding chi and will be removed by later SSAPRE phases. However, $f \leftarrow c_3 + d_2$ contains d_2, whose definition does not dominate the χ^β. According to Table 3.1, we must assign the real occurrence a new version, even though the chi represents the correct value on the right path. The optimality claim (Theorem 4) of Chow, Kennedy, et al assumes that a redundancy exists only if there is no non-phi assignment to a variable in the expression between the two occurrences: "We need only show that any redundancy remaining in the optimized program cannot be eliminated by any safe placement of computations. Suppose $\mathcal{P}$ is a control flow path in the optimized program leading from one computation, ψ_1, of the expression to another computation, ψ_2, of the same expression with no assignment to any operand of the expression along $\mathcal{P}$" [1,2]. This example frustrates that assumption, and the running example in Briggs and Cooper [34] demonstrates that this situation is not unrealistic when PRE is used with other optimizations. The optimized form we desire is in Figure 3.5(c), which would result if SSAPRE were run twice. This is referred to as a *secondary effect*. The situation is not just a product of our three-address representation, since it is conceivable that d has independent uses in other expressions.

Finally, this algorithm cannot pass the frontier defined by Knoop et al [55]. Consider the example in Figure 3.6(a). The computations $b_3 + a$ and $c_4 + a$ in blocks 4 and 5, respectively, are partially redundant, since they are each computed in block 2. Thinking strictly lexically, we cannot insert computations in block 1 for either expression, because such a computation would not be downsafe, as it has a path to exit on which the expression is not computed. However, note that in block 1, b_2 and c_2 have the same value, and therefore $b_2 + a$ and $c_2 + a$ will also have the

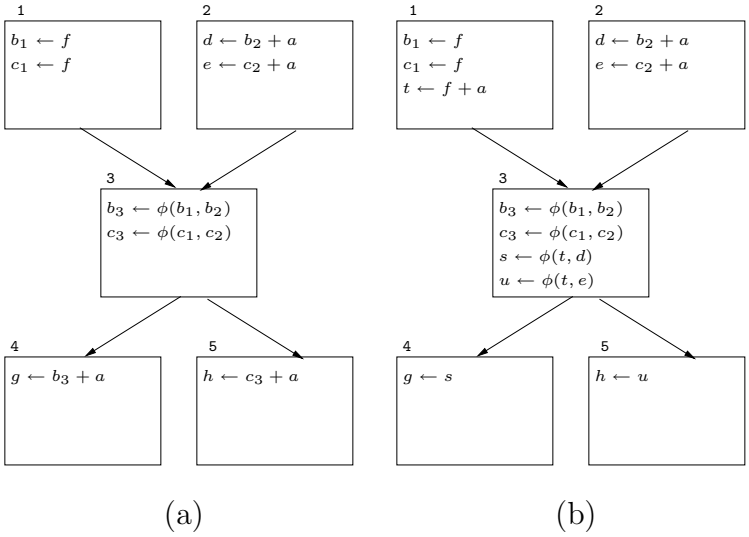


Figure 3.6. Frontier case

same value. Accordingly, only one computation need be inserted for both $b + a$ and $c + a$, and that computation would be downsafe because (if we think in terms of values and not just lexical expressions) it is used in both block 4 and block 5. The optimized version appears in Figure 3.6(b). Being blinded by lexical equivalence is only half the problem; we also need a way to recognize that the chis for expressions $b + a$ and $c + a$ have parameters from block 1 which need to be considered to be the same parameter for the sake of computing downsafety. In this way, SSAPRE cannot compete with Bodík [57].

3.3 ASSAPRE

The problems with SSAPRE stem from assumptions about lexical equivalence. From this point on, we will abandon all notions of lexical equivalence among distinct SSA variables. We will ignore the fact that some SSA variables represent the same source-level variables and consider any variable to be an independent temporary whose scope is all instructions dominated by its definition.

As already noted, SSAPRE is characterized by availability: chis are placed at the dominance frontier of a computation and thus indicate where a value is (at least partially) available. In this section, we present a new algorithm, *Anticipation*-SSAPRE (ASSAPRE) which searches backward from computations and places chis at merge points where a value is anticipated. If the algorithm discovers the definition of a variable in the expression for which it is searching, it will alter the form of that expression appropriately; for example, if it is searching for $t_3 + t_7$ and discovers the instruction $t_7 \leftarrow t_1$, then from that point on it will look for $t_3 + t_1$. (We do not need also to look for $t_3 + t_7$ since t_7 is out of scope above its definition.) When a chi is created, it is allocated a fresh hypothetical temporary, which will be the target of the phi that will be placed there if the chi is used in a transformation. The result of this phase is that instructions are assigned temporaries that potentially store earlier computations of the instructions' values. The instructions may be replaced by moves from those temporaries.

Since all variables are distinct, so are all expressions. This obviates the need for the notion of canonical expressions and the need for version numbers. Thus our algorithm has no renaming phase. Not only do the problems in SSAPRE come from ambiguities in the specification of Rename, but also our implementation experience has found Rename to be particularly prone to bugs [65]. Next, the algorithm analyzes the chi operands that are $\perp$ to determine which ones represent downsafe insertions and analyzes the chis to determine which ones can be made available, which ones can be postponed, and, from these, which ones should be made into phis. This happens in the two phases Downsafety and WillBeAvail. Finally, where appropriate, insertions are made for chi operands, phis are put in the place of chis, and redundant computations are replaced with moves from temporaries. This phase, Code Motion, subsumes the Finalize phase in SSAPRE. Thus, ASSAPRE has four phases from SSAPRE's six.

The next subsections describe the four phases of ASSAPRE in detail. We clarify each step by observing what it does on a running example. The unoptimized version

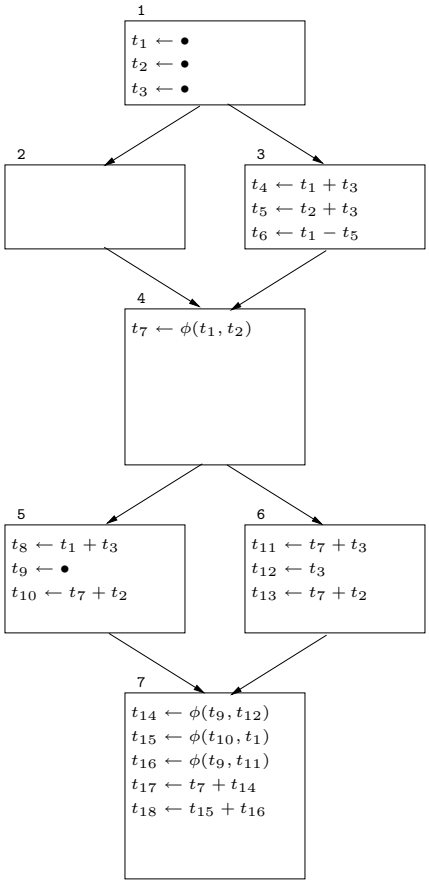


Figure 3.7. Unoptimized

is in Figure 3.7. Assignments with no right hand side are assumed to come from sources we cannot use in ASSAPRE. For example, t_1, t_2, and t_3 in block 1 may be parameters to the procedure, and t_9 in block 5 could be the result of a function call.

3.3.1 Chi Insertion

Chis, chi operands, and instructions. Chi Insertion is the most complicated of the phases, and it differs greatly from the equivalent phase in SSAPRE. Recall that a chi is a potential phi or merge point for a hypothetical temporary, and it has an expression associated with it, for which the hypothetical temporary holds a pre-computed value. The hypothetical temporary is allocated at the time the chi

is created, so it is in fact a real temporary from the temporary pool of the IR; it is hypothetical only in the sense that we do not know at the time it is allocated whether or not it will be used in the output program. The hypothetical temporary also provides a convenient way of referring to a chi, since the chi is the hypothetical definition point of that temporary. In the examples, a chi will be displayed with its operands in parentheses followed by its hypothetical temporary in parentheses. The expression it represents is written in the margin to the left of its basic block (see Figure 3.9).

As before, a chi operand is associated with a CFG edge and either names a temporary or is $\perp$. However, an important feature of our algorithm is that chis at the same block can share chi operands. Assume that if two chi operands are in the same block, are associated with the same CFG edge, and refer to the same temporary, then they are the same. Chi operands that are $\perp$ are numbered in our examples to distinguish them. The temporary to which a chi operand refers is called the *definition* of that chi operand, and if the chi operand is $\perp$, we say that its definition is *null*. Chi operands also have expressions associated with them, as explained below, but to reduce clutter, they are not shown in the examples.

Real occurrences of an expression (ones that appear in the code and are saved to a temporary) also have definitions associated with them, just as chi operands do. This is the nearest temporary (possibly hypothetical) found that contains the value computed by the instruction. If no such temporary exists, then the definition is $\perp$. In the examples, this temporary is denoted in brackets in the margin to the left of its instruction. This notion of definition induces a relation among real instructions, chis, and chi operands and is equivalent to the *factored redundancy graph* of Chow, Kennedy et al. [1, 2]

Searching from instructions. Chi Insertion performs a depth-first, preorder traversal over the basic blocks of the program. In each basic block, it iterates forward over the instructions. For each instruction on which we wish to perform PRE (in our

case, binary arithmetic operations), the algorithm begins a search for a definition. During the search, it maintains a *search expression* e, initially the expression computed by the instruction where it starts. It also maintains a reference to the *starting instruction*, i. Until it reaches the preamble of the basic block (which we assume includes the phis and chis), the algorithm inspects an instruction at each step, which we refer to as the *current instruction*. Note that the "current instruction" is that which is current in the inner loop, where as the current instruction in the outer loop we call the "starting instruction," i.

The current instruction being inspected is either a computation of e, the definition point of one of the operands of e, or an instruction orthogonal to e. In practice, we expect that this last case will cover the majority of the instructions inspected, and such instructions are simply ignored. If the instruction is a computation of e (that is, the expression computed exactly matches e), then the search is successful, and the temporary in which the current expression stores its result is given as the definition of i. When the current expression defines one of e's operands, then what happens depends on the type of instruction. If it is $\bullet$, then nothing can be done; the search fails, and i's definition is null, since a component to e is not live earlier than this instruction and consequently the search expression cannot exist earlier. If the current instruction is a move, then we *emend* the search expression accordingly: the occurrences in e of the the target of the move are replaced by the move's source. The search then continues with the new form of e. Consider the situation in Figure 3.8. The search from instruction $t_4 \leftarrow t_1 + t_3$ begins using $t_1 + t_3$ as its search expression and then hits the instruction $t_3 \leftarrow t_2$. $t_1 + t_3$ certainly cannot exist prior to that instruction, but the emendation $t_1 + t_2$ may. If a copy propagation has been performed immediately before ASSAPRE, then this should not be necessary, since moves will be eliminated. However, we make no assumptions about the order in which optimizations are applied, and in our experience such moves proliferate under many IR transformations.

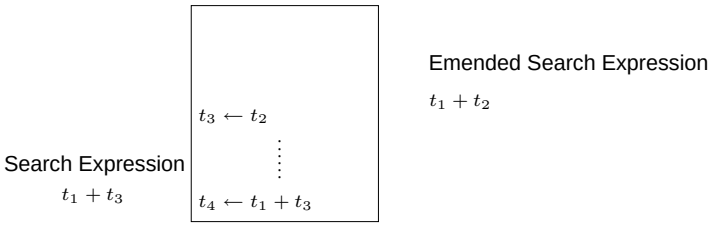


Figure 3.8. Making an emendation

If the current instruction stores the result of a PRE-candidate computation to an operand of e, then we make a *conjectural emendation* of the search expression; that is, we optimistically assume the instruction will be replaced by a move, and emend e as if that move were already in place. Since blocks are processed in depth-first preorder and instructions processed by forward iteration, we know that the instruction being inspected has already been given a definition. If that definition is not null, then we can conjecture that the instruction will be replaced by a move from the temporary of its definition to the temporary being written to, and we emend e as if that move were already present. From a software engineering standpoint, such conjectural emendations may be skipped during early development stages. However, they are necessary for optimality, since they take care of the situation in Figure 3.5. Without them, ASSAPRE would require multiple runs to achieve optimality, especially in concert with the enabling optimizations described in Briggs and Cooper [34]. If the instruction's definition is null (or if conjectural emendations are turned off), such an instruction must be treated the same way as $\bullet$, and the search fails.

When the search reaches the preamble of a block, there are three cases, depending on how many predecessors the block has: zero (in the case of the entry block), one (for a non-merge point), or many (for a merge point). At the entry block, the search fails. At a non-merge point, the search can continue starting at the last instruction of the predecessor block with no change to i or e (we know that those instructions already have been given definitions because the predecessor in this case would be a dominator). Merge points are the interesting case. There the algorithm inspects

the expressions represented by the chis already placed in that block. If one is found to match e, then that chi (or more properly, that chi's hypothetical temporary) is given as i's definition. If no suitable chi is found, the algorithm creates one; a new temporary is allocated (which becomes i's definition), and the chi is appended to the preamble and to a list of chis whose operands need to be processed, as will be described below. In either case, the search is successful.

Consider the program in Figure 3.7. From the instruction $i = t_4 \leftarrow t_1 + t_3$[3] in block 3, we search for an occurrence of $e = t_1 + t_3$ and immediately hit the preamble of the block. Since it is not a merge point, the search continues in block 1. The assignment of t_3 is relevant to e, but since that instruction is not a move or a PRE candidate, the search fails and i is assigned $\perp$. Similarly, the searches from $t_5 \leftarrow t_2 + t_3$ and $t_6 \leftarrow t_1 - t_5$ fail. For $i = t_8 \leftarrow t_1 + t_3, e = t_1 + t_3$, the search takes us to the preamble of block 4. Since it is a merge point, we place a chi there with expression $t_1 + t_3$ and allocate t_{19} as its hypothetical temporary, which also becomes the definition of $t_8 \leftarrow t_1 + t_3$. Similarly for $i = t_{10} \leftarrow t_7 + t_3$, we place a chi, allocating the temporary t_{20}; and for $i = t_{11} \leftarrow t_7 + t_3$, a chi with temporary t_{21}. For $i = t_{13} \leftarrow t_7 + t_2, e = t_7 + t_2$, when the preamble of block 4 is reached, we discover that a chi whose expression matches e is already present, and so its temporary (t_{20}) becomes i's definition. Finally, searches from the two real instructions in block 7 produce the chis with temporaries t_{22} and t_{23}, which serve as the real instructions' definitions. Figure 3.9(a) displays the program at this point.

Searching from chi operands. When all basic blocks in the program have been visited, the algorithm creates chi operands for the chis that have been made. Chi operands stand in for operands to the phis that may be created later, and since chi operands have definitions just like real instructions, this involves a search routine identical to the one above. The only complications are chi-operand sharing among chis at the same block and emendations with respect to phis and other chis. For each

[3]Recall that $\leftarrow$ denotes assigment in our language; thus = stands for equals in the mathematical and logical sense.

chi c in the list generated by the search routine and for each in-coming edge η at that block, the algorithm determines an expression for the chi operand for that edge. To do this, we begin with the chi's own expression, e, and inspect the phis at the block and the chis that have already been processed. If any write to an operand of e (that is, if one of e's operands is the target of a phi or the hypothetical temporary of a chi), then that operand must be replaced by the operand of the phi that corresponds to η or the definition of the chi operand that corresponds to η. (If such a chi operand is $\perp$, then we can stop there, knowing that the chi operand we wish to create will be $\perp$, and that it is impossible to make an expression for it.) For example, if $e = t_4 + t_6$ and the current block contains $t_6 \leftarrow \phi(t_1, t_2)$, the left chi operand will have the expression $t_4 + t_1$ and the right will have $t_4 + t_2$. We call the revised expression e'. Once an expression has been determined, the algorithm inspects the chi operands corresponding to η of all the chis at that block that have already been processed; if any such chi operand has an expression matching e', then that chi operand also becomes an operand of c. This sharing of chi operands is necessary to cross Knoop et al's frontier [55] as the examples will show. If no such chi operand is found, the algorithm creates a new one with e' as its expression, and it searches for a definition for it in the same manner as it did for real instructions, in this case i referring to the chi operand. Note that this may also generate new chis, and the list of chis needing to be processed may lengthen.

So far, this phase has generated the five chis in Figure 3.9(a). The search for definitions for their operands is more interesting. In block 4, the t_{19} chi has expression $e = t_1 + t_3$. Since none of e's operands are defined by phis, e also serves as the expression for the chi's operands. Search from the left operand fails when it hits the write to t_3 in block 1. Search from the right operand, however, discovers $t_4 \leftarrow t_1 + t_3$, which matches e; so t_4 becomes its definition. The t_{20} chi has expression $e = t_7 + t_2$. Since $t_7 \leftarrow \phi(t_1, t_2)$ writes to one of e's operands, the two chi operands will have the expressions $t_1 + t_2$ and $t_2 + t_2$. Since neither have occurred in the program, two $\perp$ chi operands are created. The t_{21} chi has expression $e = t_7 + t_3$. The phi changes this to

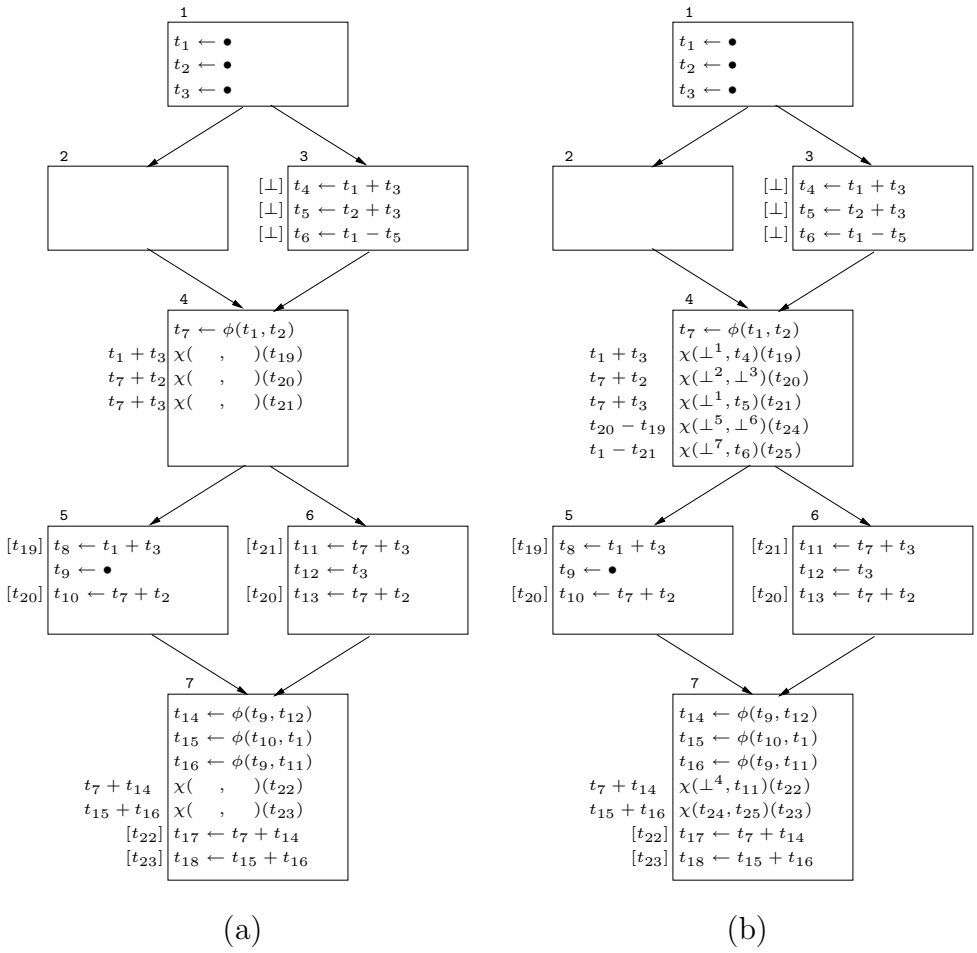


Figure 3.9. During Chi Insertion

$t_1 + t_3$ for the left edge and to $t_2 + t_3$ for the right edge. The former is identical to the expression for the left operand of the t_{19} chi, so that operand is reused. Searching from the right chi operand discovers $t_5 \leftarrow t_2 + t_3$.

Turning to block 7, the t_{22} chi has expression $t_7 + t_{14}$, which the phis change to $t_7 + t_9$ for the left edge and $t_7 + t_{12}$ for the right. On the left side, a search for a definition for the chi operand halts at $t_9 \leftarrow \bullet$ in block 5. The right hand search discovers $t_{11} \leftarrow t_7 + t_3$. The left and right chi operands for the t_{23} chi ($e = t_{15} - t_{16}$) have expressions $t_{10} - t_8$ and $t_1 - t_{11}$, respectively. On the left, $t_{10} \leftarrow t_7 + t_2$ and $t_8 \leftarrow t_1 + t_3$ in block 5 affect the search expression, which we conjecturally emend to $t_{20} - t_{19}$. When the preamble of block 4 is reached, a new chi must be placed, allocating the temporary t_{24}. On the right, $t_{11} \leftarrow t_7 + t_3$ causes the search expression to be emended to $t_1 - t_{21}$, which also requires a new chi at block 4. Finally, we search for definitions for the chi operands of the two chis recently placed in block 4. The program at the end of the Chi Insertion phase is shown in Figure 3.9(b).

Termination. Since the search for a definition for a chi operand may traverse a back edge, we now convince ourselves that the algorithm will terminate on loops; indeed, one qualification needs to be made for conjectural emendations in order to ensure termination. Consider the case when the search from a chi operand brings us back to the merge point of its chi. If the search expression has not changed, then the chi itself is the definition, and the search ends; otherwise, if no chi at that merge matches the current search expression, a new chi will be made. If operands in the search expression have been replaced only by other temporaries or constants in the input program (in other words, no conjectural emendations), then this process cannot go on forever, because there are only a finite number of temporaries or constants in the code. The danger lies when a temporary has been replaced by a hypothetical temporary. We say that a chi is *complete* if searches from all of its chi operands have been finished, and none refer to chis that are not complete. We say that a chi c *depends* on a chi operand ω if ω is an operand of c or if ω is an operand to a chi that

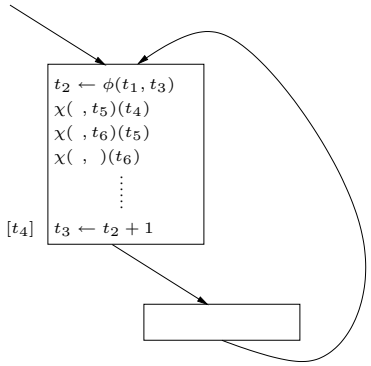


Figure 3.10. Unterminated Chi Insertion

in turn is referred to by a chi operand on which c depends. Supposing, as above, that e' is the search expression when searching for a definition of chi operand ω, then if the current instruction being inspected writes to an operand of e' but is defined by a chi c that does not depend on ω, then this search does not prolong the sequence of searches that make c complete. If c does depend on ω, we have no such guarantee. For example, if the expression in question is the increment of a loop counter, the algorithm would attempt to insert a chi for each iteration—an infinite number. To see this illustrated, consider Figure 3.10. t_1, t_2, and t_3 represent a loop-counter in the source code. If we search for an earlier occurrence of t_2+1, we will place a chi for it at the top of the block. Searching, then, for the chi operands, we emend the search expression to t_3+1 because of the phi. This leads us back to the original occurrence, $t_2 + 1$, with target t_3. If we were to emend the search expression conjecturally, we would replace t_3 with t_4, the occurrence's definition. Placing a chi, then for $t_4 + 1$, we search for an operand for the expression $t_5 + 1$, since the previous chi merges t_5 into t_4. Continuing like this would place chis at this point indefinitely. Thus:

> *when searching from a chi operand ω, conjectural emendations are forbidden if the current instruction is defined by a chi that depends on ω.*

The algorithm for this phase is summarize in Figure 3.11.

Let Δ be a mapping from instructions, chis, and chi operands to hypothetical temporaries
For each block b
    For each instruction $i = t \leftarrow t' \ \mathsf{op} \ t''$
        Let e be $t' \ \mathsf{op} \ t''$, i' be the instruction preceding i and β be b
        Loop
            If $i' = \tau \leftarrow \gamma$
                If τ appears in e
                    If $\gamma = \tau'$
                        replace τ in e with τ'
                    Else if $\gamma = e$ and we conjecturally emend
                        replace τ in e with $\Delta(i')$
                    Else if $\gamma = \bullet$ or we do not conjecturally emend
                        $\Delta(i) := \perp$; break
                $i' :=$ the instruction preceding i'
            Else if we have reached the preamble of β
                If $|pred(\beta)| = 0$
                    $\Delta(i) := \perp$; break
                Else if $|\mathsf{pred}(\beta)| = 1$
                    $\beta := \mathsf{pred}(\beta)$
                    $i' :=$ the last instruction in β
                Else if $|\mathsf{pred}(\beta)| > 1$
                    Find or create a chi c for e with its hypothetical temporary t^*
                    $\Delta(i) := t^*$
                    Add c to the chi worklist While the chi worklist is not empty
    Let c be the next chi on the worklist, e be the expression associated with c,
        and b be the block hosting c
    For each $\eta \in \mathsf{pred}(b)$
        Let e' be e transformed by the chis and phis at b with respect to η
        If a chi operand ω for e',η exits
            Set that as c's operand corresponding to η
        Else
            Create a new ω and search for a definition for using the same loop as above.

Figure 3.11. Algorithm for Chi Insertion

We have treated this algorithm as though it were working only on arithmetic operations (the non-bracketed portion of the language presented in Section 1.2.1). With a little extension, this algorithm can work on object and array loads as well, which greatly increases its potency. Care must be taken because objects and arrays can have aliases that make it difficult to know when two expressions are equivalent, a problem we explore in depth in Chapter 5. For our present purposes, it is best to rely on an underlying GVN and alias analysis, as is done in Fink et al's load elimination [3] In that case, we search for an instruction (either a load or a store) that has definitely the same value as the instruction from which we started. The search fails if we cross a store to something that is not definitely different, since that could represent a change in the expression's value. If reads kill [66], then this is true also for loads that are not definitely different.

3.3.2 Downsafety

Recall that a computation to be inserted is downsafe only if the same computation is performed on all paths to exit. Avoiding inserting computations that are not downsafe prevents lengthening computation paths and causing exceptions that would not be thrown in the unoptimized program. Since chi operands represent potential insertions, we consider downsafety to be a property of chi operands rather than of chis as in Chow, Kennedy, et al [1, 2] and Table 3.2. The second phase of ASSAPRE is a static analysis that determines which chi operands are downsafe. We are interested only in chi operands that are $\perp$ or defined by a chi, because these are potential insertion points: if a chi operand's definition is the result of a real instruction, no insertion would ever need to be made for it, but a $\perp$ chi operand would certainly need an insertion; a chi operand defined by another chi would need an insertion if no phi is made for its defining chi. The question of whether a chi operand's value is used on all paths to program exit is akin to liveness analysis for variables and can be determined by a fixed-point iteration. At the entry to a block that is a merge-point,

the chi operands of the chis there become "live." A use of their value—which happens if a real instruction has the hypothetical temporary of the chi as a definition—"kills" them. All chi operands that are not killed in that block are "live out," and must be considered "live in" for the block's successors.

The fact that chis share chi operands complicates downsafety, since if a chi operand is an operand to several chis, a use of any of their hypothetical temporaries will kill it. Moreover, the use of a chi's hypothetical temporary will kill all of that chi's chi operands. To handle this, in addition to a set of live chi operands, we must maintain on entry and exit of every block a mapping from temporaries to sets of chi operands which a use of the temporaries will kill.

What if a killing temporary for a chi operand is used as the definition to another chi operand? We could consider that chi operand to be a use occurring at the end of the corresponding predecessor block (not in the block that contains its chi). That would complicate things because if the chi operand was the only killer for the first chi operand on a path to exit, then the downsafety of the first would depend on the downsafety of the second. SSAPRE handled this by propagating a false value for downsafety from chis already found to be false to chis that depended on them. We can handle this with our map structure: if a use of t_1 will kill chi operand ω_1, chi operand ω_2 has t_1 as its definition, and a use of t_2 will kill ω_2, then we say that t_2 will also kill ω_1. The effect this has is that the use of a temporary as the definition of a chi operand cannot itself kill another chi operand, but that the fate of both chi operands are linked on subsequent paths to exit. One thing should be noted, though: if a chi appears at the beginning of a loop and is the definition of another chi there, its operand will have the temporaries of both chis as killers. If the second chi turns out not to be inserted but is the definition of a real occurrence in the block, a chi operand in the first chi may be wrongly killed and the chi wrongly thought downsafe. To correct this, when a chi operand is new at the beginning of a basic block, only the chis that have it as an operand should define a kill for it.

$$
\begin{array}{rcl}
\mathit{live\_in}(b) & = & (\bigcup_{p \in \mathsf{pred}(b)} \mathit{live\_out}(p)) \bigcup \{\text{chi operands at } b\} \\
\mathit{map\_in}(b) & = & \mathit{clear}_b(\bigcup_{p \in \mathsf{pred}(b)} \mathit{map\_out}(p)) \\
 & & \bigcup\{t \mapsto \sigma \mid \text{ chi at } b \text{ with temporary } t \\
 & & \quad \text{and set of chi operands } \sigma\} \\
\mathit{live\_out}(b) & = & \mathit{live\_in}(b) \\
 & & -\{\omega \mid \Delta(i) = t \text{ and } \mathit{map\_in}(t) = \omega \\
 & & \quad \text{for some instruction } i \in b \\
 & & \quad \text{or for some corresponding} \\
 & & \quad \text{chi operand } i \text{ in a successor}\} \\
\mathit{map\_out}(b) & = & \mathit{map\_in}(b) \\
\mathit{clear}_b(m) & = & \{t \mapsto \sigma \mid t \mapsto \sigma' \in m \text{ and } \sigma = \sigma' - \Omega \\
 & & \quad \text{where } \Omega \text{ is the set of chi operands at } b\}
\end{array}
$$

Figure 3.12. Data flow equations for Downsafety

Suppose $\mathit{live\_in}(b)$, $\mathit{live\_out}(b)$, $\mathit{map\_in}(b)$, and $\mathit{map\_out}(b)$ are the live in and out sets and mappings in and out, respectively, for a block b, and that $\Delta(i)$ finds the definition of instruction i. Then to compute downsafety, we must solve the data flow equations in Figure 3.12.

Table 3.3 lists what chi operands are downsafe in our example and explains why. As in the Figures, chi operands are identified by their definition or $\perp$. Of particular interest is $\perp^1$ because it is downsafe only because it is shared—it is killed on the left path because it belongs to the t_{19} chi and on the right because it belongs to the t_{21} chi. If these were considered separate chi operands, then an opportunity to eliminate a redundancy would be missed.

Extra care needs to be taken when optimizing a program in a language that supports precise handling of exceptions, such as Java. Instructions that could throw an exception (like any division where the denominator could be zero) must not be reordered, which could happen by the insertions and eliminations done by this algorithm. We solve this elegantly with a slight modification to the Downsafety phase. If we are checking the downsafety of a chi operand that represents an expression that could except, a use that occurs after another potentially excepting instruction

Table 3.3
Downsafety for the running example

$\perp^1$	*downsafe*: killed by $t_8 \leftarrow t_1 + t_3$ on the left path and $t_{11} \leftarrow t_7 + t_3$ on the right.
t_4	Irrelevant: defined by real instruction.
$\perp^2$	*downsafe*: killed by $t_{10} \leftarrow t_7 + t_2$ on the left path and $t_{13} \leftarrow t_7 + t_2$ on the right.
$\perp^3$	*downsafe*: killed by $t_{10} \leftarrow t_7 + t_2$ on the left path and $t_{13} \leftarrow t_7 + t_2$ on the right.
t_5	Irrelevant: defined by real instruction.
$\perp^5$	Not *downsafe*.
$\perp^6$	Not *downsafe*.
$\perp^7$	Not *downsafe*.
t_6	Irrelevant: defined by real instruction.
$\perp^4$	*downsafe*: killed by $t_{17} \leftarrow t_7 + t_{14}$.
t_{11}	Irrelevant: defined by real instruction.
t_{24}	*downsafe*: killed by $t_{18} \leftarrow t_{15} + t_{16}$.
t_{25}	*downsafe*: killed by $t_{18} \leftarrow t_{15} + t_{16}$.

should not make the chi operand considered downsafe. So when a potentially excepting instruction is encountered during Downsafety, all potentially excepting chi operands should be killed.

3.3.3 WillBeAvail

The WillBeAvail stage computes the remaining properties for chis and chi operands, namely, *canBeAvail*, *later*, and *willBeAvail* for chis and *insert* for chi operands, as listed in Table 3.5. The most important of these is *willBeAvail* because it characterizes chis that will be turned into phis for the optimized version.

We first determine whether it is feasible and safe for a chi to be made into a phi. If all of a chi's operands either have a real use or are *downsafe*, then that chi is *canBeAvail*. A chi is also *canBeAvail* if all of its chi operands that are not *downsafe* are defined by chis which also are *canBeAvail*, since, even though an insertion for that

Table 3.4
Properties for chis and chi operands

Property	Meaning	How calculated
Properties for chis		
canBeAvail	Safe insertions can make this value available	Two-level iteration over all chis, checking downsafety of chi operands and dependence among chis
later	The computation may as well be postponed	Two-level iteration over all chis, checking for operands defined by real occurrences and dependence among chis
willBeAvail	This chi should become a phi	True if a chi is *canBeAvail* but not *later*
Property for chi operands		
insert	An insertion must be made for this chi operand	True if a chi operand is an operand to a chi that is *willBeAvail* but is $\perp$ or defined by a chi that is not *willBeAvail*

Table 3.5
Properties for chis and chi operands

chi operand would not be safe, no insertion is needed if the value will be available from the defining chi.

To this end, after initializing *canBeAvail* to true for all chis in the program, we iterate through all chis. If a chi c has a chi operand that is not *downsafe* and is $\perp$ (and if *canBeAvail* has not already been proven false for c), we set *canBeAvail* to false for c. Then we make an inner iteration over all chis; for any chi that has an operand defined by c, if it has a non-*downsafe* chi operand but is still marked *canBeAvail*, then its *canBeAvail* should be cleared in the same manner. In our example, all chis are *canBeAvail* except for the ones with temporaries t_{24} and t_{25}, since they each have at least one $\perp$ chi operand that is not *downsafe*.

Next, we compute *later*, which determines if a chi can be postponed. This will prevent us from making insertions that have no benefit and would only increase register pressure. *later* is assumed true for all chis that are *canBeAvail*. Then we

iterate through all chis, and if a chi c is found for which *later* has not been proved false and which has an operand defined by a real occurrence, we reset *later* for it. To do this, we not only set *later* to false, but, similarly to how *canBeAvail* was propagated, we also iterate through all chis; if any is found that has an operand with c as a definition, that chi's *later* is reset recursively. The idea is that if the definitions of any of a chi's variables are available (either because they are real occurrences or because they are chis that cannot be postponed), the chi itself cannot be postponed. In our example (see Figure 3.9) the t_{20} chi is *later* because both of its operands are $\perp$. The t_{23} chi is also *later* because both of its operands come from chis for which *canBeAvail* is false. These computations can be postponed.

At this point, computing *willBeAvail* is straightforward. A chi will be available if it can be made available and there is no reason for it not to be—that is, if it is *canBeAvail* and not *later*. In our example, all chis are *willBeAvail* except for the ones associated with t_{20}, t_{24}, and t_{25}. This property can be computed on demand based on stored values for *willBeAvail* and *later*. From this we also can compute *insert*, which characterizes chi operands that require a computation to be inserted. A chi operand is *insert* if it belongs to a *willBeAvail* chi and is either $\perp$ or defined by a chi for which *willBeAvail* is false. Being in a *willBeAvail* chi implies that such a chi operand is *downsafe*. In our example, chi operands $\perp^1$ and $\perp^4$ are *insert*. *willBeAvail* and *insert* can be computed on demand when they are needed in the next phase.

3.3.4 CodeMotion

The earlier phases having gathered information, the final stage, CodeMotion, transforms the code by inserting phis and anticipated computations and eliminating redundant computations. The net effect is to hoist code to earlier program points.

If *willBeAvail* is true for a chi, then the value it represents should be available in a temporary in the optimized program; a phi needs to be put in its place to merge the values on the incoming paths. The operands to this new phi will be the temporaries

that hold the values from the various predecessors. If *insert* is true for any of its operands (indicating that the value it represents is not available, that is, has not been computed and stored in a temporary), then a computation for that value must be inserted at the end of the predecessor block it represents. Any real occurrence whose definition is another real occurrence or a *willBeAvail* chi is redundant, and can be replaced with a move from the temporary holding its value—if it is defined by a chi, the temporary is the target of the phi put in place of the chi; if it is defined by a real occurrence, the temporary is the one for the result of that occurrence.

Three steps complete the changes to the code: Inserting appropriate computations, creating new phis, and eliminating fully redundant computations.

To do the insertions, we iterate over all chi operands. If any is marked *insert*, then we allocate a new temporary, manufacture an instruction which computes the chi operand's expression and stores the result in the fresh temporary, append that instruction at the end of the corresponding basic block, and set the fresh temporary to be the chi operand's definition. In our example, $\perp^1$ requires us to insert $t_{26} \leftarrow t_1 + t_3$ in block 2, where t_{26} is fresh. Similarly, we insert $t_{28} \leftarrow t_7 + t_9$ at block 5 for $\perp^4$.

We then iterate over all chis. For a chi c that *willBeAvail*, we insert a phi at the end of the list of phis already present at the block. That phi merges the temporaries that define c's chi operands into c's hypothetical temporary. Because insertions have been made, all valid chi operands will have temporaries for definitions by this point. In our example, we create $t_{19} \leftarrow \phi(t_{26}, t_4)$ and $t_{21} \leftarrow \phi(t_{26}, t_5)$ in block 4 and $t_{22} \leftarrow \phi(t_{28}, t_{11})$ in block 7.

Finally, we iterate over all instructions. If any is defined by the target of another real instruction or of a *willBeAvail* chi (which by this time has been made into a phi), it is replaced with a move instruction from its definition to its target. In our example, $t_{11} \leftarrow t_7 + t_3$ in block 6 is replaced with $t_{11} \leftarrow t_{21}$, and $t_{17} \leftarrow t_7 + t_{14}$ in block 7 is replaced with $t_{17} \leftarrow t_{22}$.

An algorithmic description is found in Figure 3.13.

For each chi operand ω
    If ω is marked *insert*
        Create a new instruction storing to t, set $\Delta(\omega) = t$
For each chi c with hypothetical temporary t
    If $c = \chi(\omega_1, \ldots, \omega_n)$ is marked *willBeAvail*
        Create a new phi, $t \leftarrow \phi(\Delta(\omega_1), \ldots, \Delta(\omega_n))$.
For each instruction $i = t \leftarrow \gamma$
    If $\Delta(i) \neq \bot$
        Replace i with $t \leftarrow \Delta(i)$

Figure 3.13. Algorithm for Code Motion

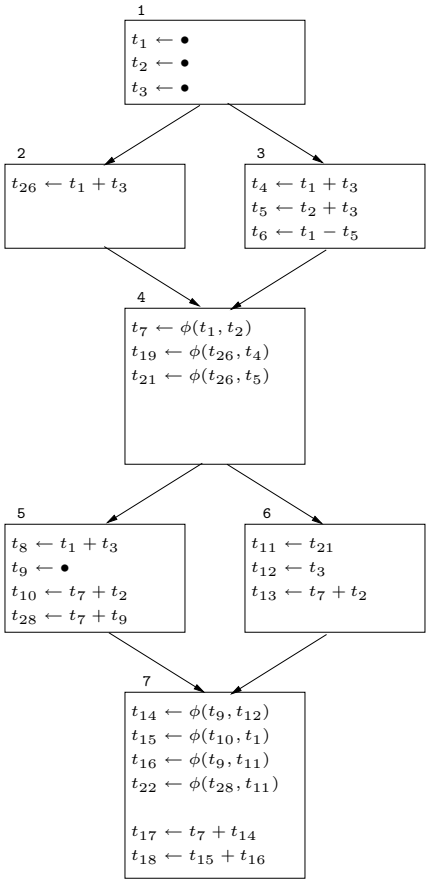


Figure 3.14. Optimized

Final program. Figure 3.14 displays the final program. The improvement can be seen by comparing the number of computations on each possible execution path: (1,2,4,5,7), (1,3,4,5,7), (1,2,4,6,7), and (1,3,4,6,7). In the unoptimized program, the number of computations are four, four, seven, seven, respectively; in the optimized program, they are four, three, six, five. The benefit varies among the possible paths, but never is a path made worse.

Chow, Kennedy, et al claim that the running time of their algorithm is linear in the size of the program multiplied by the sum of the edges and nodes in the CFG [1,2]. One caveat with ASSAPRE is that under certain conditions, the number of chis can explode—if a basic block has a large number of in-edges and a variable in one of that block's expressions has a different definition on each in-edge, the number of variant search expressions (and consequently chis) could become very large. While this is rare, we have seen some real code examples of methods with large and complicated CFGs where this algorithm is slow and space-hungry. This is one reason why the GVNPRE algorithm presented in Chapter 4 is preferable.

3.4 Experiments

Our experiments use Jikes RVM [7, 8], a virtual machine that executes Java classfiles. We have implemented the algorithm described here as a compiler phase for the optimizing compiler and configured Jikes RVM version 2.2.0 to use the optimizing compiler only and a semi-space garbage collector.

The optimizing compiler performs a series of code transformations on both SSA and non-SSA representations. Before converting to SSA form, it performs branch optimizations, tail recursion elimination, dead code elimination, and constant folding. In SSA form, it performs local copy and constant propagation, Array SSA load elimination [3], global value numbering, and redundant branch elimination. After SSA form, it repeats earlier stages and performs common subexpression elimination and code reordering for better I-cache locality and branch prediction. We placed AS-

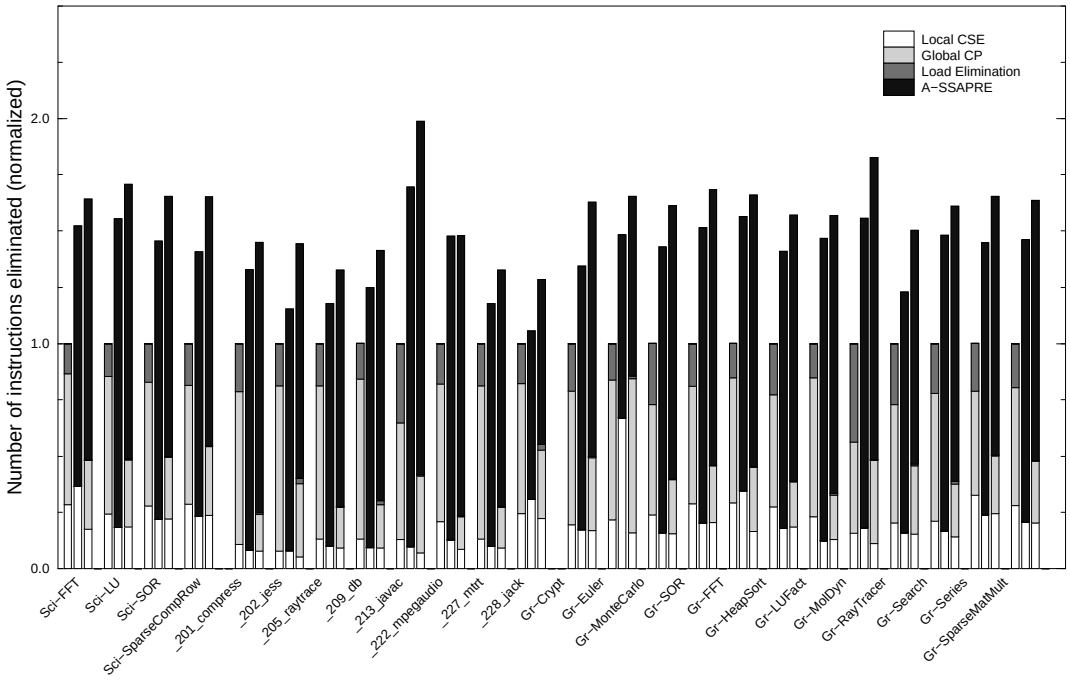


Figure 3.15. Static results for the benchmarks

SAPRE at the beginning of the SSA-based sequence. Our implementation performs the full optimization described in this chapter and operates over not only arithmetic expressions but also checks and array and object loads and stores.

We use three sets of benchmarks in our experiments: eight from the SPECjvm98 suite [11], four from SciMark [12], and twelve from the sequential benchmarks of the Java Grande Forum [13]. The runs were executed on a 733 MHz Power Macintosh with 32Kb I cache, 32Kb D cache, and 256K L2 cache running Mandrake Linux 8.2. Each benchmark is run once so that it will be fully compiled by the dynamic compiler and then (without restarting the virtual machine, so as to avoid recompiling the benchmarks) run ten times, with the elapsed time measured for each run. The time is reported in milliseconds by Java's `System.currentTimeMillis()` call. Thus we avoid including compilation time in the measurement since compilation takes place in the initial, untimed run of each benchmark. We report the best of the final nine runs with garbage collection time subtracted out. We do this to isolate program performance and because we have found that heavy optimizations such as ASSAPRE

left the heap in such a state that later garbage collection times were greater during the run of the program than if the optimizations had not been performed.

Since ASSAPRE is an expensive optimization, we study its effects on Jikes RVM optimization level O2. We were particularly interested in how it competes against other optimizations that do code motion or eliminate computations. Accordingly, the four optimization phases we are concerned with are local common subexpression elimination (Local CSE), global code placement (Global CP, which includes global common subexpression elimination and loop invariant code motion), load elimination (Load Elimination), and the optimization presented here (ASSAPRE). Given these optimization phases, we define the following three optimization levels:

O2: To mark our competition, we run the benchmarks at the pre-defined O2 level, which includes Local CSE, Global CP, and Load Elimination, but not ASSAPRE.

PRE: To measure the effect of ASSAPRE directly, we run ASSAPRE without the other phases listed except for Local CSE (to clean up a few superfluous checks generated by our hoisting potentially excepting exceptions).

BOTH: To observe synergies, we run all four phases.

Figure 3.15 presents the static results of the optimization by giving the number of instructions eliminated by each optimization at each level. The bars for each benchmark from left to right represent the number of instructions eliminated at O2, PRE, and BOTH, respectively. The numbers are normalized so that the total number of instructions eliminated at O2 sum to 1. These results should be used only to give a feel for how much motion is being done, since this ignores the fact that at the same time instructions are also being inserted for hoisting. Therefore each eliminated instruction, at least in the case of ASSAPRE, should be considered as at least one execution path being shortened by one instruction. We see that ASSAPRE can eliminate a larger number of instructions than Global CP and Load Elimination and subsumes much of what they do. In most cases, the number of instructions Load

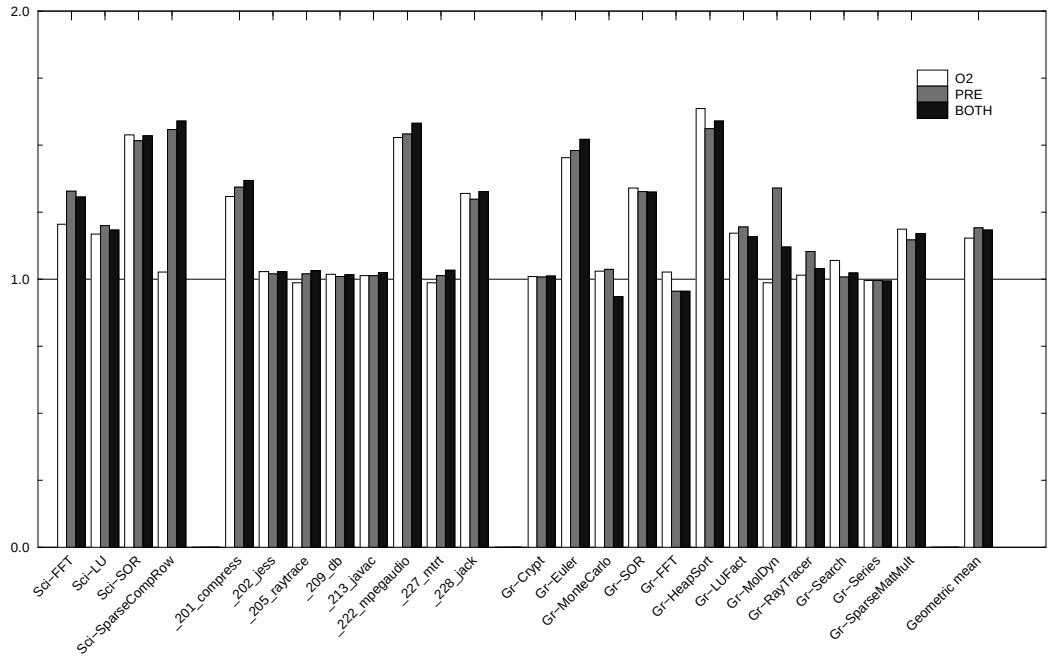


Figure 3.16. Speedup on benchmarks

Elimination eliminates at the BOTH level is negligible. However, Load Elimination and Global CP were not completely subsumed. One reason for this, we found, was that in some cases (particularly for loop invariant code motion) an instruction was being moved earlier or later in the loop without any redundant computation being eliminated or that loop invariant code motion was doing speculative motion that was not downsafe. The optimizations also had different constraints on what methods they could be applied to—since the optimizing compiler inlines very aggressively, some methods became too large to apply these optimizations practically, and there were some methods Global CP could run on that ASSAPRE could not and vice versa.

We also considered how these transformations impact performance. Figure 3.16 shows speedups for the three optimization levels over a fourth level: O2 with all four optimizations turned off. Removing redundant instructions does not always significantly impact performance. However, on certain benchmarks we achieved speedups of up to 1.6. ASSAPRE does about as well as global code placement and load elimination combined, and in many cases slices off a little more execution time. On

two benchmarks (SciMark's SparseCompRow and Grande's MolDyn), it performs significantly better than its rivals.

In summary, we have demonstrated that ASSAPRE is a complete PRE algorithm which competes with common subexpression elimination, loop invariant code motion, and load elimination, both statically and dynamically.

3.5 Conclusion

The contribution of this chapter is a simpler version of SSAPRE that makes fewer assumptions and covers more cases. The error-prone Rename phase has been eliminated, Downsafety has been recast as a version of a standard data flow problem, the algorithm no longer makes assumptions about the namespace, and a wider range of redundancies are eliminated. It is now fit to be used in conjunction with other optimizations. Perhaps the best contribution of this algorithm is didactic and experiential: by removing the dependence of SSAPRE on source-level lexical equivalence, we have moved it one step closer to GVN. The experience with an anticipation-oriented algorithm prepares us for applying anticipation to GVN as we cross-breed it with PRE in the next chapter.

4 VALUE-BASED PARTIAL REDUNDANCY ELIMINATION

> . . . and then [Men] would or could be made to mate with Orcs, producing new breeds, often larger and more cunning. There is no doubt that long afterwards, in the Third Age, Saruman rediscovered this, or learned of it in lore, and in his lust for mastery committed this, his wickedest deed: the interbreeding of Orcs and Men, producing both Men-orcs large and cunning, and Orc-men treacherous and vile.
>
> —J.R.R. Tolkien, *Morgoth's Ring*

4.1 Introduction

Recall that in Muchnick's view of value numbering (based on the ARWZ strain of GVN), neither GVN nor PRE is strictly more powerful than the other [47], as illustrated in Figure 2.1. In Chapter 2, we posed the question, can lexical PRE be extended to consider a value-based view. In other words, can ARWZ GVN be extended to consider partial redundancies. In yet other words, can there be an algorithm that is a complete hybrid of PRE and GVN, subsuming both? Answering this challenge, we have found a technique, GVNPRE, that does subsume both and eliminates redundancies that would be undetected by either working in isolation. We create a system of data flow equations that, using the infrastructure of a simple hash-based value numbering system, calculates insertion points for partially redundant values rather than partially redundant expressions. We present this in a framework that allows clear reasoning about expressions and values and implement it with an algorithm that can easily be reproduced. Our implementation subsumes GVN.

4.1.1 Overview

In this chapter, we present a new algorithm for PRE on values which subsumes both traditional PRE and GVN. Section 4.2 gives preliminaries, such as assumptions we make about the input program, precise definitions of concepts we use for our analysis, and descriptions of global value numbering infrastructure we assume already

available. Section 4.3 contains the meat of the chapter, describing the various phases of the approach both formally and algorithmically. We comment on several corner cases in Section 4.4. The algorithm is discussed in the context of other research in Section 4.5. Section 4.6 gives details on our implementation and its results. We conclude by discussing its advantages and shortcomings in Section 4.7.

4.2 Framework

In this section, we present our framework in terms of the model of expressions and values we use. Recall the assumptions about program structure from Section 1.2; this chapter uses the non-bracketed portion of the language.

4.2.1 Values and expressions

A *value* is a set of expressions grouped together by static analysis of their computational result. v ranges over values. An *expression* is similar to an operation except that if an expression involves an operator, its operands are given in terms of values. This way we can think of expressions more generally. If t_1 and t_3 are members of value v_1, and t_2 is a member of v_2, we need not think of $t_1 + t_2$ and $t_3 + t_2$ as separate expressions; instead, we think only of the expression $v_1 + v_2$.

$$e \quad ::= \quad t \mid v \ \mathsf{op} \ v \qquad\qquad \textit{Expressions}$$

4.2.2 The value table

Our goal in value numbering is to obtain three pieces of information: value numbers, available expressions, and anticipated expressions. Traditional ARWZ GVN requires only the first two. First, we partition all expressions in the program into values. We represent this partition with a map (or value table), Δ, with the following operations in Figure 4.1.

$\mathsf{add}(\Delta, e, v)$ $\equiv$ Set v to be e's value in Δ; add e to the set v.

$\mathsf{lookup}(\Delta, e)$ $\equiv$ Return e's value, null if none has been set. (For smarter value numbering, assume this function recognizes algebraic identities.)

$\mathsf{lookup\_or\_add}(\Delta, e)$ $\equiv$ Do the following:

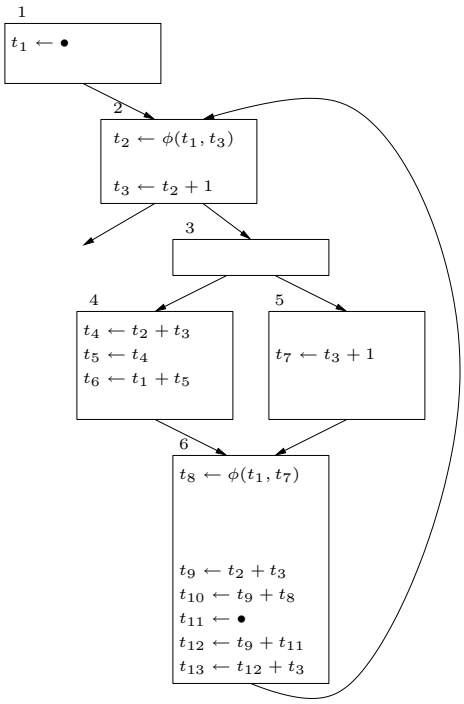
```
x := lookup(Δ, e)
if (x = null)
   v := {}
   x := add(e, v)
return x
```

Figure 4.1. Operations for the value table

We treat the value table as a black box, but assume it to be used for a simple hashing value number scheme (such as Click's [43]), recognizing expressions by their structure and, if smart enough, algebraic properties, associating them with values. It is worth noting that any notion of completeness depends on the smarts of the the function lookup. Two expressions are in the same value if lookup recognizes them to have the same value, and this is more of an engineering question than a theoretical one because there is no limit on how smart the function lookup is engineered to be. For example, if v_1 contains 1, v_2 contains 2, v_4 contains $v_3 + v_1$ for some v_3, and v_5 contains $v_4 + v_1$, then v_5 also should contain $v_3 + v_2$. To enumerate all such cases would be infinite.[1]

Figure 4.2(a) displays a running example we will refer to throughout this and the next section. Since it is large, we will often discuss only part of it at a time. For the present, consider block 4. The table in Figure 4.2(b) displays the values referenced in the block and the expressions they contain. For clarity, we assign a value the same subscript as one of its temporaries wherever possible. Values v_1, v_2, and v_3 are defined in blocks that dominate block 4, so they are in scope here. The

[1]The author thanks Di Ma for this insight.



(a) Program CFG

values

v_1: t_1
v_2: t_2
v_3: t_3
v_4: $t_4, v_2 + v_3, t_5$
v_6: $t_6, v_1 + v_4$

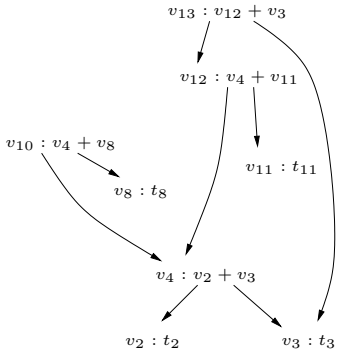
temps generated:

t_4, t_5, t_6

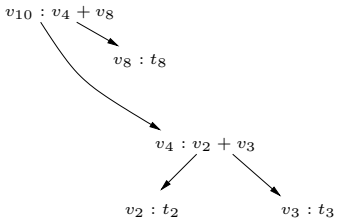
exprs generated:

$t_2, t_3, t_4, t_1, t_5, v_2 + v_3, v_1 + v_4$

(b) Values, temporaries, and expressions of block 4



(c) Anticipated expressions at block 6, not considering the kill of t_{11}



(d) Actual anticipated expressions at block 6

Figure 4.2. Running example

instruction $t_4 \leftarrow t_2 + t_3$ leads us to discover expressions $v_2 + v_3$ and t_4 as elements of the same value, which we call v_4. Because of the move $t_5 \leftarrow t_4$, t_5 is also in v_4. Finally, $v_1 + v_5$ and t_6 are in a value we call v_6. Recall that these values are global, so we are showing only part of the value table for the entire program (in fact, values v_3 and v_4 have more members, discovered in other parts of the program).

4.2.3 The available sets

The second piece of information is what expressions are available to represent the values; or, put differently, what values are already computed and stored in temporaries at a given program point. If more than one temporary of the same value is live at a point, we want to pick one as the *leader* [51], which will be used as the source of the value to replace an operation of that value with a move. To be unambiguous, we postulate the leader to be the "earliest" temporary available; that is, the temporary whose defining instruction dominates the defining instructions of all other temporaries of that value live at that point. We define the *leader set* to be the set of leaders representing available values at a program point, where "program points" occur before and after each instruction. Although by our theoretical definition, there is a leader set for any program point, we abstract the notion by looking at only the sets at the end of a basic block, AVAIL_OUT. We shall see that these are the only ones needed for our algorithm. The leaders available out of block 4 are those expressions listed first for each value in Figure 4.2(b). (This notion of availability is sometimes called *upsafety* [55].)

4.2.4 The anticipated sets

Finally, we want to know what values are anticipated at a program point; that is, will be computed or used on all paths from that point to program exit. Just as we want appropriate temporaries to represent values in the leader set, so we want appropriate expressions to represent anticipated values—that is, anticipated

values should also have a reverse-flow leader or *antileader*. An antileader can be a live temporary or a non-simple expression whose operands are represented in the antileader set. This is so that the expression could become available by an insertion at that point, or several insertions if it is a nested expression. Conversely from the leader set, we are interested only in the antileader sets at the beginning of basic blocks, ANTIC_IN.

Although our method for computing antileader sets presented later is much different, it may be more easily understood initially if we compare this to parts of the algorithm for ASSAPRE. Suppose we iterated backwards over the instructions in a basic block as in Chi Insertion 3.3.1, but instead of searching for a specific expression, we collect the expressions (including the temporaries that stand for their subexpressions) which we find as a set which will become the antileader set of the block. For an antileader set, it does not matter which expression represents a value, so long as that value is live. A temporary potentially in ANTIC_IN becomes dead if it is written to. This is a consequence of SSA form and is similar to what happens when ASSAPRE crosses the definition of a temporary in the search expression. If the assignment is from something for which we can make an expression (as opposed to $\bullet$), that expression replaces the temporary as the antileader. If the assignment is from $\bullet$, then the value is no longer represented at all (consider that if a temporary is defined by a $\bullet$, then nothing else can be in the same value). Furthermore, any other expression that has that (no longer represented) value as an operand also becomes dead. Therefore antileaders and the values they represent are killed by definitions of temporaries in the block. An antileader set can be pictured as a dag. Consider basic block 6 in Figure 4.2(a), alternately with and without the instruction $t_4 \leftarrow \bullet$. In the case where we exclude that instruction, assume t_4 to be global. Without the definition of t_4, ANTIC_IN can be represented by the dag in Figure 4.2(c). The nodes of the dag are pairs of values and the antileaders representing them; edges are determined by the operands of the antileader. If we suppose the block contains

$t_6 \leftarrow \bullet$ as it does in the program, then $v_4 : t_4$ is killed, along with all expressions that depend on v_4 also, in cascading effect. See Figure 4.2(d).

Other sets. To calculate these sets, we find two other pieces of information useful: the temporaries generated by a block, meaning those that are defined; and the expressions generated by a block, meaning those that occur as operations, operands to operations, or sources of moves. We say that the block "generates" these because they will be used as "gen" sets for the flow equations we define in the next section for computing the leader and antileader sets. These are given for block 4 in Figure 4.2(b).

4.3 GVNPRE

The GVNPRE algorithm has three steps: BuildSets, Insert, and Eliminate. BuildSets, which we describe formally and algorithmically, populates the value table and the leader and antileader sets. Insert places new instructions in the program to make partially available instructions fully available. Eliminate removes computations whose values are already available in temporaries or as constants.

4.3.1 BuildSets

Flow equations. To compute AVAIL_IN and AVAIL_OUT for a block, we must consider not only the contents of the block itself, but also expressions inherited from predecessors (for AVAIL_OUT) and anti-inherited from successors (for ANTIC_IN). For this we use a system of flow equations. As is common for flow equations, we also define the sets AVAIL_IN and ANTIC_OUT, although only AVAIL_OUT and ANTIC_IN are used later in the optimization. We have three gen sets, EXP_GEN(b) for expressions (temporaries and non-simple) that appear in the right hand side of an instruction in b; PHI_GEN(b) for temporaries that are defined by a phi in b; and TMP_GEN(b) for temporaries that are defined by non-phi instructions in b. There

are no kill sets for calculating AVAIL_OUT, since SSA form implies no temporary is ever killed. For ANTIC_IN, TMP_GEN acts as a kill set. Since an available expression must be defined in an instruction that dominates the program point in question, so in calculating AVAIL_OUT, we consider inherited expressions only from the block's dominator. In terms of flow equations,

$$\begin{aligned} \mathsf{AVAIL\_IN}[b] &= \mathsf{AVAIL\_OUT}[\mathsf{dom}(b)] && (4.1) \\ \mathsf{AVAIL\_OUT}[b] &= \mathsf{canon}(\mathsf{AVAIL\_IN}[b] \cup \mathsf{PHI\_GEN}(b) && (4.2) \\ &\qquad \cup \mathsf{TMP\_GEN}(b)) \end{aligned}$$

where canon is a procedure that, given a set of temporaries, partitions that set into subsets which all have the same value and chooses a leader from each. Of course canon would be inconvenient and inefficient to implement; instead, AVAIL_OUT$[b]$ can be calculated easily and efficiently at the same time as the gen sets, as we will show later.

For ANTIC_IN, handling successors is more complicated. If there is only one successor, we add all its antileaders to AVAIL_OUT of the current block; however, we must translate some temporaries based on the phis at the successor. For example, if t_1 is anticipated by block b, and block b has a phi which defines t_1 and has t_2 as an operand from block c, then t_2, rather than t_1, is anticipated at the end of block c, and it has a different value. For this we assume a function phi_translate which takes a successor block, a predecessor block (i.e., there is an edge from the second block to the first), and a temporary; if the temporary is defined by a phi at the successor, it returns the operand to that phi corresponding to the predecessor, otherwise returning the temporary. If there are multiple successors, then there can be no phis (because of critical edge removal), but only values anticipated by all of the successors can be anticipated by the current block. This assures that all anticipated expressions are downsafe—they can be inserted with confidence that they will be used on all paths to program exit.

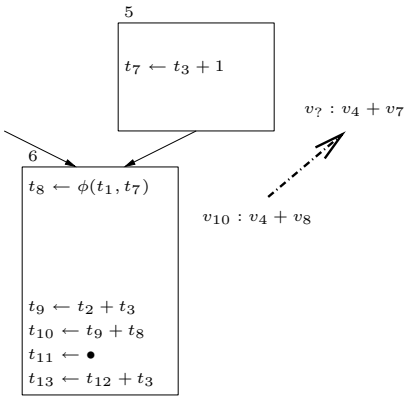


Figure 4.3. Translation through phis

The flow equations for calculating the antileader sets are

$$\mathsf{ANTIC\_OUT}[b] = \begin{cases} \{e | e \in \mathsf{ANTIC\_IN}[\mathsf{succ}_0(b)] \wedge \\ \forall\ b' \in \mathsf{succ}(b), \exists e' \in \mathsf{ANTIC\_IN}[b'] | \\ \quad \mathsf{lookup}(e) = \mathsf{lookup}(e')\} & \text{if } |\mathsf{succ}(b)| > 1 \\ \\ \mathsf{phi\_translate}(\mathsf{ANTIC\_IN}[\mathsf{succ}(b)], b, \\ \qquad \mathsf{succ}(b)) & \text{if } |\mathsf{succ}(b)| = 1 \end{cases} \tag{4.3}$$

$$\mathsf{ANTIC\_IN}[b] = \mathsf{clean}(\mathsf{canon}_{\mathsf{EXP}}(\mathsf{ANTIC\_OUT}[b] \cup \mathsf{EXP\_GEN}[b] - \mathsf{TMP\_GEN}[b])) \tag{4.4}$$

When phi_translate translates expressions through phis, it may involve creating expressions that have not been assigned values yet and therefore require new values to be created. Consider calculating $\mathsf{ANTIC\_OUT}[B_5]$ in our example. Value v_{10} is represented by $v_4 + v_8$, which is translated through the phi to $v_4 + v_7$. However, that expression does not exist in the program—it needs a new value. See Figure 4.3. Thus sometimes the value table will need to be modified while calculating these sets.

$\mathsf{canon}_{\mathsf{EXP}}$ generalizes canon for expressions. For ANTIC_IN, we do not care what expression represents a value as long as it is live, so $\mathsf{canon}_{\mathsf{EXP}}$ can make any choice

as long as it is consistent. This essentially makes the union in the formula for ANTIC_IN to be "value-wise," meaning that given two sets of expressions representing values, we want every value in each set represented, but by only one expression each. Similarly, the formula for ANTIC_OUT when there are multiple successors performs a "value-wise" intersection—only values that are represented in all successors should be represented here, but which expression represents it does not matter. clean kills expressions that depend on values that have been (or should be) killed. Because information will also flow across back edges in the graph, these anticipation sets must be calculated using a fixed-point iteration.

Algorithm. Calculating BuildSets consists of two parts. The first is a top-down traversal of the dominator tree. At each block, we iterate forward over the instructions, making sure each expression has a value assigned to it. We also build EXP_GEN, PHI_GEN, and TMP_GEN for that block.

We have already mentioned that computing canon directly is inconvenient and costly. We avoid computing it by maintaining an invariant on the relevant sets, that they never contain more than one expression for any value. Since the value leader set for the dominator will have been determined already, we can conveniently build the leader set for the current block by initializing it to the leader set of the dominator and, for each instruction, adding the target to the leader set only if its value is not already represented. Since we will often add something to a set only if its value is not already represented, we define the operation val_insert(S, e) for doing this; also, val_replace(S, e) works similarly, except that if an expression representing the value is already present, remove it and add the given expression. Using this, we never add a temporary to an AVAIL_OUT set unless its value is not yet represented. Similarly, we need only one representative in EXP_GEN for each value, so we do not add an expression for a value that has already appeared (this way EXP_GEN contains only the first appearance of a value, appropriate for an antileader). We do not calculate AVAIL_IN sets since it is trivial.

```
For each block b in a top-down traversal of the dominator tree
    EXP_GEN[b] := {}
    PHI_GEN[b] := {}
    TMP_GEN[b] := {}
    AVAIL_OUT[b] := AVAIL_OUT[dom(b)]
    For each instruction i = t ← γ in sequence of b
        if γ = φ(t1 ... tn)
            add(t, {})
            insert(PHI_GEN[b], t)
        else if γ = t'
            v := lookup(t')
            add(t, v)
            val_insert(EXP_GEN[b], t')
            insert(TMP_GEN[b], t)
        else if γ = t1 op t2
            v1 := lookup(t1)
            v2 := lookup(t2)
            v := lookup_or_add(v1 op v2)
            add(t, v)
            val_insert(EXP_GEN[b], t1)
            val_insert(EXP_GEN[b], t2)
            val_insert(EXP_GEN[b], v1 op v2)
            insert(TMP_GEN[b], t)
        else if γ = •
            v = add(t, {})
            insert(TMP_GEN[b], t)
        val_insert(AVAIL_OUT[b], t)
```

Figure 4.4. First phase of BuildSets

The algorithm for the first phase of BuildSets is in Figure 4.4. In our pseudocode, we take the liberty of some ML-style notation; for example, if we say "if $\gamma = t$", we mean "if γ is an instance of a temporary" and "let the variable t stand for that simple expression in the then clause of this branch."

The second part calculates flow sets to determine the antileader sets and conducts the fixed-point iteration. Until we conduct a pass on which no ANTIC_IN set changes, we perform top-down traversals of the postdominator tree. This helps fast convergence since information flows backward over the CFG. The algorithm is given

in Figure 4.5. To keep ANTIC_IN canonical, we remove the killed temporaries separately from ANTIC_OUT and EXP_GEN, and then do what amounts to a value-wise union.

Both phi_translate and clean process elements in such a way that for any element, other set elements on which they depend must be processed first. The key to doing this efficiently is to maintain all sets as topological sorts of the dags they model, which can be done by implementing the sets as linked lists. Figure 4.6 shows a round of this process on block 5, including ANTIC_IN$[B_5]$ before and after the insertions from S. Notice that the net effect is to replace $v_7 : t_7$ in ANTIC_OUT with $v_7 : v_3 + v_{100}$, and that although the order has changed, it is still a topological sort.

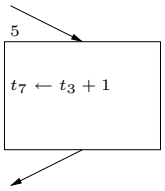
For this phase to be efficient, the fixed point iteration must converge quickly. Convergence depends primarily on CFG structure, specifically the number of back edges and the nesting level of loops. In fact, theoretically, the number of iterations can be arbitrarily large; for any $\epsilon > 0$, we can construct a CFG that requires ϵ iterations by taking a CFG that requires $\epsilon - 1$ iterations and adding an extra outer loop nesting level. In all the code we have compiled, we have found methods in the virtual machine itself (recall that Jikes RVM is itself written in Java) that required up to 97 iterations to converge. In normal benchmarks, however, the maximum number of iterations needed for any method being compiled was 26, and the average number of rounds needed for methods with more than 5 basic blocks was just over 2. (Smaller methods were excluded because they tend to be trivial and would artificially bring down the average.) Moreover, and we found that if we set 10 as the limit, there was no decrease in the number of operations eliminated in the elimination phase, implying that the extra rounds needed in extreme cases produce no useful data.

4.3.2 Insert

Insert is concerned with hoisting expressions to earlier program points. If this phase were skipped, we would be left with a traditional global value numbering

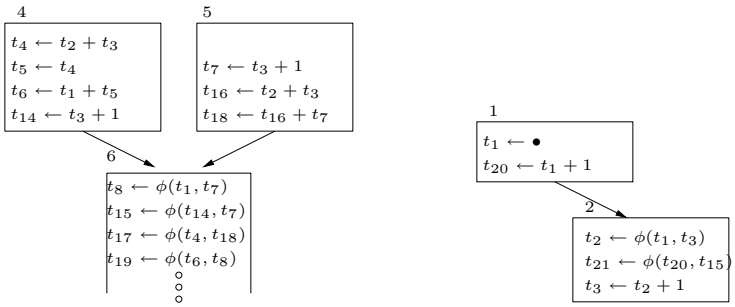
```
changed := true
while changed
    changed := false
    for each block b in a top-down traversal of the postdominator tree
        old := ANTIC_IN[b]
        if |succ(b)| = 1
            ANTIC_OUT := phi_translate(ANTIC_IN[b], b, succ(b))
        else if |succ(b)| > 1
            worklist := makelist(succ(b))
            first := remove(worklist)
            ANTIC_OUT := ANTIC_IN[first]
            while worklist is not empty
                b' := remove(worklist)
                for each e ∈ ANTIC_OUT
                    v := lookup(e)
                    if find_leader(ANTIC_IN[b], v) = null
                        remove(ANTIC_OUT, e)
        else
            ANTIC_OUT := {}
        S := ANTIC_OUT − TMP_GEN[b]
        ANTIC_IN[b] := EXP_GEN[b] − TMP_GEN[b]
        for each e ∈ S
            if find_leader(ANTIC_IN[b], lookup(e')) = null
                vinsert(ANTIC_IN[b], e)
        ANTIC_IN[b] := clean(ANTIC_IN[b])
        if old ≠ ANTIC_IN[b]
            changed := true
```

Figure 4.5. Second phase of BuildSets



$$
\begin{aligned}
\mathsf{TMP\_GEN}[5]: &\quad v_7 : t_7 \\
\mathsf{EXP\_GEN}[5]: &\quad v_3 : t_3, v_{100} : 1, v_7 : v_3 + v_{100} \\
\mathsf{ANTIC\_OUT}: &\quad v_2 : t_2, v_3 : t_3, v_4 : v_2 + v_3, v_7 : t_7, \\
&\quad v_{101} : v_4 + v_7 \\
S: &\quad v_2 : t_2, v_3 : t_3, v_4 : v_2 + v_3, v_{101} : v_4 + v_7 \\
\mathsf{ANTIC\_IN}[5]_{\text{orig}}: &\quad v_3 : t_3, v_{100} : 1, v_7 : v_3 + v_{100} \\
\mathsf{ANTIC\_IN}[5]: &\quad v_3 : t_3, v_{100} : 1, v_7 : v_3 + v_{100}, v_2 : t_2, \\
&\quad v_4 : v_2 + v_3, v_{101} : v_4 + v_7
\end{aligned}
$$

Figure 4.6. Example for the second phase of BuildSets



(a) (b)

Figure 4.7. Results of Insert

scheme. Insertions happen only at merge points. This phase iterates over blocks that have more than one predecessor and inspects all expressions anticipated there. For a non-simple expression, we consider the equivalent expressions in the predecessors. This requires some translation because of the phis at the block, for which we use phi_translate. We look up the value for this equivalent expression and find the leader. If there is a leader, then it is available. If the expression is available in at least one predecessor, then we insert it in predecessors where it is not available. Generating fresh temporaries, we perform the necessary insertions and create a phi to merge the predecessors' leaders.

Figure 4.7(a) shows the result of Insert at the join point of block 6. The anticipated expressions $v_2 + v_3$ and $v_4 + v_8$ are available from block 4, so $t_{16} \leftarrow t_2 + t_3$ and $t_{18} \leftarrow t_{16} + t_7$ are hoisted to block 5. Here the topological sort of ANTIC_IN$[B_6]$ comes in handy again, since these expressions are nested and $v_2 + v_3$ must be inserted first. Note that thus our algorithm handles second order effects without assigning ranks to expressions (compare Rosen et al [6]). Appropriate phis are also inserted.

The hoisted operations and newly created phis imply new leaders for their values in the blocks where they are placed, and these leaders must be propagated to dominated blocks. This could be done by re-running BuildSets, but that is unnecessary and costly. Instead, we assume a map, new_sets, which associates blocks with sets of

expressions which have been added to the value leader sets during Insert. Whenever we create a new computation or phi, we possibly make a new value, and we at least create a new leader for that value in the given block. We update that block's leader set and its new set. Since this information should be propagated to other blocks which the new temporaries reach, for each block we also add all the expressions in its dominator's new set into the block's own leader set and new set. This also requires us to make Insert a top-down traversal of the dominator tree, so that a block's dominator is processed before the block itself.

What we have said so far, however, is not completely safe in the sense that it in some cases it will lengthen a computational path by inserting an instruction without allowing one to be eliminated by the next phase. Since $v_3 + v_{100}$ is also anticipated in at block 6, being anti-inherited from $t_3 \leftarrow t_2 + 1$ in block 2, we insert $t_{14} \leftarrow t_3 + 1$ in block 4, as shown in Figure 4.7(a). This does not allow any eliminations, but only lengthens any trace through block 4. The value is still not available at the instruction that triggered this in block 1 because block 1 was visited first, at which time it was available on no predecessors and therefore not hoisted. To fix this, we repeat the process until we make a pass where nothing new is added to any new set. On the next pass, $v_7 : t_{15}$ is available in block 6, so we hoist $t_1 + 1$ to block 1, as shown in Figure 4.7(b). In practice, Insert converges quickly. On most benchmarks, the maximum number of required rounds for any method was 3, and on average it took only a single round. Note that we do not need predicates for determining latest and earliest insertion points, since insertions naturally float to the right place. Insertions made too late in the program will themselves become redundant and eliminated in the next phase. Figures 4.8 and 4.9 show the algorithm for Insert.

4.3.3 Eliminate

Eliminate is straightforward. For any instruction, find the leader of the target's value. If it differs from that target, then there is a constant or an earlier-defined

```
new_stuff := true
while new_stuff
    new_stuff := false
    for each block b in a top-down traversal of the dominator tree
        new_sets[b] := {}
        for each e ∈ new_sets[dom(b)]
            add(new_sets[b], e)
            val_replace(AVAIL_OUT[b], e)
        if |pred(b)| > 1
            worklist = makelist(ANTIC_IN[b])
            while worklist is not empty
                e := remove(worklist)
                if e = v1 op v2
                    if find_leader(AVAIL_OUT[dom(b)], lookup(e)) ≠ null
                        continue
                    avail := new map
                    by_some := false
                    all_same := true
                    first_s := null
                    for each b' ∈ pred(b)
                        e' := phi_translate(v1 op v2, b', b)
                        v' := lookup(e')
                        e'' := find_leader(AVAIL_OUT[b'], v')
                        if e'' = null
                            put(avail, b', e')
                            all_same := false
                        else
                            put(avail, b', e'')
                            by_some := true
                            if first_s = null
                                first_s := e''
                            else if first_s ≠ e''
                                all_same := false
                    (continued)
```

Figure 4.8. Algorithm for Insert

```
if !all_same and by_some
    for each b' ∈ pred(b)
        e' := get(avail, b')
        if e' = v1 op v2
            s1 := find_leader(AVAIL_OUT[b'], v1)
            s2 := find_leader(AVAIL_OUT[b'], v2)
            t := get_fresh_temp()
            b' := b' ∪ (t ← s1 op s2)
            v := lookup_or_add(v1 op v2)
            add(v, t)
            insert(AVAIL_OUT[b'], t)
            put(avail, b', t)
    t := get_fresh_temp()
    add(t, lookup(Δ, e))
    insert(AVAIL_OUT[b], t)
    b := t ← φ(... xi ...) ∪ b
        where xi = get(avail, bi),  bi ∈ b̄ = pred(b)
    new_stuff := true
    insert(new_sets[b], t)
```

Figure 4.9. Algorithm for Insert, continued

temporary with the same value. The current instruction can be replaced by a move from the leader to the target. The order in which we process this does not matter. The algorithm for Eliminate, along with the optimized program, are shown in Figure 4.10.

4.3.4 Complexity

In this section we discuss the complexity of GVNPRE. Suppose I is the number of instructions in the program, B the number of basic blocks, and V the number of values. In phase 1 of BuildSets, if the value table and the leader sets are implemented as linked hashsets using good hashing functions [67], insertions to the table and sets can be done in constant time, and so the complexity is linear in the number of basic blocks visited in the top-down traversal, $O(B)$. Phase 2 is iterative. The

```
For each block b
    For each i ∈ b
        if i = t ← s1 op s2
            s' = find_leader(AVAIL_OUT[b],
                             lu(t))
                if s' ≠ t
                    replace i in b with t ← s'
```

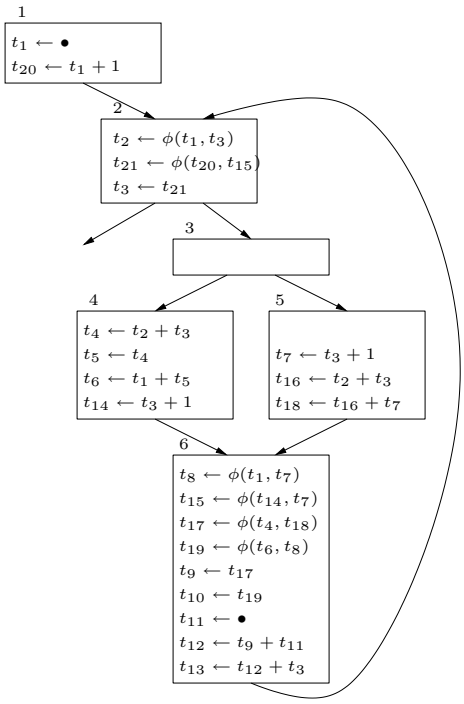


Figure 4.10. Algorithm for Eliminate and the optimized program

most expensive part of each iteration is phi translation. The translation of a single expression consists of lookups and insertions to leader sets, and so the cost of translating the entire set, which in the worst case could contain one expression for every value in the program, $O(V)$. Each iteration also involves visiting each block, so the cost of an iteration is $O(V \times B)$. Recall that the number of iterations is arbitrarily large. However, we know that in practice, bounding the number of iterations by a constant allows us to gather sufficient information for the optimization, so we assume the number of iterations is bounded by a constant.

Insert parallels the complexity of BuildSets phase 2. In a top-down traversal, it acts at each join point, which is $O(B)$. At each join point, it considers hoisting for each value anticipated there, at worst case $O(V)$. Again we have $O(B \times V)$. Finally, eliminate iterates over all instructions and does one lookup for each, $O(I)$.

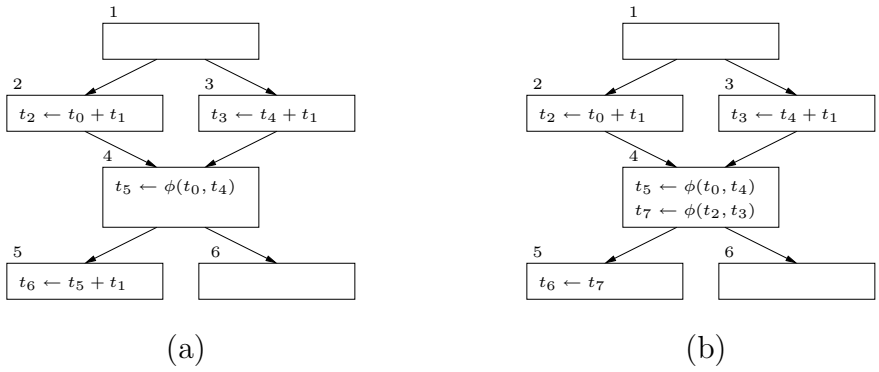


Figure 4.11. The need for partial anticipation

Thus we have $O(B + V \times B + V \times B + I)$. Each value must be defined by an instruction, except for the handful of constants and parameters to the function, so $V = O(I)$. Furthermore, since basic blocks are sets of instructions, we can assume $I \geq B$. Thus GVNPRE's worst-case complexity is $O(I + I^2 + I^2 + I) = O(I^2)$. In practice, we acquired a feel for the cost of the optimization by timing the build of the Jikes RVM image with and without GVNPRE. Jikes RVM has the option of compiling itself at the highes level of otpimization. When GVNPRE is turned on, the compilation time increases by about a factor of four.

4.4 Corner cases

In this section, we briefly describe extensions to this algorithm necessary for theoretical optimality yet giving no benefit in practice: partial anticipation and code placement. We also comment on how to move code if the programming environment demands precise handling of exceptions.

4.4.1 Partial anticipation

Consider the program in Figure 4.11(a). The instruction $t_6 \leftarrow t_5 + t_1$ in block 5 is fully redundant, although there is no available expression at that point. A no-

$$\mathsf{PA\_OUT} = \begin{cases} \mathsf{canon_{EXP}}(\bigcup_{b' \in \mathsf{succ}(b), \neg b' \gg b} (\mathsf{ANTIC\_IN}[b'] \cup \mathsf{PA\_IN}[b'])) & \\ \quad -\mathsf{ANTIC\_OUT}[b] & |\mathsf{succ}(b)| > 1 \\ \mathsf{dep\_phi\_trans}(\mathsf{PA\_IN}[\mathsf{succ}(b)], & \\ \quad \mathsf{ANTIC\_IN}[\mathsf{succ}(b)]) & |\mathsf{succ}(b)| = 1 \end{cases} \tag{4.5}$$

$$\mathsf{PA\_IN}[b] = \mathsf{dep\_clean}(\mathsf{canon_{EXP}}(\mathsf{PA\_OUT}[b] - \mathsf{TMP\_GEN}[b] - \mathsf{ANTIC\_IN}[b], \mathsf{ANTIC\_IN}[b])) \tag{4.6}$$

Other definitions:

$$\mathsf{dep\_phi\_trans}(S, S', b, b') = \{\mu'(e, S, S', b, b') | e \in S\} \tag{4.7}$$

$$\mu'(t, S, S', b, b') = \mu(t, S, b, b') \tag{4.8}$$

$$\mu'(v_1 \text{ op } v_2, S, S', b, b') = \mathsf{lookup}(\mu'(e_1, S, S', b, b')) \text{ op } \mathsf{lookup}(\mu'(e_2, S, S', b, b')), \text{ where } e_1, e_2 \in S \cup S', \mathsf{lookup}(e_1) = v_1, \mathsf{lookup}(e_2) = v_2 \tag{4.9}$$

$$\mathsf{dep\_clean}(S, S') = \{e | e \in S, (\mathsf{live}(e, S) \wedge \mathsf{live}(e, S'))\} \tag{4.10}$$

Figure 4.12. Flow equations for partial anticipation

cost phi inserted at block 4, however, would allow us to remove that computation. See Figure 4.11(b). Yet our algorithm as presented so far would not perform this insertion, because the expression is not anticipated at block 4.

To improve on this, we define *partial anticipation*. A value is partially anticipated at a program point if it is computed on at least one but not all paths to program exit. We define flow equations for calculating PA_IN and PA_OUT sets for blocks which are similar to those for ANTIC_IN and ANTIC_OUT, except that they also depend on on ANTIC_IN and ANTIC_OUT, and that PA_OUT requires a value-wise union instead of intersection. The versions of phi_translate and clean, called dep_phi_trans and dep_clean respectively, depend on ANTIC_IN. The flow equations and related definitions are in Figure 4.12.

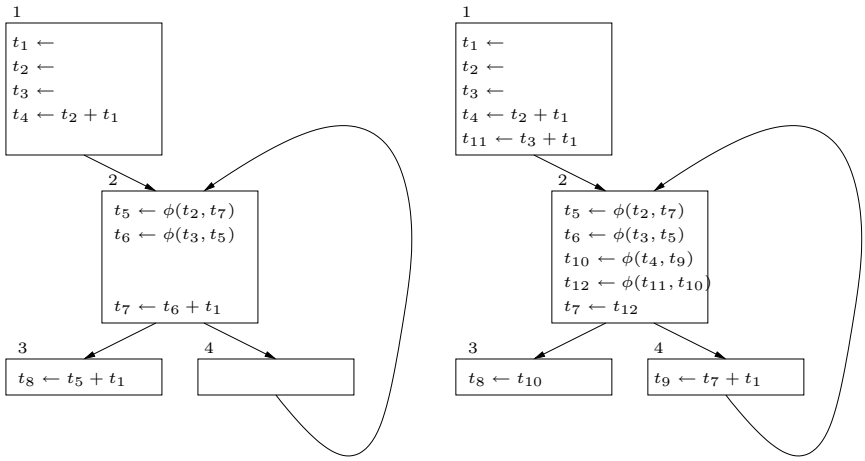


Figure 4.13. Why partial anticipation is tough

To implement partial anticipation, it would be very inefficient to union all successor full and partial anticipation sets, then choose canonical representatives, and then to delete those that are fully anticipated, as the flow equation suggests. Rather, we iterate through each expression in ANTIC_IN and PA_IN for each successor, and if the expression's value is not already in PA_OUT or ANTIC_OUT, we add it to PA_OUT.

Notice that the flow equation for PA_OUT where there is more than one successor excludes passing partially anticipated expressions across a back edge (that is, when the successor dominates the block). This is necessary for termination. Consider the program in Figure 4.13(a). In the body of the loop, a value is incremented during each iteration, but the result is rotated around t_5 and t_6, so that the value from a previous iteration is preserved. The computation $t_5 + t_1$ is redundant in all cases except when the loop body is executed exactly twice: if the loop is executed only once, its value is stored in t_4; if it is executed more than twice, its value was computed in the third-to-last loop execution. Assuming we name values with the same numbering as the temporaries which represent them, the expression $v_5 + v_1$ is fully anticipated into block 3. On a first pass, it is not anticipated into block 4, so it is partially anticipated in block 2. If we were to propagate it across the back edge

(4,2), it would become $v_7 + v_1$ in block 4, which would require a new value, say v_9. On the next pass, the new value would become partially anticipated in block 2. If we propagated partial anticipation through the back edge again on the next iteration, it would become $v_8 + v_1$ since v_7 is represented by $v_6 + v_1$ which maps to $v_5 + v_1$, representing v_8. This new expression would again need a new value; this process would recur infinitely.

Disallowing partial anticipation propagation through a back edge, however, does not prevent us from identifying and eliminating the redundancy in this case. On the second pass, block 4 anticipated v_5+v_1, making that expression now fully anticipated in block 2. Since it is partially available (t_4 from block 1), we insert $t_9 \leftarrow t_7 + t_1$ in block 4. This insertion makes the value for $v_6 + v_1$ partially available, and since it is fully anticipated at the beginning of block 1, we insert $t_{11} \leftarrow t_3 + t_1$ in block 1. The phis $t_{10} \leftarrow \phi(t_4, t_9)$ and $t_{12} \leftarrow \phi(t_{11}, t_{10})$ allow us to eliminate the computations for t_7 and t_8, shortening the single-loop iteration scenario. See Figure 4.13(b).

For Insert, we add an extra step for each join point where we iterate over partially anticipated expressions. We do not want to make insertions if something is only partially anticipated, so we create a phi only if the equivalent value exists in all predecessors We need an extra variable, *by_all*, which we set to false if we inspect a predecessor where find_leader returns null. We insert only if *by_all* is true.

In our experiments on real code, cases like this occur rarely (in one benchmark it accounted for 3% of the operations eliminated; in seven, 1% or fewer; in ten it never occurred). PA_IN sets also consume much memory, and calculating them increases the average number of rounds for convergence of BuildSets by an order of magnitude.

4.4.2 Knoop et al's frontier

Problem

In the categories of Knoop et al [55], our algorithm is code motion, not code placement, and does not cross the frontier illustrated in Figure 2.2. To extend

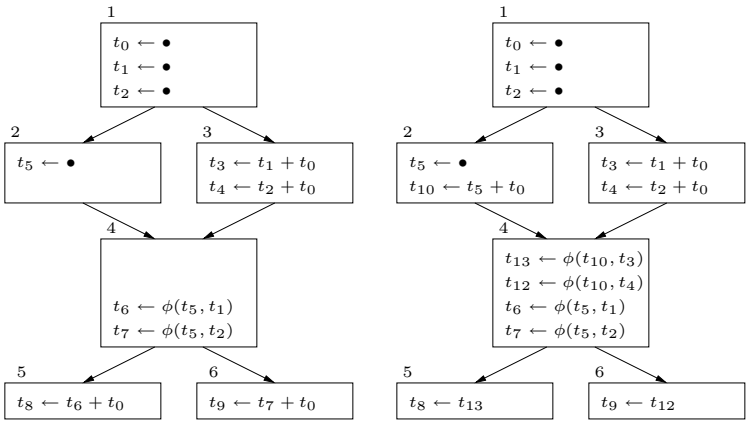


Figure 4.14. Code placement

this algorithm for code placement, partial anticipation must be used. Our goal is to eliminate redundant computations without introducing spurious computations. Therefore as our algorithm makes changes in the program, we make sure never to lengthen any program path by a computation, though we may lengthen it by moves and phis. So far, our rule for merging values (or leaders of the same value) in a phi is that such a merge is appropriate at a join point either if the corresponding expression is fully anticipated and at least partially available or if it is at least partially anticipated and fully available. In the former case, we are assured that any insertion we may need to make will allow an elimination later in all paths from that point. In the latter case, we are assured that no insertions of computations are necessary.

Although this approach is safe, it is not complete, which we see as we again turn to the frontier case. Consider the program in Figure 4.14(a). At the join point in block 3, the expression $t_6 + t_0$ is partially anticipated. It is available from block 2: translated through the phis, it becomes $t_1 + t_0$, the leader of whose value is t_3. It is not available in block 1. The expression $t_7 + t_0$ is also partially anticipated, and it also is available from block 2: translated through the phis, it becomes $t_2 + t_0$ in block 2, the

leader of whose value is t_4. It is not available in block 1. Partially anticipated and partially available, these do not qualify for optimization in the algorithm defined so far. However, they can be optimized according to our goals. Both of the expressions translate through the phis to $t_5 + t_0$. If we insert that computation in block 1 and store it in t_{10}, we can use t_{10} as the operand to two phis, one to merge it with t_3 and knock out the computation $t_6 + t_0$, the other to merge it with t_4 and knock out $t_7 + t_0$. See Figure 4.14(b).

Solution

An enhancement to our algorithm covering these cases is complicated but still elegant. Consider what is happening at the join point. The computation needed at the join point is $t_5 + t_0$. The question is, should we insert that computation? Since it is not equivalent to any fully anticipated expression, we know it will not be used in block 4. Therefore the answer is, it should be inserted if it is used in (all paths from) block 5 and (all paths from) block 6. How do we know if these will indeed be used? It will be used on block 5 if we make a phi for $t_6 + t_0$; it will be used on block 6 if we make a phi for $t_7 + t_0$. How do we know if we should indeed make these phis? We should make a phi for a given expression if it is partially available (which is true in our cases) and is fully hypothetically available (in other words, if it is already available or will be after safe insertions). Thus an expression at a join point is "fully hypothetical" if on every predecessor either it is already available or we will insert a computation for it.

In the words of a children's song, there is a hole in the bucket. Whether or not we insert a computation ultimately depends on whether or not we will insert it. To state the situation formally, suppose π stands for an expression that is partially anticipated and partially available (a "partial-partial"). Let u range over expressions which would be the right-hand side of hypothetical instructions, possibly to be inserted. Suppose that for each partial-partial π, we have a map, μ_π, which associates

$$
\begin{aligned}
\mathsf{will\_insert}(u) &= \forall\ s \in \mathsf{succ}, \mathsf{used}(u, s) && (4.11)\\
\mathsf{used}(u, s) &= \exists\ \pi \in \mu_u(s) | \mathsf{make\_phi}(\pi) && (4.12)\\
\mathsf{make\_phi}(\pi) &= \mathsf{part\_avail}(\pi) \wedge \mathsf{full\_hyp}(\pi) && (4.13)\\
\mathsf{part\_avail}(\pi) &= \exists\ p \in \mathsf{pred} | \mathsf{avail}(p, \pi) && (4.14)\\
\mathsf{avail}(\pi, p) &= \mathsf{find\_leader}(\mathsf{ANTIC\_IN}[p], && \\
&\qquad \mathsf{lookup}(\mathsf{equivalent\_expression}(\mathsf{succ}(p), p, \pi))) && \\
&\neq \mathsf{null} && (4.15)\\
\mathsf{full\_hyp}(\pi) &= \forall p \in \mathsf{pred}, \mathsf{avail\_or\_hyp}(\pi, p) && (4.16)\\
\mathsf{avail\_or\_hyp}(\pi, p) &= \mathsf{avail}(p, \pi) \vee \mathsf{will\_insert}(\mu_\pi(p)) && (4.17)\\
& && (4.18)
\end{aligned}
$$

Figure 4.15. Predicate system for insertion under partial anticipation

predecessors to expressions for hypothetical instructions. Suppose further that for each expression for a hypothetical instruction u, we have a map, μ_u, which associates successors with sets of partial-partials. We then have the system of predicates in Figure 4.15

The system is recursive, and if will_insert is true, the recursion will be infinite. We believe the converse is true. Thus we search for a contradiction to this system, and consider it to be true if no contradiction is found. In terms of implementation, will_insert should return true if it is reentered for the same argument.

Algorithm

To implement this solution, we need a small change to BuildSets: we need to build and maintain a map, *part_succ_map*, which associates an expression with the set of blocks on which the partially anticipated expression is fully anticipated but which are successors to blocks on which it is partially anticipated. Note that this might not be simply a single block; if a block has three successors and an expression is anticipated on two of the three, we need to know about both. Thus *part_succ_map*

is not a simple map associating a key with a single entry but a multi-map associating a key to a set of entries. Thus we assume another operation on this map, set_add, which takes a map, a key, and an entry; instead of overwriting any older entry for the given key, it adds that entry to the set of entries for that key.

More needs to be done to Insert. We need a map (*hyp_map*) which will associate hypothetical computations with maps (μ_u) from successors to sets of partial-partials, and a map (*part_part_map*) which will associate partial-partials with maps (μ_π) from predecessors to hypothetical computations. While doing insert for partially anticipated expressions in a given block, if an expression is also partially available (if it is a partial-partial), take note of it by making a μ_π for it; for each predecessor, μ_π should have a simple expression already available or a hypothetical computation which could be inserted. Each hypothetical computation should have its μ_u, which we populate by the successors of the current block found among the blocks at which the current partial-partial is fully anticipated. After iterating through the partially anticipated expressions, we consider what hypothetical computations should have insertions and which partial-partials consequently should phis be made for.

4.4.3 Precise handling of exceptions

Finally, we comment on how to modify this algorithm for the precise handling of exceptions. The Java programming language [68] requires that the order in which exceptions are thrown be preserved across transformations of the program. This means that instructions that could potentially throw an exception may not be reordered. To prevent this, we have to consider any potentially excepting instruction to kill all potentially excepting expressions when calculating anticipation. To adjust our algorithm, if a block contains any expression that could throw an exception, ANTIC_OUT should be purged of any potentially excepting expressions before being used to calculate ANTIC_IN. Furthermore, EXP_GEN of that block should not include

any potentially excepting expressions except for the first that occurs in the block, since that first such expression kills all others except itself.

4.5 GVNPRE in context

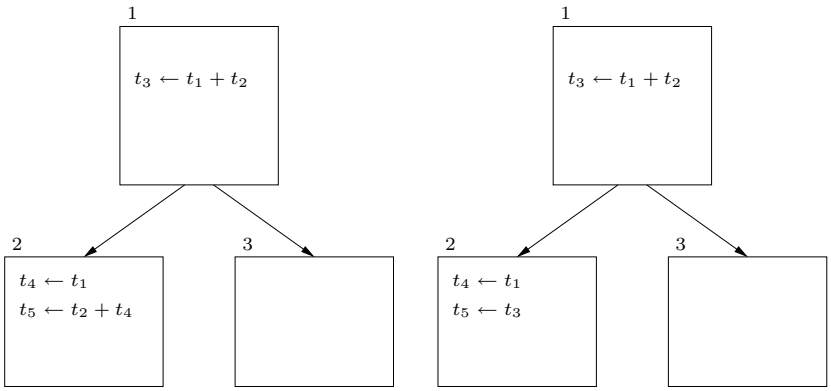
4.5.1 A comparison with Bodík

The contributions of our approach are as follows. First, our analysis considers larger program chunks than Bodík (basic blocks instead of single instructions). Second, we present a framework for expressions and values that takes full advantage of SSA, completely ignoring source-level lexical constraints and thinking solely in terms of values and expressions that represent them. Third, no graph is explicitly constructed nor is any novel structure introduced (contrast with Bodík's VNG). Instead, we achieve the same results by synthesizing well-known tools (SSA, control flow graphs, value numbers) and techniques (flow equations, liveness analysis, fixed-point iteration). In fact, our algorithm can be viewed as an extension of a simple hash-based GVN. Finally, we give an efficient and easily-reproduced implementation for this algorithm.

4.5.2 A response to our critic

This algorithm was presented in a paper at the Thirteenth International Conference on Compiler Construction [16]. One reviewer, who argued for acceptance, included comments from an outside reader (who had a penchant for citing SKR papers [48, 52–54, 56]) claiming,

> The paper shows an odd understanding of the differences and commonalities of [PRE] and [GVN]. It states that neither PRE nor GVN is strictly more powerful than the other transformation, and gives evidence for this thesis by means of an illustrating example. The thesis, however, is wrong. GVN is strictly more powerful than PRE and encompasses it. The ex-



(a) Unoptimized (b) Optimized

Figure 4.16. Why ARWZ is not "local."

> ample demonstrates that local value numbering (LVN), i.e., inside of a basic block, and PRE, a global program transformation, do not cover each other. [69]

The implication is that the comparison is not only wrong but banal—of course a local transformation is weaker than a global one. However, the survey of relevant literature in Chapter 2 shows that the source of confusion here lies in how ARWZ and SKR differ on what GVN should be. The "odd understanding" of GVN is the understanding of a highly published body of literature [38, 41, 44, 51, 57] and the standard advanced compilers textbook [47]. To be fair, Muchnick does compare PRE with what he calls "value numbering," not "*global* value numbering" [47], but referring to that notion of value numbering as *local* or suggesting that it works only within a basic block is wrong. If a value is computed in a block and recomputed in a block dominated by the first, the latter computation ought to be removed in the ARWZ view, because there is a value redundancy that is a full redundancy, as illustrated in Figure 4.16. This is clearly inter-block and hence legitimately can be called global. The weakness of ARWZ GVN is that it does not consider partial redundancies nor do hoisting, and that is what GVNPRE remedies.

The critic went on to claim

> [T]he authors [sic] conclusion "This (paper) demonstrates that performing PRE as an extension of GVN can be done simply from a software engineering standpoint, and that it is feasible as a phase of an optimizing compiler" has been known for more than 15 years. [69]

Fifteen years implies Steffen's original SKR paper at TAPSOFT'87 [48]. Besides the fact that even some of the latest SKR papers are not exactly implementation-oriented [55], they also concede "major obstacles opposing to its widespread usage in program optimization" [52]. GVNPRE, on the other hand, is synthesized from known and widely-used tools and algorithms.

4.5.3 GVNPRE and GCC

From a software engineering perspective, GVNPRE has had something of a success story as a compiler phase implemented in the GNU Compiler Collection [70]. Dan Berlin, a researcher at IBM's T.J. Watson Research Center, wrote,

> Having gone through hell implementing and maintaining SSAPRE (it's also over 3000 lines of code), I looked upon the relative simplicity of GVN-PRE with glee. In fact, I'm considering implementing it... [67]

After working on the proposed implementation, he said,

> ...I'd like to point out that my GVN-PRE implementation can already properly rebuild GCC with itself turned on. It took me almost 2 years to do this with SSAPRE due to the hard-to-understand nature of it. Thanks a bunch for making my life easier. [67]

4.6 Experiments

Our experiments use Jikes RVM [7–9], a virtual machine that executes Java class-files. We have implemented the algorithm described here as a compiler phase for the

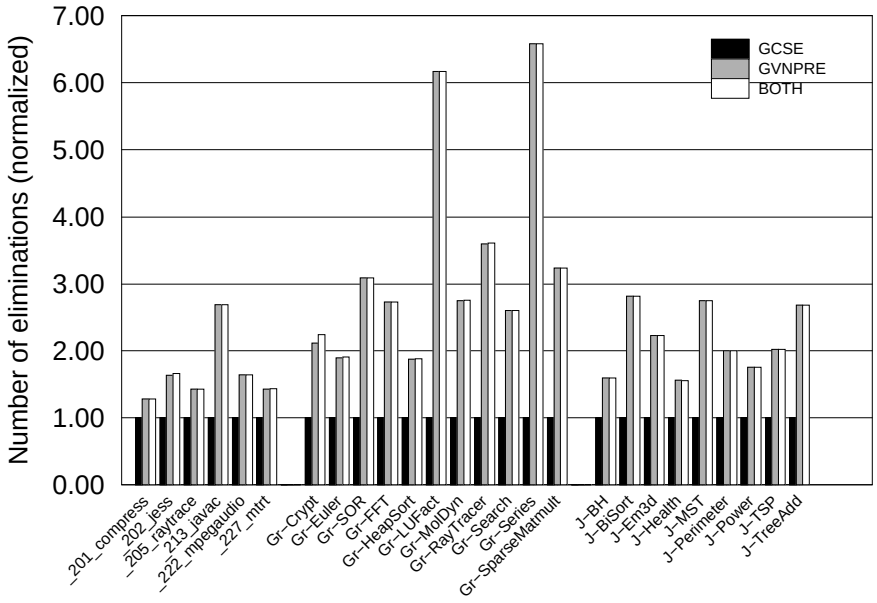


Figure 4.17. Static eliminations

optimizing compiler and configured Jikes RVM version 2.3.0 to use the optimizing compiler only and a generational mark-sweep garbage collector. The optimizing compiler performs a series of code transformations on both SSA and non-SSA representations. It already has global common subexpression elimination (GCSE) in SSA, which relies on GVN. Placing our phase before these two shows that GVNPRE completely subsumes GCSE in practice. (Jikes RVM's LICM is not used because it performs unsafe speculative motion and forward propagation which GVNPRE does not.) GCSE is equivalent to GVN as it is presented in this dissertation.

Figure 4.17 shows static eliminations performed by each optimization level. The left column in each set represents the number of intermediate representation operations eliminated by GCSE normalized to one, and the second those eliminated by GVNPRE. In each case, GVNPRE eliminates more, sometimes over six times as much, although the optimization also inserts operations and in some cases may later eliminate an operation it has inserted. The third bar shows the number of GVNPRE eliminations plus the number of operations GCSE can eliminate after GVNPRE has

run. Apart from a sprinkling of cases where GCSE picks up an instruction or two, perhaps the by-product of other transformations on the IR, GCSE does nothing in addition to GVNPRE.

Our performance results are in Figures 4.18 and 4.19. The runs shown in Figure 4.18 were executed on a 733 MHz Power Macintosh with 32Kb I cache, 32Kb D cache, and 256K L2 cache running Macintosh OS X. Those in Figure 4.19 were executed on a 1600MHz Intel Pentium III with 512 MB of RAM and 256 KB cache, running Red Hat Linux. We use three sets of benchmarks: six from the SPECjvm98 suite [11], eleven from the sequential benchmarks of the Java Grande Forum [13], and nine from the JOlden Benchmark Suite [14]. (Some benchmarks from these suites were omitted for implementation issues.) As in Chapter 3, we time only the running of the application (not compilation), and take the best of ten runs. First we considered the benchmarks run with Jikes RVM's O2 optimization level stripped down so that load elimination (to be addressed in the next chapter), global common subexpression elimination, and loop invariant code motion are turned off. This is our baseline. The three bars represent the results of three optimization levels normalized to our baseline: GCSE, like our baseline except with global common subexpression elimination turned on; GVNPRE, like our baseline except with the algorithm presented here turned on; and BOTH, with global common subexpression elimination, loop invariant code motion, and GVNPRE turned on. We also show the geometric mean.

On the Macintosh, these optimizations consistently made a modest improvement on almost all benchmarks, with our GVNPRE having a slight edge. However, on the Pentium we note that results are mixed for all optimization levels; often the optimizations make hardly any impact, and sometimes even degradation is incurred. The most obvious reason is that by keeping a value in a variable to eliminate a later computation, we could be increasing the live range of that variable (or even introducing a new one). The increase of register pressure could result in more register spills at compile time and therefore more memory access at run time. A write and

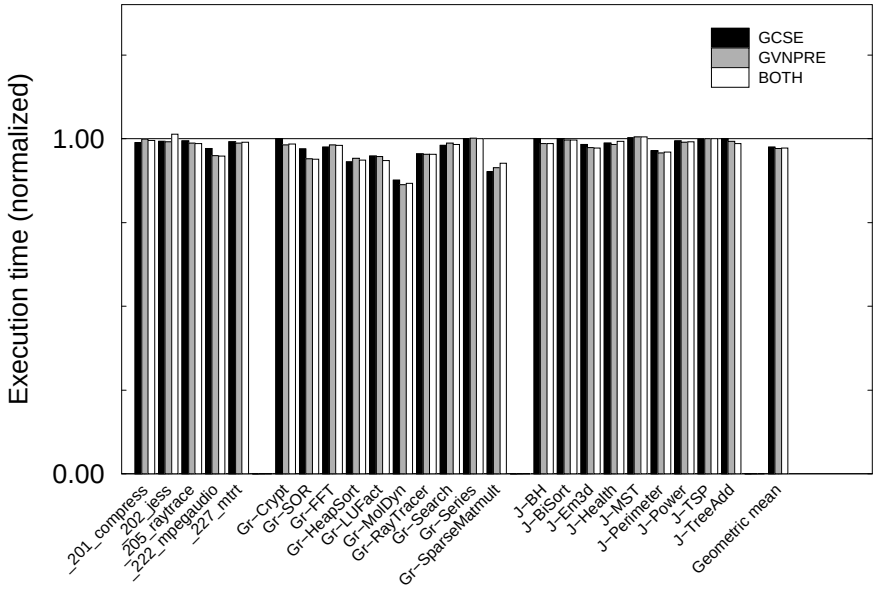


Figure 4.18. Performance results on PowerPC

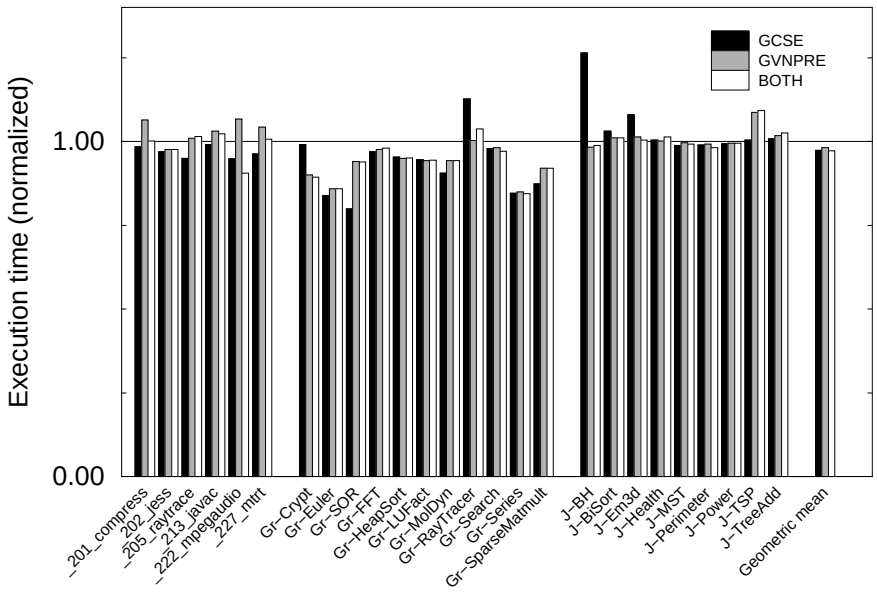


Figure 4.19. Performance results on Intel

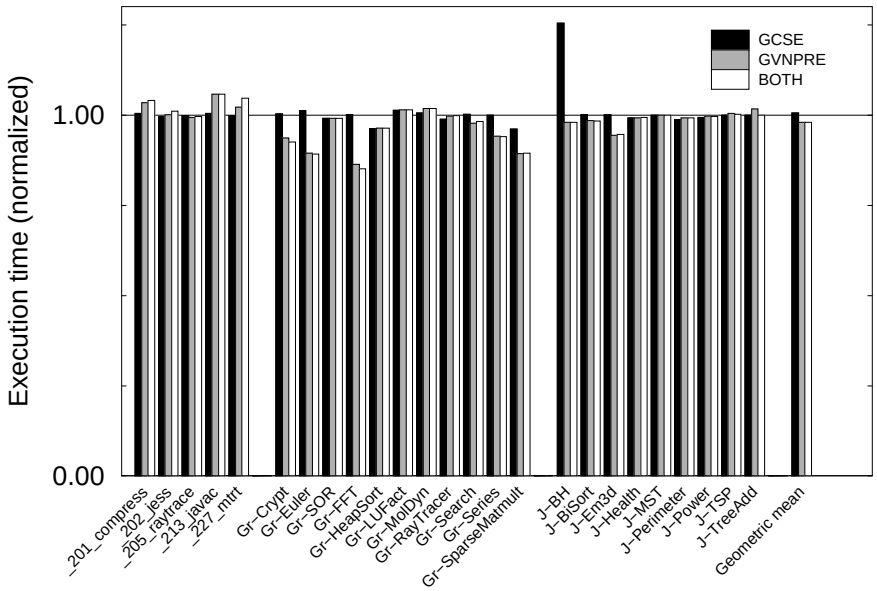


Figure 4.20. Performance results for the pseudo-adaptive framework on Intel

subsequent read are more expensive than a simple arithmetic expression. This is especially critical in cases where the register allocator is tuned more for compile time performance than run time (such as in linear scan allocation [71], which Jikes RVM uses) or when the register set is small, such as on the Pentium. Any transformation that eliminates instructions by reloading the value from a variable (such as GCSE) may increase live ranges, but doing hoisting (as in GVNPRE) is even more dangerous. Notice that GCSE results in serious degradation for BH on the Pentium, but GVNPRE brings the execution time back down. Conversely, GCSE does not effect the execution time of TSP, but GVNPRE degrades it.

To explore this further, we focused our attention on frequently executed code. Jikes RVM contains an adaptive framework which initially compiles all code with a base compiler, but on the fly observes what methods are "hot" and recompiles them with an optimizing compiler. We used a "pseudo-adaptive" variation on this framework which precompiles previously-identified hot methods with optimizations (in our case, the various optimization levels described above), but base compiles all

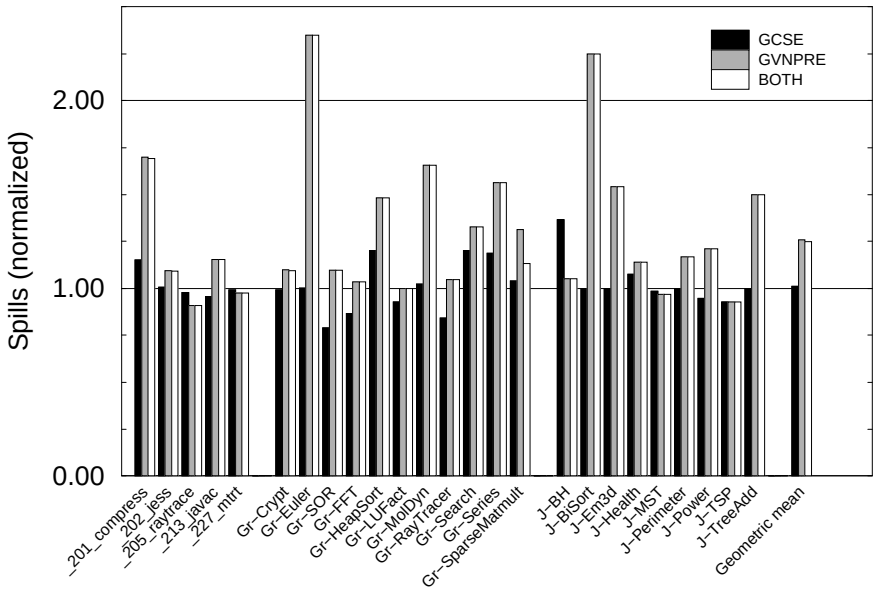


Figure 4.21. Number of spills for hot methods on Intel

other code. Figure 4.20 shows normalized execution time on the Pentium as in Figure 4.19, except with the optimizations applied only to hot methods. _222_mpegaudio has been removed because its method names are obfuscated in such a way that makes them difficult to specify for precompilation. The first thing we notice is that in general the variations over optimization levels is more muted, but that is especially the case for GCSE—the geometric mean indicates that on the average case, it no longer has an advantage over the baseline, although GVNPRE clearly does. This suggests GVNPRE is more effective on the code that matters. The case of degradation for GVNPRE on TSP has been alleviated, but GCSE still seriously worsens BH.

How confidently can we implicate register pressure? To explore this, we also studied the number of spills produced by hot methods, which we display in Figure 4.21. The bars represent the number of spills occurring during the compilation of the same set of hot methods for each optimization level, including our base, to which the numbers of the other levels are normalized. To begin, optimizations like these

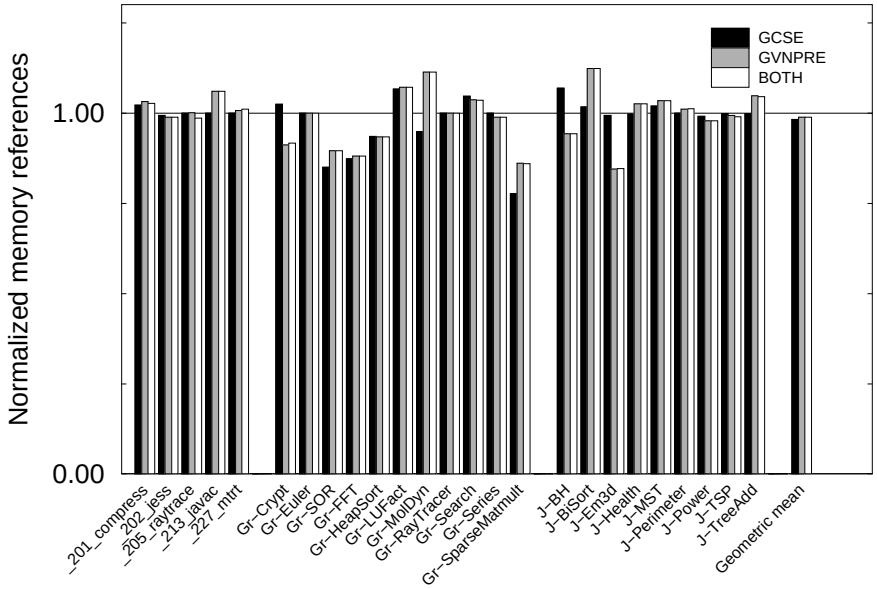


Figure 4.22. Number of dynamic memory accesses on Intel

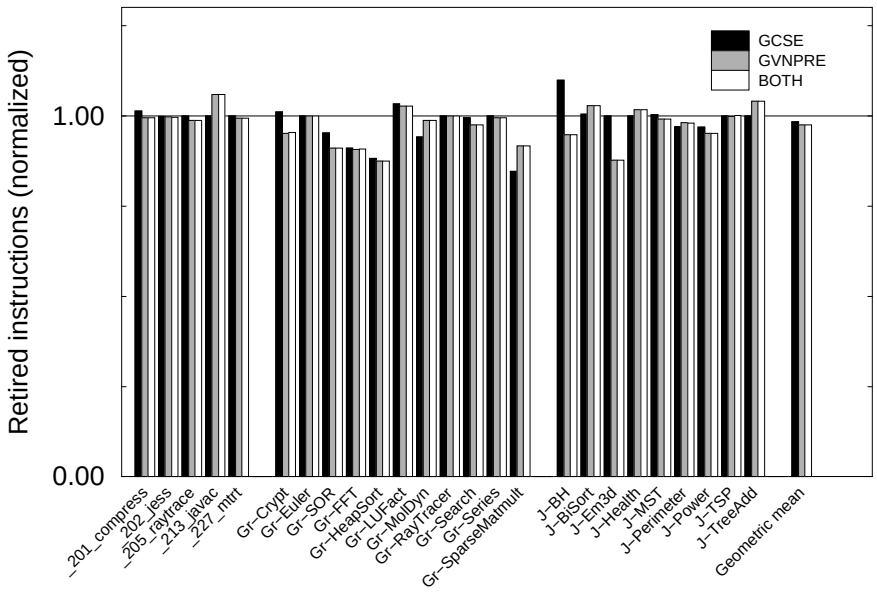


Figure 4.23. Number of retired instructions on Intel

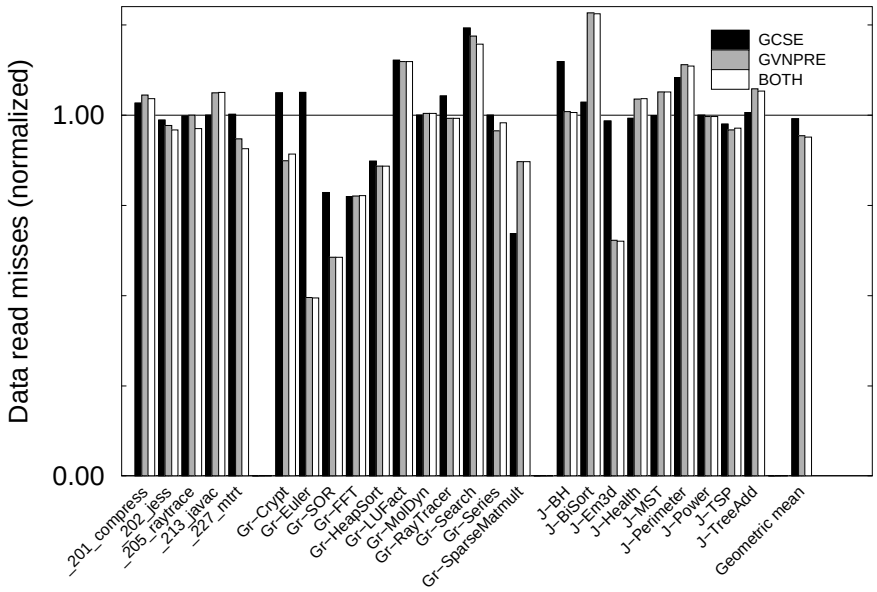


Figure 4.24. Number of data read misses on Intel

indeed tend to increase spills, often severely, and, looking at the geometric mean, GVNPRE does so more severely than GCSE. On the other hand, there are cases where either GCSE or GVNPRE decreases spills. Is the number of spills a predictor of performance? Not exactly, since there are cases such as Euler where GVNPRE results in a dramatic increase of spills and yet manages a speedup in Figure 4.20. However, in cases where there is degradation, we typically also find an increased number of spills (_201_compress and _213_javac for GVNPRE and, most strikingly, BH for GCSE).

We also considered the number of dynamic memory references and number of retired instructions, and data read misses, as shown in Figures 4.22 and 4.23, respectively. These were obtained using the performance counters available on the Pentium [72], using performance counter codes DATA_MEM_REFS, INST_RETIRED, and DATA_READ_MISS, respectively. These were measured for the entire application, not just the running of the hot methods (though not for compilation, of course), but still using the "pseudo-adaptive" framework for determining which methods

to opt compile. Although both GCSE and GVNPRE improve memory references slightly, the results are more sporadic than those for retired instructions. This suggests that while the effect on loads and stores to memory are hard to predict, these optimizations are indeed decreasing the programs' computational needs, and GVN-PRE usually more so than GCSE, as observed in Crypt, Search, BH, Em3d, and Power. The number of data read misses, however, varies greatly, as shown in Figure 4.24, although on the average, the more optimized levels reduce the number of misses.

4.7 Conclusions

We have presented an algorithm in which PRE and GVN are extended into a new approach that subsumes both, describing it formally as a data flow problem and commenting on a practical implementation. This demonstrates that performing PRE as an extension of GVN can be done simply from a software engineering standpoint, and that it is feasible as a phase of an optimizing compiler. An area of incompleteness is that this approach does not cover instructions for object and array loads. Such an extension is the topic of the next chapter.

5 LOAD ELIMINATION ENHANCED WITH PARTIAL REDUNDANCY ELIMINATION

What happens to a dream deferred?
...
Maybe it just sags
like a heavy load.

—Langston Hughes

5.1 Motivation

Our experiments on the combined GVNPRE technique showed that performance improvements were modest at best. A significant restriction on GVNPRE's power is that it operates only on simple (for example, arithmetic) operations and not on loads and stores from memory. As the disparity between processor speed and memory speed widens, it is becoming more critical to optimize memory access instructions. The goal of the present chapter is to extend the work in the previous chapter for object and array loads. This section discusses the challenges of this task.

First, we extend our language to include the bracketed portion of the language presented in Chapter 1. This includes the *getfield* (`gf`) operation and *putfield* (`pf`) instruction, and implies we should also extend the notion of expressions from Chapter 4 for getfields:

$$e \quad ::= \quad \ldots \mid \texttt{gf}\ x\ v \qquad\qquad \textit{Expressions}$$

x ranges over field names. The expression corresponds to the getfield operation. There is no expression for putfields since a putfield does not generate a value but a side effect. However, if t_1 and t_2 have values v_1 and v_2, respectively, an instruction like `pf` $x\ t_1\ t_2$ implies that the expression `gf` $x\ v_1$ has value v_2, even if no corresponding getfield operation appears in the program. We also note that the principles applied

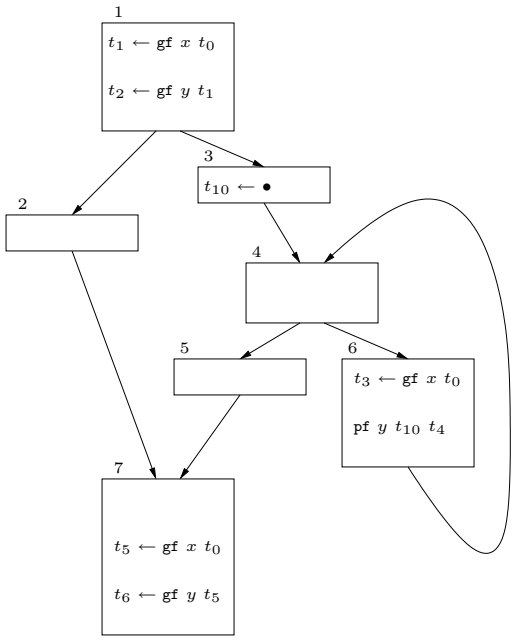


Figure 5.1. Unoptimized program with object references

here to object references can also be applied to array references (which we in fact do in our implementation and experiments). Instead of a field name and a pointer, array instructions would have a pointer and an (integer-valued) index.

At first glance, this looks like a simple extension, but in fact it presents a more difficult problem. Consider the example in Figure 5.1. If we blindly plug the new language constructs into the old value numbering scheme, we get the value table shown in Figure 5.2(a) and the transformed program in 5.2(b). This transformation is wrong because it replaces `gf` y t_5 in block 7 with a reload from t_2 which, although valid, for example, on trace (1,2,7), uses an outdated value for any trace that passes through block 6, where field y of value v_1 is updated. Depending on how careless the implementation is, it could even determine that t_4 and the expression for whatever operation produced it is also part of the same value, causing further errors.

The essence of the problem is that our SSA assumptions—that any expression will always have the same static value—do not hold true in the presence of object

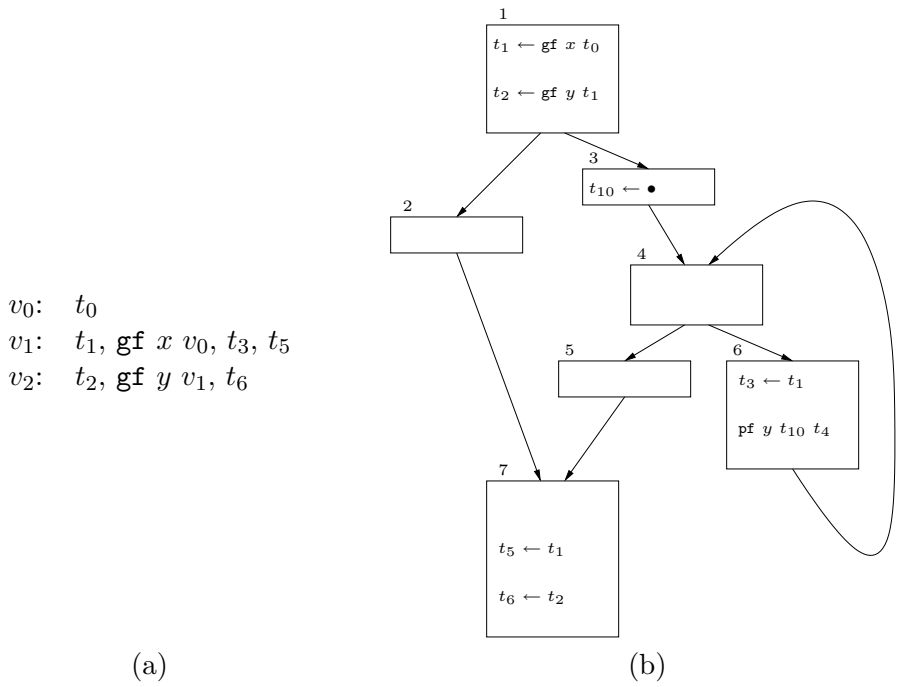
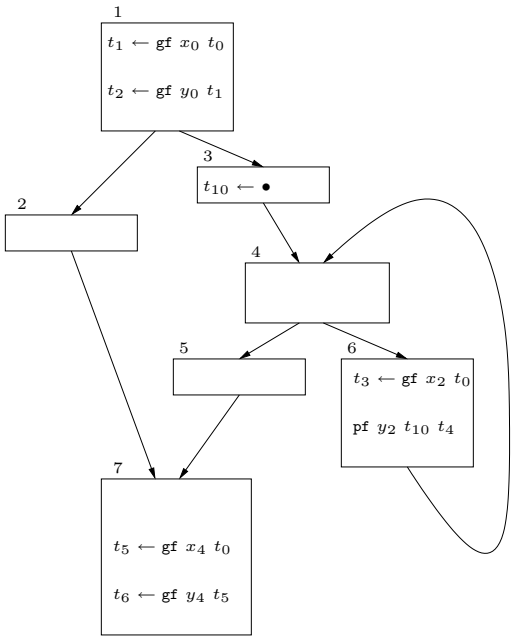


Figure 5.2. Program naively optimized

references. Since the contents of fields may change, gf y v_1 may have a different value at the end of block 6 from what it has at the end of block 2.

Suppose we enhance our IR to keep track of the states of object fields by giving version numbers to fields similar to the version numbers of SSA form (this approach will be described in detail below). In such a case, getfields, putfields, and join points (as well as some other instructions, like method calls, known to us only as $\bullet$) would define new states for the field at hand. If an instruction $t_2 \leftarrow$ gf x t_1 redefined x_1 to x_2 and the expression gf x_1 v_1 was available, say, as value v_2, we would say that expression gf x_2 v_1 as well as t_1 were in v_2. On the other hand, if we found an instruction pf x t_1 t_3 under the same circumstances, where t_3 has value v_3, we could say that expression gf x_2 v_1 was in v_3, different from gf x_1 v_1.

Our example program, enhanced with versioned fields, appears in Figure 5.3(a). Applying our algorithm would yield the value table seen in Figure 5.3(b), which although accurate, is imprecise because it fails to recognize that field x is never



(a)

v_0:	t_0
v_1:	t_1, gf x_0 v_0, gf x_0 v_0
v_2:	t_2, gf y_0 v_1
v_3:	t_3, gf x_2 v_0
v_4:	t_4, gf y_3 v_3
v_5:	t_5, gf x_4 v_0, gf x_5 v_0
v_6:	t_6, gf y_4 v_5, gf y_5 v_5

(b)

Figure 5.3. Program enhanced with versioned fields

updated, so all getfields on x for reference v_0 produce the same value. In fact, using the information in the table, we discover no opportunities for optimization.

The challenge in eliminating redundant getfields is that we must determine not only when a field has been read for a value but also whether or not that field has been changed. Two things complicate determining this: aliasing and back edges. First, suppose we have the code sequence

$t_3 \leftarrow$ gf x t_0

pf x t_1 t_2

$t_4 \leftarrow$ gf x t_0

If t_0 and t_1 have the same value (say, v_0), then we know that the expression gf x v_0 is available at the third instruction, and the operation can be replaced by t_2. If they are found to be different values (say, v_0 and v_1), however, that is not a strong enough condition to claim that gf x v_0 is available in t_3 since t_1 could still

alias t_0; for example, they both could be results of function calls (which would be represented by $\bullet$ in our language), and dynamically it is possible that the functions returned the same object reference. In other words, two expressions being in the same value is a *definitely-same (DS)* relationship; for precise load optimization, we also need to know what expressions are in a *definitely-different (DD)* relationship, for which we cannot rely on them simply having different values.

Second, knowing what values are the same and available, with any degree of precision, requires propagating information across back edges. This implies the values must be calculated using a fixed point iteration rather than a single top-down pass as in GVNPRE. Fink et al presented a load elimination algorithm (LE) which uses a fixed point iteration, not to determine value numbers, but to determine, for each versioned field, what are the values for which the dereference of that field is available, and to eliminate fully redundant getfields [3]; we describe this algorithm in Section 5.2. Notice that this uses value numbers already computed. However, it handles only full redundancy; it cannot optimize the instruction $t_6 \leftarrow \texttt{gf}\ y\ t_5$.

What we would like is a value-numbering that discovers in our example that all references to field x are of the same value, but that this is not true for field y. We feed this into our GVNPRE which will hoist a getfield for y into block 5. The value table and the optimized program are shown in Figure 5.4.

How do we calculate this? The pitfall is that we have a chicken-and-egg problem. In order to partition the expressions into values precisely, we need the results of the fixed point iteration, but to do the fixed point iteration, we need global value numbers. It is not a simple thing to let the fixed point iteration refine the value table, since merging values is a difficult thing to do in a hashing value numbering: the entire hash table would have to be traversed to update all the references to the values that have been merged. Furthermore, for expressions to have values as subexpressions requires values to be stable. Value-numbering schemes that split congruence classes, such as that in Chapter 3 of Simpson's dissertation [45], are not

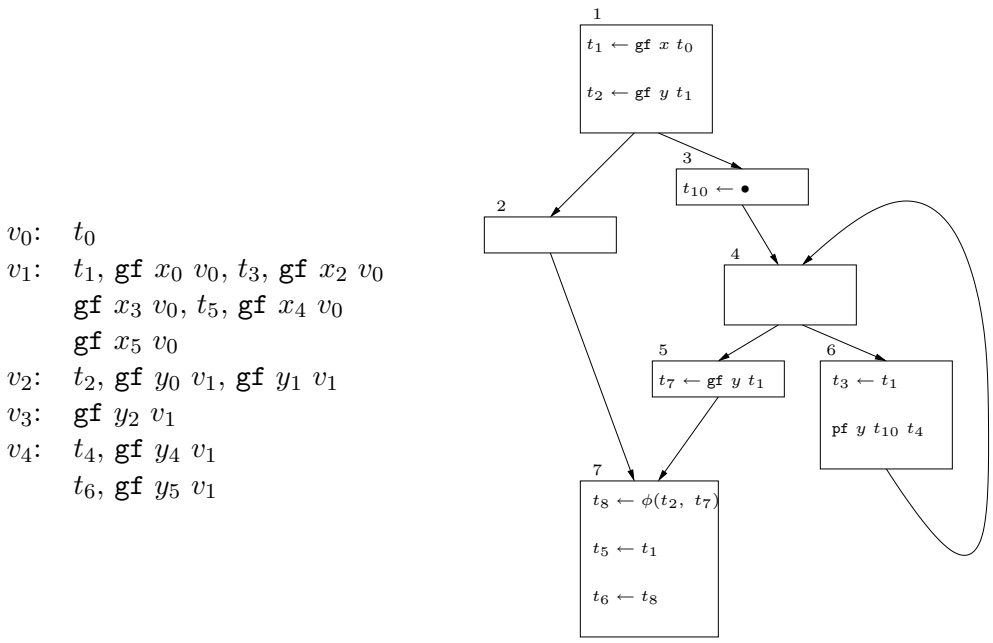


Figure 5.4. Desired value table and optimized program

desirable here because they do not simultaneously compute availability and cannot use our notion of values as subexpressions.

The rest of the chapter. In this chapter, we present an approximate solution by extending Fink et al's LE for PRE using the sets from our GVNPRE. In addition to the problem statement of PRE on object references given above, our contributions are a succinct re-presentation of Fink et al's LE (Section 5.2), a simple extension for PRE as an approximate solution to the problem (Section 5.3), comments on an implementation of this extension (Section 5.4), and ground work for a complete solution to the problem (Section 5.5).

5.2 Load elimination

Fink et al's LE is a relatively straight forward algorithm, once the preliminary concepts are well established. It has been implemented in Jikes RVM, and our results

section in this chapter (Section 5.4) shows its impact. The original presentation is difficult to digest [3]. We give here what we believe to be a simplified explanation, one that is also amenable to the extension we have in mind. This section contains clarifications of the preliminary concepts, the algorithm, and a brief discussion.

5.2.1 Preliminaries

LE requires three things: an IR that captures field states, GVN, and alias analysis.

To capture field states, Fink et al developed an extension of SSA called *Extended Array SSA* (EASSA) form [3], based on an earlier extension by Knobe and Sarkar [40]. Field names are given version numbers like variables in SSA form. These versioned fields are called *heap variables*. Understanding heap variables requires inverting the natural way to think of objects and their fields. Normally, we would think of field references as being "object-major": first we find the object in memory, and then search for the appropriate field in that object. For an instruction `pf` x t_1 t_2, one might say, "we are writing to the object to which t_1 points, specifically the field x of that object." A heap variable view is "field-major": first we identify the field in question and then consider which is the relevant object. For the above instruction, we would say, "we are writing to the field x, specifically that field in the object to which t_1 points." The distinction is important because we track how the state of the *field* changes (across all objects having such a field) rather than how an object (as a collection of fields) changes.

New field states are defined by variations on the phi functions of SSA. EASSA uses three types of phis for field states. The first, *control phis*, merge states at join points just as SSA phis do, and they will appear the same way in our examples. Second, *use phis* introduce a new field state after a read from a field; these appear in code examples as $u\phi$, and they take the previous state as an argument. Finally, *definition phis* introduce a new field state after a write to a field; they appear in

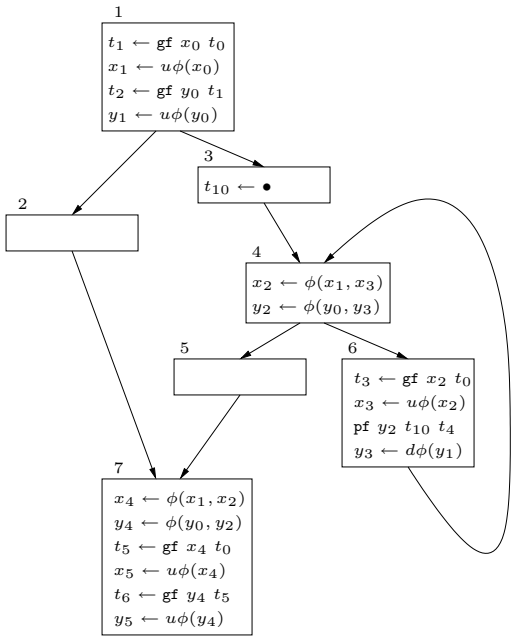


Figure 5.5. Program in Extended Array SSA form

code as $d\phi$, and they take the previous state as an argument. See our example, unoptimized, translated into EASSA form in Figure 5.5, and compare it with Figure 5.3(a).

LE also needs a value-numbering scheme that gives a value (or value number) for any variable name. The lookup function of GVNPRE will do nicely. One optimization level discussed in Section 5.4 shows the results of feeding GVNPRE into an LE implementation originally designed for an ARWZ GVN.

Finally there must be available information to determine *DD* and *DS*. The original LE presentation considered these to be predicates over variable names, but considering them to be predicates on values works just as well, since that reduces *DS* to equality. *DD* is tricky, in particular because it is not an equivalence relation. Precise determination is impossible statically, so we assume each pair of values is definitely different unless proven otherwise. Fink et al give two observations for determining two values to be different:

Table 5.1
Lattice equations for load elimination

Phi	equation	
$x_0 = \phi(x_1, \ldots, x_n)$	$\mathcal{L}(x_0) = \sqcap(\mathcal{L}(x_1), \ldots, \mathcal{L}(x_n))$	*Just take the intersection*
$x_1 = u\phi(x_0)$	$\mathcal{L}(x_1) = \mathcal{L}(x_0) \sqcup v$	*where v is the value referenced in the corresponding instruction*
$x_1 = d\phi(x_0)$	$\mathcal{L}(x_1) = v\sqcup \{w \mid w \in \mathcal{L}(x_0), DD(w, v)\}$	*where v is the value referenced in the corresponding instruction*

> 1. Object references that contain the results of distinct allocation-sites must be different.
>
> 2. An object reference containing the result of an allocation-site must differ from any object reference that occurs at a program point that dominates the allocation site. (As a special case, this implies that the result of an allocation site must be distinct from all object references that are method parameters). [3]

This could made more precise by using alias analysis schemes that consider the class hierarchy, such as Type-Based Alias Analysis [73].

5.2.2 Algorithm

Load elimination is based on solving a system of lattice operations. Each heap variable x_i has an associated lattice element, $\mathcal{L}(x_i)$, which is the set of value numbers $\{\ldots v_j \ldots\}$ such that `gf` x_i v_j has been computed. The algorithm has three steps: building and solving the set of data flow equations, identifying candidates for removal and storage to a temporary, and performing the replacements. Fink et al refer to the last as a *scalar replacement*.

Flow equations. To set up the flow equations, we walk through the code and for each phi over heap variables add an equation according to those found in Table 5.1.

$$
\begin{array}{rcll}
\mathcal{L}(x_1) & = & \mathcal{L}(x_0) \sqcup v_0 & = & \{v_0\} \\
\mathcal{L}(y_1) & = & \mathcal{L}(y_0) \sqcup v_1 & = & \{v_1\} \\
\mathcal{L}(x_2) & = & \mathcal{L}(x_0) \sqcap \mathcal{L}(x_3) & = & \{v_0\} \\
\mathcal{L}(y_2) & = & \mathcal{L}(y_0) \sqcap \mathcal{L}(y_3) & = & \bot \\
\mathcal{L}(x_3) & = & \mathcal{L}(x_2) \sqcup v_0 & = & \{v_0\} \\
\mathcal{L}(y_3) & = & \{w | w \in \mathcal{L}(y_2), DD(w, v_3)\} \sqcup v_1 & = & \{v_3\} \\
\mathcal{L}x_4 & = & \mathcal{L}(x_1) \sqcap \mathcal{L}(x_2) & = & \{v_0\} \\
\mathcal{L}y_4 & = & \mathcal{L}(y_1) \sqcap \mathcal{L}(y_2) & = & \bot \\
\mathcal{L}x_5 & = & \mathcal{L}(x_3) \sqcup v_0 & = & \{v_0\} \\
\mathcal{L}y_5 & = & \mathcal{L}(y_3) \sqcup v_5 & = & \{v_5\}
\end{array}
$$

Figure 5.6. Lattice equations and solutions for our example

The equations and their results after the propagation can be seen in Figure 5.6. We use the value numbering from Figure 5.5(b).

Replacement analysis. Next we identify a *UseRepSet*, the set of uses of heap variable we will replace (all of these will be getfield instructions). This set should contain all uses of a heap variable and value number such that the value number is in the heap variable's lattice entry:

$$UseRepSet = \{t_k \leftarrow \texttt{gf}\ x_i\ t_j \mid \mathsf{lookup}(t_j) \in \mathcal{L}(x_i)\}$$

In our example, $t_3 \leftarrow \texttt{gf}\ x_2\ t_0$ and $t_5 \leftarrow \texttt{gf}\ x_4\ t_0$ are in the *UseRepSet*.

Second, we identify a *DefRepSet*, the set of definitions of heap variables for which we will replace a use of that heap variable for the referenced value number (these may be getfields or putfields). Formally:

$$DefRepSet = \{\texttt{gf}\ x_i\ t_j \text{ or } \texttt{pf}\ x_i\ t_j\ t_k \mid \exists\ \texttt{gf}\ x_\ell\ t_m \in UseRepSet$$
$$\text{such that } \mathsf{lookup}(t_j) = \mathsf{lookup}(t_m)\}$$

Although technically the *DefRepSet* should contain the entire *UseRepSet*, we can leave those elements out for practical purposes. In our example, the *DefRepSet* includes $\texttt{gf}\ x_0\ t_0$.

Replacement transformation. Finally, for each field-value pair represented in the *UseRepSet*, choose a new temporary. Note that this is per field, not per heap

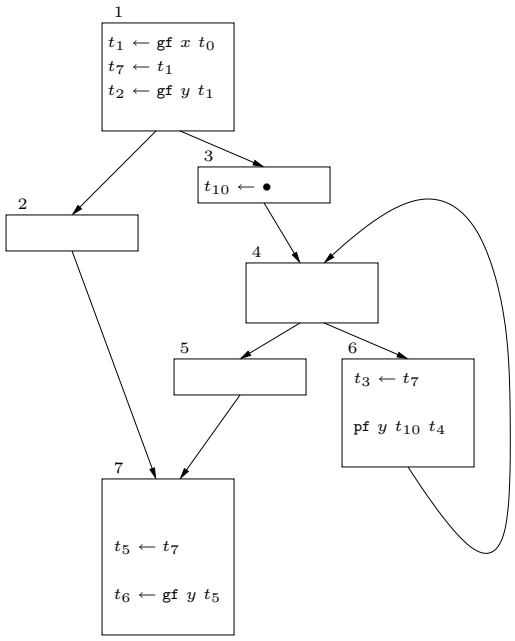


Figure 5.7. Optimized by Fink et al's load elimination

variable. For each member in the *DefRepSet*, add a store to the appropriate temporary immediately after the instruction. For $t_k \leftarrow \texttt{gf}\ x_i\ t_j$ in the *DefRepSet*, if the temporary for $(x, \mathsf{lookup}(t_j))$ is t_ℓ, insert an instruction $t_\ell \leftarrow t_k$. For $\texttt{pf}\ x_i\ t_j\ t_k$, similarly insert $t_\ell \leftarrow t_k$. For each member in the *UseRepSet*, replace the instruction with a move from the appropriate temporary. For $t_k \leftarrow \texttt{gf}\ x_i\ t_j$ in the *UseRepSet*, replace it with $t_k \leftarrow t_\ell$. Figure 5.7 shows the result of this in our running example.

5.2.3 Discussion

Note that all definition points for an expression that is used somewhere redundantly are in the *DefRepSet* (unless we exclude the ones that are also in the *UseRepSet*), not only those that will actually be used for a replacement. This is haphazard, but in our cost model it is harmless because it only introduces new moves. The jarring consequence is that it breaks SSA. Since several runs of the algorithm may be necessary to eliminate all the fully redundant loads, SSA would

have to be rebuilt. The root of the problem is that the algorithm regresses to lexical equivalence since it groups expressions by a form of lexical similarity (whether they are in the form `gf` x t, as long as each t is in the same value), not by the static value they compute, and then assigns a variable name to that group. To preserve SSA, each expression in the group would require its own variable (which would actually be unnecessary, considering each would be part of a value that already has a leader that is a variable or constant), and we would need to collect the relevant join points for placing of new phis to merge these variables.

5.3 LEPRE

We come now to the main portion of the chapter. We present a simple extension to Fink et al's load elimination to cover cases where a load is partially redundant. The essence of the idea is to let the lattice equations for phis compute a union rather than an intersection; since we will hoist instructions to predecessors where a certain value is not available, we want to consider a value available at a phi if it is available on at least one predecessor. However, that is not enough for safe code motion, because we will make such insertions only if the value is *fully anticipated* at that program point, that is, it will be used in a replacement on all paths to program exit. Unfortunately, expanding the *DefRepSet* to include phis only gives us *single-path anticipation* (the value is anticipated somewhere), not *all-paths anticipation*, which we want. To accommodate this, we must extend EASSA form and compute a second lattice for anticipation; the two lattices will interact.

5.3.1 Extending EASSA

Phis are useful to mark places where availability merges. Since anticipation is the backward-flow analogue of availability, merge points for anticipation are the branch points in forward flow of the program. To capture the merging of anticipation, we introduce a new construct, a *psi*, which is placed on branch points in the CFG.

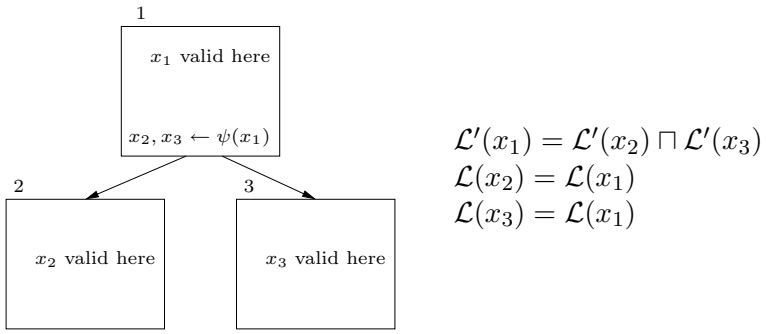


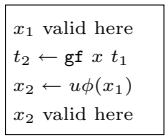
Figure 5.8. Psi illustrated

The psi takes as an argument the current heap variable for a field and returns a set of new heap variables (new versions for the field), one for each successor. See the example in Figure 5.8. Building EASSA this way is simple; in a practical EASSA implementation, there are already other kinds of instructions that require the definitions of heap variables for all valid fields (calls, for example); all we need to do is include basic block headers among those instructions that rename all valid fields.

5.3.2 Lattice operations

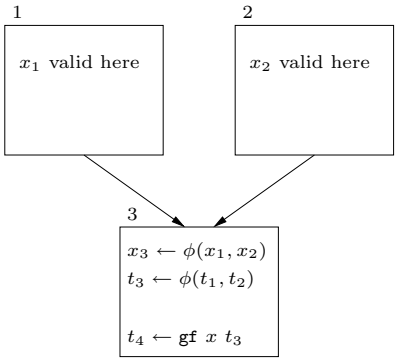
The modified lattice operations are in Table 5.2. To account for this new variety of "phi," we need an extra kind of flow equation for the original lattice. A psi simply propagates the current value numbers from its argument's lattice cell to those of its definitions, as illustrated by the two equations defining $\mathcal{L}(x_2)$ and $\mathcal{L}(x_3)$ in Figure 5.8. In the reverse direction, the value for x_1 in the anticipation lattice is defined by the intersection of the predecessor elements, $\mathcal{L}'(x_2)$ and $\mathcal{L}'(x_3)$ in our example. This works just as phis do for the availability lattice in the original set of equations.

The equations for use phis and def phis are similar to those for the original lattice, except the flow goes in the opposite direction. In Figure 5.9, the use phi attached to the getfield adds v_1 to the set of values anticipated in the sequent. In the case of



$\mathcal{L}'(x_1) = v_1 \sqcup \mathcal{L}'(x_2)$

Figure 5.9. Illustration of use phi



if $\mathcal{L}'(x_3) = \{v_2\}$, then

$\mathcal{L}'(x_1) = \{\mathsf{phi\_translate}(v_3, 3, 1)\} = \{v_1\}$

$\mathcal{L}'(x_2) = \{\mathsf{phi\_translate}(v_3, 3, 2)\} = \{v_2\}$

Figure 5.10. Illustration for control phis

a def phi (not shown), we again must purge values not definitely different from the one used at the given instruction.

Control phis are handled for anticipation just as psis are handled for availability, except that the value must be transfered through the phis at the block. In Figure 5.10, $\mathcal{L}'(x_1)$ and $\mathcal{L}'(x_2)$ should be $\{v_1\}$ and $\{v_2\}$, respectively. The function $\mathsf{phi\_translate}(v_3, 3, 1)$, where b_1 is the predecessor that corresponds to t_1, returns v_1; similarly with b_2, v_2. But the most important change is that to control phis for the original lattice. Here is where the lattices interact. As before, we begin with the intersection of the lattice elements from the predecessors. In addition, we add any value that is anticipated (that is, it is present in the heap variable's other lattice element) whose translated value is also available on at least one predecessor. We add a further restriction—on all predecessors the value must transfer to something that is available. This is so we have the necessary value available if we need to make

Table 5.2
Additional and revised lattice equations for LEPRE

Phi	equation	
$x_0 = \phi(x_1, \ldots, x_n)$	$\mathcal{L}'(x_i) = \{v \mid \exists w \in \mathcal{L}(x_0),$ $v = \mathsf{phi\_translate}(w, b, b_i)\}$	*where $b_1, \ldots, b_n$ are the predecessors of b corresponding to $x_1, \ldots, x_n$, respectively*
$x_1 = u\phi(x_0)$	$\mathcal{L}'(x_0) = \mathcal{L}'(x_1) \sqcup v$	*where v is the value referenced in the corresponding instruction*
$x_1 = d\phi(x_0)$	$\mathcal{L}'(x_0) = v\sqcup$ $\{w \mid w \in \mathcal{L}'(x_1), DD(w, v)\}$	*where v is the value referenced in the corresponding instruction*
$x_1, \ldots, x_n = \psi(x_0)$	$\mathcal{L}'(x_0) = \sqcap\mathcal{L}'(x_1) \ldots \mathcal{L}'(x_n)$	*just take the intersection*
$x_0 = \phi(x_1, \ldots, x_n)$	$\mathcal{L}(x_0) = \sqcap(\mathcal{L}(x_1), \ldots, \mathcal{L}(x_n))$ $\sqcup\{v \mid v \in \mathcal{L}'(x_0)$ and $\exists\mathcal{L}(x_i) \in \{\mathcal{L}(x_1), \ldots, \mathcal{L}(x_n)\}$ such that $\mathsf{phi\_translate}(v, b, b_i) \in \mathcal{L}(x_i)$ and $\forall b_j \in \{b_1, \ldots, b_n\}$, $\mathsf{phi\_translate}(v, b, b_j) \neq \mathsf{null}\}$	*where $b_1, \ldots, b_n$ are the predecessors of b corresponding to $x_1, \ldots, x_n$, respectively*
$x_1, \ldots, x_n = \psi(x_0)$	$\mathcal{L}(x_i) = \mathcal{L}(x_0)$	*For $i \in 1 \ldots n$*

an insertion on any predecessor. Consider the example in Figure 5.11 (we show the EASSA annotations for the field y only, for brevity). Since $\mathcal{L}'(y_1) = \{v_1\}$ and $\mathcal{L}(y_2) = \{v_1\}$ (where $\mathsf{lookup}(t_1) = v_1$), we would say that $\mathcal{L}(y_1) = \{v_1\}$ except that `gf` y v_1 is not insertable in block 1 since the value v_1 has no representative (*leader* in our terminology) there. This example also illustrates the need for several rounds of this optimization (just like the original load elimination), since in the first round, `gf` x t_0 will be hoisted to block 1, after which the insertion for y will be possible.

5.3.3 Insertion

We must add another phase, between replacement analysis and replacement transformation, a phase for the insertion of new instructions. For each control phi $x_0 \leftarrow \phi(x_1, \ldots, x_n)$, we must consider the value numbers in $\mathcal{L}(x_0)$. For a value number v, consider if $v_i \in \mathcal{L}(x_i)$ for all $x_i \in \{x_1, \ldots, x_n\}$, where $v_i = \mathsf{phi\_translate}(v, b, b_i)$ and b_i is the predecessor block corresponding to x_i. If $v_i \notin \mathcal{L}(x_j)$, corresponding to

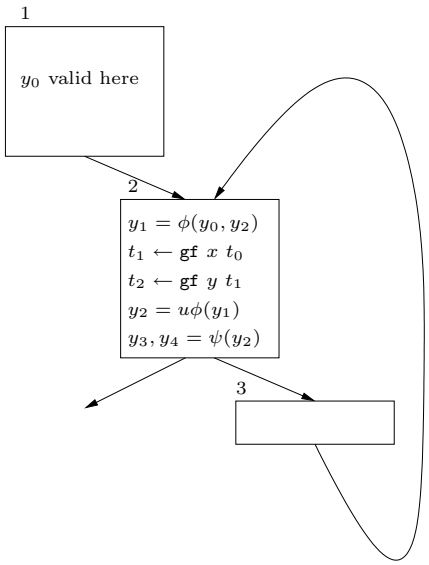


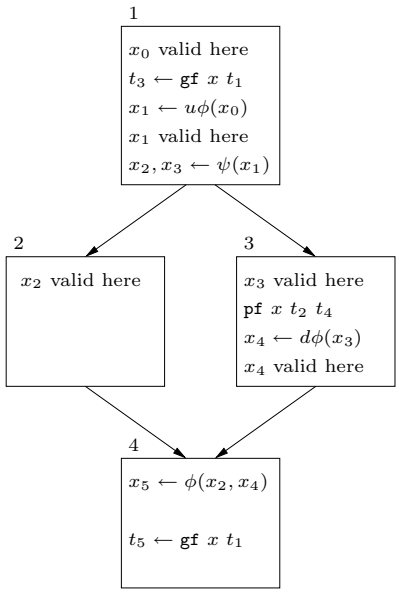
Figure 5.11. Need for insertablility on all preds

block b_j, then append to b_j the instruction $t_\ell \leftarrow$ gf x t_m, where t_ℓ is the appropriate temporary for (x, v_m) from the Replacement Transformation phase of load elimination, $v_m = \mathsf{phi\_translate}(v, b, b_j)$, and t_m is the leader of v_m in b_j. On the other hand, if $v_i \in \mathcal{L}(x_j)$, make sure the instruction for v is in the *DefRepSet*. In either case, insert a move, $t' \leftarrow t_\ell$ where t' is the appropriate temporary for (x, v).

5.3.4 Comparison with Fink et al

We now apply our algorithm to two examples in which Fink et al's algorithm would fail to eliminate redundancy, for comparison. In Figure 5.12, we assume $\mathcal{L}(x_3) = \{v_1\}$ initially. If v_1 and v_2 were definitely different, then $\mathcal{L}(x_4)$ would be $\{v_1, v_2\}$; to find $\mathcal{L}(x_5)$, we intersect $\mathcal{L}(x_4)$ with $\mathcal{L}(x_2)$, and get $\mathcal{L}(x_5) = \{v_1\}$. In that case, the getfield in block 4 could be removed. However, assuming that v_1 and v_2 are not definitely different, the def phi associated with the putfield in block 3 knocks v_1 out of $\mathcal{L}(x_4)$. Accordingly, $\mathcal{L}(x_5) = \bot$. The getfield in block 4 is still partially redundant, but Fink et al have no means for dealing with this.

Assume $v_1 = \{t_1\}$, $v_2 = \{t_2\}$, and $\neg DD(v_1, v_2)$.



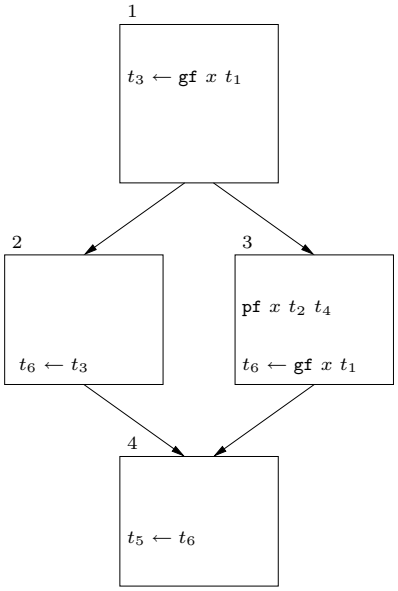
$\mathcal{L}(x_1) = \bot$ and $\mathcal{L}(x_3) = \{v_1\}$

$\mathcal{L}(x_2) = \mathcal{L}(x_1) \sqcup v_1 = \{v_1\}$

$\mathcal{L}(x_4) = \bot$

$\mathcal{L}(x_5) = \{v_1\} \sqcap \bot = \bot$

Figure 5.12. Source example and equations from Fink et al

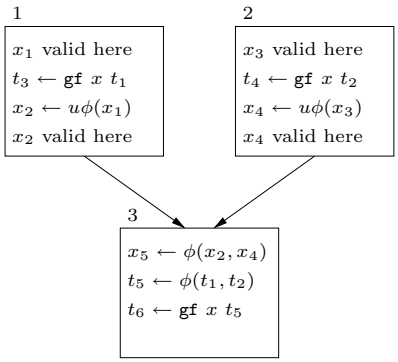


$\mathcal{L}'(x_5) = \{v_1\}$

$\mathcal{L}(x_4) = \bot$

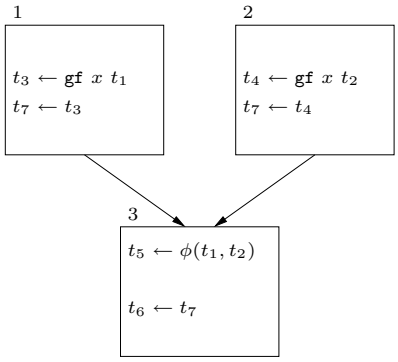
$\mathcal{L}(x_5) = (\{v\} \sqcap \bot) \sqcup \{v_1\} = \{v_1\}$

Figure 5.13. Equations from our algorithm with optimized version



$$\mathcal{L}(x_2) = \{v_1\}$$
$$\mathcal{L}(x_4) = \{v_2\}$$
$$\mathcal{L}(x_5) = \{v_1\} \sqcap \{v_2\} = \bot$$

Figure 5.14. Source example and equations from Fink et al



$$\mathcal{L}'(x_5) = \{v_5\}$$
$$\mathcal{L}(x_5) = (\{v_1\} \sqcap v_2) \sqcup \{v_5\} = \{v_5\}$$

Figure 5.15. Equations from our algorithm with optimized version

Using the algorithm presented in this chapter, we first note that $\mathcal{L}'(x_5) = \{v_1\}$. Then, since phi_translate$(v_1, 4, 2) = v_1$ and $v_1 \in \mathcal{L}(x_2)$ (that is, the equivalent value number is available on a predecessor), we find $\mathcal{L}(x_5) = \{v_1\}$. In the insertion stage, we insert a getfield in block 3 and a move in block 2, allowing us to eliminate the computation in block 4. See Figure 5.13.

In Figure 5.14, we have an example where Fink et al fails to remove even a full redundancy. The value equivalent to the instruction gf x t_5 has been computed on each path to the occurrence, but since the object reference has a different value from the one in block 3, Fink et al's analysis finds $\mathcal{L}(x_5) = \bot$. Using our algorithm, however, we find $\mathcal{L}'(x_5) = \{v_5\}$, since phi_translate$(v_5, 3, 1) = v_1$ and

phi_translate$(v_5, 3, 2) = v_2$. In the insertion stage, we insert no computations but only moves. Then the computation in block 3 can be removed.

5.3.5 A comparison with Bodík

Bodík gives examples that include array accesses [57], but does not discuss aliasing. One would assume that value threads would need to be cut when crossing an instruction that writes to an array that could be the same as the array used in the current value thread, similar to the provision for ASSAPRE in chi insertion at the end of Section 3.3.1. Although Bodík's approach is more precise than the ad hoc LEPRE, LEPRE has the advantage of using the standard and relatively simple lattice approach instead of the value name graph, and it does not require considering each instruction as its own basic block.

5.4 Experiments

Our experiments use Jikes RVM [7–9], a virtual machine that executes Java class-files. We have implemented the algorithm described here as a compiler phase for the optimizing compiler and configured Jikes RVM version 2.3.0 to use the optimizing compiler only and a generational mark-sweep garbage collector. The optimizing compiler performs a series of code transformations on both SSA and non-SSA representations. It already has LE.

We use three sets of benchmarks: five from the SPECjvm98 suite [11], ten from the sequential benchmarks of the Java Grande Forum [13], and seven from the JOlden Benchmark Suite [14]. First we considered the normal O2 level optimization of Jikes RVM, with the addition of the GVNPRE algorithm from the previous chapter. This is our baseline. Our intention is to show the affect of the LEPRE algorithm from this chapter; however, LEPRE requires a stronger value numbering than the one provided by Jikes RVM and used by default by LE. So we consider a level called VNPRELE, which shows the effect of running the old LE except with GVNPRE's

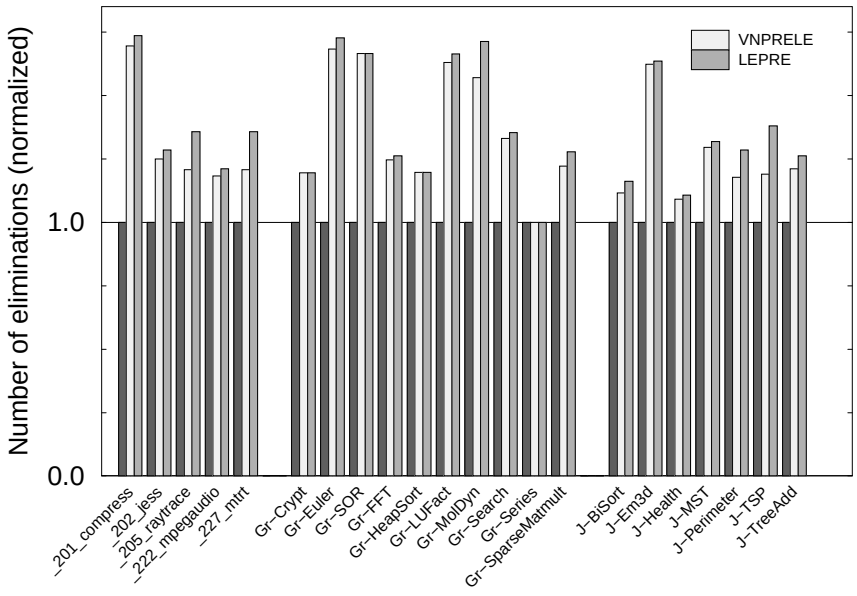


Figure 5.16. Static eliminations

value numbering rather than the default. The other level demonstrates the effects of optimizing a program with LEPRE. Figure 5.16 shows static eliminations performed by each optimization level, with the first bar representing the number of instructions eliminated by LE followed by the number eliminated by VNPRELE and LEPRE, all normalized to the LE level. We make two observations. First, the number of instructions eliminated changes noticeably by simply giving a smarter value numbering to old LE. Second, the increase of eliminations when PRE is added is only modest. There appears to be little opportunity to hoist loads to earlier program points.

Our performance results in Figure 5.17 show runs executed on a 1600MHz Intel Pentium 3 with 512 MB of RAM and 256 KB cache, running Red Hat Linux. Recall from Chapter 4 that optimizations that reload values from temporaries can have the negative effect of lengthening live ranges and bringing out more spills, and that hoisting can aggravate this further. However, the extra eliminations using VNPRELE appear to have little effect, positive or negative. Not surprisingly, the few eliminations gained by LEPRE do not affect performance much either.

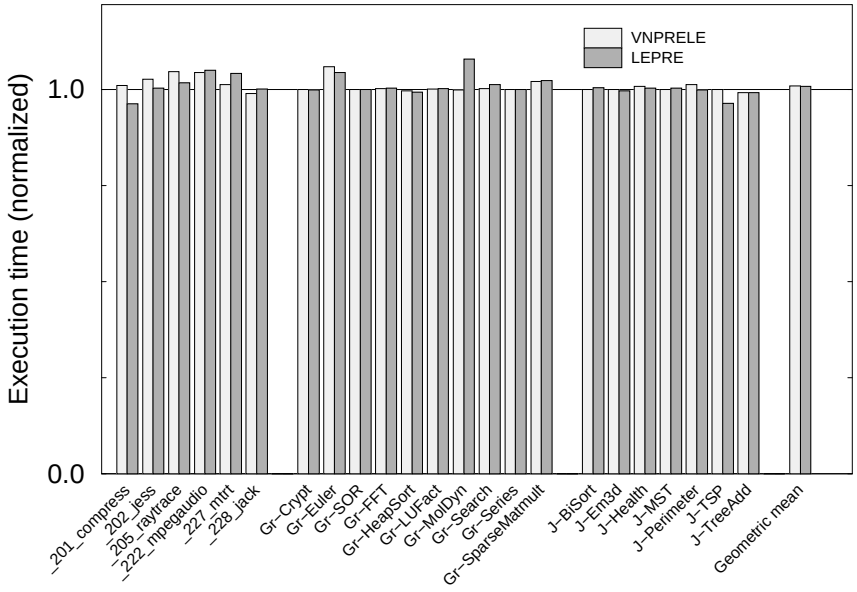


Figure 5.17. Performance results

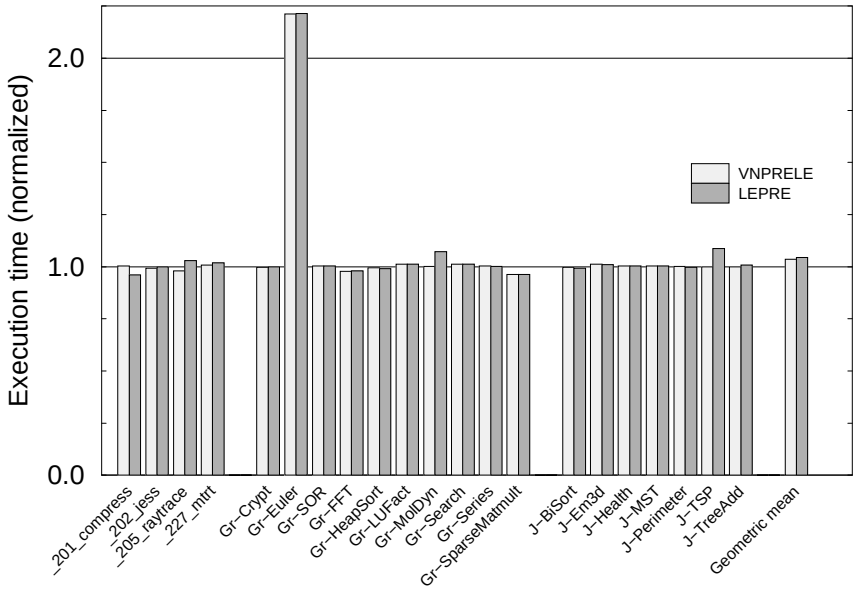


Figure 5.18. Performance results for the pseudo-adaptive framework

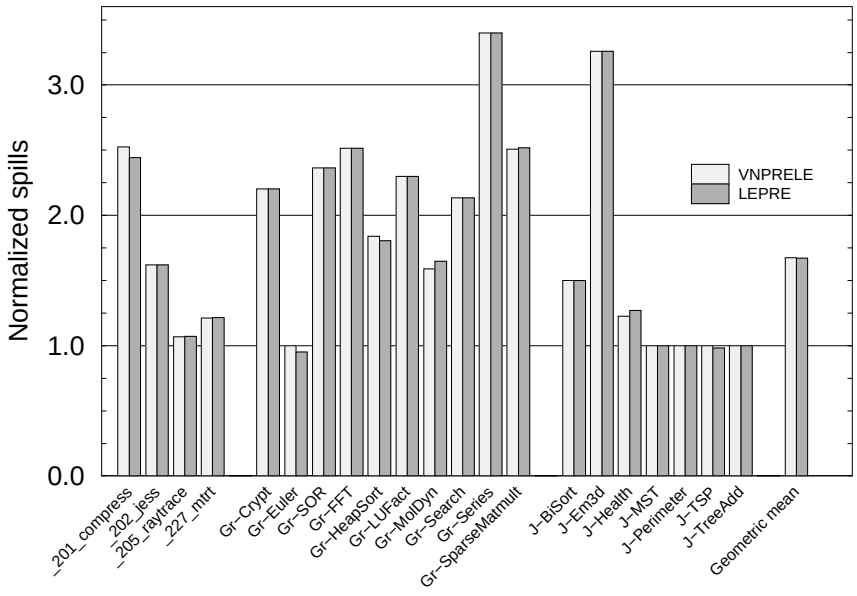


Figure 5.19. Number of spills for hot methods

As in Chapter 4, we considered the effect of running these optimizations only on hot code and measuring both performance and spills, which results appear in Figures 5.18 and 5.19, respectively. While the extra eliminations do produce more spills in most benchmarks, they do not appear to translate into a performance hit. Most benchmarks appear flat apart from the anomalous degradation of Euler, a case when in fact we reduce the number of spills.

To round out the picture, we also considered the affects on dynamic memory references, retired instructions, and data read misses, as we did earlier. See Figures 5.20, 5.21, and 5.22. Again, these in the framework where the optimizations are performed only on the hot methods. Most of these numbers are fairly flat as well, although we do gain insight into the behavior of MolDyn in the LEPRE level. MolDyn takes a hit even from whole-program optimization in Figure 5.17, which recurs in the optimization of hot methods in Figure 5.18. We can relate that to increased memory references and retired instructions. While MolDyn on the LEPRE level does not increase spills excessively, a few spills in critical areas could cause this degradation.

Euler's poor behavior can now be explained by the sharp increase of data read misses in Figure 5.22; data read misses are flat on most other benchmarks. We conclude from this that more study needs to be done on measuring the opportunities for this type of optimization, which we address in the future work section of Chapter 6.

Using the technique described in the previous chapter of timing the building of the Jikes RVM image, we found that feeding GVNPRE into Fink et al's LE increased the build time by a factor of three, but this is likely due to the increased number of times that GVNPRE is performed, since the information needs to be updated for each iteration of LE. Using LEPRE was only slightly more costly; it increased the running time over LE by a factor of about 3.2.

5.5 A precise proposal

We conclude this chapter by sketching an algorithm for a precise value-numbering on object and array references. Congruence-splitting value numbering techniques

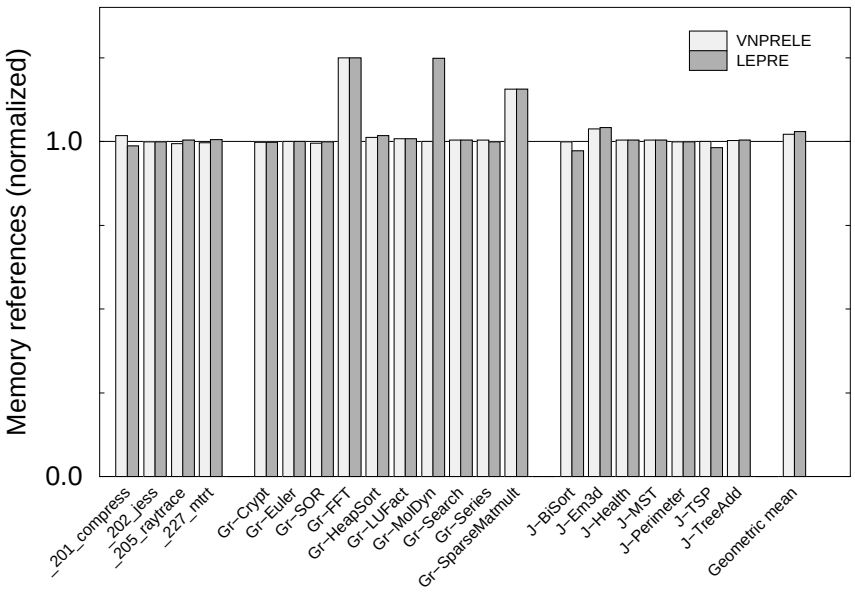


Figure 5.20. Number of dynamic memory accesses

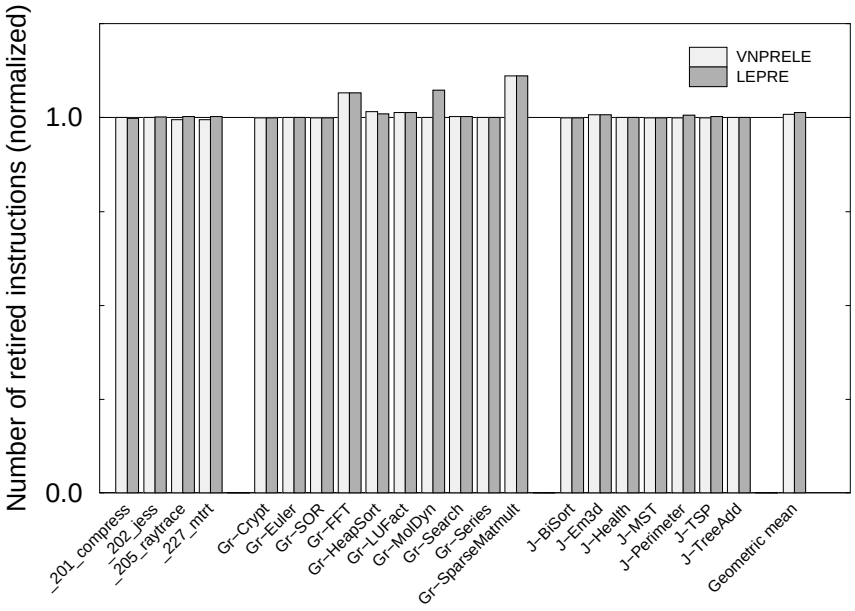


Figure 5.21. Number of retired instructions

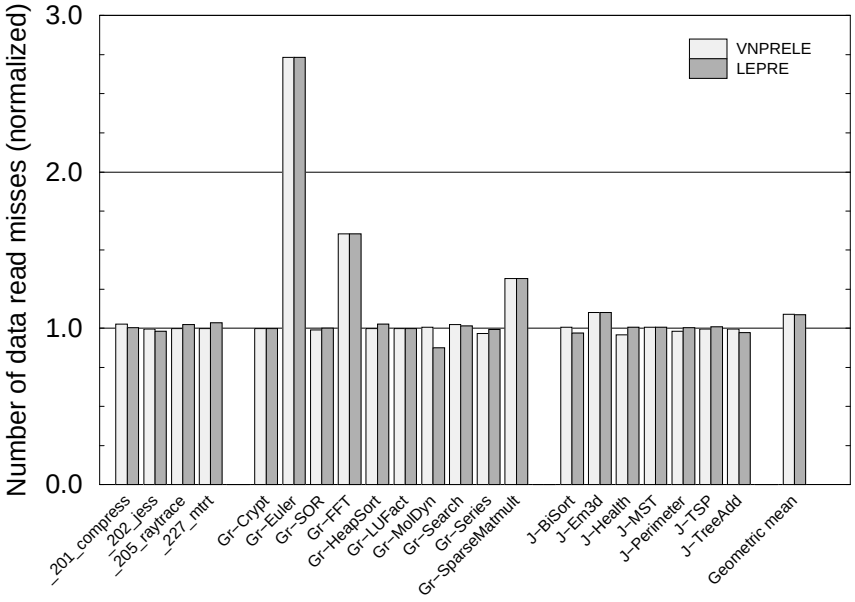


Figure 5.22. Number of data read misses

initially consider all expressions with the same operator to have the same value (or congruence class) and splits classes as it finds contradictions. Consider the code sequence

$t_1 \leftarrow \texttt{gf}\ x\ t_0$
$t_2 \leftarrow \texttt{gf}\ x\ t_1$

We can assume that the getfield together with a field constitute this approach's idea of operator; the expressions of these two instructions initially would be in the same value, but a getfield for the field y would not. t_0 would initially be in its own class. The first pass of the technique would assume t_1 had a different value from t_0 (since they are in separate classes) and thus would split the class that contained both $\texttt{gf}\ x\ t_0$ and $\texttt{gf}\ x\ t_1$.

Several things we can notice immediately. First, as mentioned earlier, our notion of expressions having values as subexpressions will not do at all. It searches for occurrences of variables in other expressions, not value classes, and since value classes are continually splitting, an expression synthesized from values is not well-defined. Second, the initial split of all expressions based on operator is much too pessimistic. The code sequence

$t_2 \leftarrow t_1 + t_2$
$\texttt{pf}\ x\ t_3\ t_2$
$\texttt{pf}\ y\ t_4\ t_2$

proves that expressions as diverse as $t_1 + t_2$, $\texttt{gf}\ x\ t_3$, and $\texttt{gf}\ y\ t_4$ can have the same static value—even our imprecise application of GVNPRE to EASSA form as in Figure 5.3 could figure that out.

Is there a way to take advantage of both approaches and apply it here? First we notice that the problem with the application of GVNPRE to EASSA form is that it separates expressions that actually have the same value into different values—some values should be merged. The key insight is that the values produced are actually

subsets of the sets of expressions we are looking for; they are core parts of congruence classes that should never be split. With this in mind, we propose a *multi-level value numbering* where a hash-based value numbering as in GVNPRE is used first to find what we call *values*, followed by a partitioning value numbering that finds *congruence classes*, which are then sets of values, just as values are sets of expressions. Recall that partitioning algorithms consider expressions to have variable names as subexpressions; what they really need is a lower level of naming (traditionally, variable names instead of values or congruence classes); in our case, since we have values as an intermediate naming level between variables and congruence classes, we can keep our notion of expressions and have the partitioning algorithm use our values rather than variable names.

Initially we assume that all values with the same type are in the same partition, and we split as we find contradictions. We find contradictions by looking at the various phis in EASSA form. Suppose a heap variable is defined by a def phi, $x_2 \leftarrow d\phi(x_1)$. Then if two values containing `gf` x_1 v_1 and `gf` x_2 v_2 are in the same partition and v_1 and v_2 are not definitely different, then that partition must be split. For a control phi like $x_3 \leftarrow \phi(x_1, x_2)$, if values for the expressions `gf` x_1 v_1 and `gf` x_2 v_1 are not in the same partition, then `gf` x_3 v_1 must be split from either of those partitions.

6 FUTURE WORK AND CONCLUSION

The future is you . . . probably.

—Strong Bad

We have four primary areas of future work: implementing the multi-level global value numbering proposed in Chapter 5, formulating partial redundancy elimination for Click's "sea of nodes" IR, quantifying the amount of redundancy eliminated, and considering the effects of register pressure. We discuss these and conclude with a summary of the accomplishments of this dissertation.

6.1 Future work

6.1.1 Multi-level GVN

The proposed precise solution to the problem of value numbering for object and array loads in Section 5.5 is a sketch. Several details must be filled in. To begin, the discussion mentions only how to consider various phis—do other kinds of instructions need to be considered? Moreover, it is difficult to foresee what parts of an algorithm are going to be difficult before the algorithm is implemented. Jikes RVM would serve as a good platform for implementation, building on top of the implementation for GVNPRE already in place.

6.1.2 The sea of nodes

Click designed an IR that organized instructions around data dependencies rather than code order; it is described in detail in a paper [38] and his dissertation [42] and is implemented in Sun's HotSpot Java Virtual Machine [74]. In brief, each instruction is represented by a node; the operator in particular is considered the content of

the node, while the temporary being defined is considered a label to that node (if temporaries are regarded at all in his IR), and the nodes have edges to the nodes that generate their inputs. Control flow is governed by regions represented by nodes special for that purpose, and instruction nodes have special edges to the regions in which they belong. In terms of a CFG, a region stands for a basic block and the basic blocks it dominates. Since everything—instructions and blocks—is represented by nodes in the graph, the representation can be thought of as a "sea of nodes."

Consider the program in Figure 6.1 and its translation into the sea of nodes. Move instructions are unnecessary—we simply eliminate the node and connect the edges leading to it to its outgoing edge. This greatly simplifies global value numbering. In the present example, the instruction $t_3 \leftarrow t_2 + t_1$ is partially redundant, since it will be the same as $t_5 \leftarrow t_4 + t_1$ on the previous loop iteration. We would like to hoist $t_0 + t_1$ to block 1. How can we do this in the sea of nodes? Again we need availability and anticipation information—this time for regions rather than basic blocks. The nodes available out of a region are the nodes pointing to the region and the nodes available out of the region's dominator. The nodes which are anticipated in for a region are those nodes that point to the region or are anticipated out of a postdominating region—except that we need to recognize what nodes are killed because they depend on a node we cannot optimize, like the $\bullet$ in our IR and the '?' in the sea of nodes as we represent it. If an anticipated node is found to be partially available, we would need to manipulate the graph by replacing the node by a phi and putting a node in a parent region, as we have in Figure 6.1(c).

A simple value-numbering is manifest simply from the construction of the IR; however, it does not recognize algebraic identities, let alone object references that have the same value, if we include those in the graph. We propose to develop an algorithm that will associate nodes in the graph into congruence classes or values and perform PRE, including on object and array loads. Infrastructure for this research will be more difficult to find, unless access is given to HotSpot's source code or Jikes RVM is modified to use a sea of nodes IR.

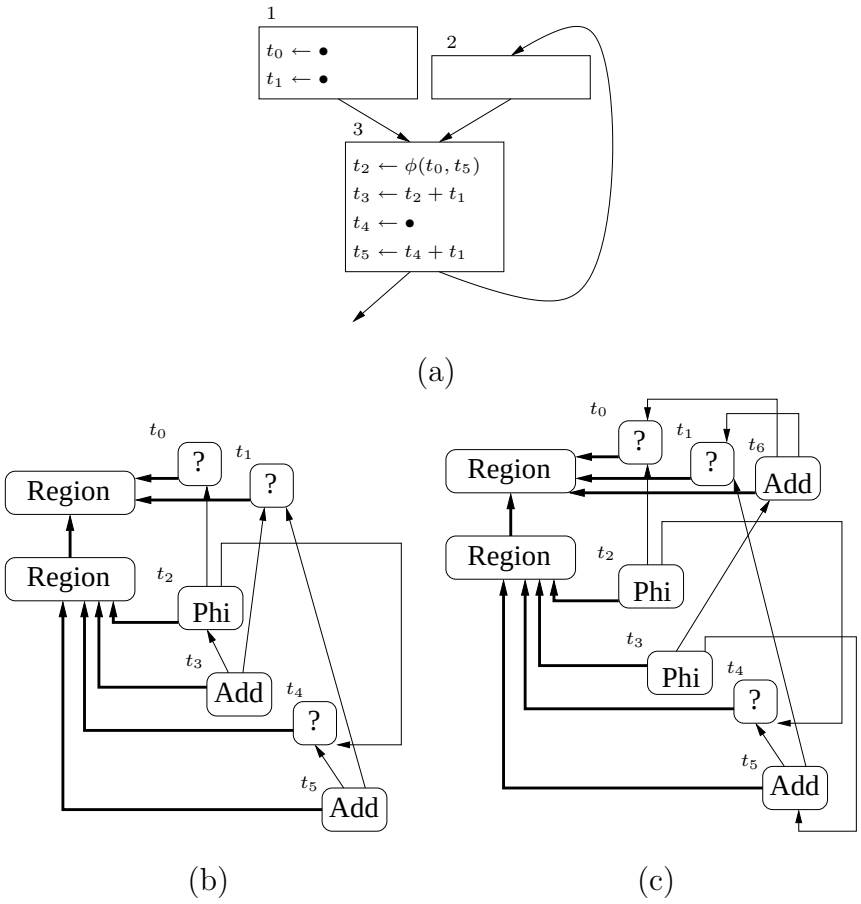


Figure 6.1. Example using Click's "sea of nodes."

6.1.3 Redundancy percentage

Let us make a few observations on our experimental results. First, our static numbers tell how many times we have improved at least one path, giving no insight to how many times the improved paths are used dynamically. Second, the disappointing performance results suggest that many real programs do not have enough redundancy to benefit from optimizations like this. We further observe that algorithms like LEPRE are approximations, and even algorithms like GVNPRE that have a certain form of theoretical completeness still deal with only static, not dynamic, redundancy. We propose to study how much run-time redundancy typical applications have, and how much of that redundancy is removed by our and similar algorithms. This would require dynamic profiling: how often is a computed value already present in the registers or memory? Jikes RVM has profiling infrastructure, but it would require modification to monitor this sort of data. A similar study was done by Bodík et al [75].

6.1.4 Register pressure

As stated elsewhere, one factor handicapping the effectiveness of PRE is register pressure. In particular, we have noticed a fair amount of performance degradation on Pentium, an architecture with comparatively few registers. This situation has been investigated previously; for example, Simpson's dissertation contains a chapter on the relief of register pressure [45]. In that work, Simpson proposed using heuristics to insert operations after PRE or related optimizations but before register allocation in hopes that these inserted operations, while introducing new redundancy, will free more registers. We propose developing heuristic approaches to limit the hoisting of instructions in the first place. Moreover, our experiments were in a compiler that uses a fairly simple register allocation [71]. Perhaps we would see more improvement if conventional, more powerful register allocations were used.

6.2 Conclusion

The goal stated in the introduction was to present practical, complete, and effective algorithms for removing redundancy. Our algorithms ASSAPRE and GVNPRE have demonstrated themselves to be practical from a software engineering perspective in that they simplify the task of compiler construction. We claim that LEPRE is also practical in that it is a fairly straightforward extension to a known approach. A highlight is the completeness of GVNPRE, in how it subsumes older PRE and GVN techniques, unifying them in a single analysis. LEPRE is not complete, but provides experience and intuition for a proposed complete solution. Our experimental results have shown that there are indeed programs that benefit from these optimizations and others like them; however, the effect they have varies among benchmarks and architectures. While the contribution towards effectiveness is only modest, we have provided compiler construction tools with strong theoretical attributes while raising a new question: within current trends of software and architecture, how much will programs benefit by redundancy elimination, no matter how accurate or aggressive?

LIST OF REFERENCES

LIST OF REFERENCES

[1] Fred Chow, Sun Chan, Robert Kennedy, Shin-Ming Liu, Raymond Lo, and Peng Tu. A new algorithm for partial redundancy elimination based on SSA form. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 273–286, Las Vegas, Nevada, June 1997.

[2] Robert Kennedy, Sun Chan, Shin-Ming Liu, Raymond Lo, Peng Tu, and Fred Chow. Partial redundancy elimination. *ACM Transactions on Programming Languages and Systems*, 21(3):627–676, May 1999.

[3] Stephen Fink, Kathleen Knobe, and Vivek Sarkar. Unified analysis of array and object references in strongly typed languages. In *Proceedings of the Static Analysis Symposium*, pages 155–174, Santa Barbara, California, July 2000.

[4] Alfred V. Aho, Ravi Sethi, and Jeffrey D. Ullman. *Compilers—Principles, Techniques, and Tools*. Addison Wesley, 1986.

[5] Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck. Efficiently computing static single assignment form and the control dependence graph. *ACM Transactions on Programming Languages and Systems*, 13(4):451–490, October 1991.

[6] B. K. Rosen, M. N. Wegman, and F. K. Zadeck. Global value numbers and redundant computations. In *Proceedings of the 15th Symposium on Principles of Programming Languages*, pages 12–27. ACM Press, 1988.

[7] B. Alpern, C. R. Attanasio, J. J. Barton, M. G. Burke, P.Cheng, J.D. Choi, A. Cocchi, S. J. Fink, D. Grove, M. Hind, S. F. Hummel, D. Lieber, V. Litvinov, M. F. Mergen, T. Ngo, J. R. Russell, V. Sarkar, M. J. Serrano, J. C. Shepherd, S. E. Smith, V. C. Sreedhar, H. Srinivasan, and J. Whaley. The Jalapeño virtual machine. *IBM System Journal*, 39(1):211–244, February 2000.

[8] Michael G. Burke, Jong-Deok Choi, Stephen Fink, David Grove, Michael Hind, Vivek Sarkar, M. Serrano, V. Sreedhar, H. Srinvasan, and J. Whaley. The Jalapeño dynamic optimizing compiler for Java. In *The Proceedings of the Java Grande Conference*, pages 129–141, San Fransisco, California, June 1999.

[9] IBM Research. The Jikes Research Virtual Machine. http://www-124.ibm.com/developerworks/oss/jikesrvm/.

[10] Bowen Alpern, Mark Wegman, and Kenneth Zadeck. Detecting equality of variables in programs. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 1–11, San Diego, California, January 1988.

[11] Standard Performance Evalution Corporation. SPECjvm 98 Benchmarks, 1998. http://www.spec.org/osg/jvm98/.

[12] Roldan Pozo and Bruce Miller. Scimark 2.0. http://math.nist.gov/scimark2/.

[13] EPCC. The Java Grande Forum Benchmark Suite. http://www.epcc.ed.ac.uk/javagrande/index_1.html.

[14] Architecture and Language Implementation Laboratory. Jolden benchmark suite. http://www-ali.cs.umass.edu/DaCapo/benchmarks.html. Described in [76].

[15] Thomas VanDrunen and Antony L Hosking. Anticipation-based partial redundancy elimination for static single assignment form. *Software—Practice & Experience*, 34(14), nov 2004. To appear.

[16] Thomas VanDrunen and Antony L Hosking. Value-based partial redundancy elimination. In *Proceedings of the International Conference on Compiler Construction*, Barcelona, Spain, March 2004. LNCS 2985.

[17] Thomas VanDrunen and Antony L Hosking. Corner cases in value-based partial redundancy elimination. Technical Report CSD-TR#03-032, Purdue University Department of Computer Sciences, 2003.

[18] John Cocke. Global common subexpression elimination. In *Proceedings of a Symposium on Compiler Optimization*, University of Illinois at Urbana-Champaign, 1970.

[19] John Cocke and J. T. Schwartz. *Programming Languages and Their Compilers: Preliminary Notes*. New York : Courant Institute of Mathematical Sciences, New York University, 1970.

[20] Jeffrey Young. Gary Kildall: The DOS that wasn't. *Forbes*, July 1997.

[21] Gary A. Kildall. A code synthesis filter for basic block optimization. Technical Report TR# 72-06-02, University of Washington Computer Science Group, University of Washington, Seatle, Washington, June 1972.

[22] Gary A. Kildall. A unified approach to global program optimization. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 194–206. ACM Press, 1973.

[23] Etienne Morel and Claude Renvoise. A global algorithm for the elimination of partial redundancies. In *Proceedings of the Second International Symposium on Programming*, pages 147–159, Paris, France, April 1976.

[24] Etienne Morel and Claude Renvoise. Global optimization by supression of partial redundancies. *Communications of the ACM*, 22(2):96–103, 1979.

[25] Karl-Heinz Drechsler and Manfred P. Stadel. A solution to a problem with Morel and Renvoise's "Global optimization by suppression of partial redundancies". *ACM Transactions on Programming Languages and Systems*, 10(4):635–640, October 1988.

[26] Arthur Sorkin. Some comments on "A solution to a problem with Morel and Renvoise's 'Global optimization by suppression of partial redundancies' ". *ACM Transactions on Programming Languages and Systems*, 11(4):666–668, 1989.

[27] D. M. Dhamdhere. A fast algorithm for code movement optimisation. *SIGPLAN Notices*, 23(10):172–180, 1988.

[28] Dhananjay M. Dhamdhere. Practical adaptation of the global optimization algorithm of Morel and Renvoise. *ACM Transactions on Programming Languages and Systems*, 13(2):291–294, April 1991.

[29] Jens Knoop, Oliver Rüthing, and Bernhard Steffen. Lazy code motion. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 224–234, San Francisco, California, July 1992. *SIGPLAN Notices* 27(7), July 1992.

[30] Jens Knoop, Oliver Rüthing, and Bernhard Steffen. Optimal code motion: Theory and practice. *ACM Transactions on Programming Languages and Systems*, 16(4):1117–1155, July 1994.

[31] Dhananjay M. Dhamdhere, Barry K. Rosen, and F. Kenneth Zadeck. How to analyze large programs efficiently and informatively. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 212–223, 1992.

[32] Karl-Heinz Drechsler and Manfred P. Stadel. A variation of Knoop, Rüthing, and Steffen's "Lazy code motion". *ACM SIGPLAN Notices*, 28(5):29–38, May 1993.

[33] Michael Wolfe. Partial redundancy elimination is not bidirectional. *SIGPLAN Notices*, 34(6):43–46, 1999.

[34] Preston Briggs and Keith D. Cooper. Effective partial redundancy elimination. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 159–170, Orlando, Florida, June 1994.

[35] Vineeth Kumar Paleri, Y. N. Srikant, and Priti Shankar. A simple algorithm for partial redundancy elimination. *SIGPLAN Notices*, 33(12):35–43, 1998.

[36] Oliver Rüthing, Jens Knoop, and Bernhard Steffen. Sparse code motion. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 170–183, 2000.

[37] Antony L. Hosking, Nathaniel Nystrom, David Whitlock, Quintin Cutts, and Amer Diwan. Partial redundancy elimination for access path expressions. *Software—Practice and Experience*, 31(6):577–600, 2001.

[38] Cliff Click and Michael Paleczny. A simple graph-based intermediate representation. In *Papers from the Workshop on Intermediate Representations*, pages 35–49, January 1995.

[39] Andrew W. Appel. Simple generational garbage collection and fast allocation. *Software–Practice and Experience*, 19(2):171–183, February 1989.

[40] Kathleen Knobe and Vivek Sarkar. Array SSA form and its use in Parallelizaiton. In *popl*, pages 107–120, San Diego, California, January 1998.

[41] Cliff Click and Keith D. Cooper. Combining analyses, combining optimizations. *ACM Transactions Programming Languages and Systems*, 17(2):181–196, 1995.

[42] Cliff Click. *Combining Analyses, Combining Optimizations*. PhD thesis, Rice University, Houston, Texas, 1995.

[43] Cliff Click. Global code motion, global value numbering. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 246–257, La Jolla, California, June 1995. *SIGPLAN Notices* 30(6), June 1995.

[44] Preston Briggs, Keith D. Cooper, and L. Taylor Simpson. Value numbering. *Software—Practice and Experience*, 27(6):701–724, 1997.

[45] Loren Taylor Simpson. *Value-driven redundancy elimination*. PhD thesis, Rice University, 1996.

[46] Kenneth Cooper and Taylor Simpson. SCC-based value numbering. Technical Report CRPC-TR 95636-S, Rice University, Houston, Texas, October 1995.

[47] Steven Muchnick. *Advanced Compiler Design and Implementation*. Morgan Kaufmann, 1997.

[48] Bernhard Steffen. Optimal run-time optimization—proved by a new look at abstract interpretation. In *Proceedings of the International Joint Conference on Theory and Practice of Software Development*, pages 52–68, Pisa, Italy, 1987. LNCS 249.

[49] Bernhard Steffen. *Abstrakte Interpretationen beim Optimieren von Programmlaufzeiten. Ein Optimalitätskonzept und seine Anwendung*. PhD thesis, Christian-Albrechts-Universität Kiel, 1987.

[50] Patrick Cousot and Radhia Cousot. Abstract interpretation: a unified lattice model for static analysis of program by construction or approximation of fixpoints. In *Proceedings of the Symposium on Principles of Programming Languages*, Los Angeles, California, 1977.

[51] Karthik Gargi. A sparse algorithm for predicated global value numbering. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 45–56, Berlin, Germany, June 2002.

[52] Oliver Rüthing, Bernhard Steffen, and Jens Knoop. Detecting equalities of variables—combining efficiency with precision. In *Proceedings of the Static Analysis Symposium*, pages 232–247, Venice, Italy, 1999.

[53] Bernhard Steffen, Jens Knoop, and Oliver Rüthing. The value flow graph—a program representation for optimal program transformations. In *Proceedings of the European Symposium on Programming (ESOP)*, pages 389–405, Copenhagen, Denmark, 1990. LNCS 432.

[54] Bernhard Steffen, Jens Knoop, and Oliver Rüthing. Efficient code motion and an adaptation to strength reduction. In *Proceedings of the International Joint Conference on Theory and Practice of Software Development*, Brighton, United Kingdom, 1991. LNCS 494.

[55] Jens Knoop, Oliver Rüthing, and Bernhard Steffen. Code motion and code placement: Just synonyms? In *Proceedings of the European Symposium on Programming*, pages 154–169, Lisbon, Portugal, 1998. LNCS 1381.

[56] Jens Knoop, Oliver Rüthing, and Bernhard Steffen. Expansion-based removal of semantic partial redundancies. In *Proceedings of the International Conference on Compiler Construction*, pages 91–106, Amsterdam, The Netherlands, 1999. LNCS 1575.

[57] Rastislav Bodík and Sadun Anik. Path-sensitive value-flow analysis. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 237–251, San Diego, California, January 1998.

[58] Rastislav Bodík, Rajiv Gupta, and Mary Lou Soffa. Complete removal of redundant expressions. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 1–14, Montreal, Quebec, June 1998.

[59] Rastislav Bodík. *Path-Sensitive, Value-Flow Optimizations of Programs.* PhD thesis, University of Pittsburgh, 1999.

[60] Rajiv Gupta and Rastislav Bodík. Register pressure sensitive redundancy elimination. In *International Conference on Compiler Construction*, pages 107–121, Amsterdam, Netherlands, March 1999. LNCS 1575.

[61] Dhananjay M. Dhamdhere. E-path_pre—partial redundancy elimination made easy. *ACM SIGPLAN Notices*, 37(8):53–65, August 2002.

[62] David Whitlock. *The BLOAT book*, 1999.
Availible at http://www.cs.purdue.edu/vandrutj/bloat.

[63] William Strunk and E. B. White. *The Elements of Style.* Macmillan, New York, 1972.

[64] Andrew W. Appel. *Modern Compiler Implementation in Java.* Cambridge University Press, Cambridge, UK, 1998. Editions for ML and C also available.

[65] Programming for Persistent Systems Research Group.
bytecode level optimization and analysis tool for java.
Available for download, http://www.cs.purdue.edu/s3/projects/bloat/, June 1999. See programmer's comment in source code.

[66] William Pugh. Fixing the Java memory model. In *The Proceedings of the Java Grande Conference*, pages 89–98, San Fransisco, California, June 1999.

[67] Dan Berlin. Private correspondence, March 2004.

[68] Tim Lindholm and Frank Yellin. *The Java Virtual Machine Specification.* Addison-Wesley, 1999.

[69] Report 041-R1. Anonymous reviewer's comments for the program committee, December 2003. The International Conference on Compiler Construction.

[70] The GNU compiler collection project. http://gcc.gnu.org.

[71] Massimiliano Poletto and Vivek Sarkar. Linear scan register allocation. *ACM Transactions on Programming Languages and Systems*, 21(5):895–913, 1999.

[72] Intel Corporation, P.O. Box 6937, Denver, Colorado. *IA-32 Intel Architecture Software Developer's Manual, Volume 3: System Programming Guide*, 2004. http://www.intel.com/design/pentium4/manuals/253665.htm.

[73] Amer Diwan, Kathryn S. McKinley, and J. Eliot B. Moss. Type-based alias analysis. *ACM SIGPLAN Notices*, 33(5):106–117, 1998.

[74] Sun Microsystems. *The Java HotSpot Virtual Machine*, v 1.4.1, d2 edition, September 2002.

[75] Rastislav Bodík, Rajiv Gupta, and Mary Lou Soffa. Load-reuse analysis: Design and evaluation. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 64–76, Atlanta, Georgia, May 1999.

[76] Brendon Cahoon and Katherine McKinley. Data flow analysis for software prefetching linked data structures in Java. In *Proceedings of the International Conference on Parallel Architectures and Compilation Techniques*, pages 280–291, Barcelona, Spain, September 2001.

VITA

VITA

Thomas VanDrunen grew up in Oak Brook, Illinois, which is famous for being home to McDonald's Corporation, being the Polo capital of the world, and being founded by the father of the producer of the broadway musical *Hair*. Thomas attended high school at Timothy Christian High School (Elmhurst, Illinois) and received his B.S. from Calvin College (Grand Rapids, Michigan) in 1998, his M.S. from Purdue University in 2000, and his Ph.D. from Purdue University in 2004. His computer science interests include compilers, the design and theory of programming languages, virtual machines, object oriented programming, software engineering, and computer science instruction. When he's not doing computer science Thomas enjoys reading theology, studying ancient Greek, and brewing beer. He is also interested in Mozart, the history of the Protestant Reformation, musical versifications of the Book of Psalms, and college football. He has composed a ballad on the life of Benjamin Franklin.