

# ELEC2100: Signals and Systems

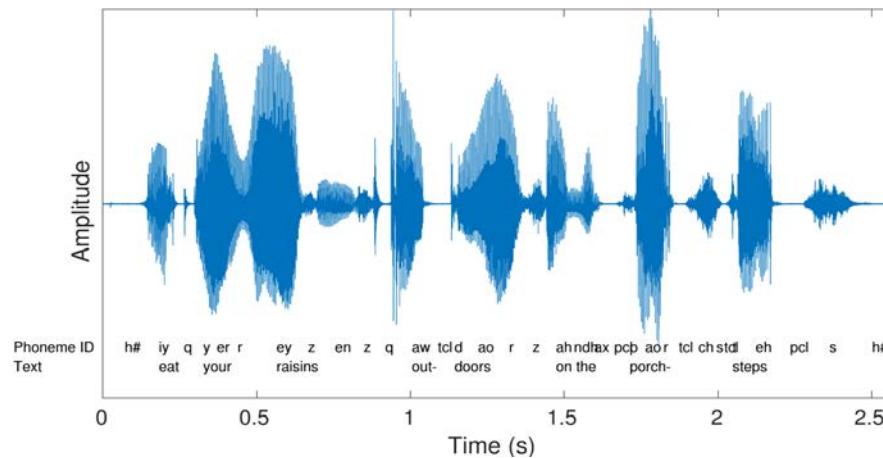
## Lecture 15

### Wireless Communication (Application)

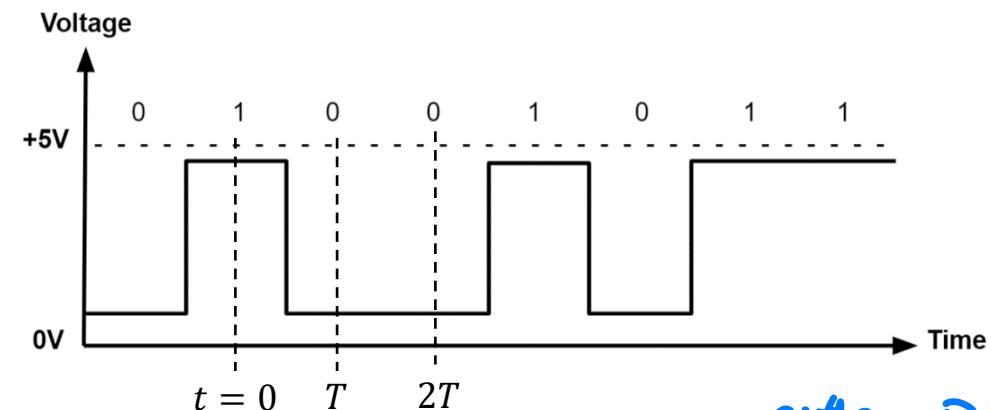
- I. Amplitude Modulation and Wireless Communications
- II. Use of Radio Spectrum
- III. I/Q Channels and QAM

# I. Amplitude Modulation and Wireless Communications

- Communication is to transmit some information signals over a distance. The information signal can be an analog waveform (e.g. a telephone speech signal over the subscriber loop), or a string of symbols/pulses representing 0 and 1 bits in **digital communications**



Analog Speech Signal  $x_{speech}(t)$



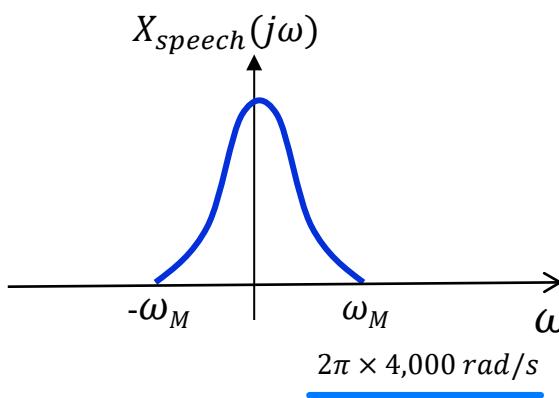
Digital Communication Signal  $x_{digit}(t) = \sum \sigma_i r(t - iT)$

- The information signal to be transmitted in its original form is called the **baseband signal**. The analog speech signal and digital communication signal above are baseband signals. Sometimes we can transmit the baseband signal as it is. In wireless communications, in general we cannot.

# Baseband Signal and Bandwidth

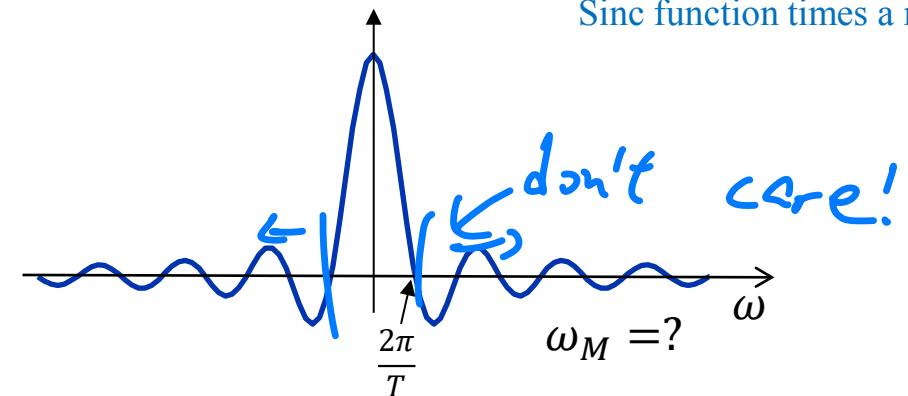
how much in freq. domain

- **Bandwidth** of a (baseband) signal refers to the width of frequency range that its spectrum occupies.
- An information signal is often **bandlimited**, meaning that its bandwidth is finite, or approximately bandlimited. For a baseband signal, usually that means there is a **maximum frequency** above which its spectrum is zero or negligible.
- For the analog speech signal the maximum frequency is  $f_M \cong 4$  KHz.
- For the digital communication signal which uses rectangular pulses to represent bits, the spectrum is a sinc function. You need to define  $f_M$  beyond which you can ignore the spectrum, and this max frequency is proportional to the symbol rate  $\frac{1}{T}$ . We may do **pulse shaping** to limit this max frequency.



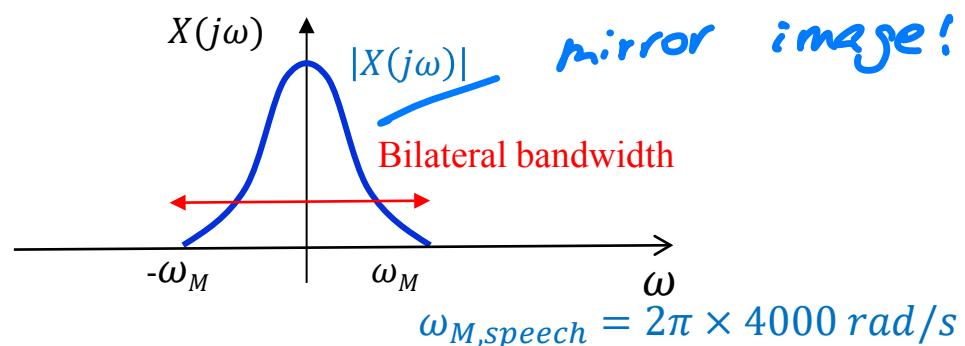
$$X_{digit}(j\omega) = \sum \sigma_i F\{r(t)\} e^{-j\omega iT} = \frac{\sin \frac{\omega T}{2}}{\omega} (\sum \sigma_i e^{-j\omega iT})$$

Sinc function times a random sum



## Unilateral and Bilateral Bandwidth

- For a real signal, its spectrum is conjugate symmetric:  $X(j\omega) = X^*(-j\omega)$ . This means that while the maximum frequency defines the **unilateral bandwidth**, the **bilateral bandwidth**, which is the total width of the spectrum including negative frequencies, is  $2 \times f_M$ .
- Often, we need to understand from context whether the unilateral or bilateral bandwidth is being referred to.
- For example, for the baseband telephony analog speech signal, the maximum frequency and the unilateral bandwidth is 4 KHz, and the bilateral bandwidth is 8 KHz. For CD music, the maximum frequency is 20KHz and the bilateral bandwidth is 40 KHz



Telephony speech signal:

Unilateral bandwidth = maximum frequency =  $f_M = 4$  KHz.

Bilateral bandwidth =  $2 \times$  Unilateral bandwidth = 8 KHz

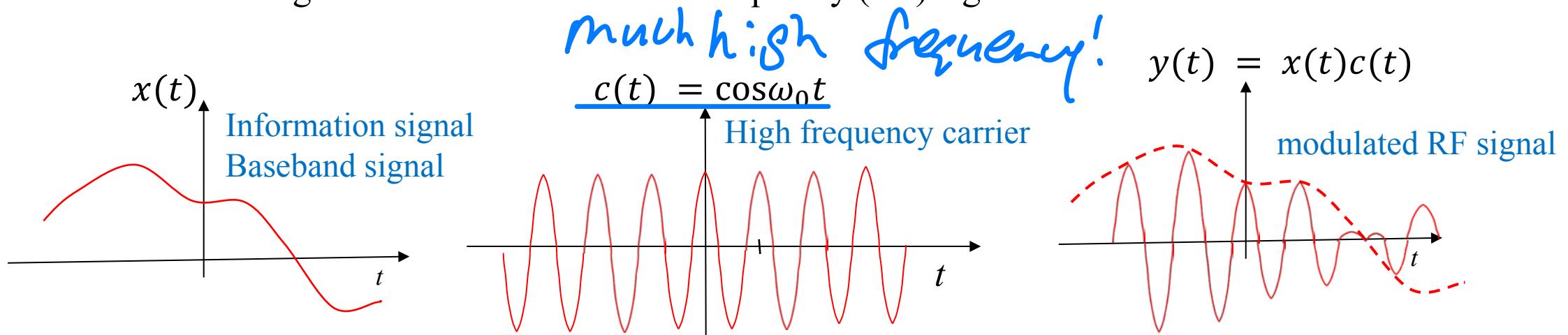
(for CD music:  $f_M = 20$  KHz, and bilateral bandwidth = 40 KHz)

While mathematically it is more convenient to use angular frequency, it is more intuitive to talk about bandwidth in ordinary frequency

# Radio Carrier and Amplitude Modulation

- For wireless communications, it is impossible to transmit low frequency baseband signals because the antenna dimension is proportional to the wavelength involved (wavelength  $\lambda = c/f$ ), so transmitting and receiving at low radio frequencies requires huge antennas.
- Hence, we first ***multiply*** the baseband signal by a radio carrier to shift its frequencies.

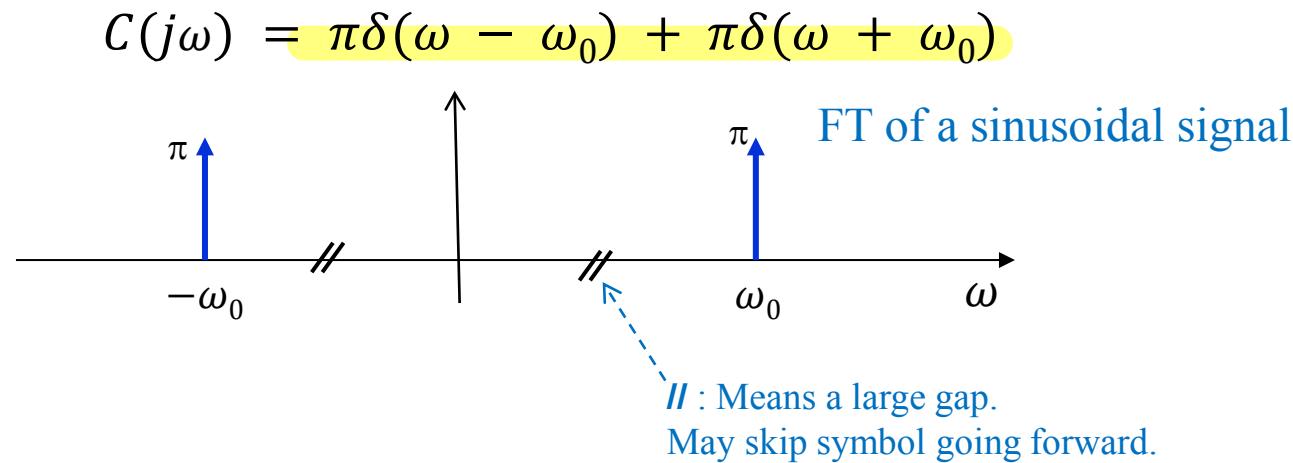
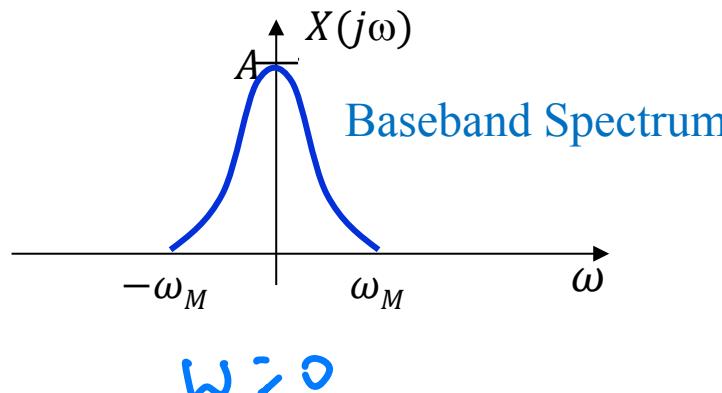
**Example 4.21:** In ***Amplitude Modulation*** (AM), we ***multiply*** the information signal  $x(t)$  by a higher frequency sinusoid ***carrier*** to produce a ***modulated signal*** before transmission. The modulated signal is also called a Radio Frequency (RF) signal.



- Multiplication of two signals is also called ***mixing***.

To understand what happens, we need to go to the frequency domain.

The spectrums of the baseband signal and the carrier are as shown below:



What is the FT, or spectrum, of  $y(t)$  the modulated signal?  $Y(j\omega) = ?$

Multiplication in time domain corresponds to convolution in frequency domain:

$$y(t) = x(t)c(t) \Leftrightarrow Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)$$

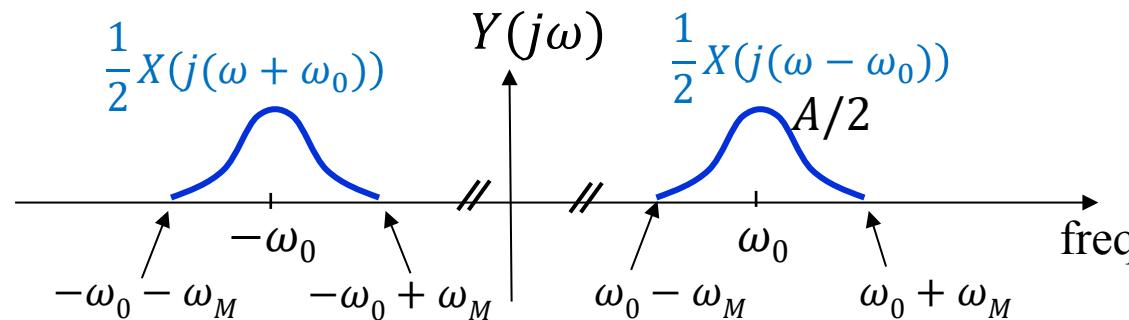
↑

But  $C(j\omega)$  consists of two impulses in frequency:  
 $C(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

$$\Rightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_0)) + \frac{1}{2} X(j(\omega + \omega_0))$$

$X(j\omega) * \delta(\omega - \omega_0)$      $X(j\omega) * \delta(\omega + \omega_0)$

This means that the FT of  $y(t)$  consists of two shifted copies of the FT of  $x(t)$



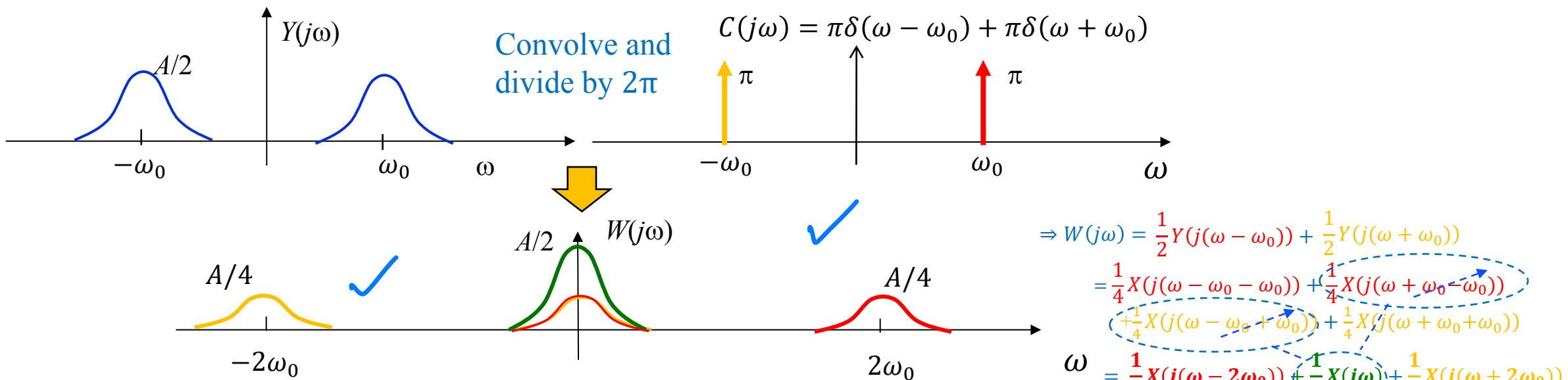
The modulated signal is now at a much higher radio frequency (RF) range and suitable for transmission as radio waves.

## Recovering $x(t)$ - Demodulation

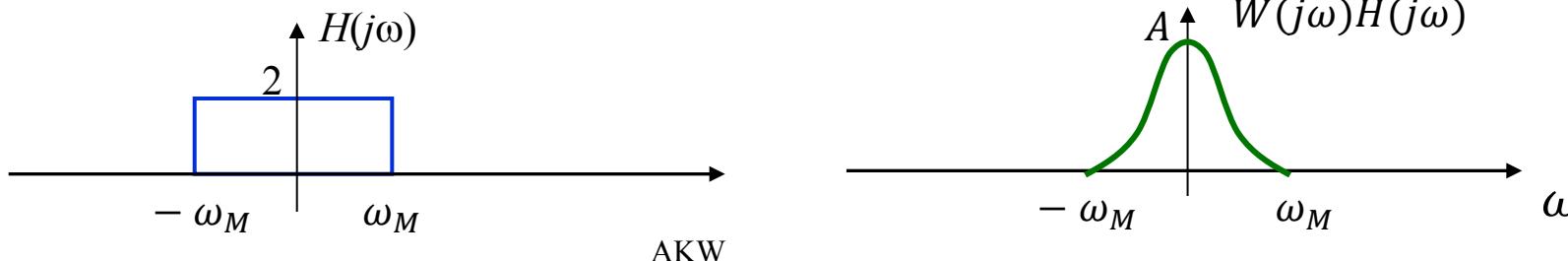
**Example 4.22:** The process of recovering  $x(t)$  from  $y(t)$  is known as demodulation.

- One way for demodulation is to multiply  $y(t)$  by  $c(t)$  again.
- Let  $w(t) = y(t)c(t)$ . Then  $W(j\omega) = \frac{1}{2\pi} Y(j\omega)^* C(j\omega)$ :

跟上一頁有？不同

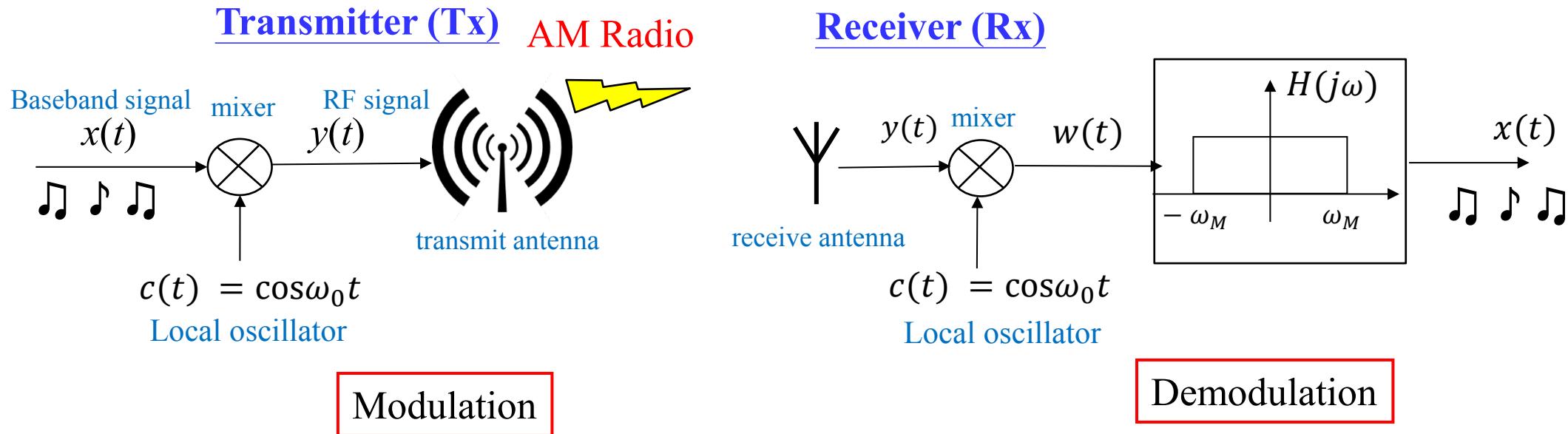


Then we apply an ILP with a scaling factor of 2 to  $w(t)$ , we recover  $x(t)$ .



# An AM Modulation/Demodulation System

- The diagram below illustrates the AM modulation and demodulation process.



- Basically all radio communication systems use radio carriers and modulation.

- Going back to the mathematics of demodulation, we observe that:

$$\begin{aligned}
 \cos^2 A &= \frac{1}{2}(1 + \cos 2A) \quad \leftarrow \quad \cos(A)\cos(B) = \frac{1}{2}(\cos(A+B) + \cos(A-B)) \\
 w(t) = y(t)c(t) &= x(t)\cos^2\omega_0 t \quad \downarrow \frac{1}{2}x(t)(1 + \cos 2\omega_0 t) \quad \text{and let } A=B \\
 &= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_0 t
 \end{aligned}$$

So  $w(t)$  has one part that is the original baseband signal,  
and another that is shifted to high frequency by  $2\omega_0$  and is removed by low-pass filtering

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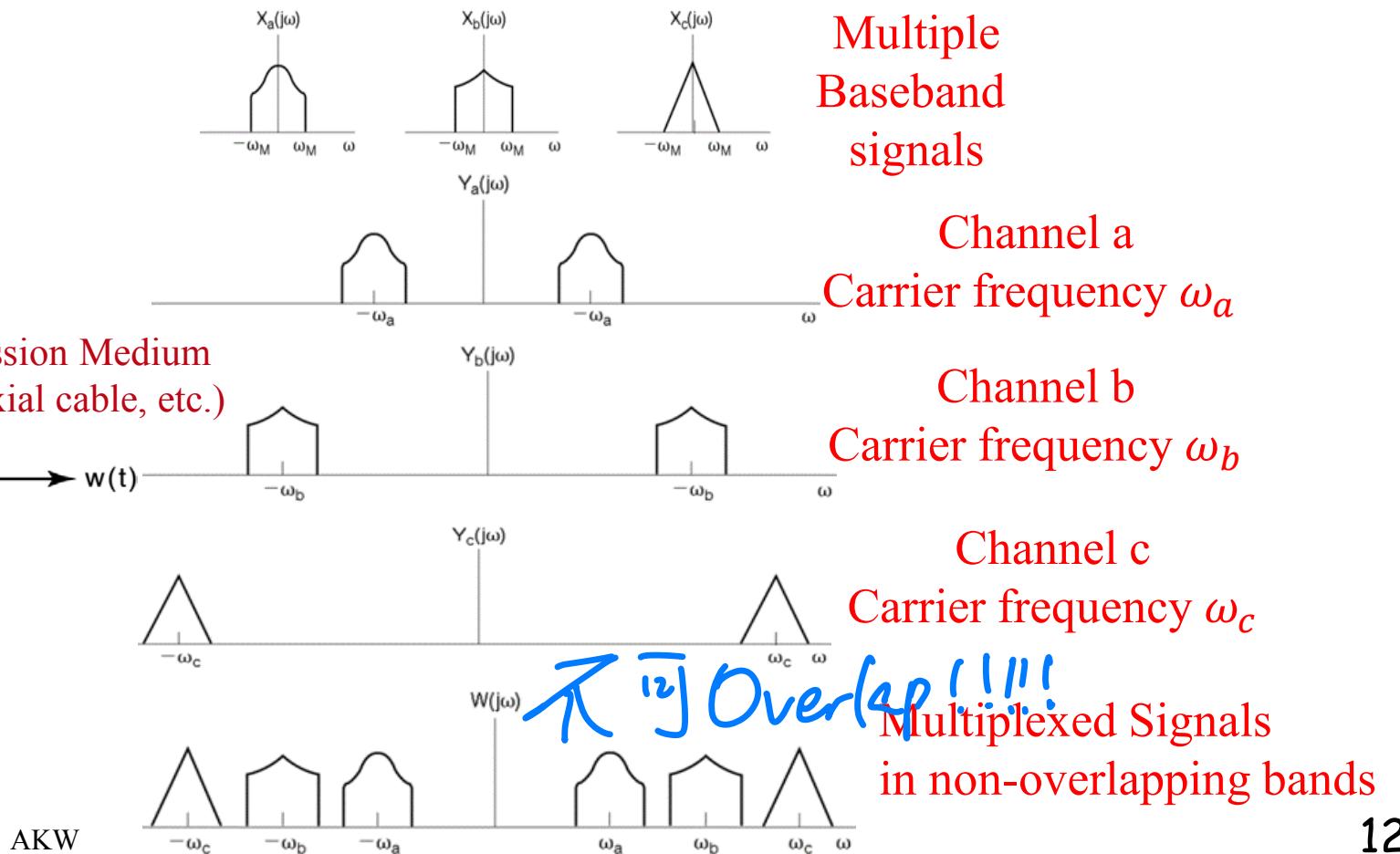
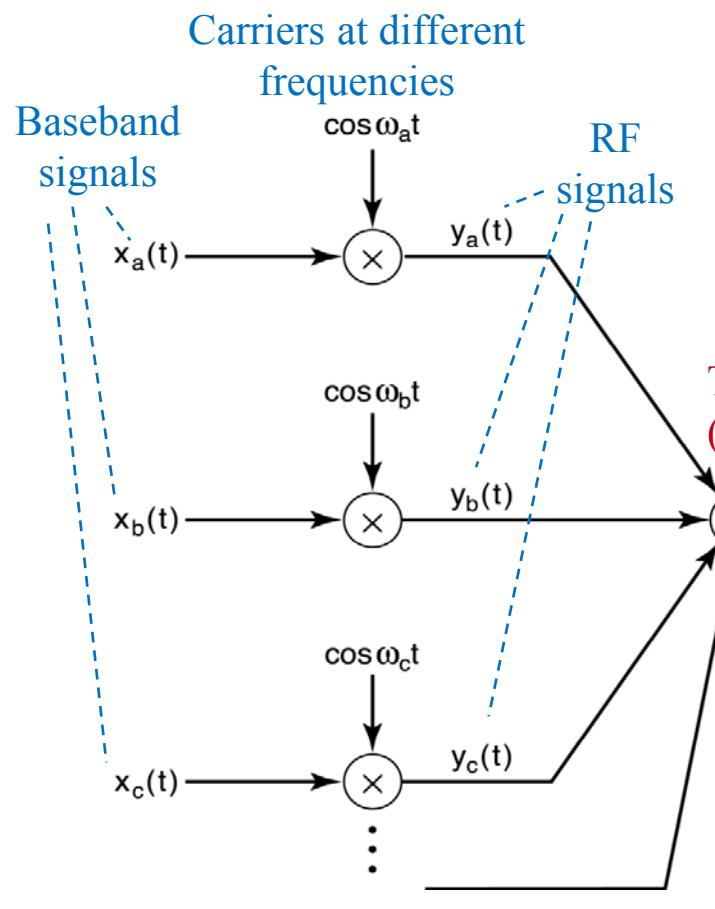
### Wireless Communications

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## II. Use of Radio Spectrum

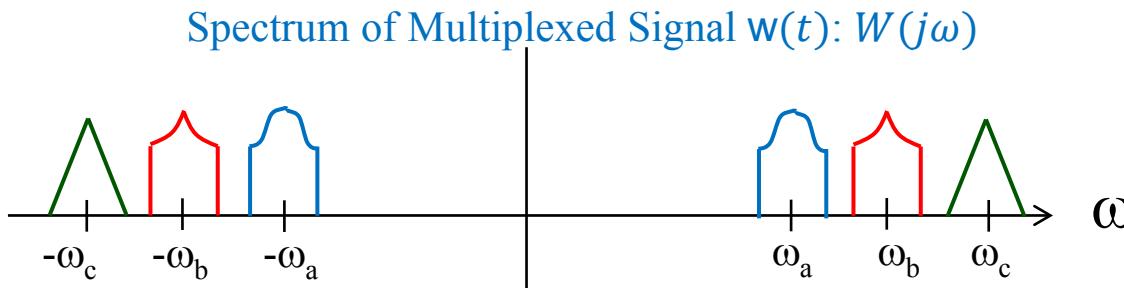
key idea in FDM

- Another important reason for modulation is for *Frequency Division Multiplexing* (FDM) which allows multiple signals to be transmitted over the same medium at the same time.
- In FDM, we modulate different baseband signals using different carrier frequencies. The resulting RF signals can be transmitted together at the same time because they remain separate in the frequency domain..

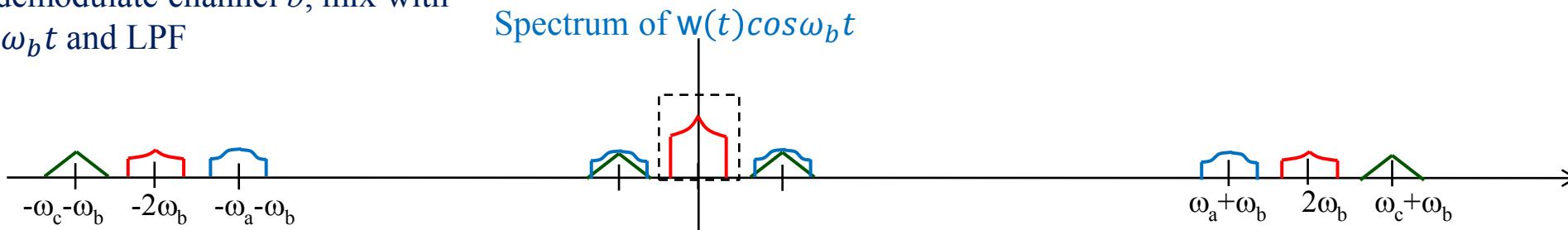


# FDM Demodulation

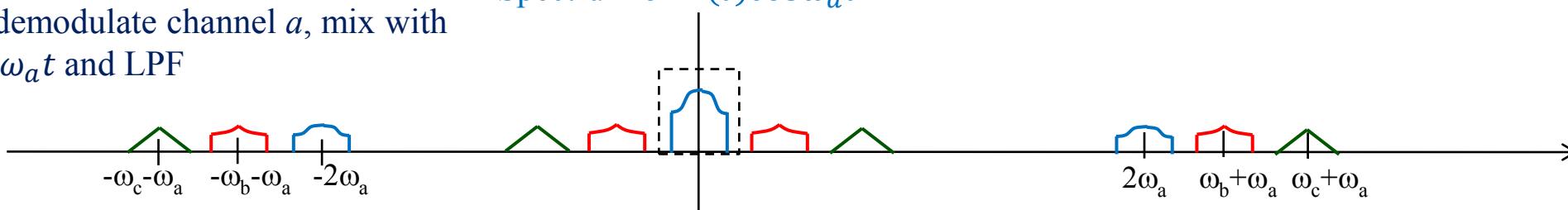
- To demodulate a specific channel within the multiplexed signal, one way is to mix the multiplexed signal with the corresponding carrier frequency and then LPF.



To demodulate channel  $b$ , mix with  $\cos \omega_b t$  and LPF



To demodulate channel  $a$ , mix with  $\cos \omega_a t$  and LPF



## Example 1 – AM Radio Broadcasting

- The table below is a partial list of the AM radio channels in Hong Kong.
- Different channels use different carrier frequencies. Sometimes the name of the channel identifies the carrier frequency.

Partial List of AM Radio Channels in Hong Kong

AM Channel	Frequency
RTHK3	576 KHz
RTHK5	783 KHz
RTHK6	675 KHz
RTHK Putonghua	621 KHz
AM864	864 KHz
Metro Plus	1044 KHz

## Example 2 – FM Radio Broadcasting

- In FM (Frequency Modulation) radio, the baseband signal  $x(t)$  is used to vary (modulate) the frequency of the radio carrier:

$$\text{Transmitted Signal} = \cos((2\pi f_c + \beta x(t))t) \quad ? \quad ? \quad ?$$

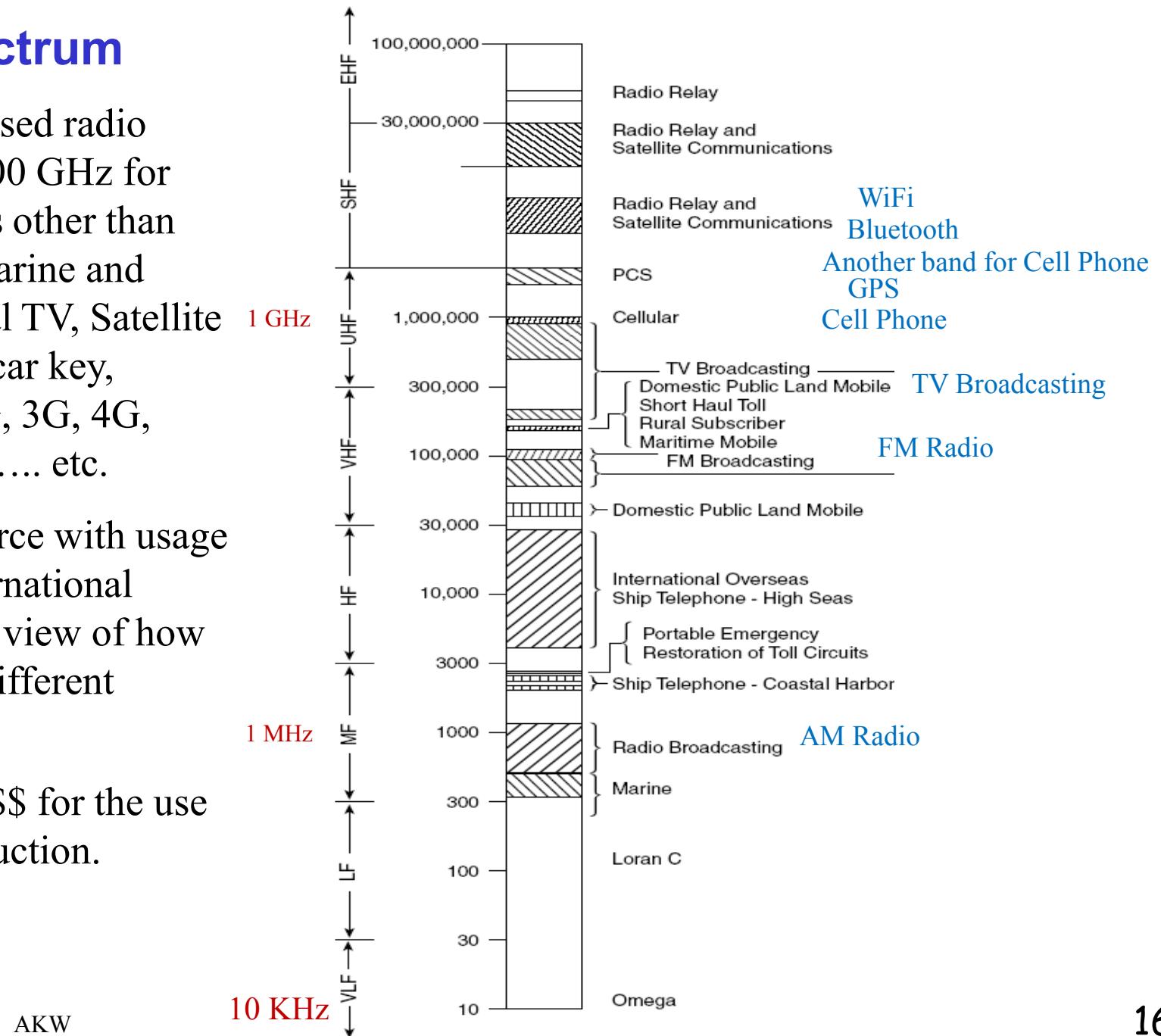
- FM radio is more immune to noise and distortion, and is better for the broadcast of music.
- Again, different FM channels use different carrier frequencies.
- While an AM radio station typically takes  $\sim 10$  KHz in bandwidth, an FM radio station takes  $\sim 2$  MHz. The carrier frequency  $f_c$  for FM radio is in the range of tens of MHz, compared to hundreds of KHz for AM radio.

### Partial List of AM and FM Radio Channels in Hong Kong

FM Channel	Frequency
RTHK1	92.5-94.4 MHz
RTHK2	94.6-96.9 MHz
RTHK4	97.6-98.9 MHz
Supercharged 881	88.1-89.5 MHz
Ultimate 903	90.3-92.1 MHz
Metro Info	99.7-102.1 MHz
Metro Finance	102.4-106.3 MHz

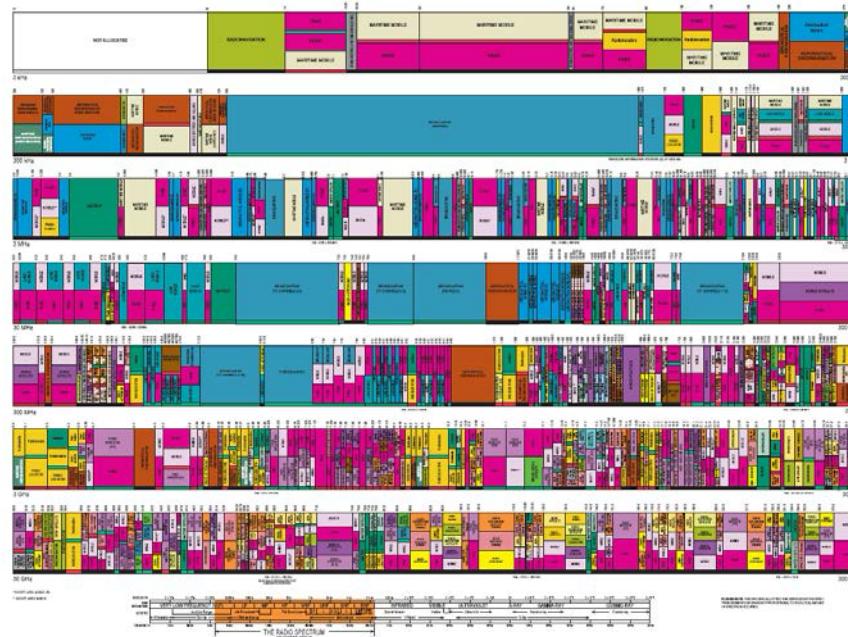
# Use of Radio Spectrum

- For about 120 years, humans have used radio waves in the range of 10 KHz to 100 GHz for many communications applications other than AM and FM radio broadcasting: marine and aviation communications, terrestrial TV, Satellite TV, walkie talkie, cordless phone, car key, garage door opener, cell phone (2G, 3G, 4G, 5G), WiFi, Bluetooth, RFID, GPS..... etc.
- Radio spectrum is a valuable resource with usage governed by governments and international treaties. To the right is a high-level view of how the radio spectrum is allocated to different applications in the US.
- Often one has to pay large sum of \$\$ for the use of specific spectrums – spectrum auction.



# Spectrum Allocation

- The detailed frequency allocation table for different applications is extremely complex. It also varies from country to country (see US table below).



- Further, each application may have several to hundreds of channels (e.g., different TV/radio channels) sharing the allocated spectrum.
- Today, every bit of the usable radio spectrum is used! Because spectrum is so valuable, different applications and different channels are often closely packed with little room between them. (**Hence we need good LPF or BPF to separate different channels.**)

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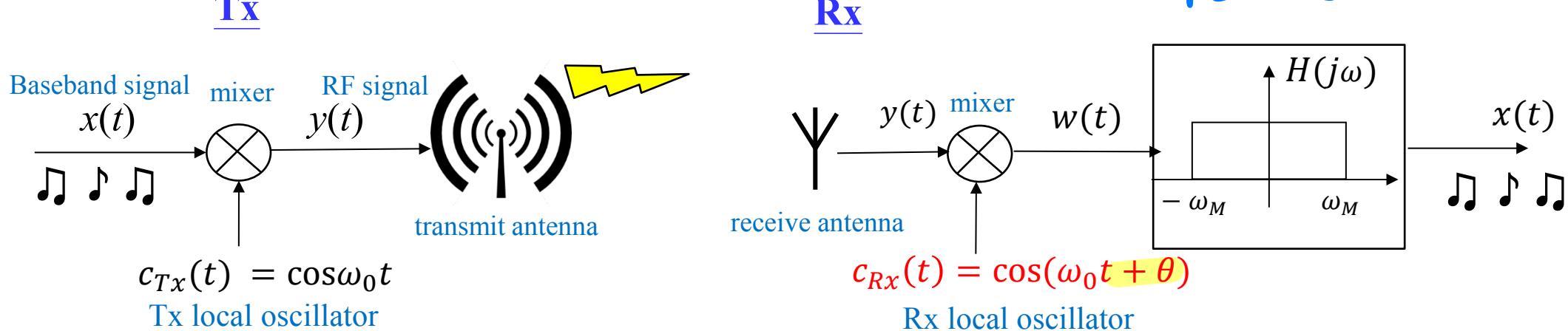
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### III. I/Q Channels

- Let us look at the AM modulation and demodulation process again. What if the Rx local oscillator has a **phase offset of  $\theta$**  compared to the Tx local oscillator?



- Looking at the math:

$$\cos(A)\cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B))$$

$$w(t) = x(t)c_{Tx}(t)c_{Rx}(t) = x(t) \cos \omega_0 t \cos(\omega_0 t + \theta) = \frac{1}{2}x(t)\cos\theta + \frac{1}{2}x(t) \cos(2\omega_0 t + \theta)$$

- If the phase offset is  $90^\circ$ , the output of the low pass filter will actually become zero because  $\cos\theta = 0$ !  
Hence, your radio receiver has to make sure that it **keeps the phase offset fairly small**.

## Two Information Channels for the Same Carrier

- On the other hand, the previous slide implies that we can use two carriers at the same frequency but  $90^\circ$  out of phase (cosine and sine waves) to transmit two information signals at the same time.
- By convention, the carrier lagging in phase by  $90^\circ$  is called the I (In-Phase) channel, and the carrier leading in phase by  $90^\circ$  is called the Q (Quadrature Phase) channel.



I/Q Transmitter

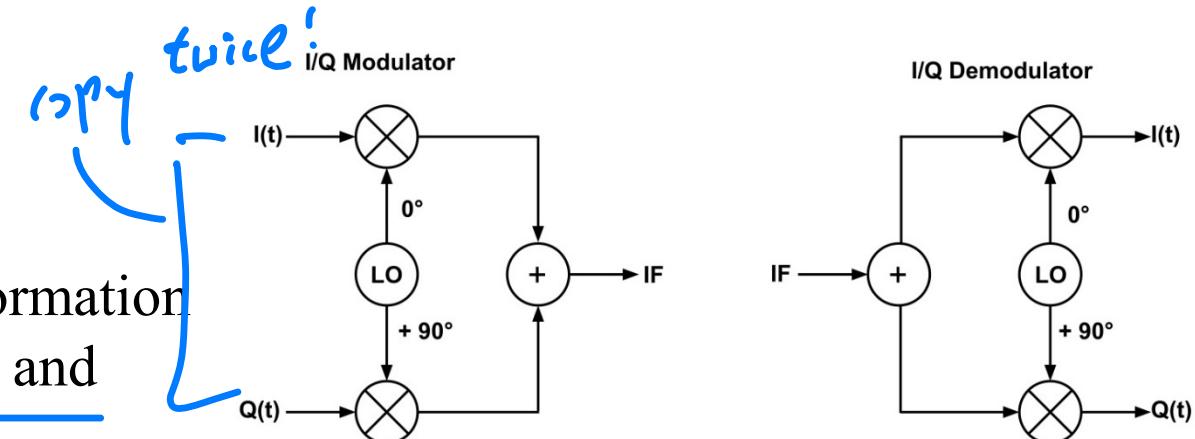
$$x_I(t) \cos(\omega t) + x_Q(t) \cos\left(\omega t + \frac{\pi}{2}\right)$$

Now I simply use  $\omega$  instead of  $\omega_0$  for convenience

$$\text{or } x_I(t) \cos(\omega t) - x_Q(t) \sin(\omega t)$$

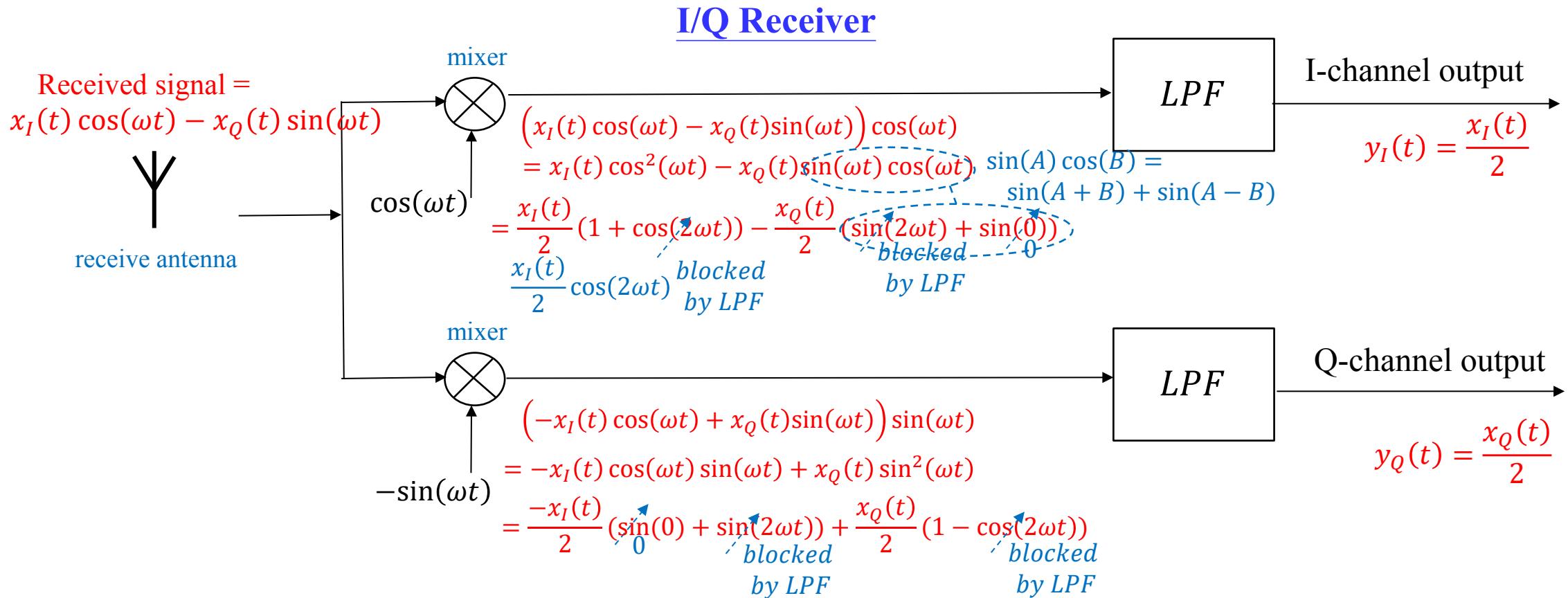
$x_I(t), x_Q(t)$ , are two information signals.

We can think of them as one complex-valued information signal, with  $x_I(t)$  being the real part of the signal and  $x_Q(t)$  being the imaginary part.



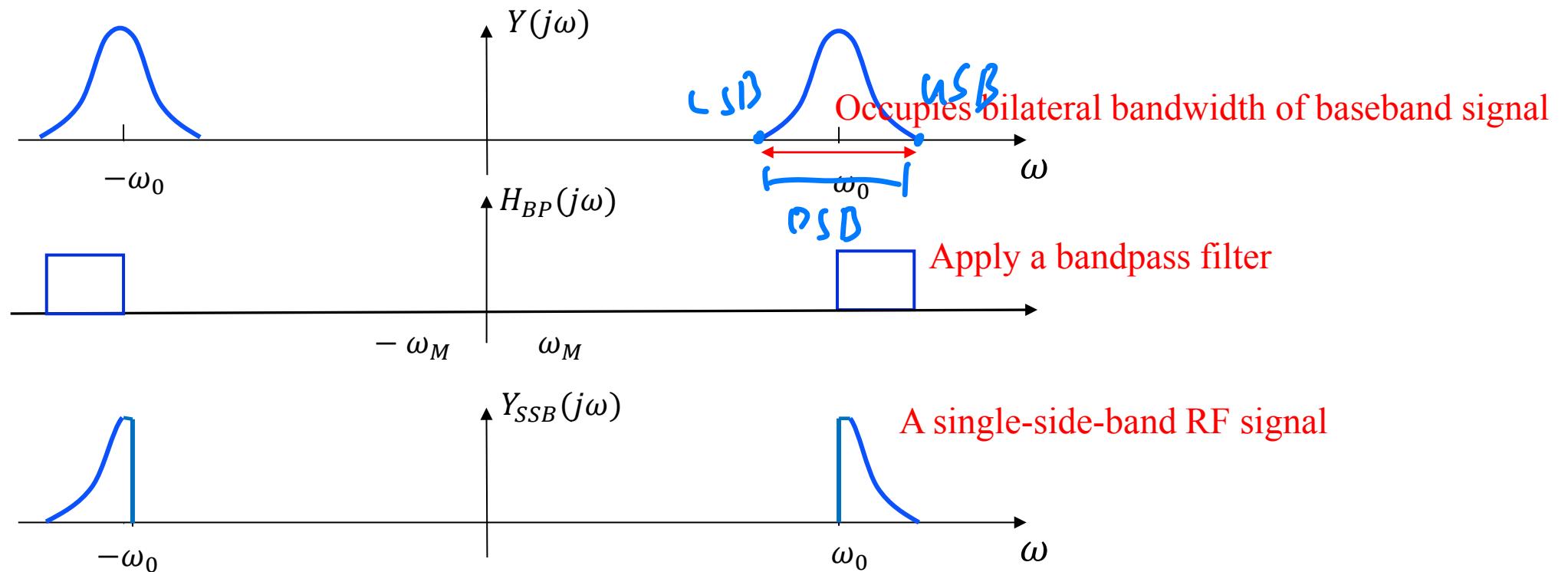
# I/Q Channel Receiver

- At the receiver, we mix the received signal separately with the in-phase carrier and quadrature carrier to recover the I-channel and Q-channel information signals. For each of the I/Q receiver, the signal with carrier that is  $90^\circ$  out of phase will not produce any output!



# Why Two Information Channels for the Same Carrier? ??!

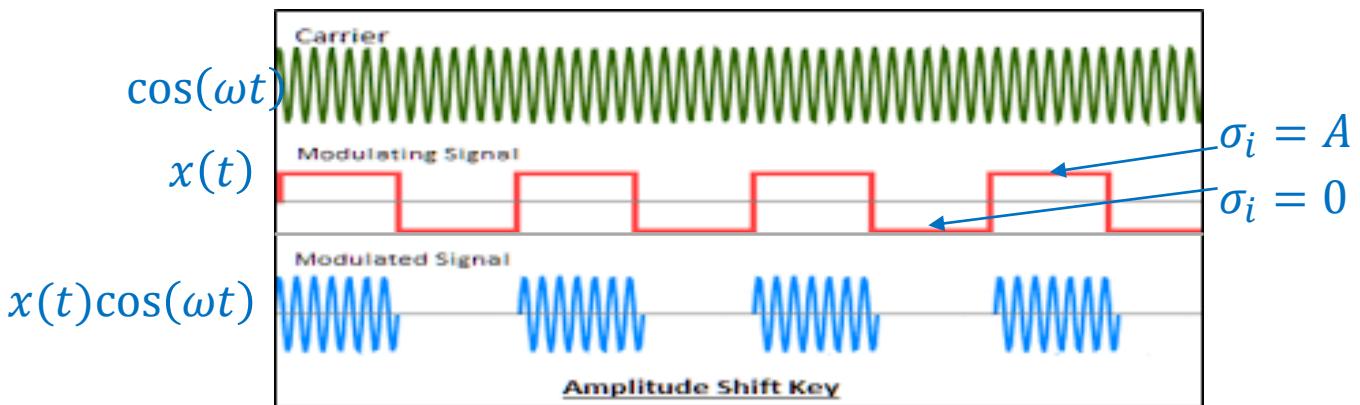
- Note that the RF signal  $y(t)$  occupies twice as much bandwidth compared to the baseband signal  $x(t)$
- In *Single-Side-Band* transmission, we use a bandpass filter to cut off the redundant spectrum in the RF signal so that we can save transmission bandwidth. This was done in traditional TV broadcasting.



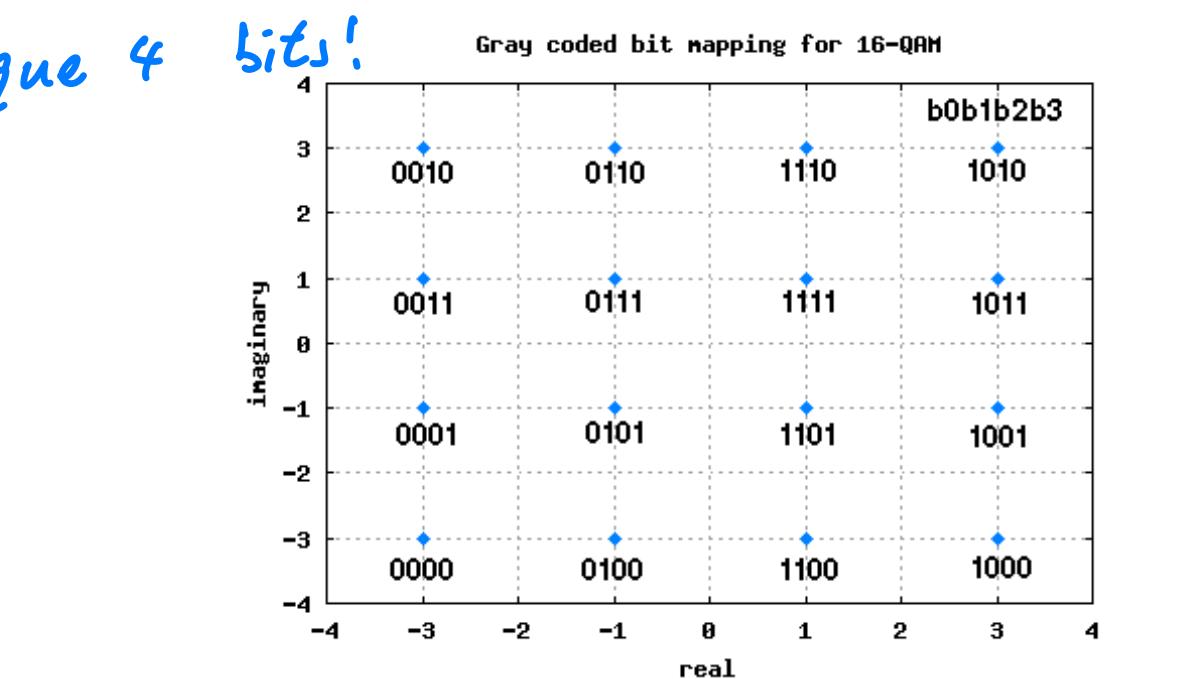
- In I/Q communication, we instead use the doubled spectral width to transmit two signals!

# I/Q Communication and QAM

- In *on-off keying*, which is the simplest form of *Amplitude Shift Keying* (ASK) we send a pulse for a “1” bit and no pulse for a “0” bit; i.e., the information signal  $x(t)$  is a sequence of on-off pulses and  $\sigma_i = \{0, A\}$ ;  $\sigma_i$  is real and equals either 0 or  $A$ .



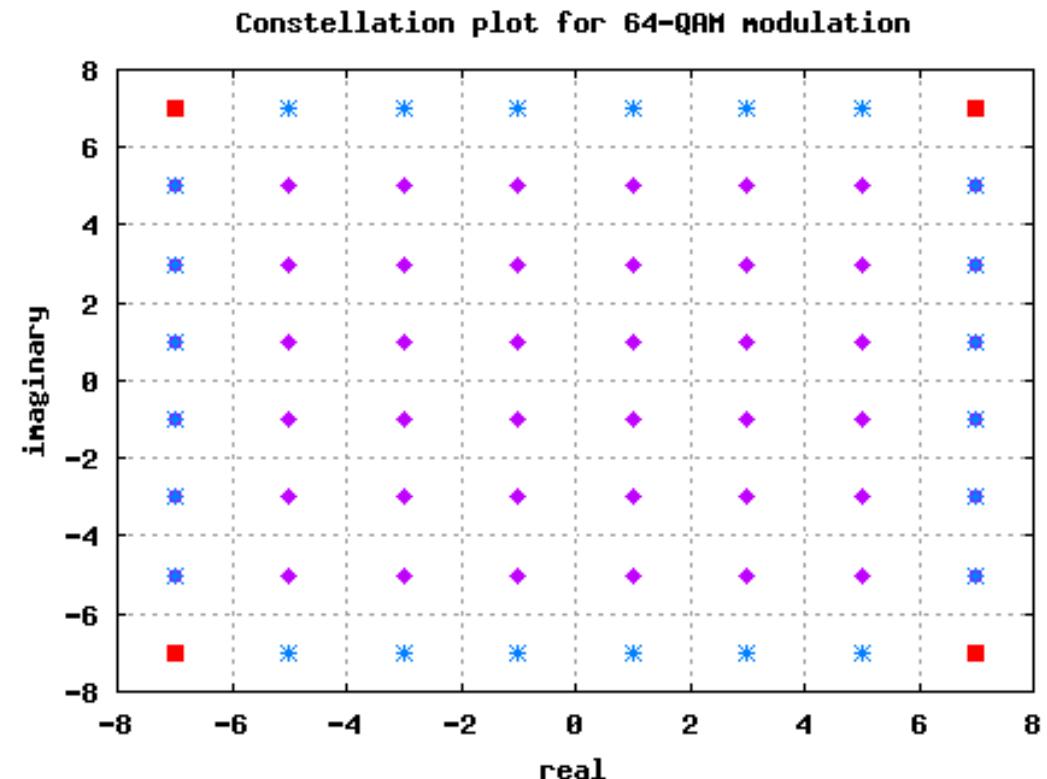
- In *Quadrature Amplitude Modulation* (QAM),  $\sigma_i$  is viewed as complex. In 16-QAM,  $\underline{\sigma_i = \{\alpha_i + j\beta_i; \alpha_i, \beta_i = -3, -1, 1, 3\}}$  - the real and imaginary parts of  $\sigma_i$  may each take one of four possible values so  $\sigma_i$  has 16 possible values and indicates 4 bits.



# Signal Constellation

noise!

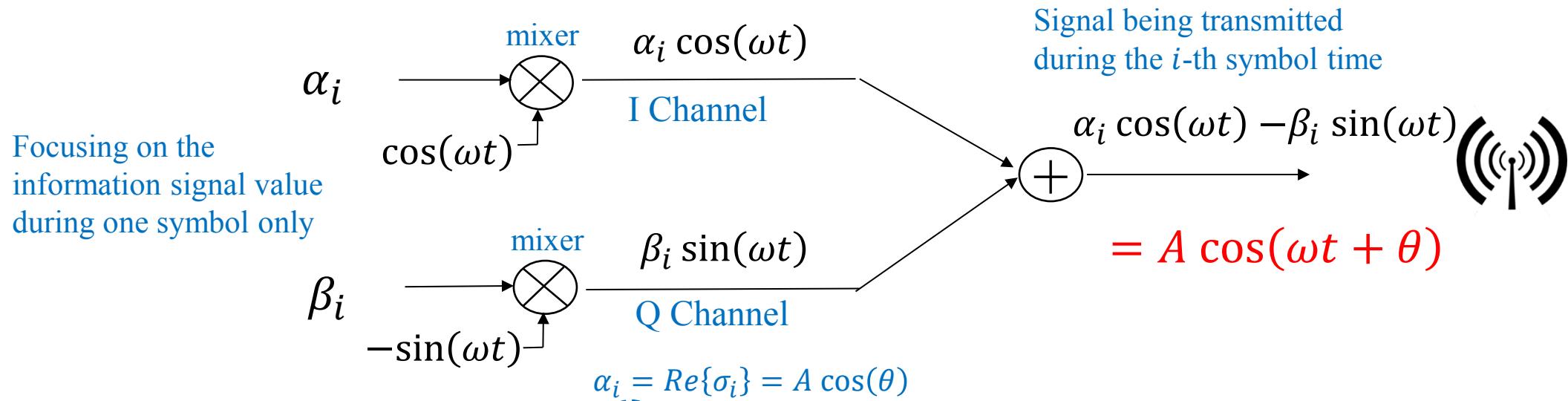
- In 64 QAM,  $\sigma_i$  takes on one of 64 values to indicate 6 bits.
- So why don't we use 128-QAM, or 256-QAM, to transmit more bits per symbol?
- The answer is that communication channels invariably add noise. If we use too many possible signal values without increasing the power in the signal, the bit error rate will increase.
- The map indicating the possible symbol values on the complex plane is called a signal *constellation*.
- Next, we show that QAM is in effect an amplitude plus phase modulation.



Small noise  $\Rightarrow$  confusion!

## I/Q Channels - Two Waves = One Wave

- So what does a complex  $\sigma_i$  mean?
- During each symbol time, to transmit  $\sigma_i$ , we modulate the in-phase carrier by  $\alpha_i = \text{Re}\{\sigma_i\}$  and the quadrature carrier by  $\beta_i = \text{Im}\{\sigma_i\}$ . Then we transmit the sum:



- Signal transmitted for the  $i$ -th symbol is  $\alpha_i \cos(\omega t) - \beta_i \sin(\omega t)$ . We can re-express it as  $A \cos(\theta) \cos(\omega t) - A \sin(\theta) \sin(\omega t)$  where  $A$  and  $\theta$  are the magnitude and phase of the symbol  $\sigma_i$ .
- Using the trigonometric identity  $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$ , we see that the transmit signal is simply  $A \cos(\omega t + \theta)$ ! The transmitted signal is simply one wave with an amplitude and phase as specified by  $\sigma_i$ !

## Complex Representation of Modulated Signal

- The transmitted signal for the  $i$ -th symbol is  $\alpha_i \cos(\omega t) - \beta_i \sin(\omega t) = A \cos(\omega t + \theta)$ .
- A more concise representation of the transmitted signal is  $\sigma_i e^{j\omega t}$ .

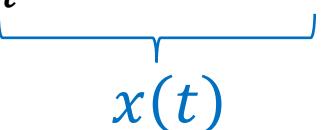
We can think of the transmitted RF signal  $A \cos(\omega t + \theta)$  as the real part of  $\sigma_i e^{j\omega t}$ ,

we can also think of  $\sigma_i e^{j\omega t}$  as twice the positive frequency part of  $A \cos(\omega t + \theta)$ ,

$$\text{since } A \cos(\omega t + \theta) = \frac{1}{2} (\sigma_i e^{j\omega t} + \sigma_i e^{-j\omega t})$$

- Over all symbols, the transmitted RF signal can be presented mathematically by:

$$\text{transmitted RF signal} = \sum_i \sigma_i r(t - iT) e^{j\omega t}$$



The complex sinusoid  
representing the  
I- and Q- carriers

The information signal  
that convey bits

# Summary - QAM

- QAM is the basis of most modern wireless and wired digital communication systems.
- Describing QAM signals and their manipulation require the use of complex numbers.
- In two lectures, we will discuss OFDM (Orthogonal Frequency Division Multiplexing), which is to apply QAM to a huge number harmonic frequencies (i.e., orthogonal) so that we can transmit a huge number of bits by each information symbol.
- OFDM is the basis of most advanced communication systems:
  - Digital subscriber loop
  - Cable modem
  - 4G/5G mobile and WiFi
  - Coherent optical transmission

64-QAM QAM64  
Quadrature Amplitude  
Modulation

