EXAMPLE

Consider the following continuous-time signal $x(t)$:

$$x(t) = \begin{cases} 2t, & \text{for } -2 \le t < 0 \\ 4 - t^2, & \text{for } 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Questions:

1. Is $x(t)$ an even signal, an odd signal, or neither? Justify your answer mathematically by checking if $x(-t) = x(t)$ or $x(-t) = -x(t)$.
2. If $x(t)$ is neither even nor odd, perform the following transformations:

- Apply a time shift by 1 unit to the right and plot the resulting signal.
- Apply a time scaling with $\alpha = 2$ to the resulting shifted signal, and plot the final signal.

Step 1: Check if $x(t)$ is even or odd:
We will apply the definitions of even and odd signals:

- Even signal: A signal is even if $x(-t) = x(t)$.
- Odd signal: A signal is odd if $x(-t) = -x(t)$.

Let's examine $x(t)$ for both cases:

1. For $t \in [-2,0)$:
   $x(t) = 2t$

$$x(-t) = 2(-t) = -2t \neq 4 - t^2 \quad for\ \ t > 0 \qquad x(-t) = -2t \neq -4 + t^2 \quad for\ t > 0$$

2. For $t \in [0,2]$:
   $x(t) = 4 - t^2$

$$x(-t) = 4 - (-t)^2 = 4 - t^2 \neq 2t \quad for\ \ t < 0 \qquad x(-t) = 4 - t^2 \neq -2t \quad for\ t < 0$$

Here, $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$. Both parts of the signal don't satisfy the conditions for evenness or oddness.

Conclusion: $x(t)$ is neither even nor odd.

EXAMPLE

$$\boldsymbol{x(t-t_0)}$$

Consider the following continuous-time signal $x(t)$:

$$x(t) = \begin{cases} 2t, & \text{for } -2 \le t < 0 \\ 4 - t^2, & \text{for } 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Questions:

1. Is $x(t)$ an even signal, an odd signal, or neither? Justify your answer mathematically by checking if $x(-t) = x(t)$ or $x(-t) = -x(t)$.
2. If $x(t)$ is neither even nor odd, perform the following transformations:

- Apply a time shift by 1 unit to the right and plot the resulting signal.
- Apply a time scaling with $\alpha = 2$ to the resulting shifted signal, and plot the final signal.

Step 2: Apply the transformations:

- Shift by 1 unit to the right:

Shifting the signal 1 unit to the right means replacing t with $t - 1$ in $x(t)$. The new shifted signal $x_{\text{shifted}}(t)$ is:

$$x_{\text{shifted}}(t) = \begin{cases} 2(t-1), & \text{for } -1 \le t < 1 \\ 4 - (t-1)^2, & \text{for } 1 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$$

As a result, the signal is now non-zero for the range $t \in [-1,1)$ and $t \in [1,3]$.

- Time scale by $\alpha = 2$:

Next, we apply time scaling by a factor of $\alpha = 2$,
$|\boldsymbol{\alpha}| > \mathbf{1}$ means ***time compression***⇒everything happening faster

$$x_{\text{scaled}}(t) = \begin{cases} 2(2\mathrm{t}-1), & \text{for } -0.5 \le t < 0.5 \\ 4 - (2t-1)^2, & \text{for } 0.5 \le t \le 1.5 \\ 0, & \text{otherwise} \end{cases}$$