ELEC 4110
--- Small-Scale Signal Variations

$\Downarrow$

LTI

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$

→ if this move, scattering environment will be changed!

linear operator

① EM Theory (Ray Tracing)

- inconvenient

② System Approach

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

LTV!

- linear
- time variant (induced by scatters!)

$$x(t) = \int_{-\infty}^{\infty} x(t-\tau)\,\delta(\tau)\,d\tau$$

linear system: if input is superposition of $\delta(\tau)$, output is also superposition $h(\tau)$

multipath, delay dimension σ_τ

- Delay spread large: impulse long tail
- Coherence BW $B_c \approx \frac{1}{\sigma_\tau}$

$$x(t) \to \boxed{h(t,\tau)} \to y(t)$$

(green's function)

(mobility)

- σ_v (Doppler spread) 2D dimension!

related

- T_c (Coherence time)

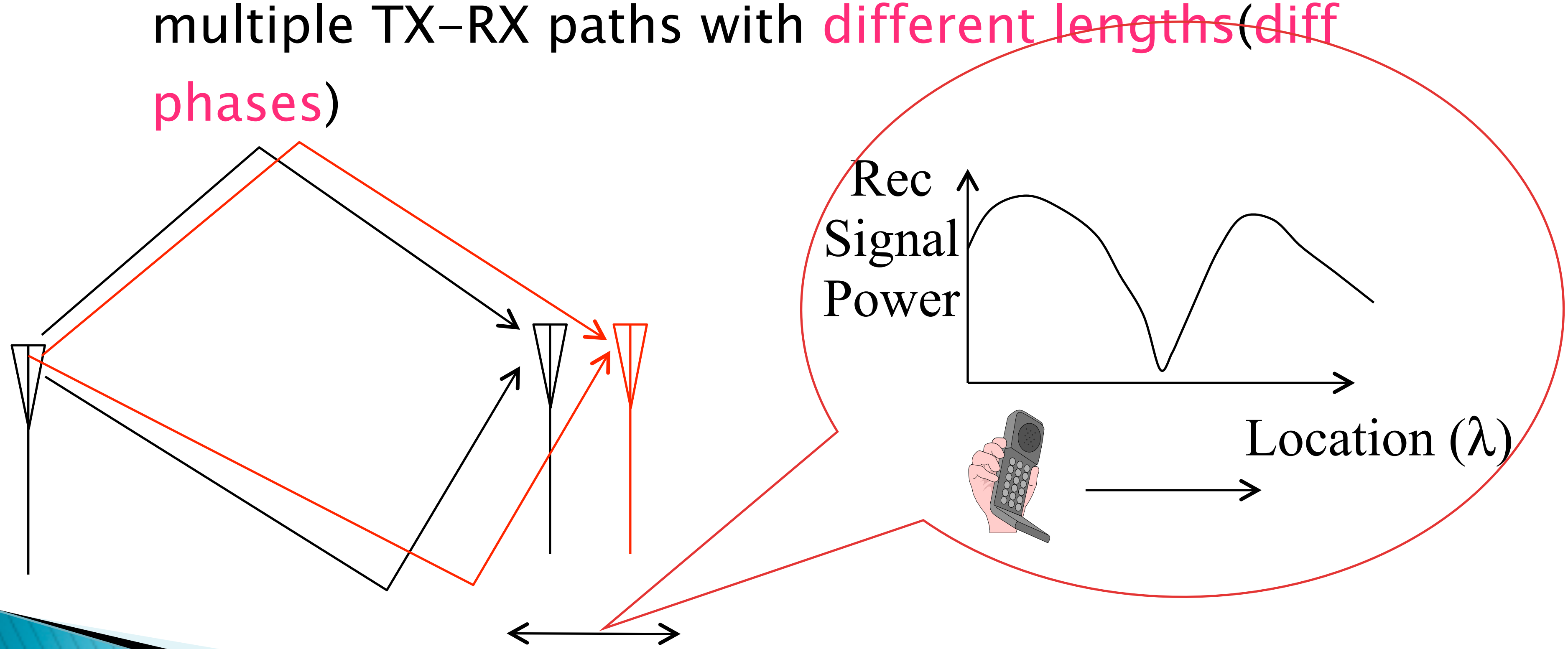
- one freedom:

coherence time:

Time that the impulse response for a channel keeps unchanged!

Small Scale Fading

- Physical cause:
  - constructive and destructive interference between multiple TX-RX paths with different lengths(diff phases)



Level 1: Multipath Fading (Cont.)

i : delay spread

- Effects: Small scale Fading

::: coherence bandwidth

  - Attenuation, received signal powervaries widely over ~30 dB (dynamic range driven by the pdf of Rayleigh fading)
  - Varies rapidly within a distance of a few wavelengths
  - If carrier frequency is 1 GHz, wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8\, m/s}{1\times10^9\, Hz} = 0.3m \cong 1\,ft$$

Wavelength is ~1 ft for a 1 GHz carrier

B_c : coherence bandwidth, within the window, they will be highly correlated.

$y(t) = \alpha(t)s(t) + n(t)$

voice ⇓ distored (single "resolvable" symbol) ← $[\ \sigma_\tau \ll T_s\ ]$ ⇕ $[\ BW_{tx} \ll B_c\ ]$

(freq flat fading) → multipath

↓

when transmit a signal with certain bandwidth → all frequency components encounter same fading coefficient α_i

1000 multipath but effects are so small!

Across the freq. spectrum will encounter different fading! ⇒ selectivity

↑

(freq selective freq.)

multipath

$[\ BW_{tx} \gg B_c\ ]$

⇕

$[\ \sigma_\tau \gg T_s\ ]$

$$y(t) = \sum_i \alpha_i(t)s(t-\tau) + n(t)$$

Example, two diffs

multiple echos! ⇐ $\left[\frac{BW_{tx}}{B_c}\right]$ or $\left[\frac{\sigma_\tau}{T_s}\right]$ multiple resolvable symbols, with large delay spread

Multipath Rayleigh Fading

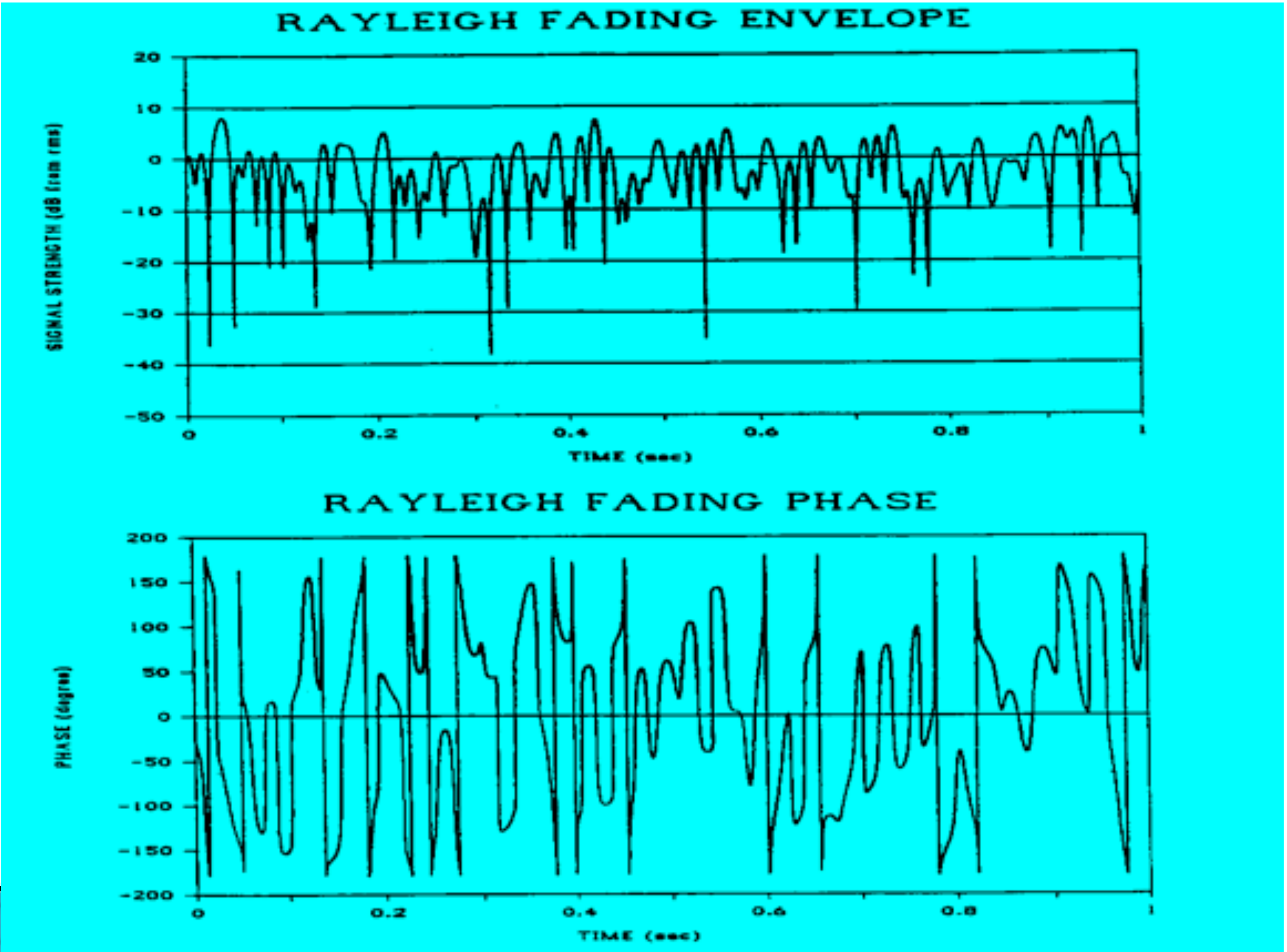
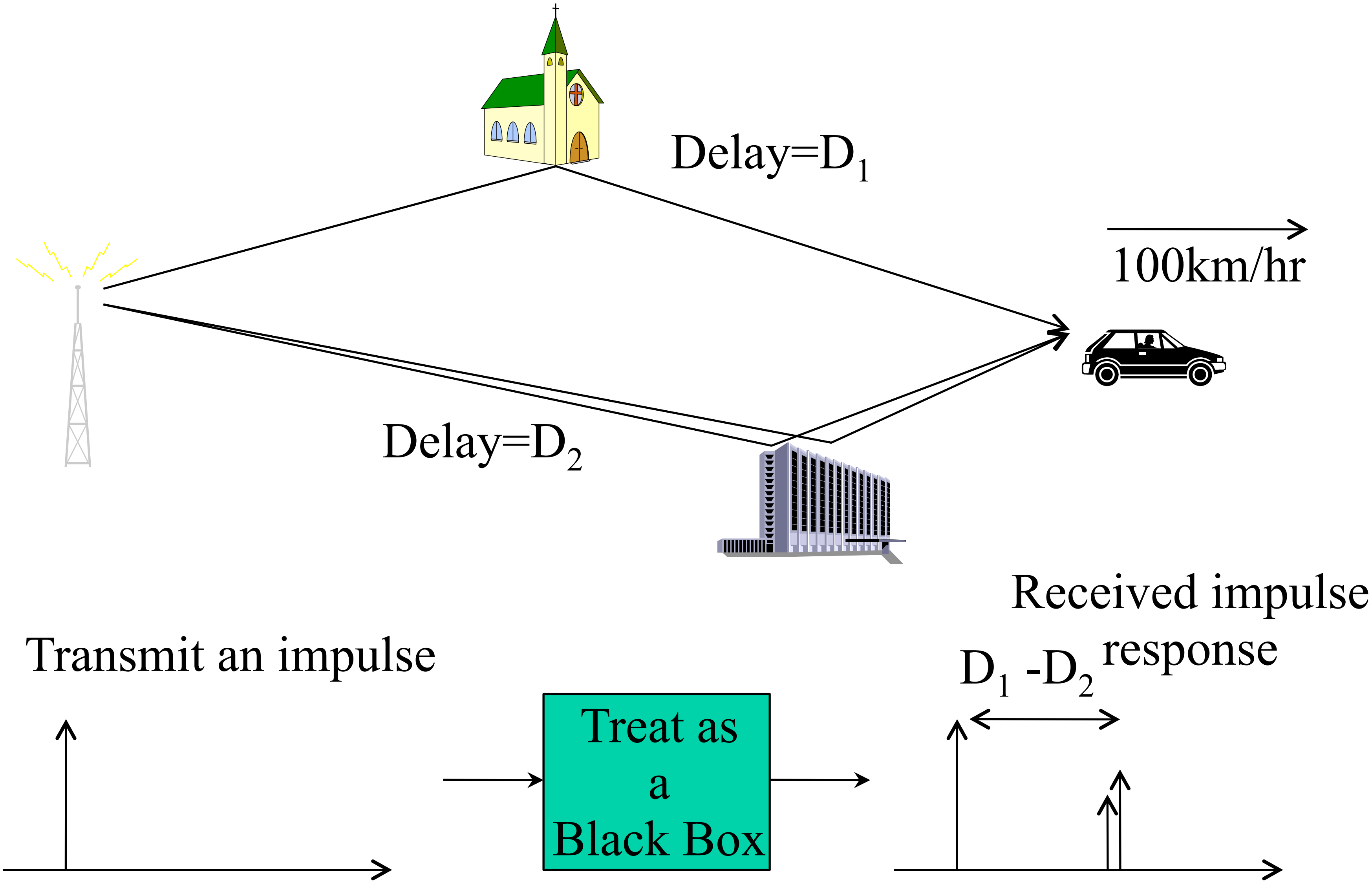


Figure 1.1: Typical Profile of the received signal's Rayleigh fading envelope and phase. Vehicular MS speed of 30 mph, carrier frequency of 900 MHz.

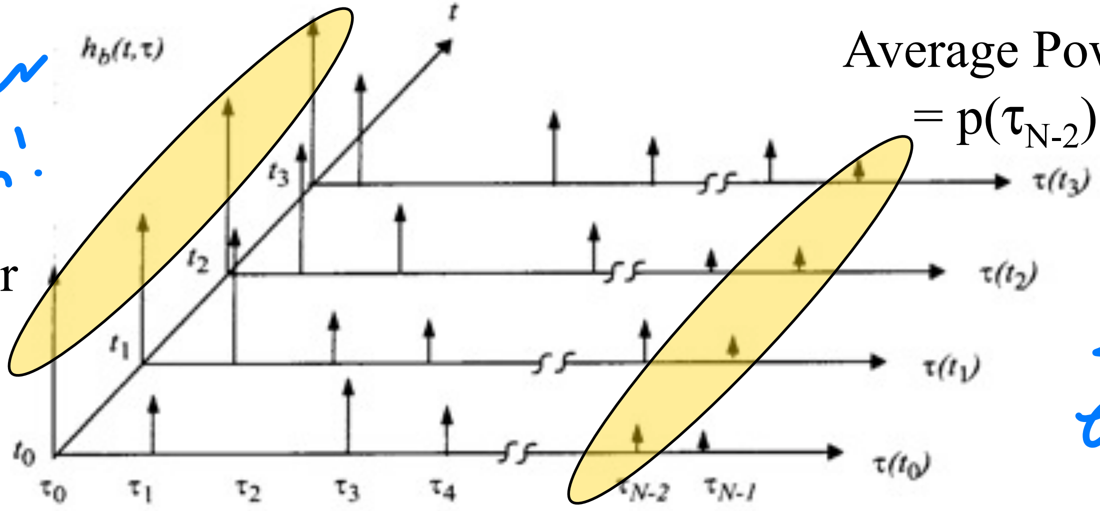
Physics of Multipath Fading



General Fading Channel Model

- Time-varying Multi-path Channel model

Average power for a path!



$h(t,\tau)$ ↓ fully characterize the channel!

Figure 4.4
An example of the time varying discrete-time impulse response model for a multipath radio channel.

- Linear Time Varying Systems (Two dimensions)
  - Multipath Response (tau)
  - Time Variation (t)

linear, but time variant!

$$y(t) = \int_0^t h(t,\tau)x(t-\tau)d\tau$$

summing across time, but depends on τ

Monday, 7 October, 13

tell some partial parameters

Part I) Multipath-Dimension of Fading

time varying. scattering in time t!

$$h(t,\tau) = \sum_i a_i(t)\delta(\tau - \tau_i)$$

delay!

FT for τ variable

$$H(t,\nu) = \sum_i a_i(t)\exp(j2\pi\nu\tau_i)$$

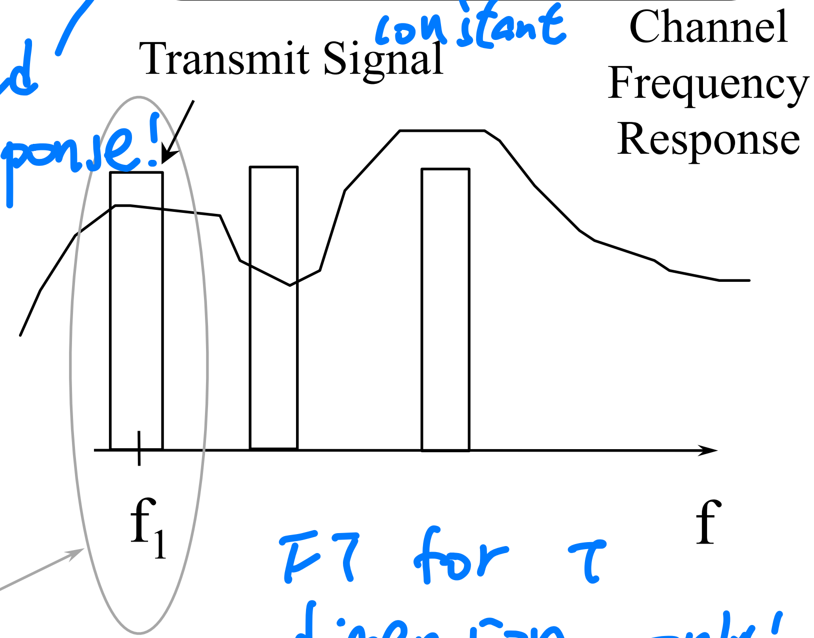
constant

weighted freq. response!

Channel Impulse Response ✓

if t is fixed
$\Rightarrow$ can do FT

τ



FT for τ dimension only!

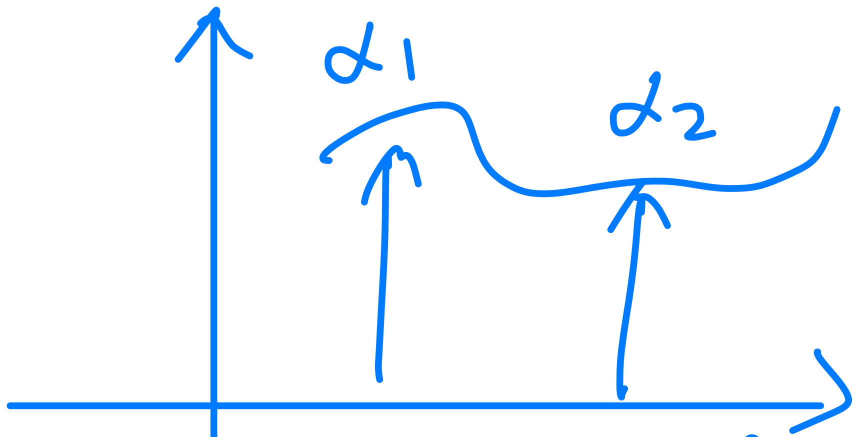
Narrow Band Transmission

then very like constant!

Equivalent Model for Narrowband:

$$y(t) = \alpha\, x(t),$$
$$t \in [0,T]$$

$$\alpha(t) = H(t, f_1)$$
$$= \sum_i a_i(t)\exp(j2\pi f_1\tau_i)$$



α_1, α_2 are random!

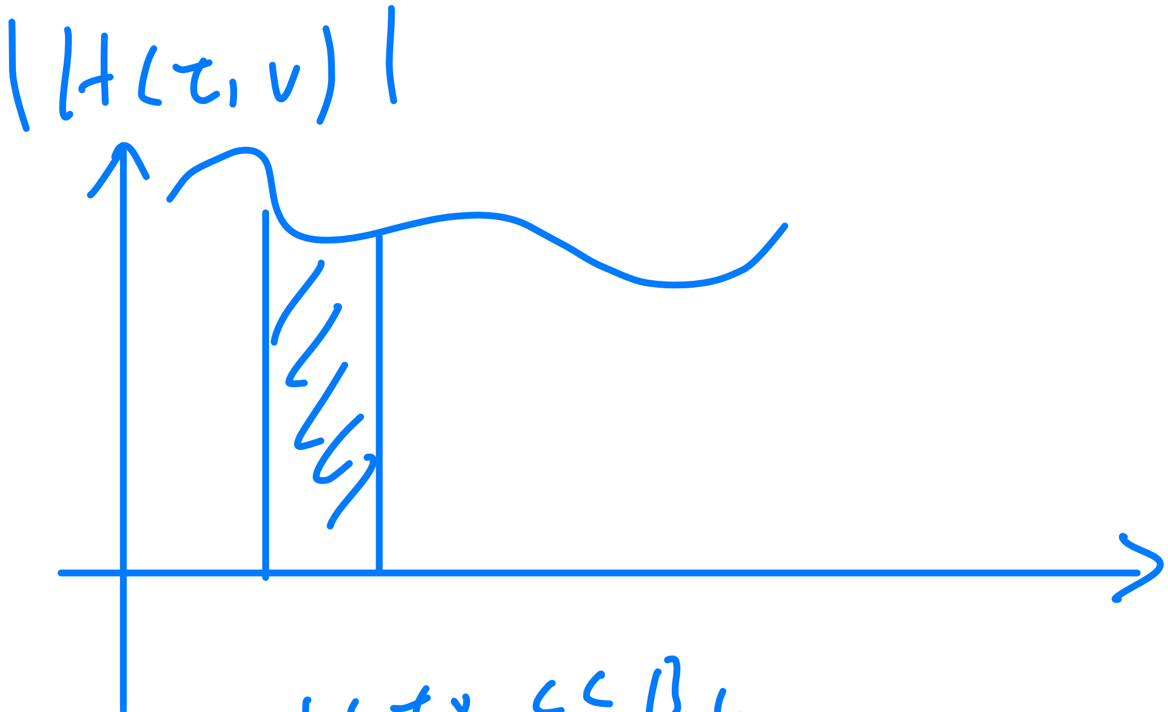
if $|f_1 - f_2| \downarrow$

$\Rightarrow \alpha_1, \alpha_2$ are very correlated!

if $|f_2 - f_1| > B_c$

then $E(\alpha_1 \alpha_2) = \text{small}$

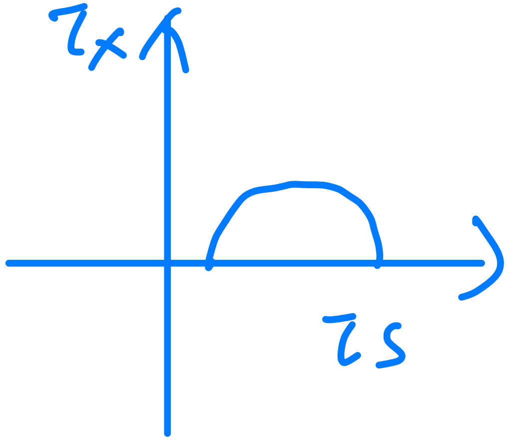
$\Delta f \gg B_c$



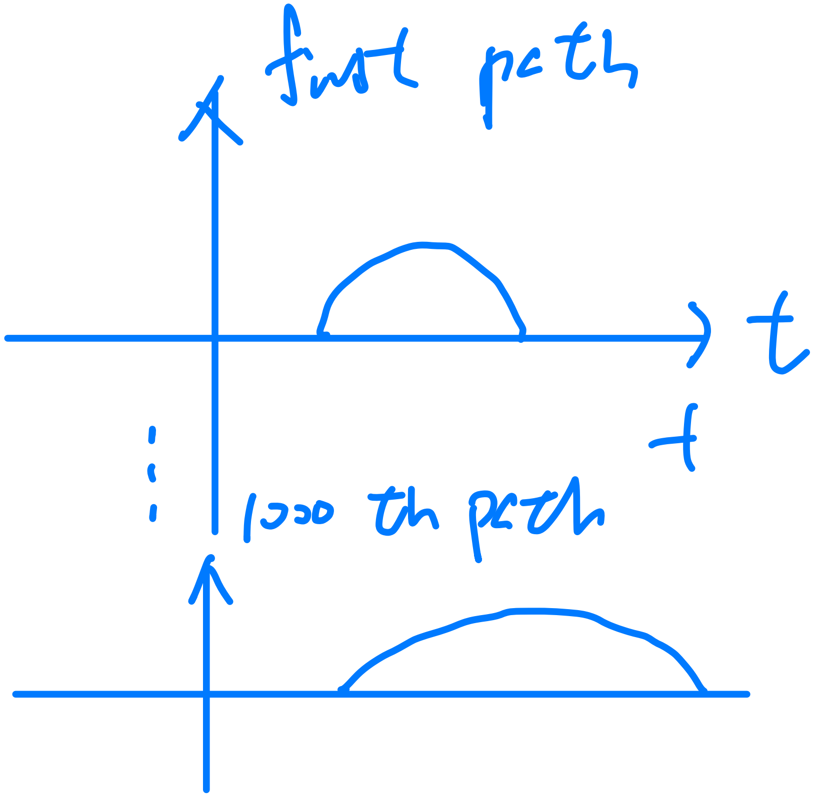
$W_{tx} \ll B_c$

frequency flat fading

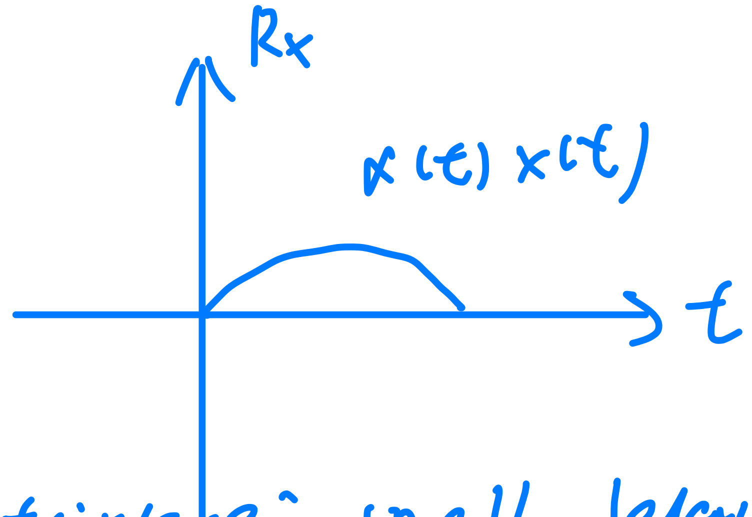
$W_K \ll B_c \iff (\tau_s \gg \sigma_\tau)$



\# of resolvable symbol = 1



→



staircase: small delay spread → distorted!

Echo button on KTV

⇒ add artificial multipath

⇒ delay spread < symbol duration

否则: delay spread 太大, hear multiple voice!

delay spread

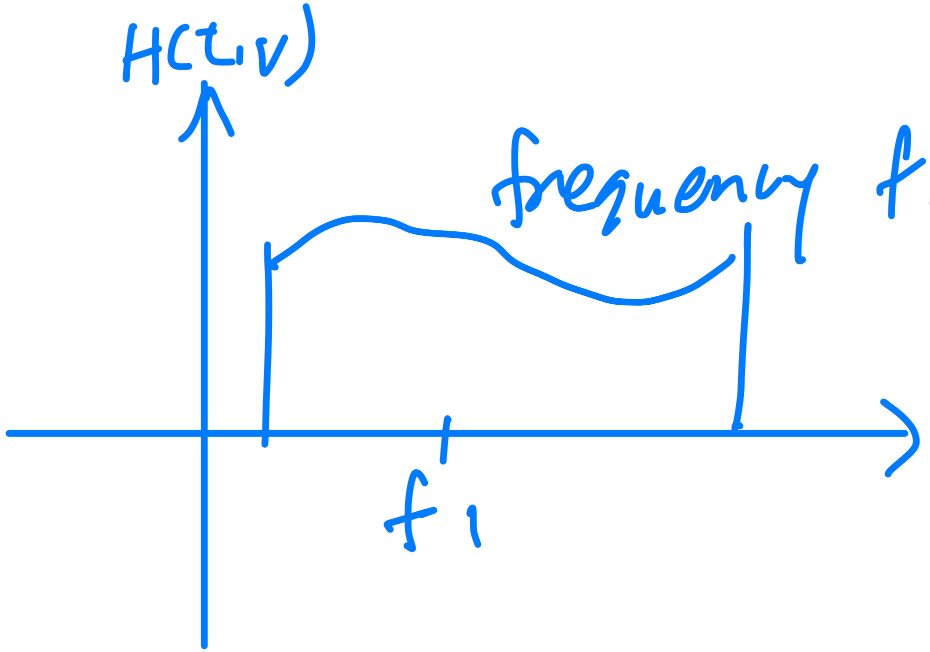
$$\left\lceil \frac{\sigma_\tau}{T_s} \right\rceil = \left\lceil \frac{W_{tx}}{B_c} \right\rceil$$

multiple resolvable signals!

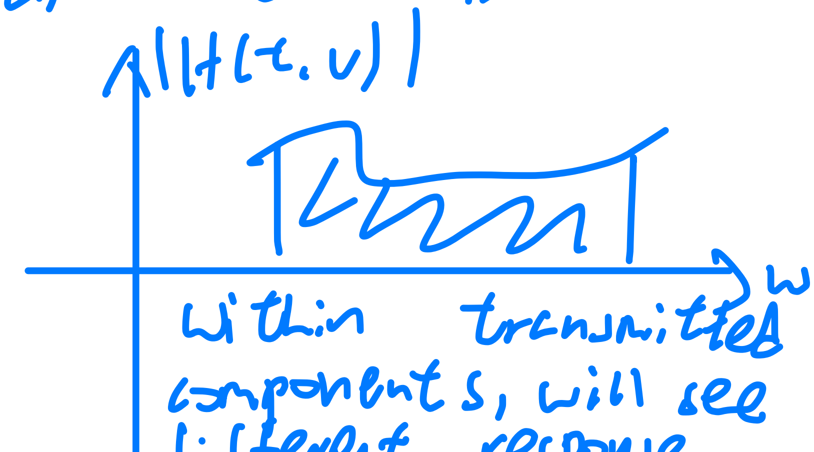
symbol duration

if $\sigma_\tau < T_s$, ceiling $= 1$

# of resolvable symbols = number of echo!



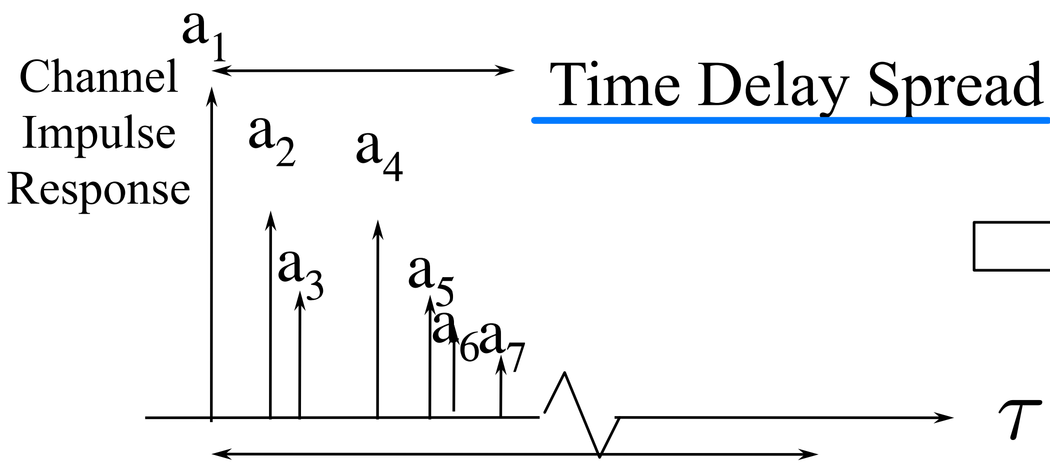
if $W_{tx} \gg B_c$



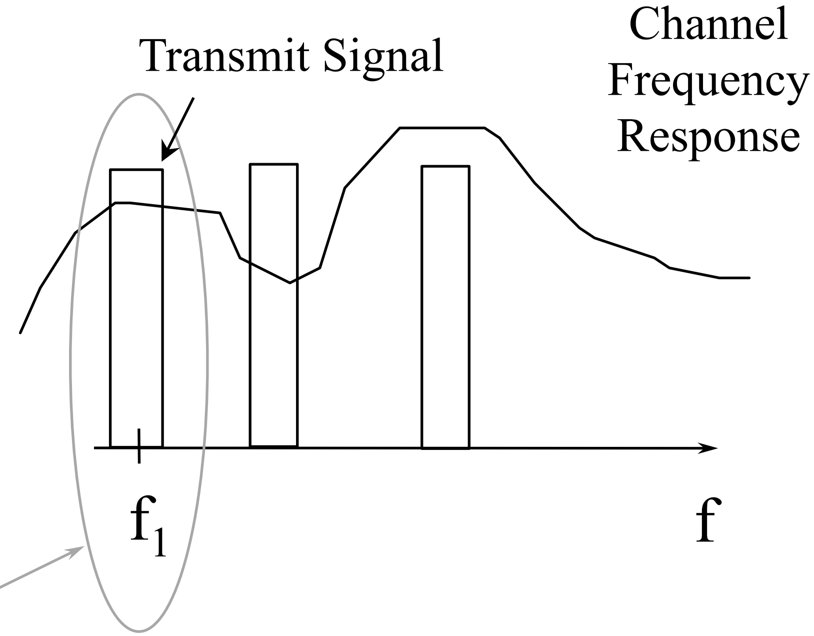
within transmitted components, will see different response introduced by the channel!

$h(t,\tau) = \sum_i a_i(t)\delta(\tau - \tau_i)$

$H(t,\nu) = \sum_i a_i(t)\exp(j2\pi\nu\tau_i)$



Symbol Period >> Time Delay Spread



Narrow Band Transmission

complex R.V.

Equivalent Model for Narrowband:

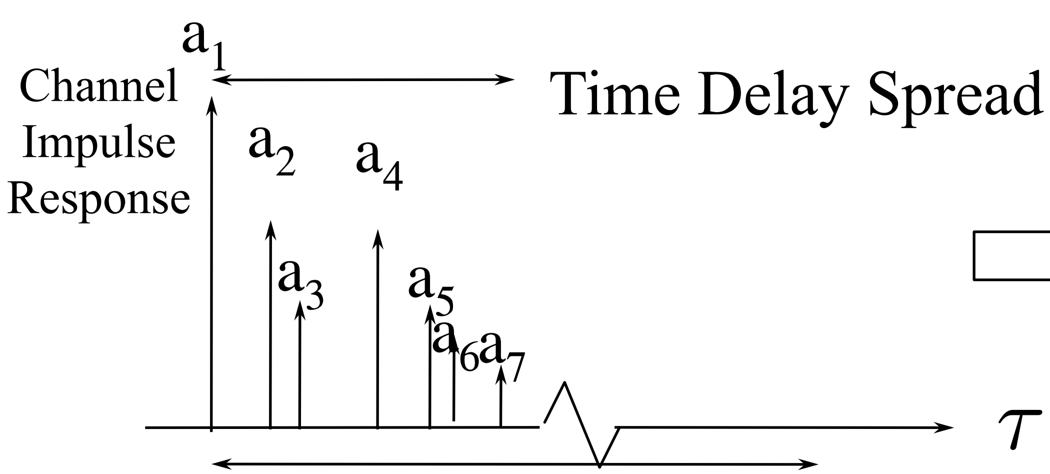
$y(t) = \alpha\, x(t),$

$t \in [0,T]$

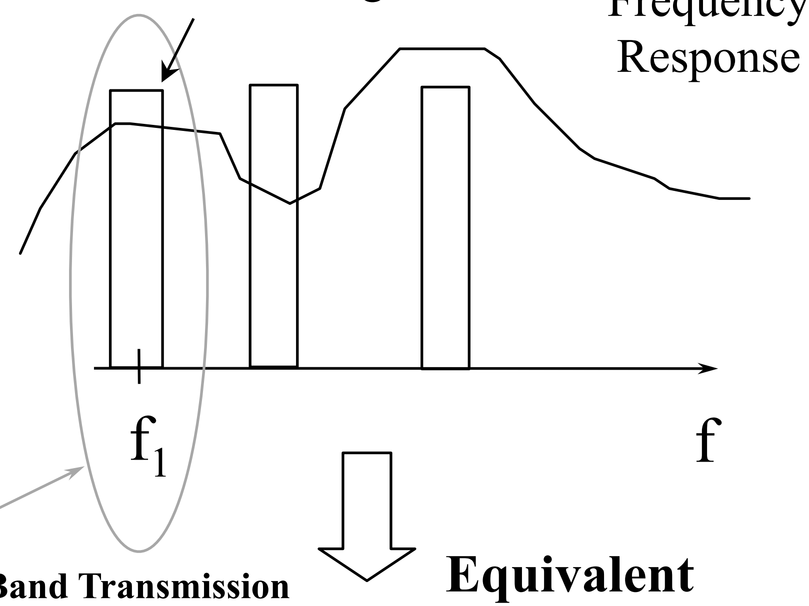
also have noise

$$\alpha(t) = H(t, f_1)$$
$$= \sum_i a_i(t)\exp(j2\pi f_1 \tau_i)$$

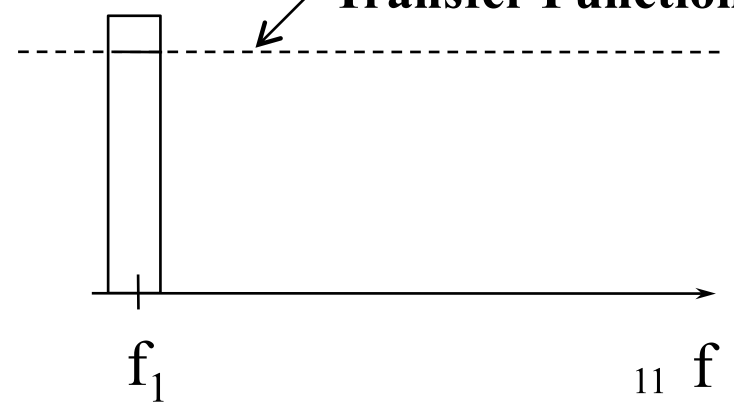
$h(t,\tau) = \sum_i a_i(t)\delta(\tau - \tau_i)$ $\longleftrightarrow$ $H(t,\nu) = \sum_i a_i(t)\exp(j2\pi\nu\tau_i)$



Symbol Period >> Time Delay Spread



Equivalent Transfer Function



Equivalent Model for Narrowband:

$$\alpha(t) = H(t, f_1) = \sum_i a_i(t)\exp(j2\pi f_1 \tau_i)$$

$y(t) = \alpha\, x(t)$, $t \in [0,T]$

No Line of Sight Path (Rayleigh Fading)

Brute force to add up. not good :(

$$\alpha = \sum \mathrm{Re}(a_i e^{j\omega_1 \tau_i}) + j\sum \mathrm{Im}(a_i e^{j\omega_1 \tau_i})$$

By Central Limit Theorem

$$= \alpha_I + j\alpha_Q$$

sum of many R.V.

Independent zero mean Gaussian random variable

(α_I, α_Q) $\Updownarrow$ rotation (r, θ) attenuation (scaling)

independent Gaussian!

$$= re^{j\theta}$$

$$f_{\alpha_I,\alpha_Q}(\alpha_I,\alpha_Q) = f_{\alpha_I}(\alpha_I) f_{\alpha_Q}(\alpha_Q) = \frac{1}{2\pi\sigma^2} e^{-\left(\alpha_I^2+\alpha_Q^2 / 2\sigma^2\right)}$$

$\theta \sim U[-\pi, \pi]$

$$f_{R\Theta}(r,\theta) = f_\Theta(\theta) f_R(r) = \frac{1}{2\pi} \frac{r}{\sigma^2} e^{-\left(r^2/2\sigma^2\right)}$$

where $\theta \in (-\pi,\pi], r \in [0,\infty)$

Phase is Uniform

Magnitude is Rayleigh

= complex fading!

Rayleigh Fading

$f_{\alpha_I,\alpha_Q}(\alpha_I,\alpha_Q)$: Independent Gaussian with mean σ^2

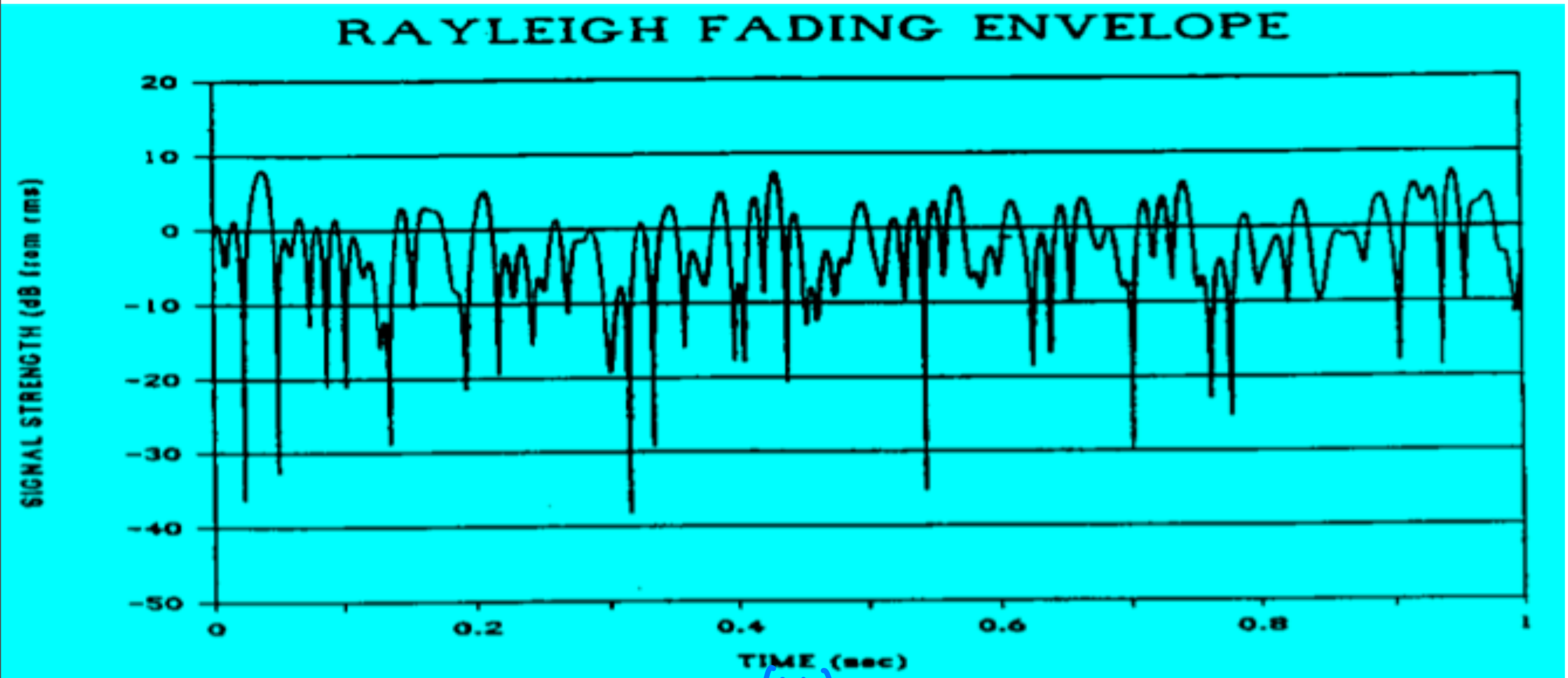
$$f_\Theta(\theta) = \frac{1}{2\pi} \quad \text{if } \theta \in [0,2\pi) \quad : \text{ Uniform Phase}$$

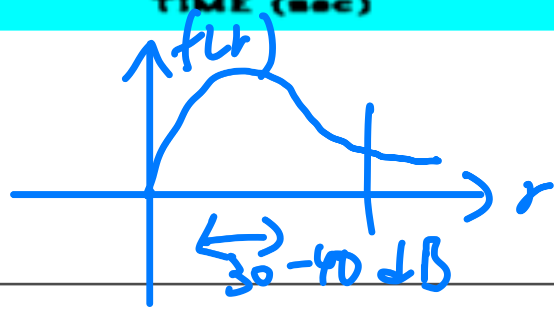
$$f_R(r) = \frac{r}{\sigma^2}\exp(-\frac{r^2}{2\sigma^2}) \quad \text{if } r > 0 \quad : \text{ Rayleigh Amplitude}$$

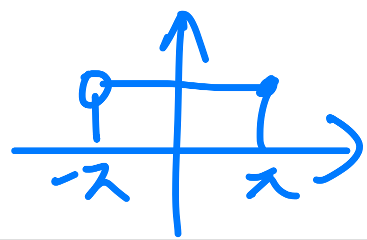
$$f_P(p) = \frac{1}{P_o}\exp(-\frac{p}{P_o}) \quad \text{if } p > 0 \quad : \text{ Exponential Channel Power Gain}$$

where $p = r^2$ and $P_{\mathrm{o}} = 2\sigma^2$ is mean channel power gain

Rayleigh Fading Sample Path







Rayleigh Fading Sample Path

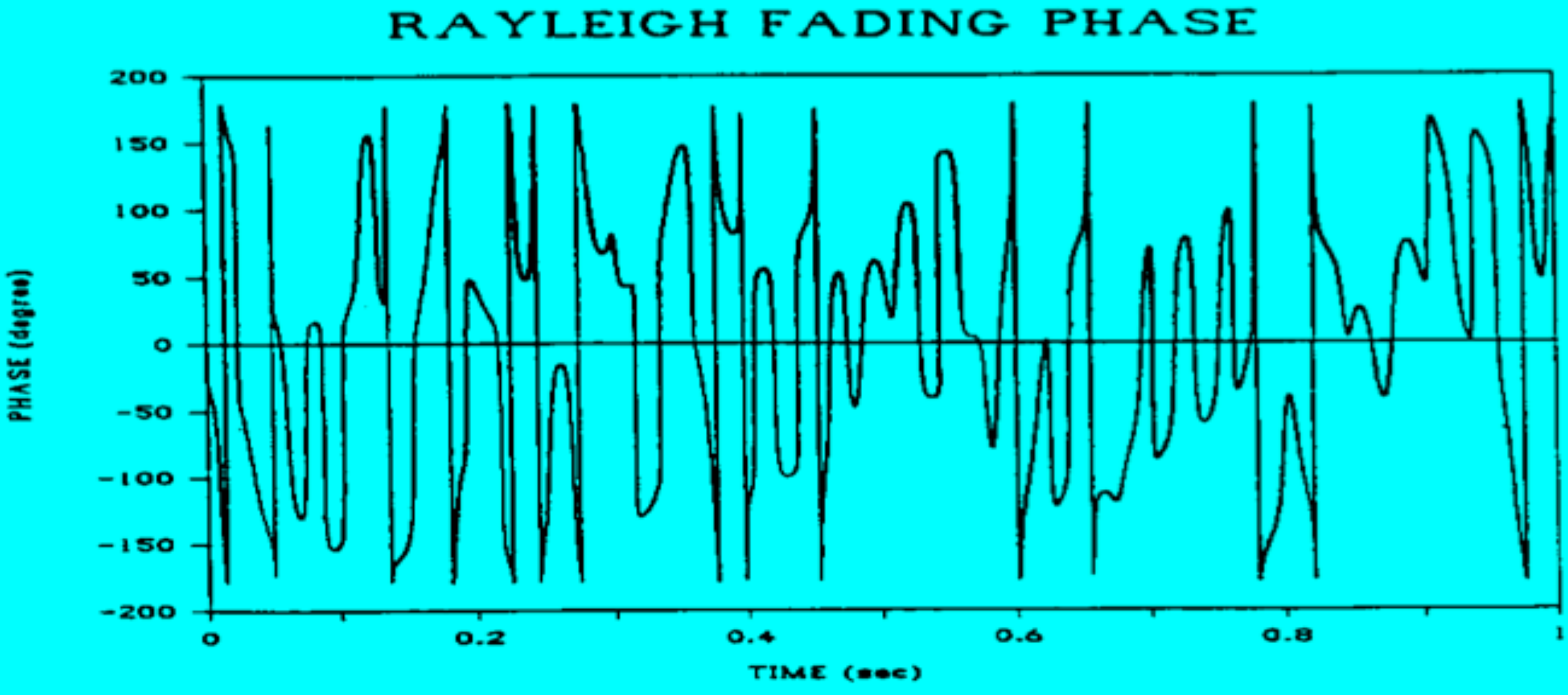


Figure 1.1: Typical Profile of the received signal's Rayleigh fading envelope and phase. Vehicular MS speed of 30 mph, carrier frequency of 900 MHz.

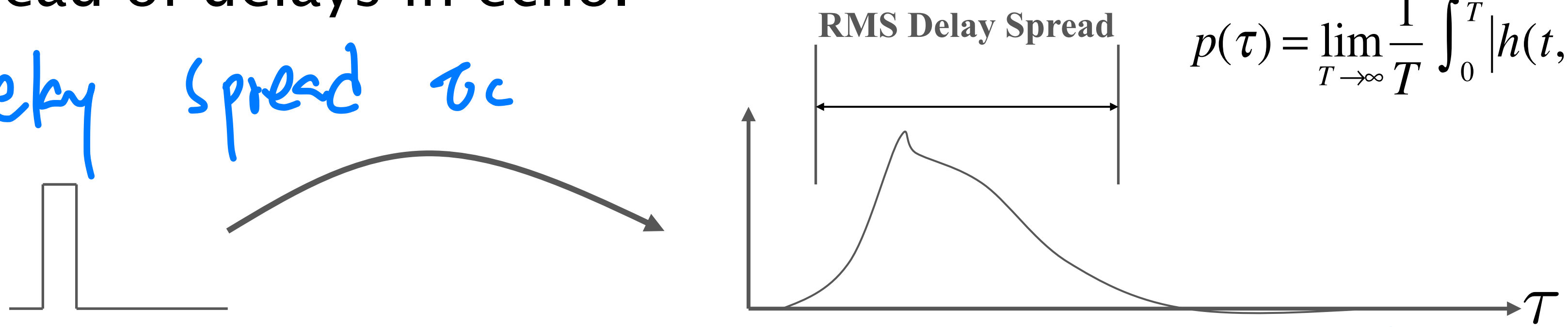
Microscopic Fading – Multipath Dimension

- Delay Spread (σ_τ):
  - spread of delays in echo.

Power Delay Profile

$$p(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |h(t,\tau)|^2 dt$$

Delay spread τ_c



⇒ if σ_c longer, takes longer channel memory!

- Coherence Bandwidth ($B_c = \frac{1}{50\sigma_\tau}$):
  - min separation of frequency for uncorrelated fading (0.5 correlation). $E[\alpha(t,f_1)\alpha(t,f_2)] = f(|f_1 - f_2|)$
- Typical values
  - Indoor: Bc ~ 1MHz
  - Outdoor: Bc ~ 100 kHz.

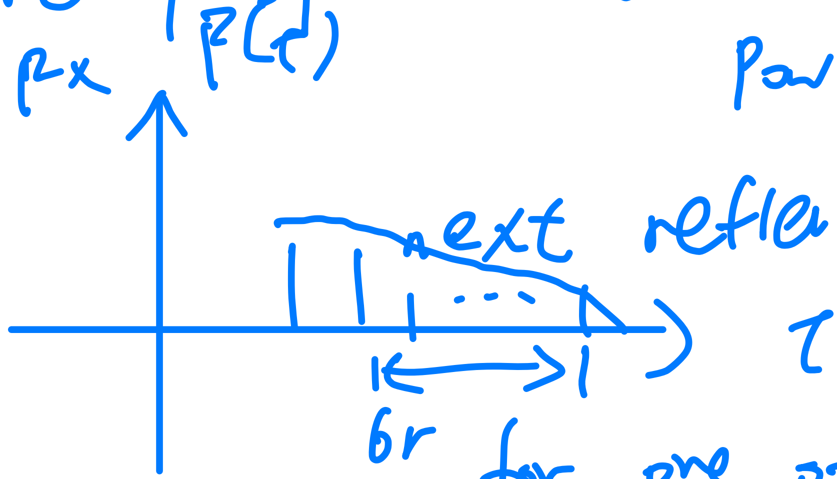
whether is good or bad?

$B_c \propto \frac{1}{\sigma_\tau}$

inversely proportional to delay spread

Delay spread τ_c:

Power-delay profile



for one of impulse response

Not Gaussian, this is response

if delay = 1ms

$\Downarrow$

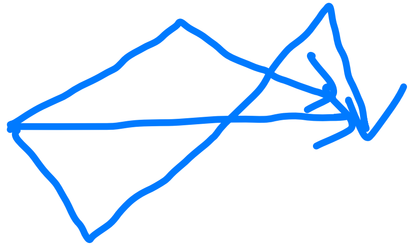
only within 1ms window,
then see significant change

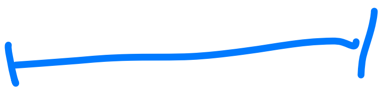
larger $\delta\tau \Rightarrow$ more multipath

impulse $\Rightarrow$ channel gives me longer response

$$\sigma_\tau^{indoor} \ll \sigma_\tau^{outdoor}$$

much more reflection



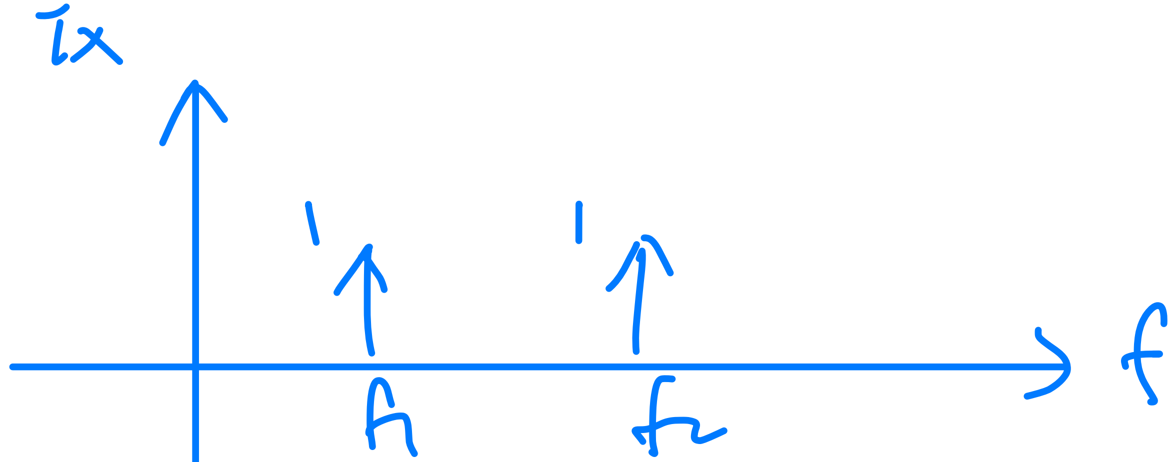


σ_τ depends on dimensions of the transmition

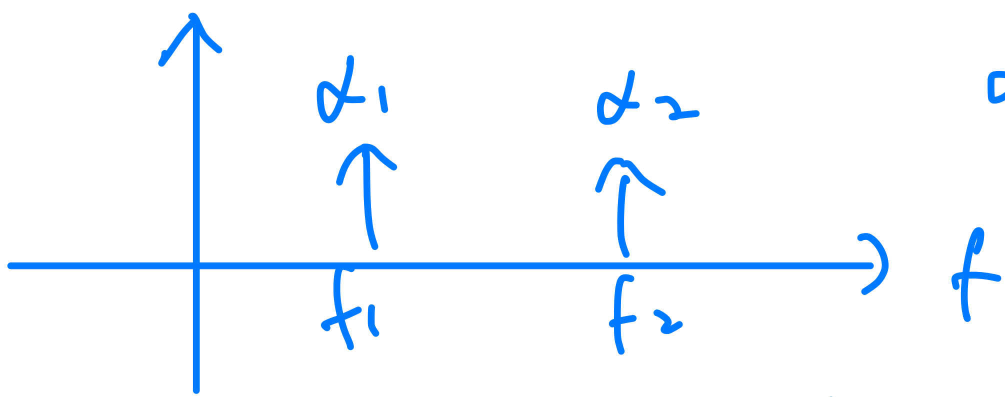
$\Rightarrow$ depends on space

$$\tau = \frac{d}{c} = \frac{10m}{3\times10^8} = \frac{1}{3}\times10^{-7}$$

Coherence BW (equivalent parameter)



↓ different frequency



α_1, α_2 are random, complex fading coefficient

$E(\alpha_1 \alpha_2)$

if $|f_2 - f_1| >$ $\Rightarrow$ $E(\alpha_1 \alpha_2) \approx$ small

-minimum seperation needs if want to see uncorrelated fading

x_i = male

$y_i \in \{-1, 1\}$

red T-shirt

$\theta = \frac{1}{N} \sum_i x_i y_i$

even high correlation,

cannot prove!

(I) Multipath Fading Summary

- Narrowband Transmission (Delay Spread >> Symbol Duration OR TxBW << Coh BW)
  - Frequency Flat Fading

$y(t) = \alpha(t)x(t) + n(t)$ where $\alpha(t)$ is a complex Guassian process.

- Wideband Transmission
  - Frequency Selective Fading

$$\left\lceil \frac{\sigma_\tau}{T_s} \right\rceil$$

  - Number of Resolvable path $= L = \cancel{\frac{T_s}{\sigma_\tau}} = \left\lceil \frac{W_{tx}}{B_c} \right\rceil.$

$y(t) = \sum_i \alpha_i(t)x(t-\tau_i) + n(t)$ where $\alpha_i(t)$ is independent complex Guassian process.

- multipath channel is just like a LTI system

Time variation

Part II) Time-Varying Dimension of Fading

- need parameter to characterize time-variation dimension!

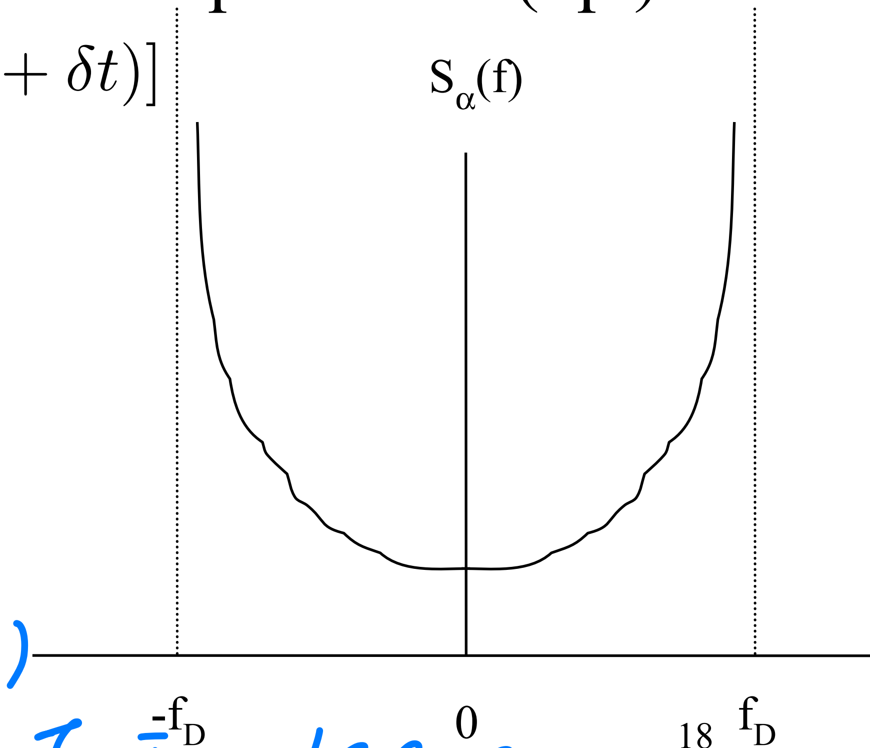
Time Varying Nature of Rayleigh Channel

- Cause for Time-varying nature
  - movements of mobile or objects in the environment

$\alpha_I(t)$ or $\alpha_Q(t)$

- Treat $\alpha_I(t)$ and $\alpha_Q(t)$ as Gaussian random processes (r.p.)
- $E[\alpha(t)] = 0 \quad R_\alpha(\delta t) = E[\alpha(t)\alpha(t+\delta t)]$

$$S_\alpha(f) = \mathcal{F}[R_\alpha(\delta t)]$$



$$S_\alpha(f) \propto \frac{1}{\sqrt{f_D{}^2 - f^2}}$$

$$R_\alpha(\tau) \propto J_0(2\pi f_D \tau)$$ ≈ 0.1 (small)

solve for τ, τ_c, coherence

$\Rightarrow \tau \propto \frac{1}{f_D}$

Physics of Doppler Effect

- At time t=0, the car observes a zero-crossing
- At time $t=1/f_c$, zero-crossing happens again at the same position, but the car has moved
- The vehicle observes a zero-crossing again at a position x unit away at time t=t'. Hence,

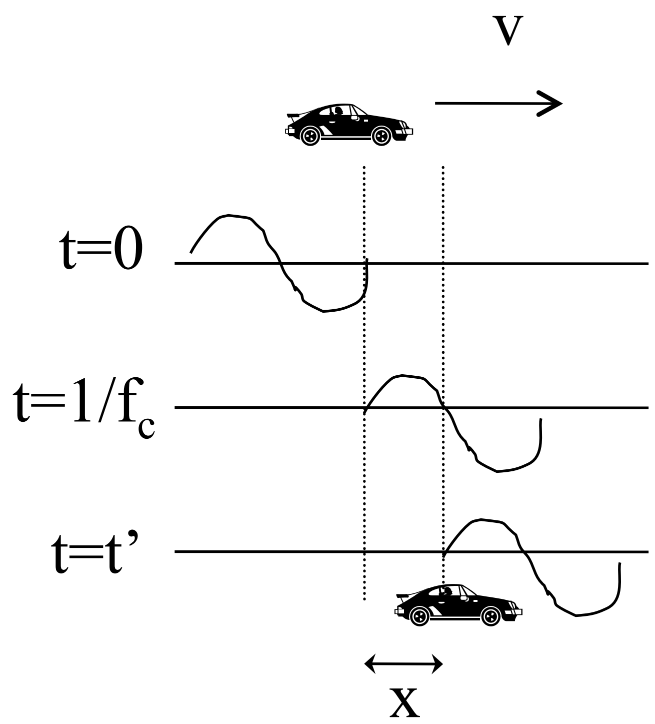
$$1/t' = f_c - f_D$$

- If v is the speed of the car, x=vt'
- As c (speed of light) is the speed of the wave,
- Hence,

$$c/f_c + x = ct'$$

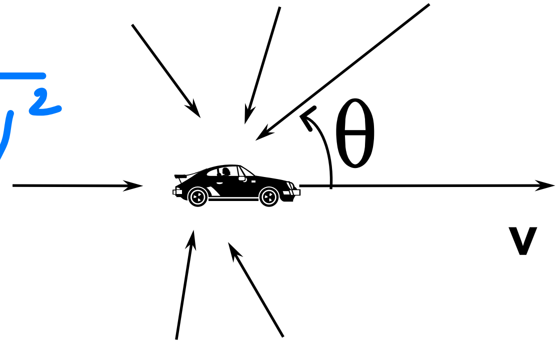
$$c/f_c = (c-v)t'$$

$$\Rightarrow 1/t' = \left(1 - \frac{v}{c}\right) f_c \Rightarrow f_D = \frac{v}{c} f_c$$

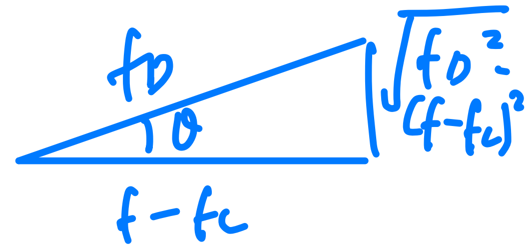


Physics of Doppler Effect

$$\sin\theta = \frac{\sqrt{f_D^2-(f-f_c)^2}}{f_D}$$



$$\cos\theta = \frac{f-f_c}{f_D}$$

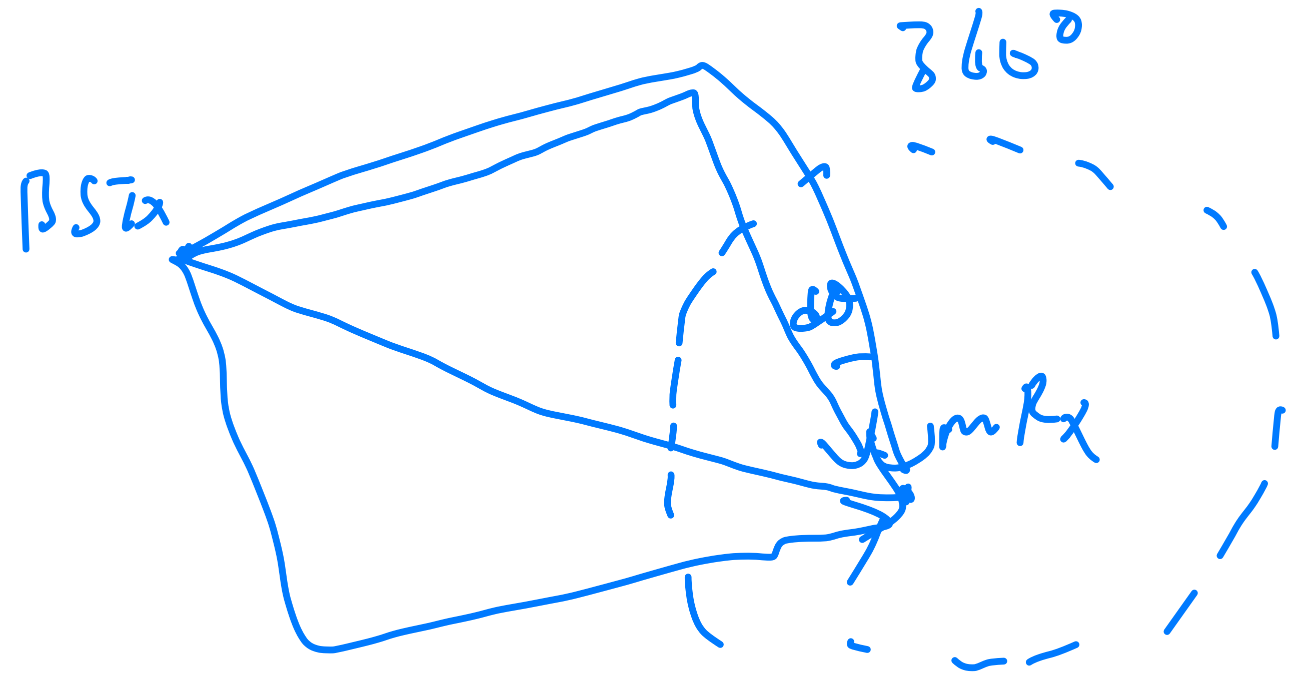


- Maximum Doppler Frequency is $f_D=(v/c)f_c$
- Doppler frequency ranges from $-f_D$ to f_D, depending on the **angle of arrival**

symmetrical

$$f(\theta) = f_c + \underline{f_D \cos\theta}, \text{ hence } f(\theta) = f(-\theta)$$

$$|df| = |-f_D \sin\theta d\theta| = \sqrt{f_D^{\,2} - (f - f_c)^2}\,|d\theta|$$



θ is uniformly distributed!

if BS Tx is low, not 360°,

⇒ new York: 30°

Physics of Doppler Effect

- fraction of power at angles in [θ,θ+dθ] : P(θ)dθ
- Antenna gain at angle θ : G(θ) and uniform angle of arrival

Power

$$S_\alpha(f)\left|df\right| = P(\theta)\left|d\theta\right|$$

← Power from $p(\theta)$ to $p(d\theta)$

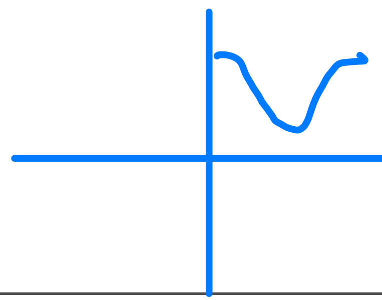
$$= (P_T/2\pi)[G(\theta)+G(-\theta)]\left|d\theta\right|$$

$$S_\alpha(f) = \frac{P_T}{2\pi\sqrt{{f_D}^2-(f-f_c)^2}}[G(\theta)+G(-\theta)]$$

independent to θ

- With omni-directional antenna, Power Spectral Density for the baseband equivalent model is

$$S_\alpha(f) = \frac{K}{\sqrt{{f_D}^2-f^2}}$$



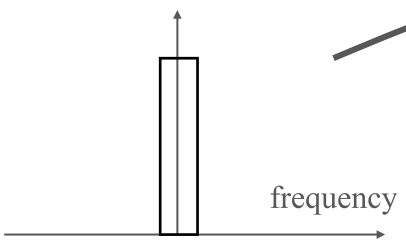
u-shape spectrum centered in f_c

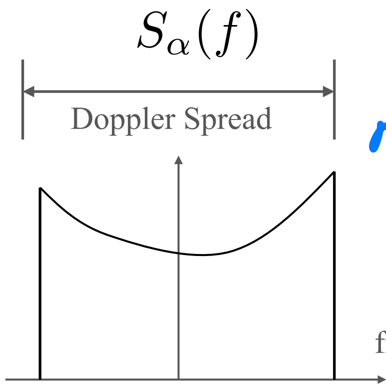
Microscopic Fading – Time Dimension

Time variation (mobility)

δf

- Doppler Spread ($f_d = \frac{v}{\lambda}$):
  - spread of frequency due to mobility





if relative motion between Tx and Rx ⇒ shift of received frequency!

⇒ uncorrelated fading

If T_C, δt will changing fast

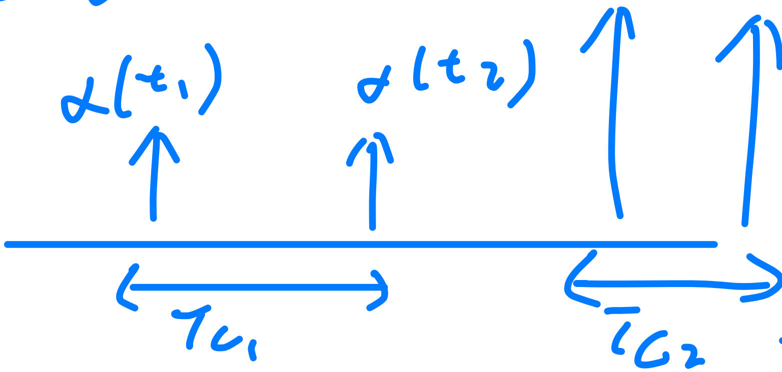
moving away: red shift $f_c - \delta f$

- Coherence Time ($T_C \approx \frac{9}{16\pi f_D}$):
  - min separation of time for uncorrelated fading (0.5 correlation).

$$E[\alpha(t)\alpha(t+\delta t)] = J_0(2\pi f_D|\delta t|)$$

- Typical Values
  - Pedestrian (~ 5 km / hr) →fd ~ 14 Hz (at 2.4 GHz)
  - Vehicular (~ 100 km/hr) →fd ~ 300 Hz (at 2.4 GHz)

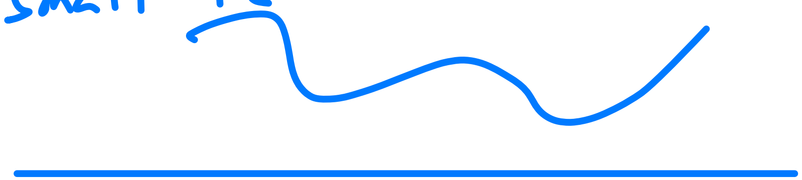
Large τ_c



$\Rightarrow$ Doppler spread $\uparrow$
mobility $\downarrow$

$\Rightarrow$ Doppler spread $\downarrow$
mobility $\uparrow$
speed $\uparrow$
change more significantly!

Small τ_c



vehichiar cellular

for MTR / train $\Rightarrow$ mobility $\uparrow$ $\Rightarrow$ speed $\uparrow$

$\Rightarrow$ smaller τ_c

fast speed

(time flat fading (slow fading))

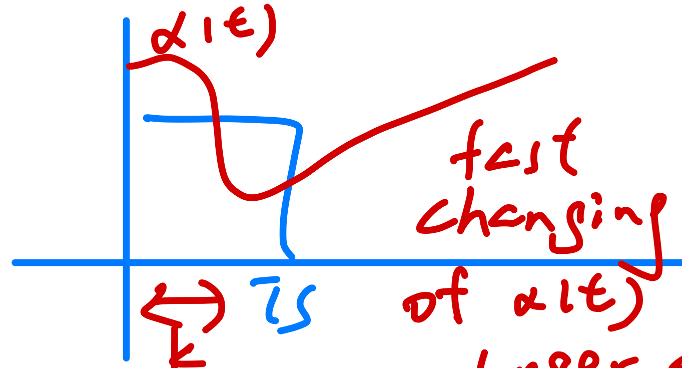
$T_c \gg \tau_s$

symbol duration << coherence time

all the waveform encounter similar fading!

(time selective fading / fast fading)

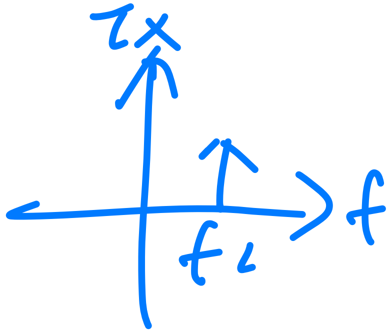
$T_c \ll \tau_s$



fast changing of $\alpha(t)$

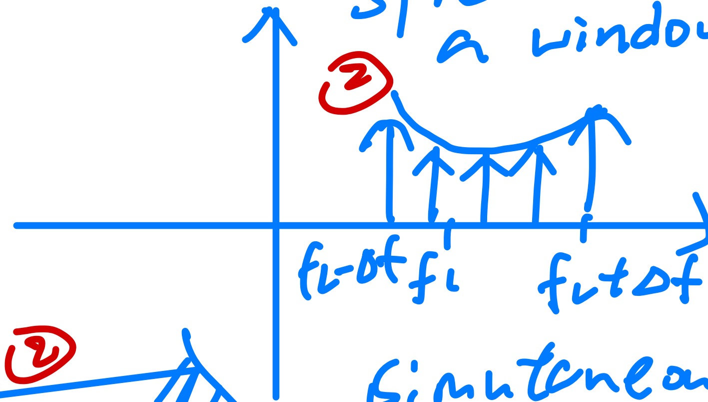
no longer a constant!

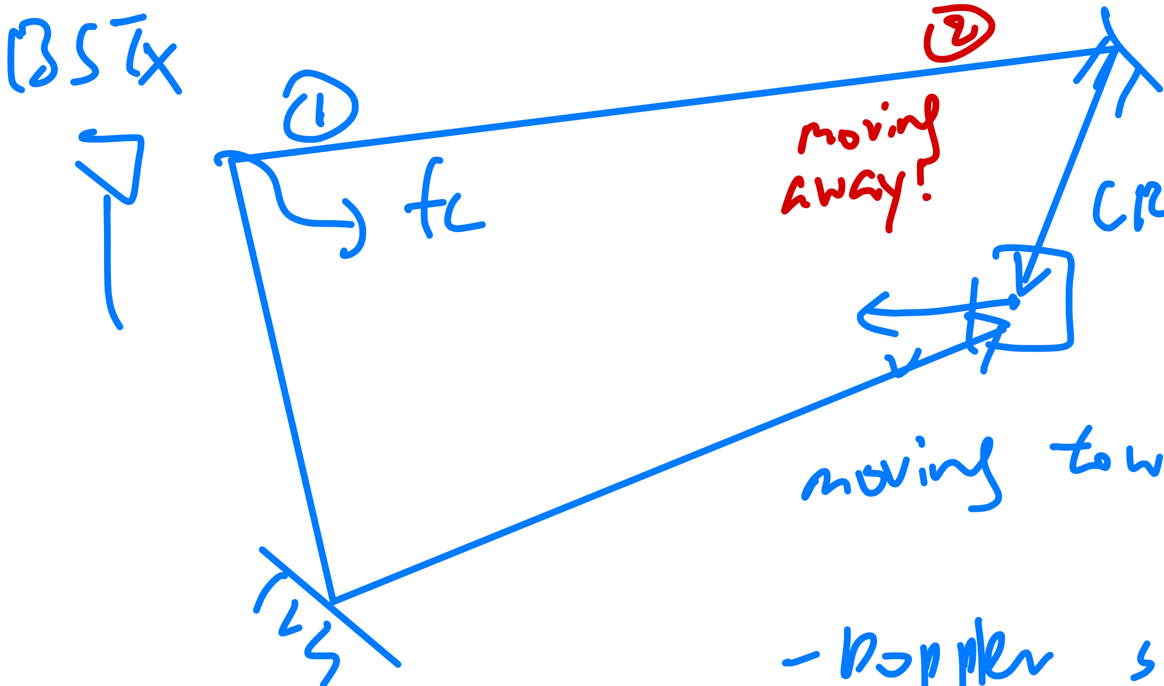
Doppler spread



Doppler's spread

spread over a window!





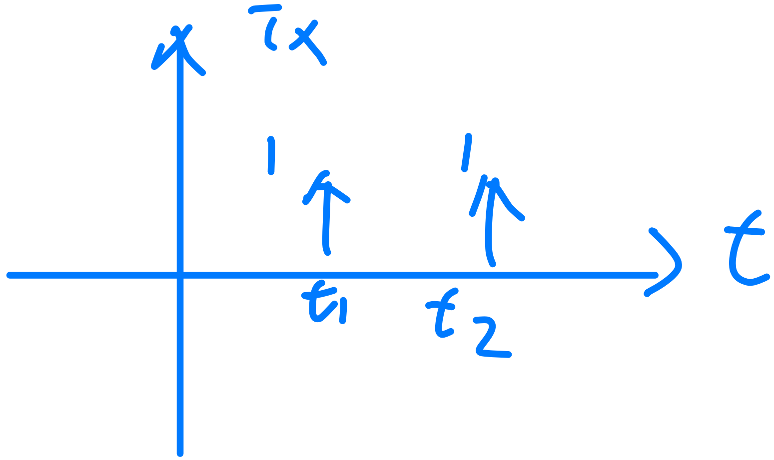
moving towards to Tx

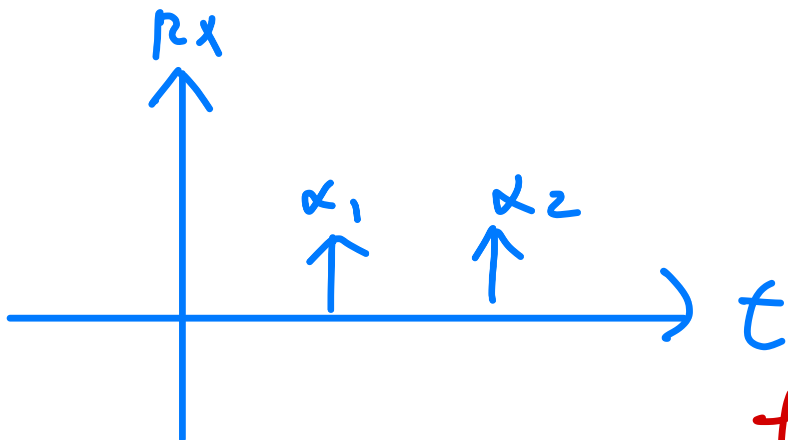
Simultaneously red shift and blue region!

- Doppler spread cannot be easily compensated!

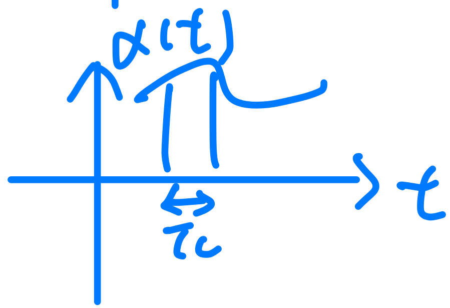
if moving speed ↑ ⇒ window wider!

② coh. time (τ_c) $\propto \frac{1}{\delta f}$





time selectivity!



If $\Delta t >$ "τ_c"

then $E(\alpha, \alpha_2) \approx$ small!

low correlation

Summary of Fading Parameters

Classification of Fading Channels

$$\boxed{\text{Number of Resolvable Multipaths} = \frac{T_s}{\sigma_T} \approx \frac{W_{tx}}{B_c}}$$

$T_s \sim 1\,\mu s$
$T_c \sim 50\,ms$
$T_c \gg T_s \Rightarrow$ slow fading!

Ts (Signal Parameter)

Coh Time (Tc)

Wtx (Signal Parameter)

communication

Slow + Frequency Selective Fading (Tc >> Ts and Wtx >> Bc)

2 ☆ trouble

Slow + Frequency Flat Fading (Tc >> Ts and Wtx << Bc)

1 ☆ trouble

slow fading

Coh BW (Bc)

fast fading

Fast + Frequency Selective Fading (Tc << Ts and Wtx >> Bc)

3 ☆ trouble!

Fast + Frequency Flat Fading (Tc << Ts and Wtx << Bc)

2 ☆ trouble

Only consider top 2!

Slow fading is much more common!

Effect of Flat Fading on BPSK

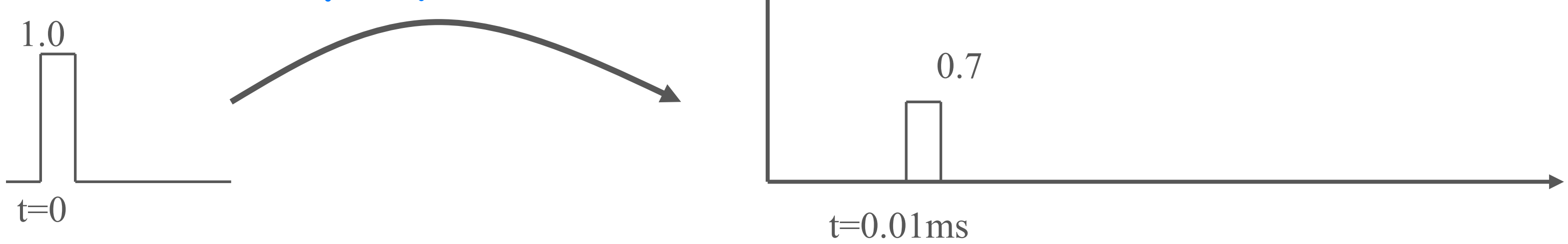
Flat Fading Channels

- "Narrowband Transmission"
  - coherence bandwidth of channel > signal BW.
- "Single path" channel model:

Random process RP_2

$$\underbrace{y(t)}_{\text{received signal}} = \underbrace{\alpha}_{\text{complex fading}} \underbrace{x(t)}_{\text{transmit signal}} + \underbrace{n(t)}_{\text{noise}}$$

RP_1 ~ constant! (fading)



BER Analysis on Flat Fading

- Consider the uncoded performance:
- Assuming BPSK and coherent demodulation, the conditional error probability (conditioned on fading α) is given by:

$$P_e(\alpha) = Q\left(\sqrt{\frac{2|\alpha|^2 E_s}{N_0}}\right) \text{ where } E_s = \int_0^{T_s} s^2(t)dt$$

- The average error probability is given by

$$\overline{P}_e = E[P_e(\alpha)] \approx \left(\frac{1}{E_s / N_0}\right) \text{ for large SNR}$$

BPSK AWGN

$y(t) = s(t) + n(t)$

① $P_e = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$

for large $\frac{E_s}{N_0}$:

$\approx \frac{1}{4\left(\frac{E_s}{N_0}\right)}$

② BPSK flat fading

$y(t) = \alpha s(t) + n(t)$

$P_e = E_\alpha\left(P_e(\alpha)\right) = E_\alpha\left[Q\left(\sqrt{\frac{2|\alpha|^2 E_s}{N_0}}\right)\right]$

condition on α

E_s

α: complex gaussian fading

$(p = |\alpha|^2)$

$= E_{|\alpha|}\left[Q\left(\sqrt{\frac{2|\alpha|^2 E_s}{N_0}}\right)\right]$

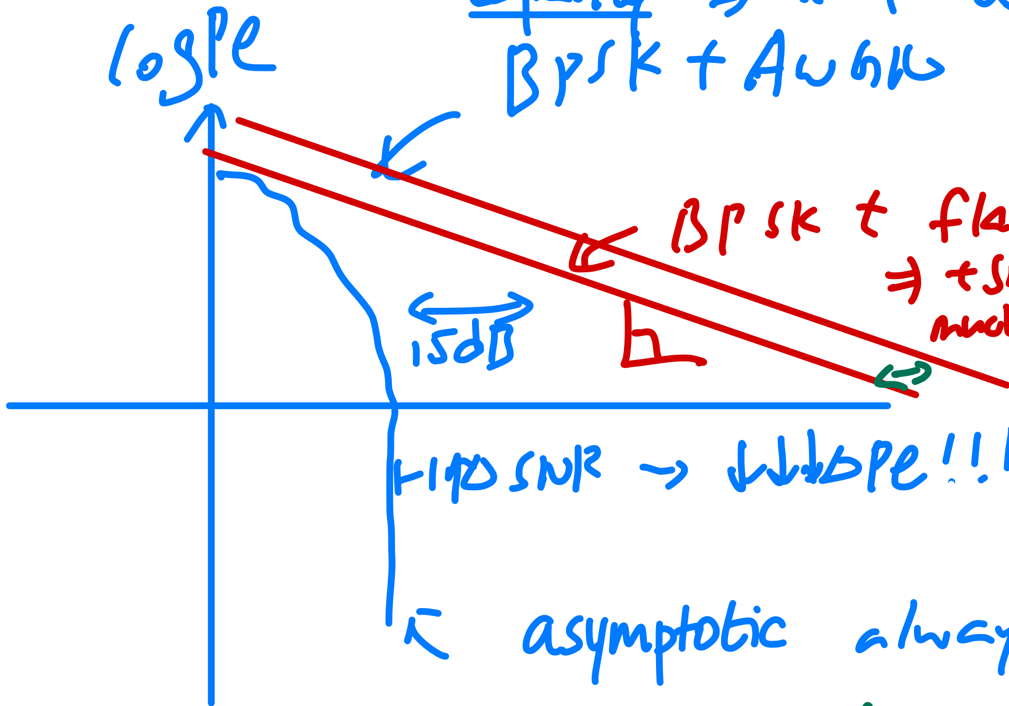
$|\alpha| \propto$ Raleigh distribution

$p = |\alpha|^2 \propto$ -ve exponential distribution ! $= E_p\left[Q\left(\sqrt{\frac{2pE_s}{N_0}}\right)\right]$

Coverage ⇒ need reduce

capacity ⇒ # of users ↓↓

BPSK + AWGN



⇒ $P_e \approx \frac{1}{4}\left(\frac{1}{E_s/N_0}\right)$

$\log P_e \approx \text{const} - \log\left(\frac{E_s}{N_0}\right)$

BPSK + flat fading

⇒ +SNR, ↓Pe, rate much less :(

+10 SNR → ↓↓↓Pe!!! ⇒ Good

← asymptotic always vertical

- If we another modulation scheme other than MPSK, just shift the line horizontally

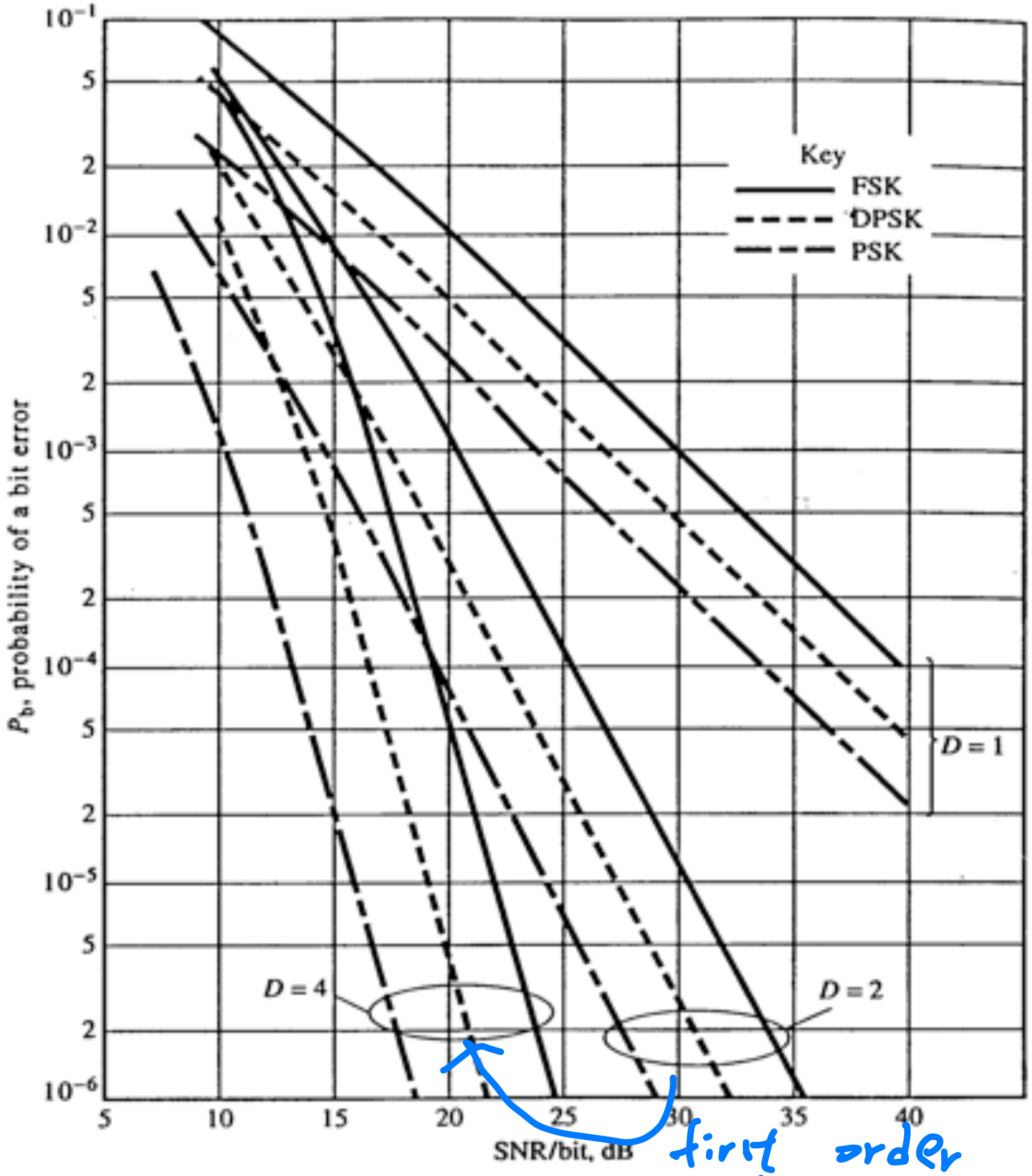
1☆ trouble: SNR penalty :(

voice application $P_e \approx 10^{-2}/10^{-3}$

video streaming ⇒ Pe even less!

Effect of Flat Fading Channels

- ►Flattening of BER Curves
- ►At BER = 10^(-3), the SNR penalty = 15dB!!
- ►Solution → Diversity



first order solution

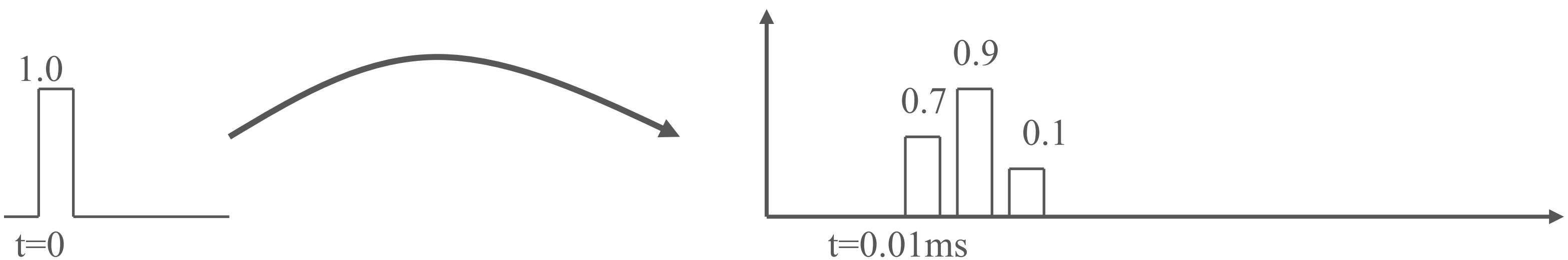
Frequency Selective Fading Channels

- Wideband Transmission:
  - Coherent BW < Signal BW.
- "multipath" channel model.

trouble!!!

$$y(t)=\alpha_1 x(t-\tau_1)+\ldots+\alpha_L x(t-\tau_L)+n(t)$$

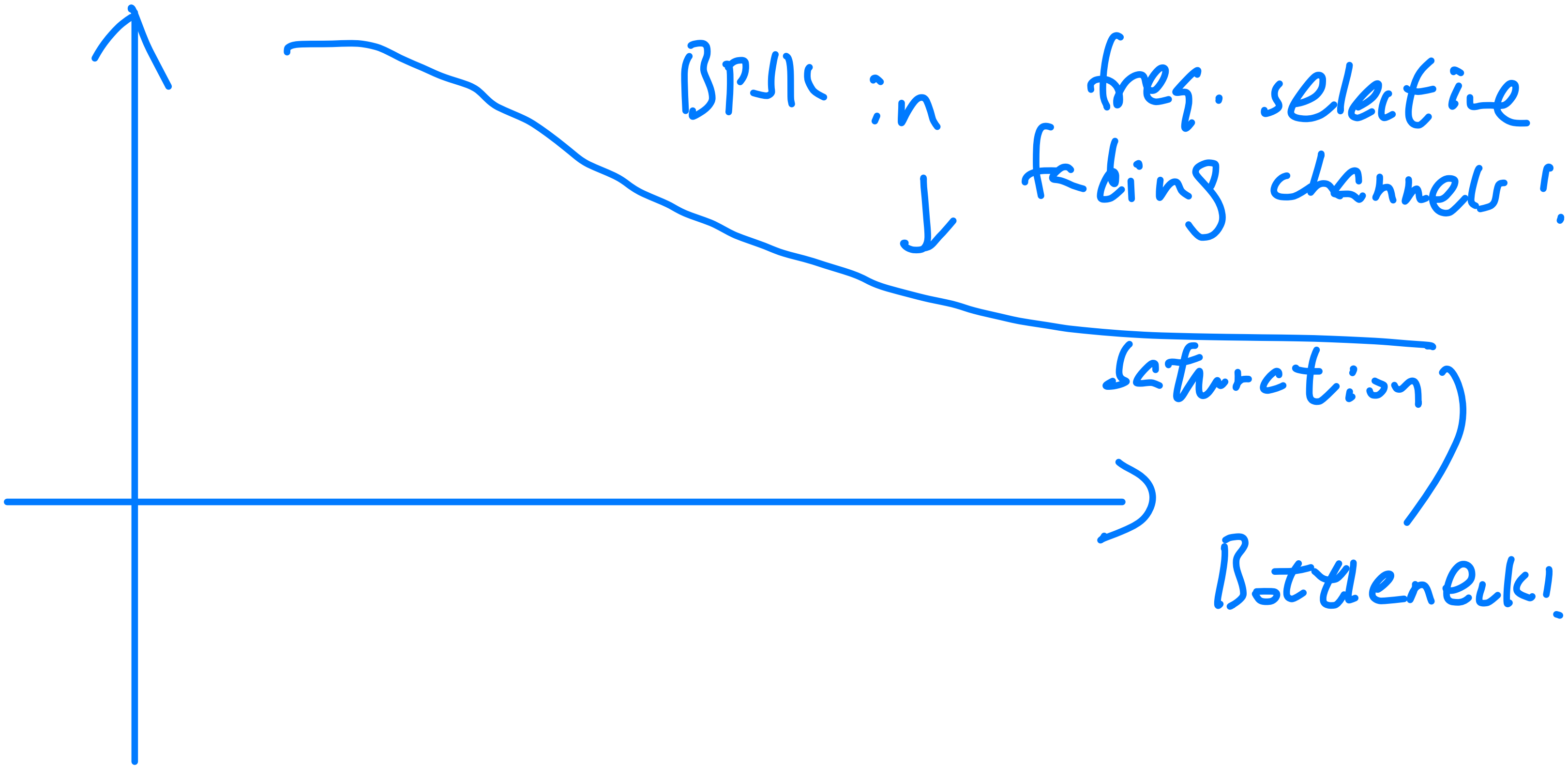
multiple fading echos!



L: resolvable signals!

- "equivalently"

$Wtx >> Btx \Rightarrow$ different fading response!



Effect of Frequency Selective Fading

- Multipath → Inter–symbol interference (ISI)
- In addition to flattening of BER curves, we have irreducible error floor.
- Solution
  - Diversity → take care of the flattening
  - Equalization → take care of error floor.

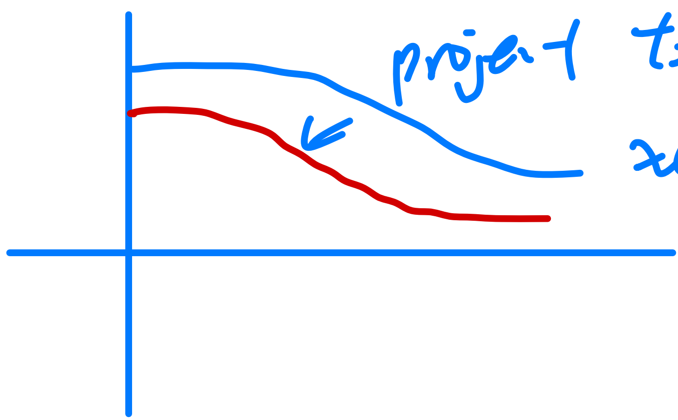
$y_4 = \alpha_1 \text{"}d\text{"} + \alpha_2 \text{"}c\text{"} + \alpha_3 \text{"}\hat{b}\text{"} + \alpha_4 \text{"}\hat{a}\text{"}$

good

$r_4 = y_4 - (\hat{\alpha}_2 \text{"}\hat{c}\text{"} + \hat{\alpha}_3 \text{"}\hat{b}\text{"} + \hat{\alpha}_2 \text{"}\hat{a}\text{"})$

"SIC" ~ error propagation

- cannot do perfect estimation accuracy only up to 2 decimal places
- If make decision error ⇒ add up more interference



project to null space

zero-forcing.

Can use multiple for SRC

optimal: try to decode all jointly based on $y_1, y_2, \dots$

$O(k^L)$ — # of resolvable symbols

exponential complexity!!!

Indoor propagation: $B_c \sim 1$ MHz

- wifi have low operating cost.

	Transmition Bandwidth: B_{WFT}	no control! B_c	#multi path L	
2G (GSM)	200kHz	~100kHz large delay spread!	2 copies	equalization
3G (GSM)	5MHz	~100kHz	50	?
4G (GSM)	20MHz	100kHz	100	? ?

bruteforce equalization not really work :(!

Tx → "build-in" structure

~ RX ~ low complexity equalization

wide band.

more serious frequency selection fading problem!

[☆] e.g. SS ~ RAKE Rx

chips structure!

$O(L)$ linear!

similar!

e.g. 2 <u>OFDM</u> ~ FFT ~ $O(N \log N)$

$$y_4 = \alpha_1 \text{"d"} + \alpha_2 \text{"c"} + \alpha_3 \text{"b"} + \alpha_4 \text{"a"} + \text{noise}$$

good term

$O(1)$

interference $O(P_s)$

$$P_e \sim SINR = \frac{|\alpha_1|^2 P_s}{(|\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2) P_s + N_0}$$

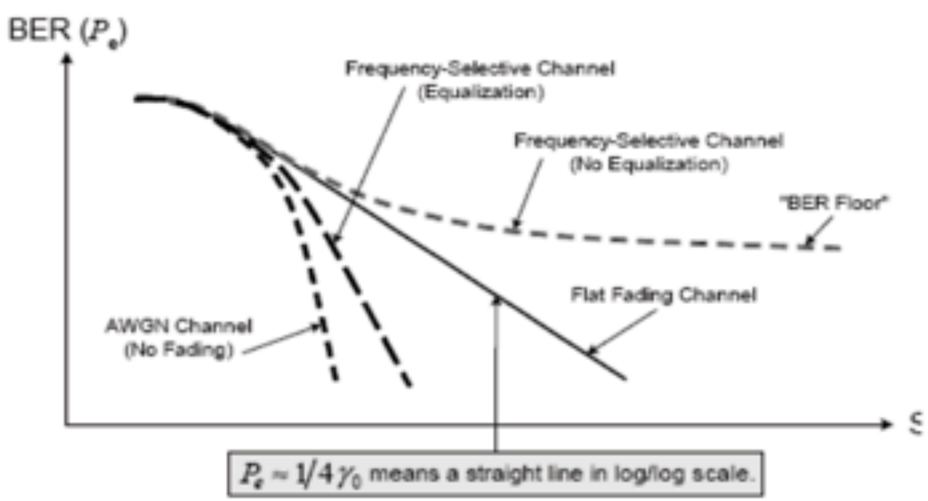
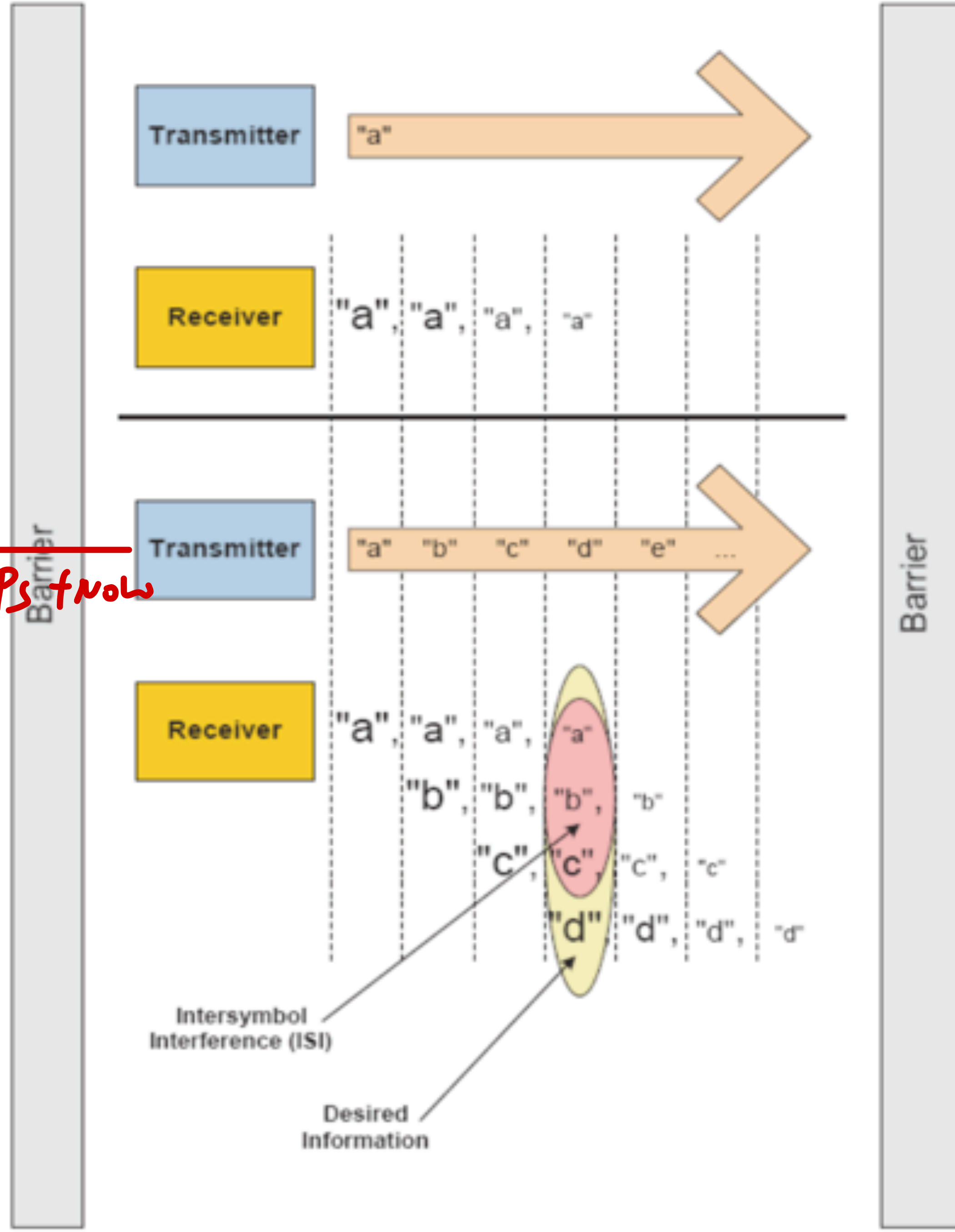
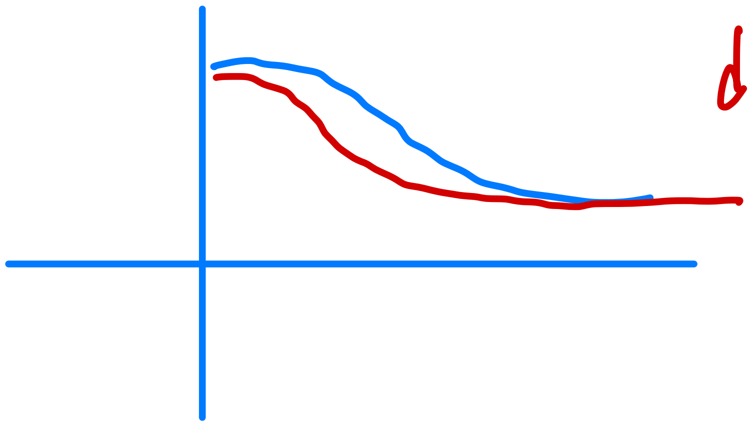


Figure 4.1 BER curves of BPSK in AWGN and flat fading char





diversity no-use :(

$$P_e \sim SNR \to \frac{|\alpha_1|^2 P_s}{N_0 W}$$ for small P_s (noise-limited region)

$$\to \frac{|\alpha_1|^2}{|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2}$$ for large P_s

$\to \beta_0$, get saturated :(, P_e also get saturated!

e.g.:

"equalization" → "SIC" ~ error propagation,
formal

remove ISI e.g.: zero-forcing,
project interference to null space
→ cost extra communication resources!

4-th time slot → demodulated previous signals + channel estimation for $\alpha_1, \alpha_2, \alpha_3$

⇒ regenerate the signal

"joint detection"
$= O(k^L)$
# of resolvable symbols!

$r_4 = y_4 - (\hat{\alpha}_2 \text{"}\hat{c}\text{"} + \hat{\alpha}_3 \text{"}\hat{b}\text{"} + \hat{\alpha}_4 \text{"}\hat{a}\text{"})$

hard to perfect channel estimation :(!
only up to several decimals
→ maybe add up interference !!!

flat fading

Fast and Slow Fading

time selective fading!

Very Fast Fading (Very rare in practical systems)

- Coherence time < Symbol period
- Channel variations faster than baseband signal variations

(not discussed :)

Fast Fading

Δt changes very fast

- Coherence time ~ 10 to a few hundred symbol periods

Slow Fading

frame order

- Coherence time ~ a thousand or more symbol

Δt changes slowly!

Other Types of Fading

- Non-LOS (Non Line of Sight) channel
  - e.g. Digital Cellular Outdoor Channel
  - α_I and α_Q are independent **zero-mean** Gaussian random variables
  - $|\alpha|$ is Rayleigh and phase is Uniform distributed
- LOS Channel

  see the receiver!

  dominant term!
  - e.g. Indoor Channel
  - α_I and α_Q are independent Gaussian random variables with mean $A/\sqrt{2}$
  - $|\alpha|$ is Ricean distributed and phase is Uniform distributed

$$p_R(r) = \frac{r}{\sigma^2}\exp\left[-\frac{r^2 + A^2}{2\sigma^2}\right] I_o\left(\frac{Ar}{\sigma^2}\right) \quad \text{for } r > 0$$

if $A = 0$ $\Rightarrow$ Rayleigh

generation of Rayleigh

Monday, 7 October, 13

Fading (Summary)

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

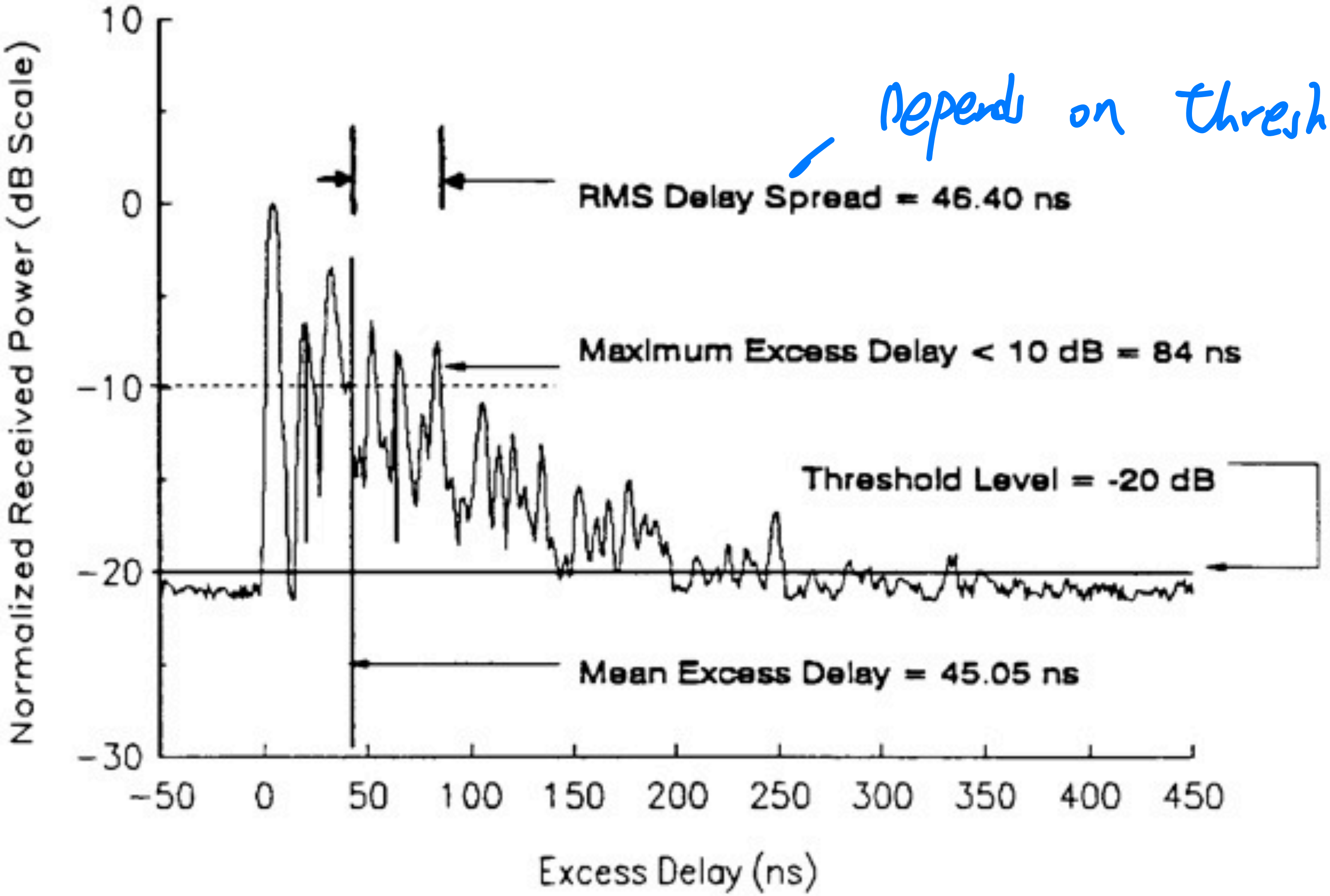
Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Fading (Summary)



Fading (Summary)

delay spread diffs

outdoor

Table 4.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]