Ch8: Digital Modulation

Information source and input transducer → Source Coding → Channel Coding → Modulator

what should be inputted into the channel

- Questions to be answered:
  - **Modulation: Why we need it?**
  - **Digital Modulation: Information modulates the carrier.**
  - **Correlator Implementation of MF**

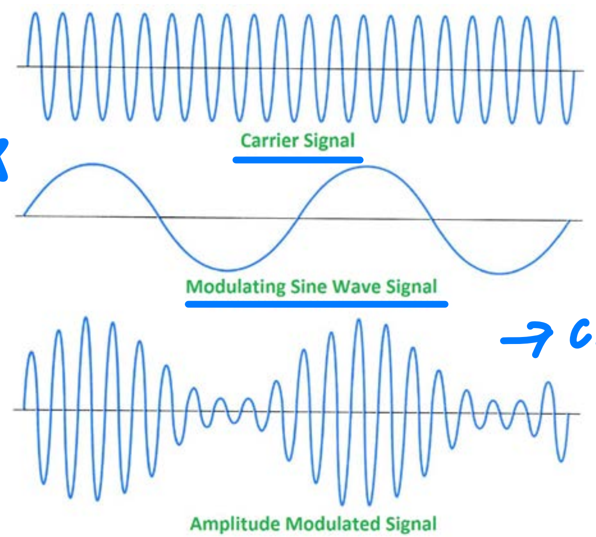
Ch8: Digital Modulation

Modulator → Channel → Demodulator (Matched Filter)

Information sink and output transducer ← Source Decoding ← Channel Decoding ← Demodulator (Matched Filter)

Ch8: Digital Modulation

x : information :

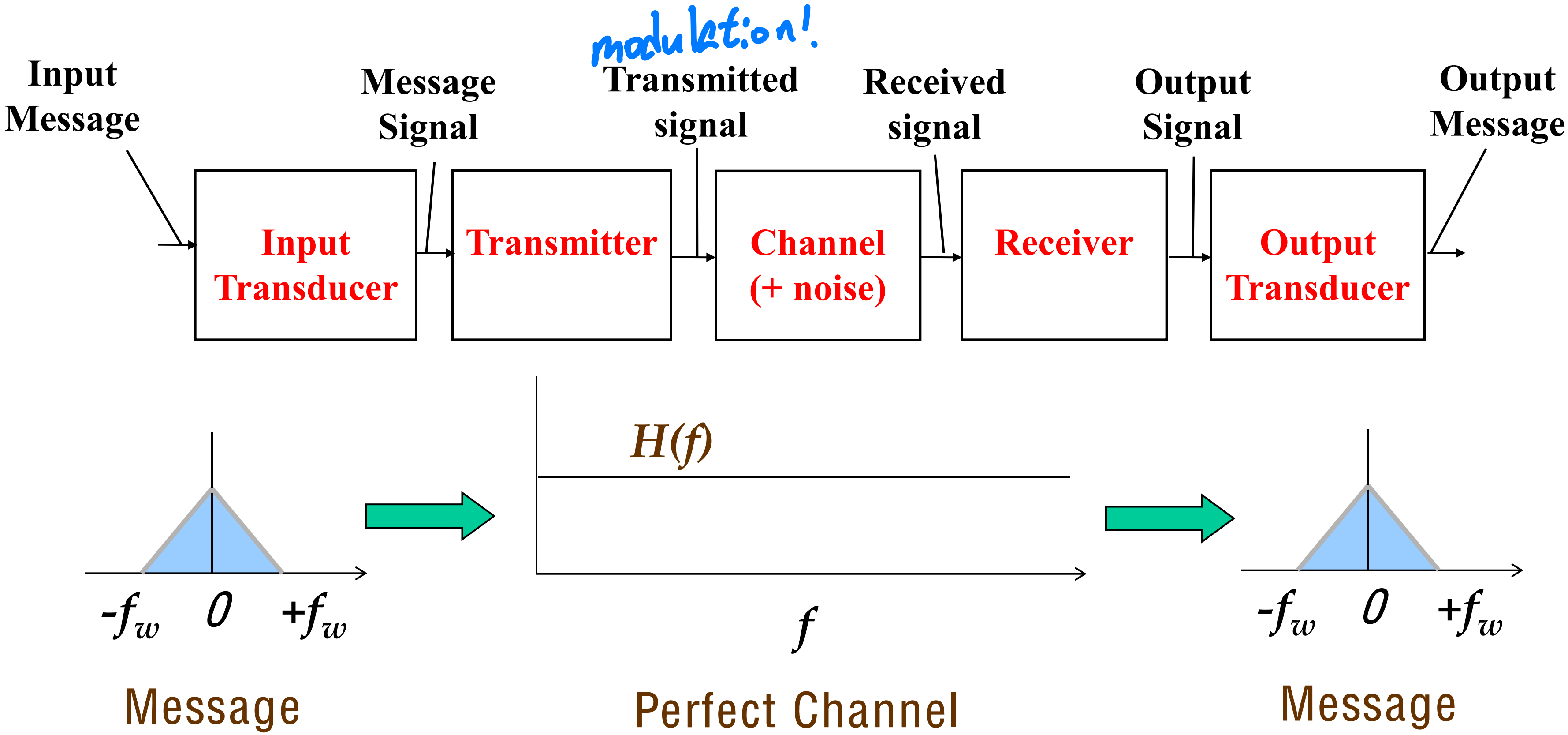


→ channel

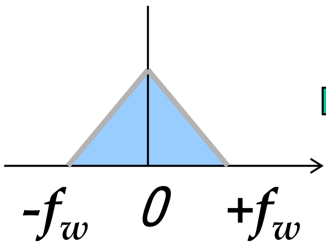
- ❑ **Modulation**
- ❑ Digital Modulation
- ❑ Correlator Implementation of MF

Communication Systems

- **Communications** involves the **transmission of information** from one point to another.

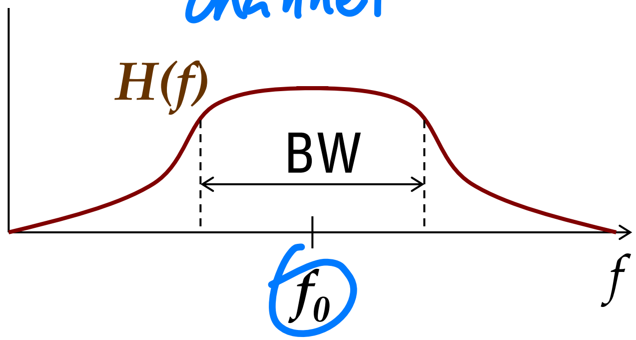


Modulation: Why we need it?



Message

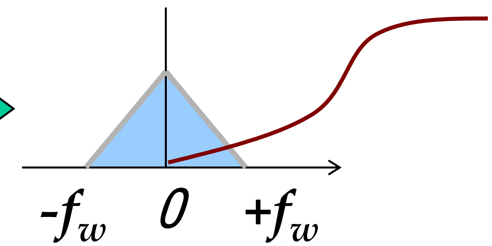
channel



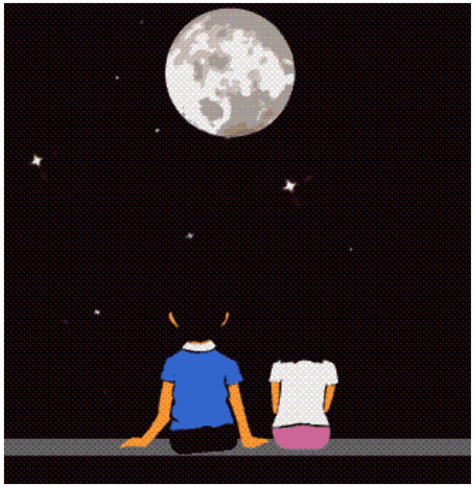
Channel

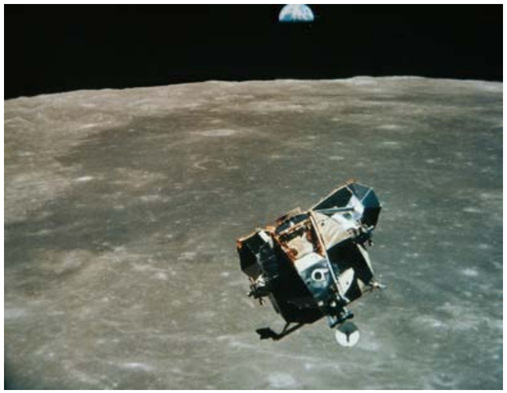
pass band = f_0

weak signal!



Message





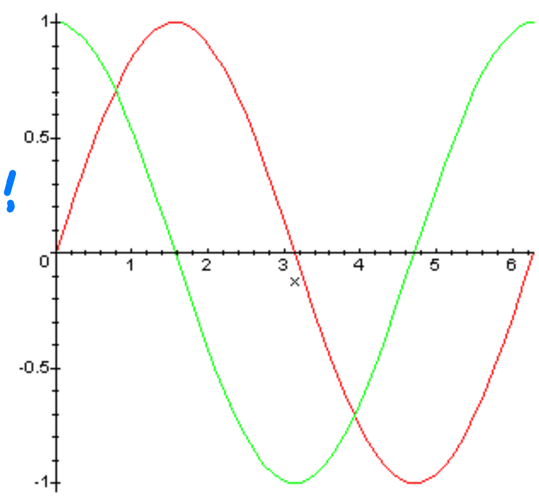
Modulation: Definition & Types

sometimes be changed due to this message!

- **Modulation**: A process by which **some parameters** of **a carrier** is **varied in one-to-one correspondence with** the **message so that the message can be recovered at the receiver.**
- Types of Modulations:
  - Analog Modulation — modulate Analog signal
    - Amplitude Modulation
    - Angle Modulation
    - …
  - Digital Modulation — modulate digital bits!
    - ASK
    - PSK
    - FSK
    - QAM
    - ……

$s_1(t) = A$ for "1"

$s_0(t) = -A$ for "0"



Linear Modulation

- Message signal: *m(t)*
- Carrier: $\boxed{A_c \cos(2\pi f_c t)}$
- Modulated carrier:

message! want recover m(t)

$$\boxed{x_c(t) = A_c m(t) \cos(2\pi f_c t)}$$

- Demodulation: Coherent demodulation

reconstruct the waveform!

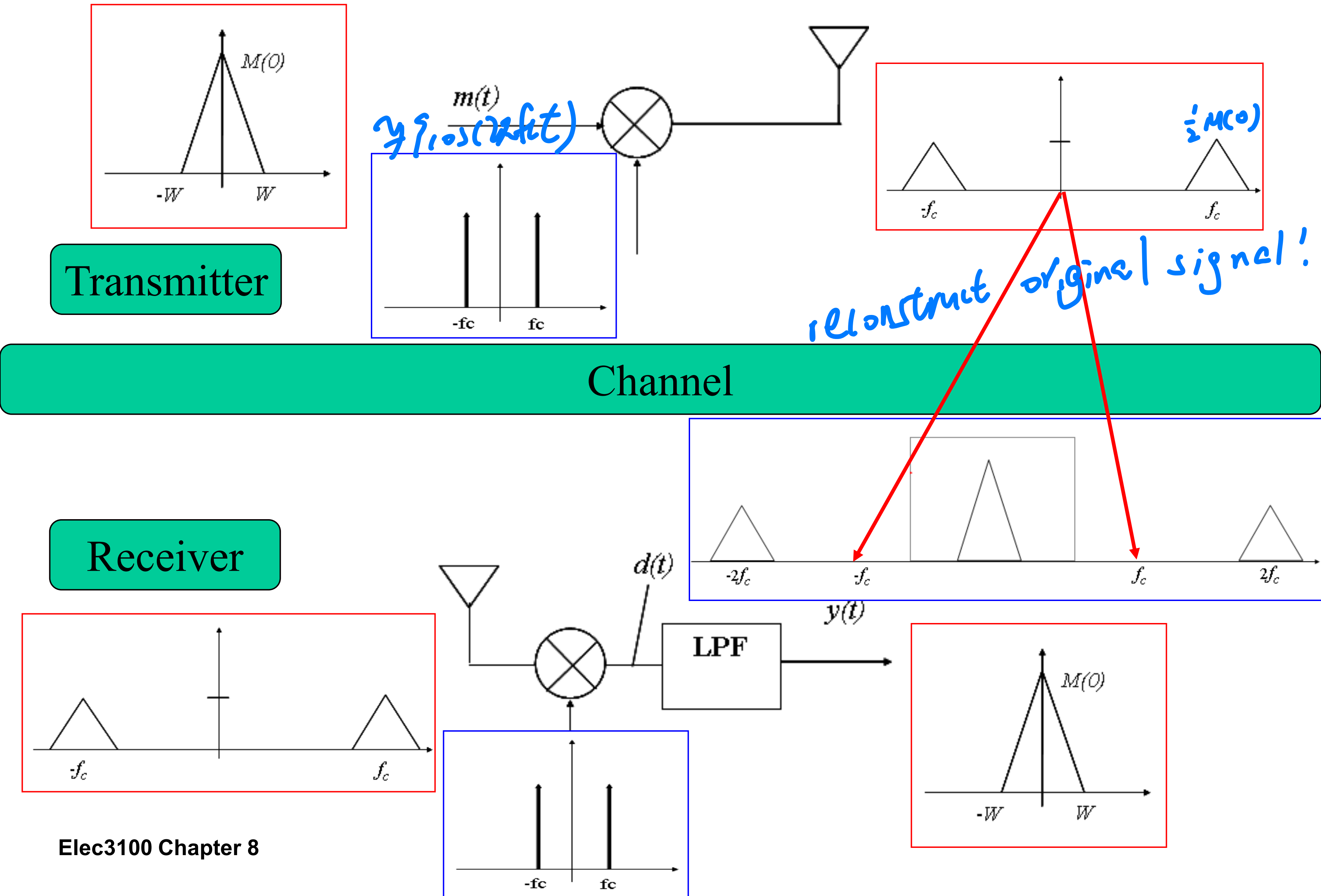
- Demodulated signal:

$$\boxed{[x_c(t)\cos(2\pi f_c t)]_{LP} = [0.5A_c m(t) + \cancel{0.5A_c m(t)\cos(4\pi f_c t)}]_{LP}}$$

Baseband highband

- LPF: *m(t)*

Frequency Domain



Form of Angle Modulation

other modulation (phase/frequency!)

$$x_c(t) = \underline{A_c} \cos(\omega_c t + \phi(t))$$

FM: $\frac{d\phi(t)}{dt} = k m(t)$

Amplitude modulation: $A_c \to A_c m(t)$

PM: $\phi(t) \to k_c m(t)$

- If we let $\psi(t) = \omega_c t + \phi(t)$ then $\psi(t)$ is known as the **instantaneous phase** of $x_c(t)$.

$m(t)$ = message!

- That is, $\psi(t)$ is a time varying function that describes the phase of the carrier.

Phase and Frequency Deviation

- The derivative of $\psi(t)$ provides us with the **instantaneous frequency** of the carrier. That is,

$$\omega_c(t) = \frac{d\psi(t)}{dt} = \omega_c + \boxed{\frac{d\phi(t)}{dt}}$$

- Thus,

$$\psi(t) = \omega_c t + \phi(t) \rightarrow \text{phase modulation (PM)}$$

$$\omega_c(t) = \omega_c + \boxed{\frac{d\phi(t)}{dt}} \rightarrow \text{FM}$$

Frequency: How quickly phase change

$\phi(t)$ is referred to as the **phase deviation.**

$\frac{d\phi(t)}{dt}$ is referred to as the **frequency deviation.**

- These two quantities describe the instantaneous phase and frequency variations of our angle modulated carrier.

Phase and Frequency Modulation

- In general, there are two types of angle modulation that make use of modulation of the instantaneous phase or frequency. They are:
  1. **Phase Modulation** (PM)
  2. **Frequency modulation** (FM)

- For PM, $\phi(t) = k_p m(t)$ where k_p is the **phase deviation constant** with units rad/unit of m(t).

Angle Modulation: FM

- Message signal: *m(t)*
- Carrier: $\boxed{A_c \cos(2\pi f_c t)}$
- Modulated carrier:

Frequency deviation carries information

change the frequency!

$$x_c(t) = A_c \cos\left[\omega_c t + \boxed{2\pi f_d \int_0^t m(\tau)d\tau}\right]$$

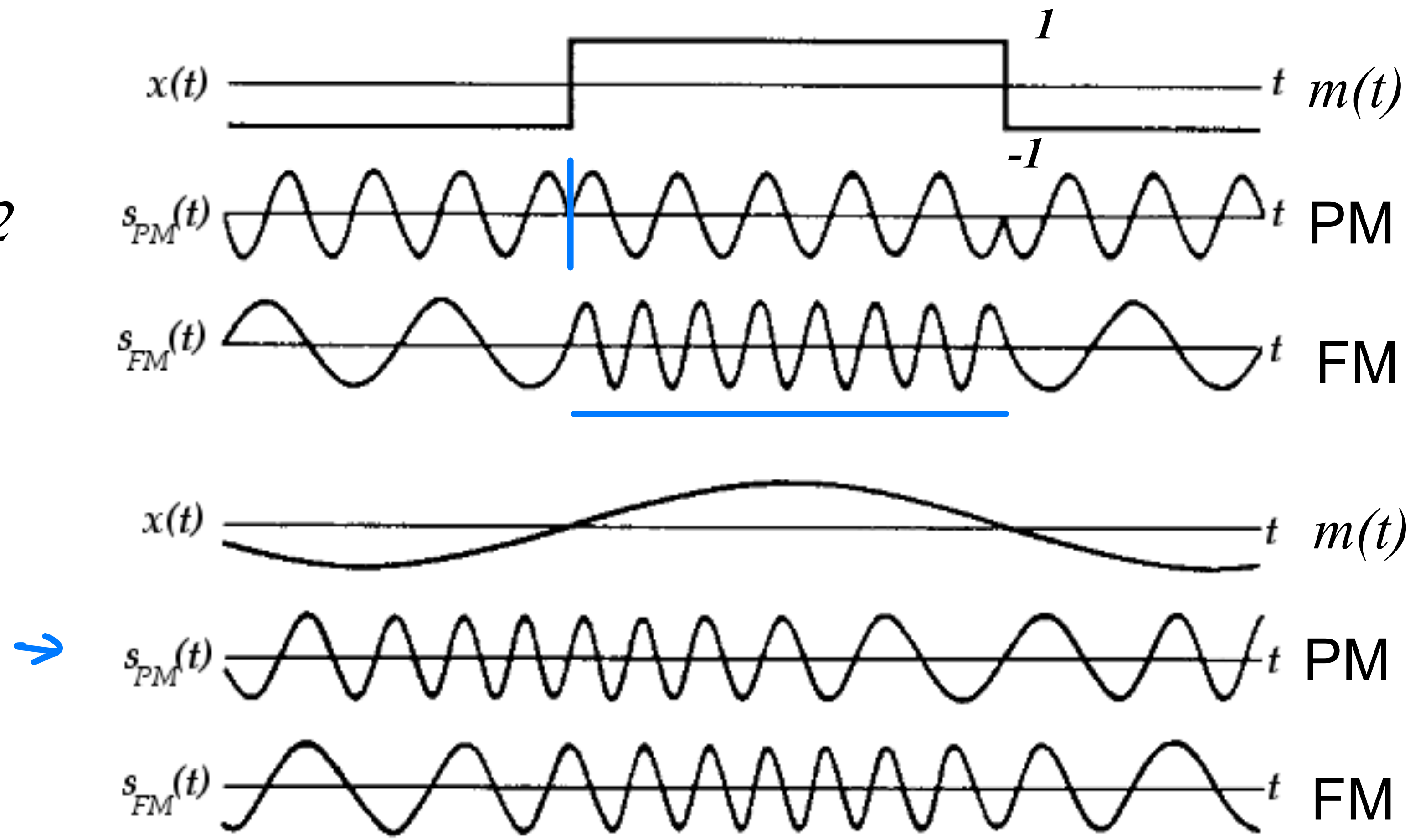
$$\frac{d\phi(t)}{d(t)} = m(t)$$

- Single tone: $m(t) = A_m \cos(2\pi f_m t)$

$$\boxed{x_c(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t]}$$

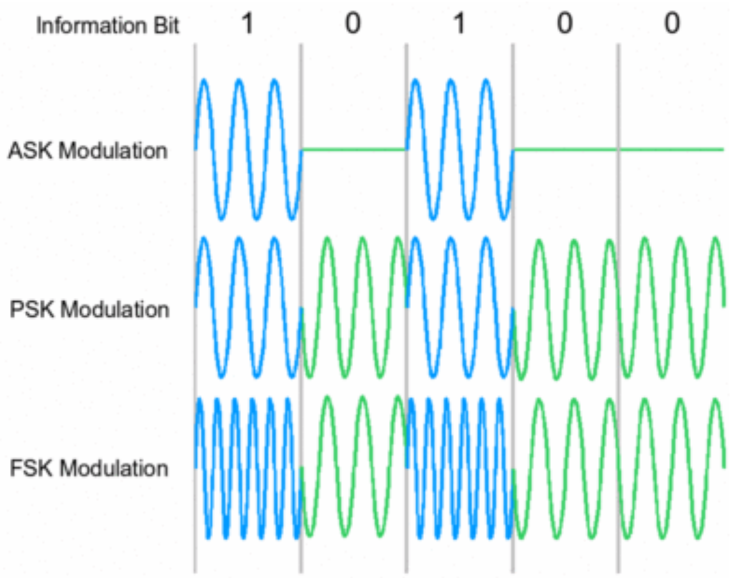
FM and PM Waveforms

$k_p = \pi/2$



Ch8: Digital Modulation

- ❑ Modulation
- ❑ **Digital Modulation**
- ❑ Correlator Implementation of MF



Digital Modulation Schemes

large bandwidth! received distorted waveform, even x noise

Tx Bits: A, -A 1 0 1 1 0 1

Antipodal baseband signal

Amplitude

ASK

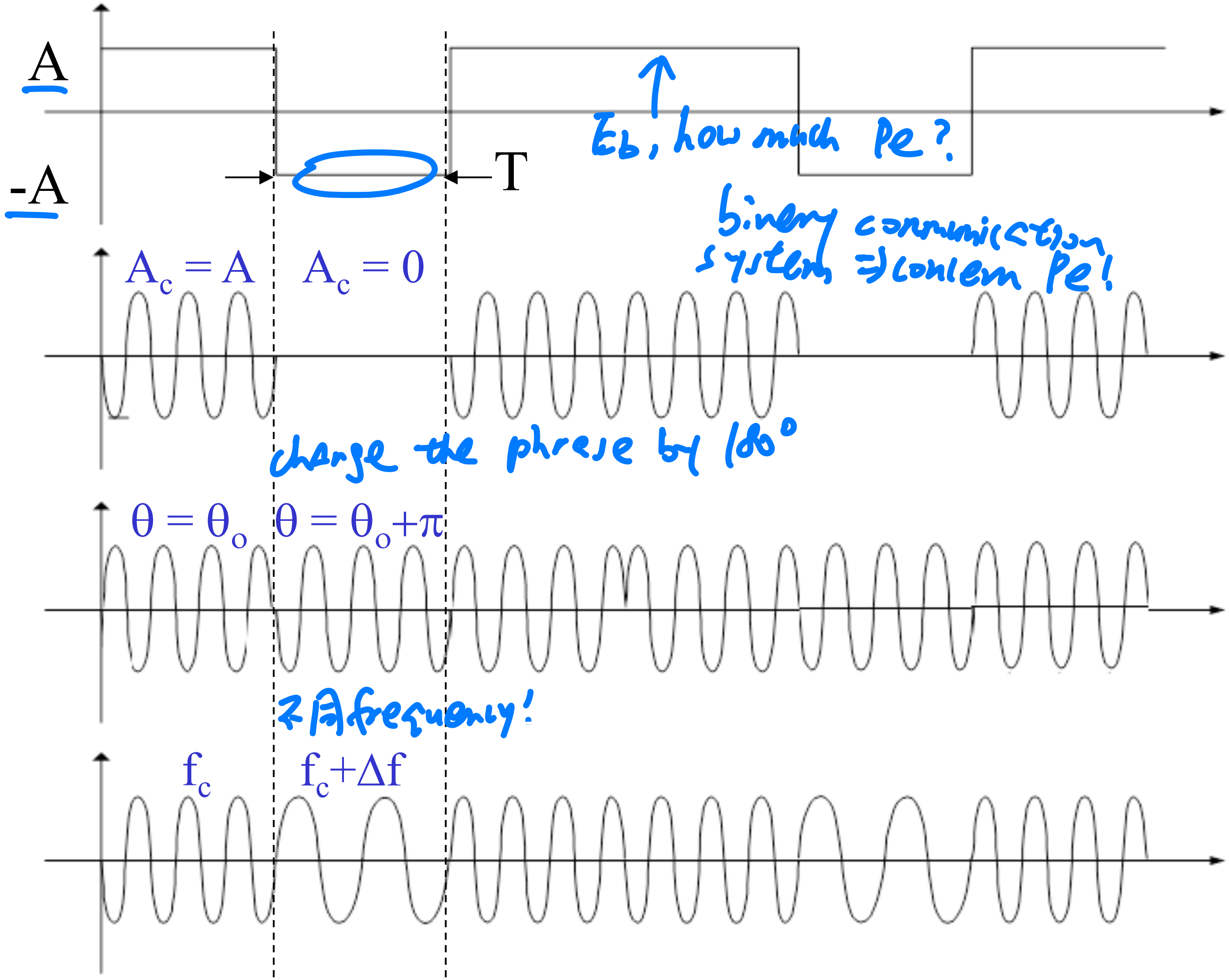
shift keying

PSK

phase

FSK

frequency!



General Signals

7.1:

- 1. Antipodal Signaling (Already studied)

Assume optimal receiver!

$$s_1(t) = A \qquad t \in [0,T]$$
$$s_0(t) = -A \qquad t \in [0,T]$$

$$P_e = Q\left[\sqrt{\frac{E_g}{2N_o}}\right] = Q\left[\sqrt{\frac{E_1 + E_0 - 2\rho_{10}\sqrt{E_1 E_0}}{2N_0}}\right]$$

$$E_1 = \int_0^T s_1^2(t)dt = A^2T \qquad \underline{E_0 = \int_0^T s_0^2(t)dt = A^2T}$$

$$E_b = \frac{1}{2}[E_1 + E_0] = \frac{1}{2}\left[A^2T + A^2T\right] = A^2T \qquad \text{energy per bit}$$

$$\rho_{10}\sqrt{E_1 E_0} = \int_0^T s_1(t)s_0(t)dt = -A^2T$$

$$\boxed{P_e = Q\left[\sqrt{\frac{4A^2T}{2N_o}}\right] = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]}$$

General Signals

2. Non-Return to Zero (NRZ)

increase error rate!!!

$$s_1(t) = A \qquad t \in [0,T]$$
$$s_0(t) = 0 \qquad t \in [0,T]$$

$$P_e = Q\left[\sqrt{\frac{E_g}{2N_o}}\right] = Q\left[\sqrt{\frac{E_1 + E_0 - 2\rho_{10}\sqrt{E_1E_0}}{2N_0}}\right]$$

$$E_1 = \int_0^T s_1^2(t)dt = A^2T \qquad E_0 = \int_0^T s_0^2(t)dt = 0$$

orthogonal signal

$$\rho_{10} = 0 \Rightarrow \frac{E_1+E_0}{2} = E_b$$

$$\Rightarrow Q\sqrt{\frac{E_b}{N_0}}$$

$$E_b = \frac{1}{2}\left[A^2T + 0\right] = \frac{A^2T}{2}$$

energy per bit

$$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t)dt = 0$$

$$\boxed{P_e = Q\left[\sqrt{\frac{A^2T}{2N_o}}\right] = Q\left[\sqrt{\frac{E_b}{N_o}}\right]}$$

General Signals

3. Amplitude Shift Keying (ASK)

$$s_1(t) = A\cos(\omega_c t + \theta_c) \qquad t \in [0,T]$$

$$s_0(t) = 0 \qquad t \in [0,T]$$

$$E_1 = \int_0^T s_1^2(t)\,dt = \frac{A^2T}{2} \qquad E_0 = \int_0^T s_0^2(t)\,dt = 0$$

$$E_b = \frac{1}{2}\left[\frac{A^2T}{2} + 0\right] = \frac{A^2T}{4}$$

energy per bit

Same!

$$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t)\,dt = 0$$

$$\boxed{P_e = Q\left[\sqrt{\frac{A^2T}{4N_o}}\right] = Q\left[\sqrt{\frac{E_b}{N_o}}\right]}$$

- 4. <u>Phase Shift Keying (PSK)</u> or BPSK → binary!

VIP

Could also be expressed as extra π phase shift

$$s_0(t) = -A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$
$$s_1(t) = A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$

$$E_1 = E_0 = \int_0^T s_1^2(t)dt = \frac{A^2T}{2}$$

$$E_b = \frac{1}{2}\left[\frac{A^2T}{2} + \frac{A^2T}{2}\right] = \frac{A^2T}{2}$$

energy per bit

correlation!

-ve ⇒ ↓ Pe :) Good!

$$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t)dt = \boxed{-\frac{A^2T}{2}}$$

$$P_e = Q\left[\sqrt{\frac{E_1 + E_0 - 2\rho_{10}\sqrt{E_1E_0}}{2N_0}}\right]$$

General!!!

$$\boxed{P_e = Q\left[\sqrt{\frac{2A^2T}{2N_o}}\right] = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]}$$

General Signals

special

5. Frequency Shift Keying (FSK)

duration to send one bit!

$$s_0(t) = A\cos(\omega_1 t + \theta_c) \quad t \in [0,T]$$
$$s_1(t) = A\cos(\omega_2 t + \theta_c) \quad t \in [0,T]$$

Suppose $f_2 > f_1$ 不同 frequency!

$s_0(t)\ s_1(t)$ orthogonal!

Let $\Delta f \triangleq f_2 - f_1 = \frac{n}{T} = nR$ **Multiple of data rate**

$\Rightarrow \int s_0(t)s_1(t) = 0$

Frequency separation

Source data rate

General Signals

- Now,

$$E_g = \int_0^T [\overset{s_1(t)}{A\cos(\omega_2 t + \theta_c)} - \overset{s_0(t)}{A\cos(\omega_1 t + \theta_c)}]^2 dt$$

$$= \int_0^T A^2 \cos^2(\omega_2 t + \theta_c)dt + \int_0^T A^2 \cos^2(\omega_1 t + \theta_c)dt$$

$$-2A^2 \int_0^T \cos(\omega_2 t + \theta_c)\cos(\omega_1 t + \theta_c)dt$$

$$= \underbrace{A^2 T}_{E_1 + E_0} - \cancel{A^2 \int_0^T \cos[(\omega_2 - \omega_1)t]dt}$$

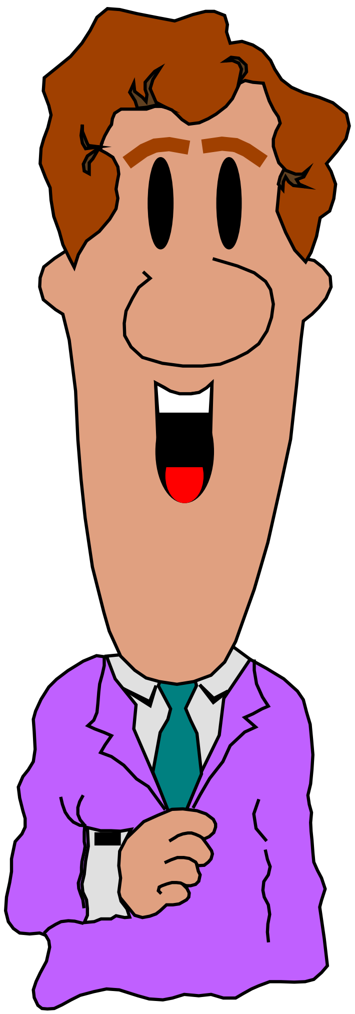
Pe depends on this!

$\to 0$ Since $\omega_2 - \omega_1 = \dfrac{2\pi n}{T}$

$$P_e = Q\left[\sqrt{\frac{E_g}{2N_0}}\right] = Q\left[\sqrt{\frac{A^2 T}{2N_0}}\right] = Q\left[\sqrt{\frac{E_b}{N_0}}\right]$$

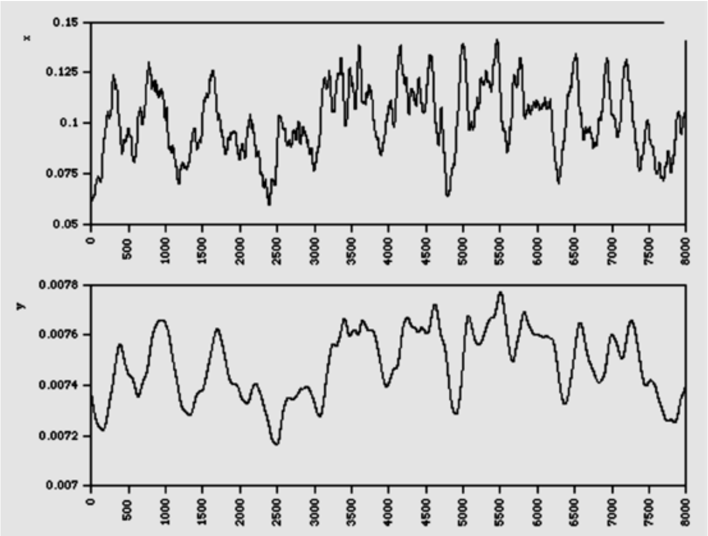
UPDATE

- Have introduced the optimum receiver for digital communications - **Matched Filter**.
- Derived P_e for various digital modulation schemes.
- **Will show that the implementation of optimum receiver**
  - **Correlator Receiver**.

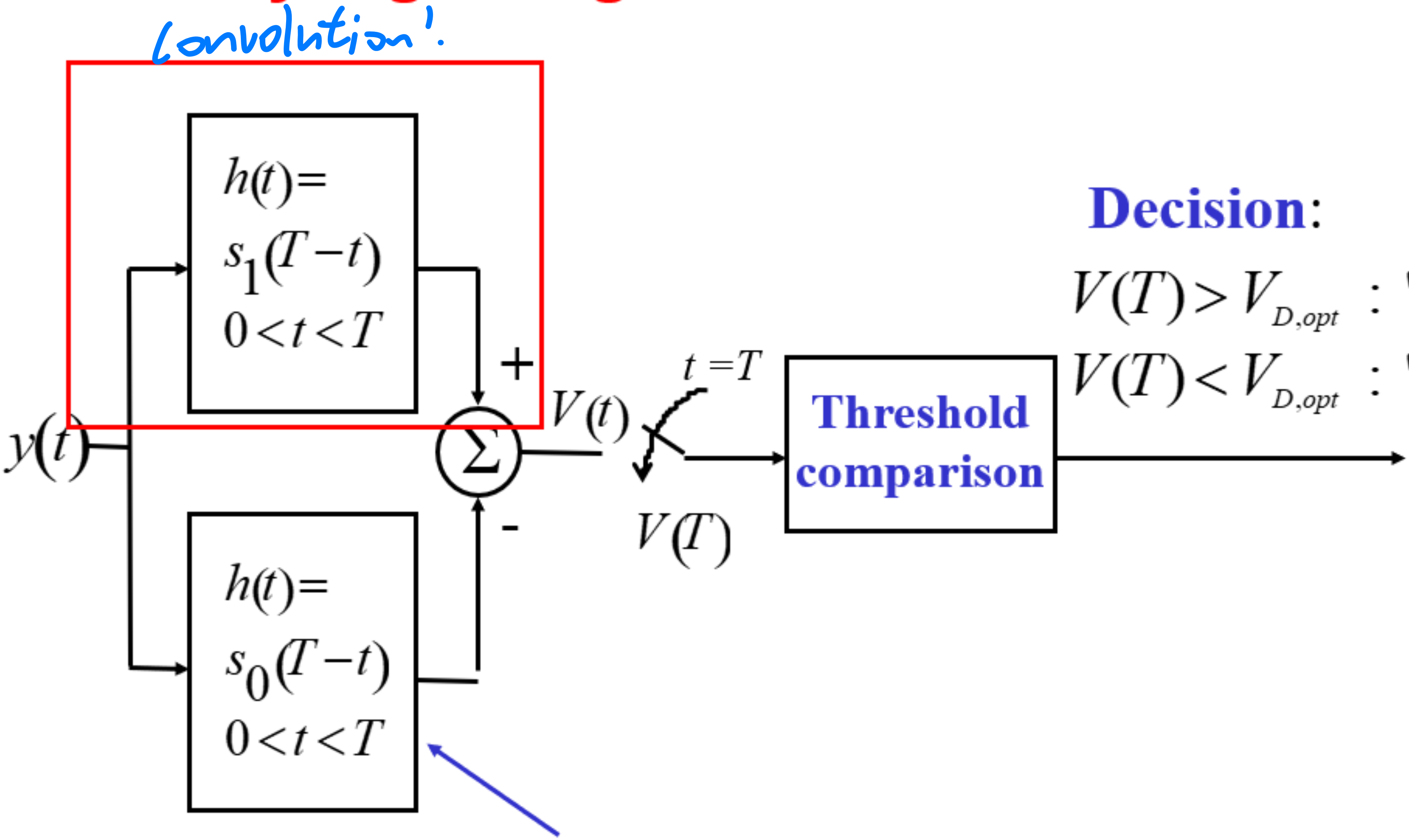


Ch8: Digital Modulation

- ❑ Modulation
- ❑ Digital Modulation
- ❑ **Correlator Implementation of MF**



Optimum (Matched filter) receiver for binary signaling in white Gaussian noise



Decision:

$V(T) > V_{D,opt}$: "1"

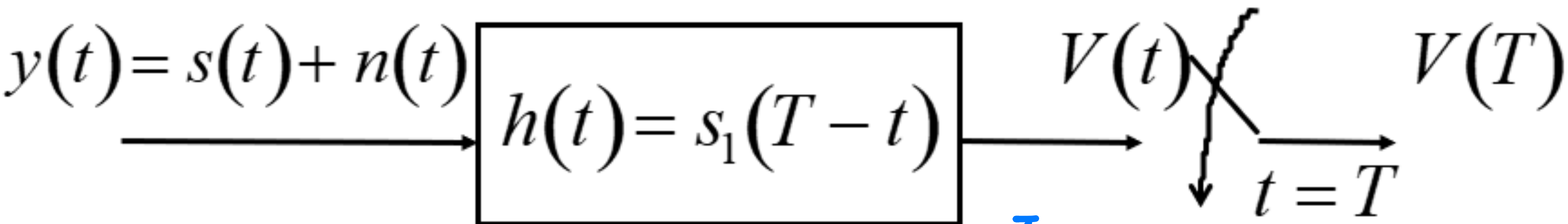
$V(T) < V_{D,opt}$: "0"

2 Matched Filters (each matched to $s_1(t)$ and $s_0(t)$)

Correlator Receiver

- We know that the optimum receiver structure consists of a matched filter so that

"upper half"



"upper half"

上一頁!

before sample!

$$V(t) = h(t) * y(t) = \int_0^T s_1(T-\tau)\, y(t-\tau)\, d\tau \quad \begin{cases} h(t) = s_1(T-\tau) & 0 \le t \le T \\ h(t) = 0 & \text{else} \end{cases}$$

Sampling at T

$$V(T) = \int_0^T s_1(T-\tau)\, y(T-\tau)\, d\tau$$

Change of variable $\alpha = T - \tau$

correlation!

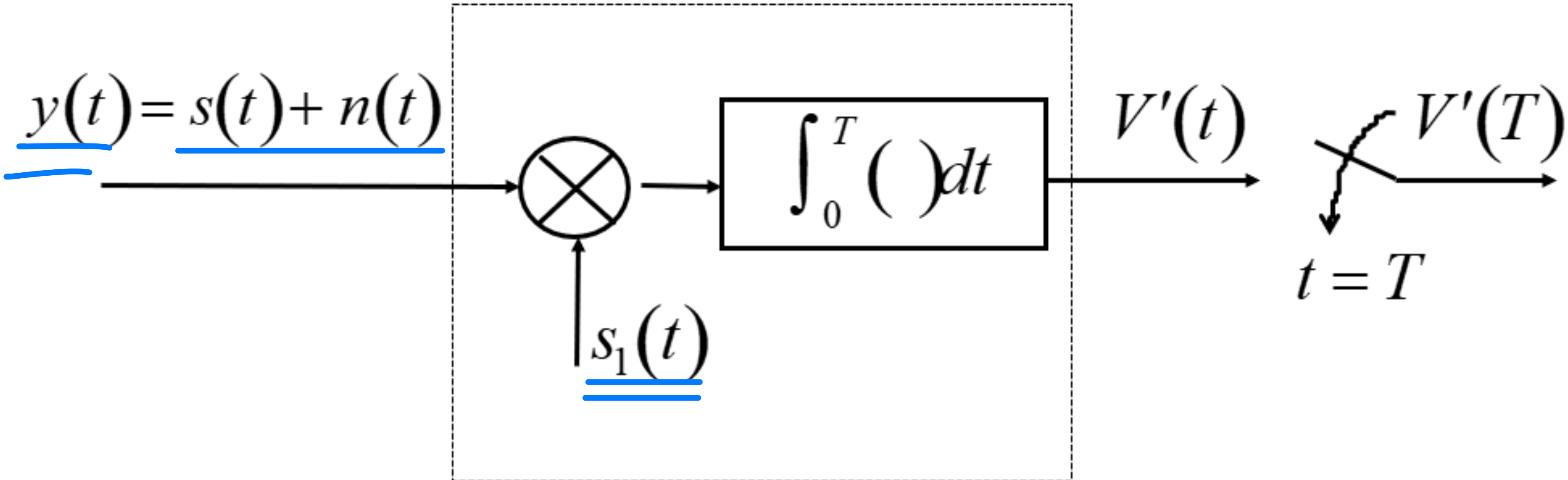
$$V(T) = \int_0^T s_1(\alpha)\, y(\alpha)\, d\alpha$$

Multiply and integration
Over a period of T

(correlated implementation of one period!

- We can therefore also **implement the matched filter as**

Multiplication + Integrate-and-Dump



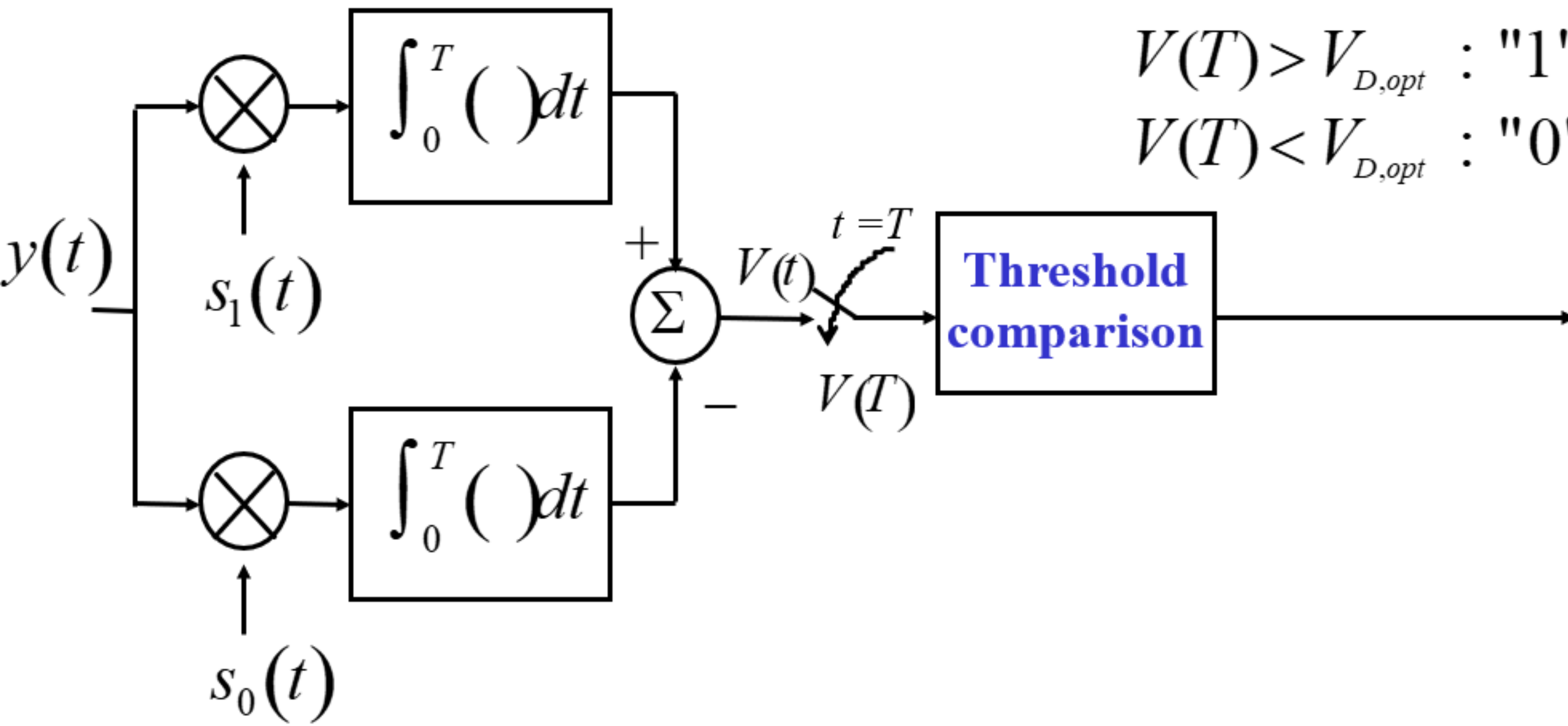
correlator!

Correlator Receiver: $V'(T) = \boxed{\int_0^T s_1(\alpha) y(\alpha) d\alpha}$

Implementation of Matched Filter By Correlator Receiver

Decision:

$$V(T) > V_{D,opt} \quad : \text{"1"}$$
$$V(T) < V_{D,opt} \quad : \text{"0"}$$



Example : ASK Receiver

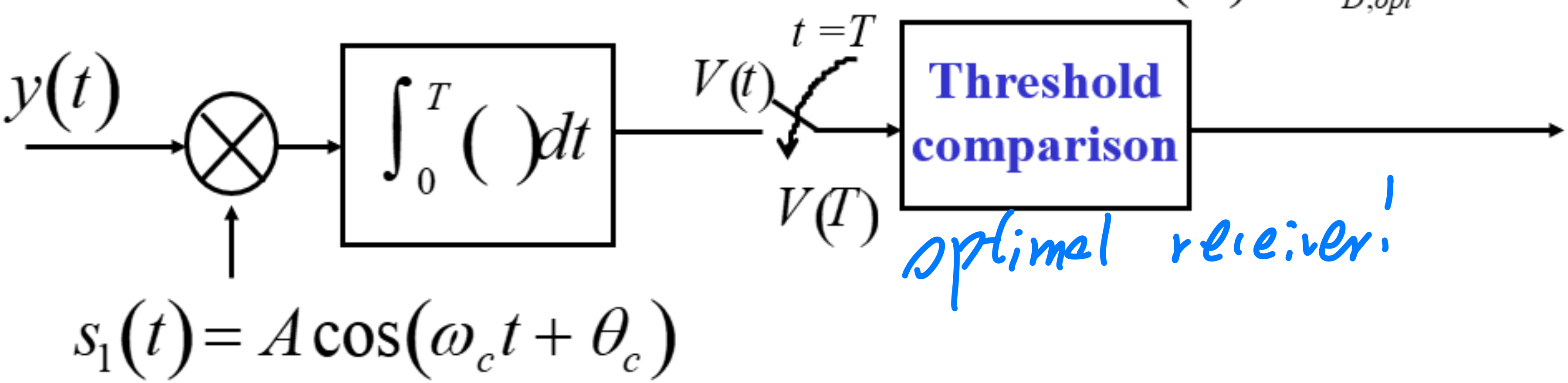
$$s_1(t) = A\cos(\omega_c t + \theta_c) \qquad t \in [0, T]$$

$$s_0(t) = 0 \qquad t \in [0, T]$$

Decision:

$$V(T) > V_{D,opt} \quad : \text{"1"}$$

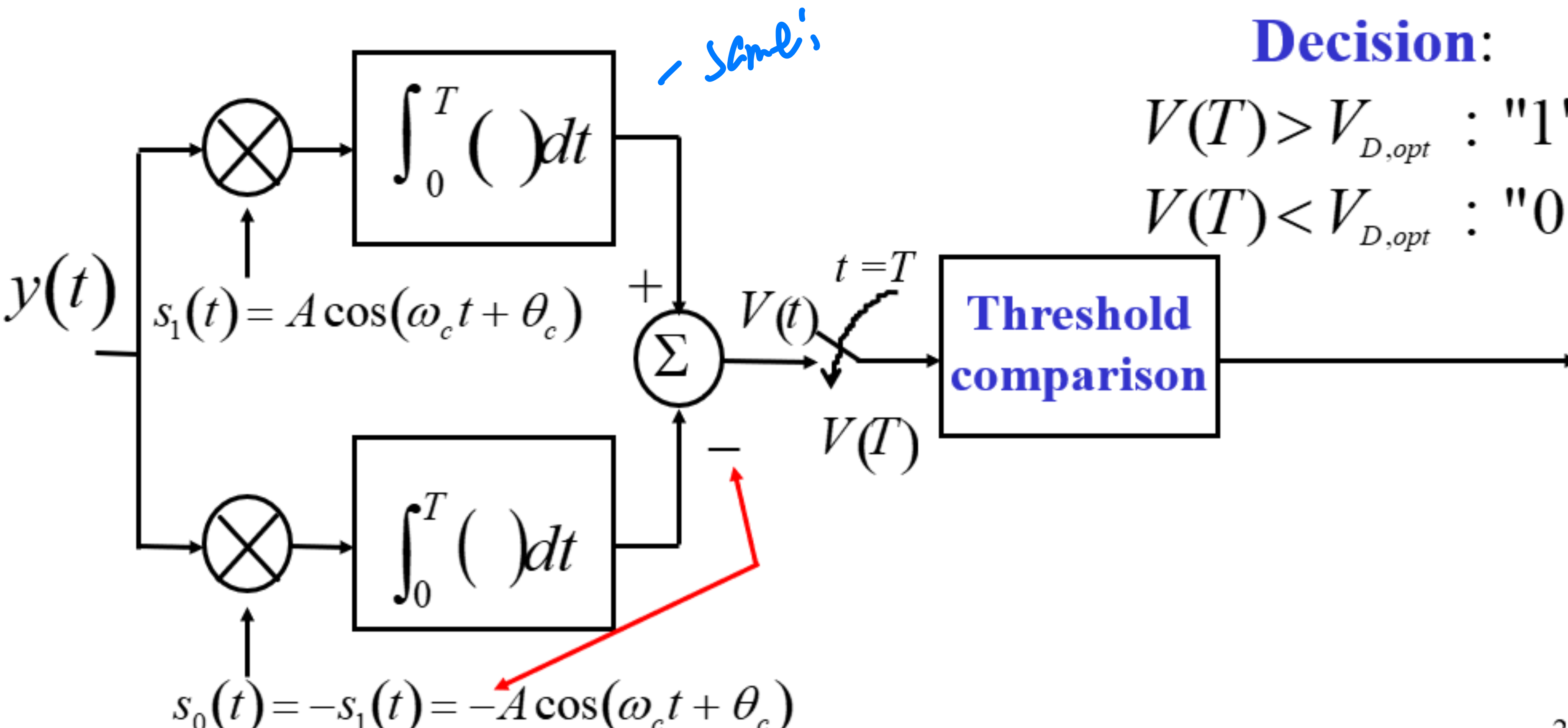
$$V(T) < V_{D,opt} \quad : \text{"0"}$$



optimal receiver!

Example : BPSK Receiver

$$s_0(t) = -A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$
$$s_1(t) = A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$



Decision:

$V(T) > V_{D,opt}$: "1"

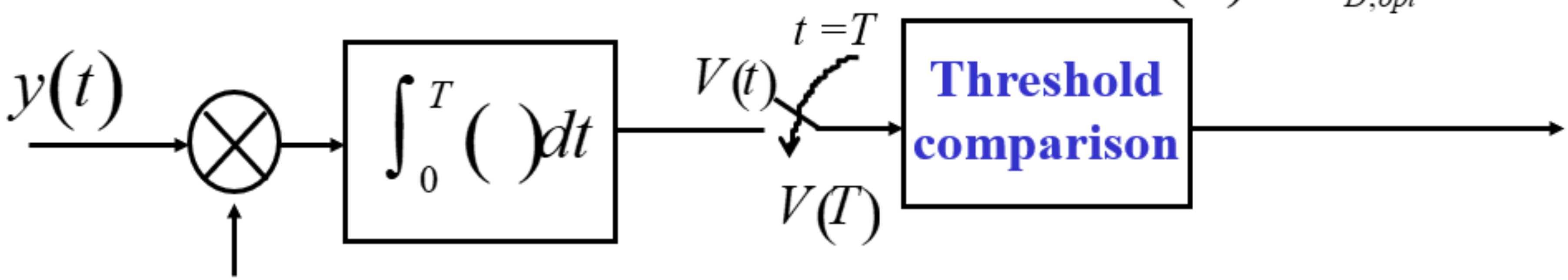
$V(T) < V_{D,opt}$: "0"

Example : BPSK Receiver

$$s_0(t) = -A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$
$$s_1(t) = A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$

Decision:

$$V(T) > V_{D,opt} \quad : \text{"1"}$$
$$V(T) < V_{D,opt} \quad : \text{"0"}$$



$$s_1(t) = A\cos(\omega_c t + \theta_c)$$

Example : BPSK Receiver

$$s_0(t) = -A\cos(\omega_c t + \theta_c) \quad t \in [0, T]$$

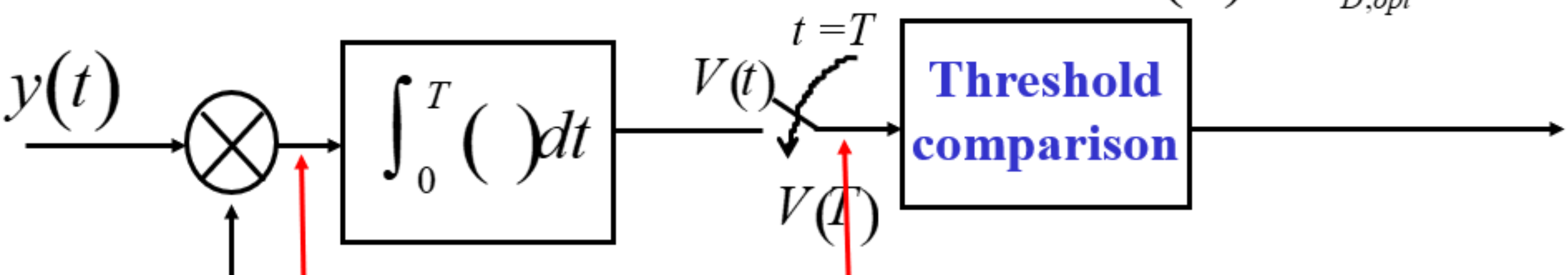
$$s_1(t) = A\cos(\omega_c t + \theta_c) \quad t \in [0, T]$$

Decision:

$$V(T) > V_{D,opt} \quad : \text{"1"}$$

$$V(T) < V_{D,opt} \quad : \text{"0"}$$

$s_1(t)$



$$s_1(t) = A\cos(\omega_c t + \theta_c)$$

ASK: (B)PSK: $\Big\}$ $A^2\cos^2(\omega_c t + \theta_c)$

ASK: (B)PSK: $\Big\}$ $A^2T/2$

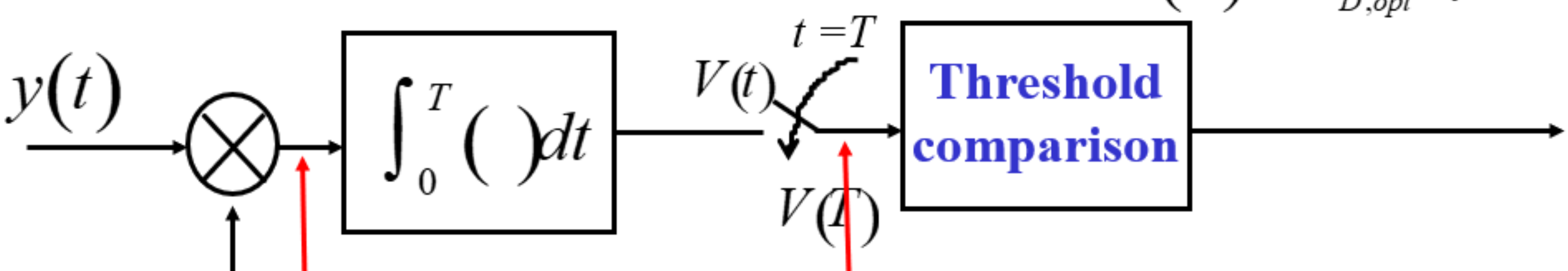
Example : BPSK Receiver

$$s_0(t) = -A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$
$$s_1(t) = A\cos(\omega_c t + \theta_c) \quad t \in [0,T]$$

Decision:

$$V(T) > V_{D,opt} \;:\; \text{"1"}$$
$$V(T) < V_{D,opt} \;:\; \text{"0"}$$

$s_0(t)$



$$s_1(t) = A\cos(\omega_c t + \theta_c)$$

ASK: 0
(B)PSK: $-A^2\cos^2(\omega_c t + \theta_c)$

ASK: 0
(B)PSK: $-A^2T/2$

$s_{o1}(T) - s_{o0}(T)$
is doubled in PSK

why BPSK Pe is smaller, Although similar implementation!!!

Summary

- Key things to remember
  - Digital modulation uses a finite set of possible signals to send signals
  - We have considered binary digital modulations only
  - Possible to detect digital signals with an Optimum receiver that minimizes the BER
  - Uses the optimized threshold and matched filter
  - Optimum receiver is nearly always assumed and therefore the formula for BER can be written as

optimal receiver!

$$\boxed{P_e = Q\left[\sqrt{\frac{E_g}{2N_0}}\right]}$$

depends on this want to be negative!

$$E_g = E_1 + E_0 - 2\rho_{10}\sqrt{E_1 E_0}$$

$\int_0^T s_0(t)s_1(t)dt$

Summary

- Implications of the formula affect Eg!
  - The shape of the signals affect the BER
  - No other parameter affects the BER when optimum receiver is used
  - The pulses can be anti-podal (best) so the correlation coefficient is -1 or orthogonal or anything else
  - However anti-podal is always the best

$$P_e = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$$

  - Orthogonal is not as good in terms of BER, but simple!

$$P_e = Q\left[\sqrt{\frac{E_b}{N_o}}\right]$$:(