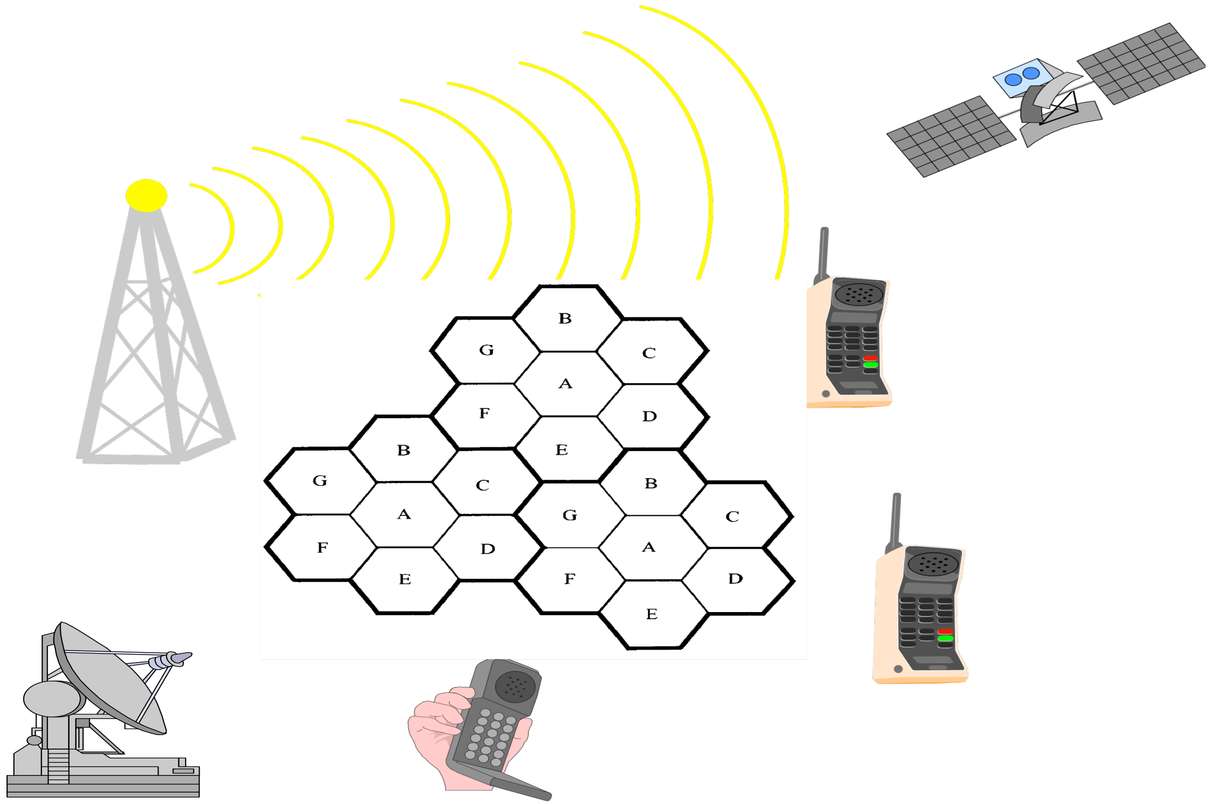
ELEC 4110 Digital Communications and Wireless Systems



Prof Vincent LAU

Department of Electronic and Computer Engineering

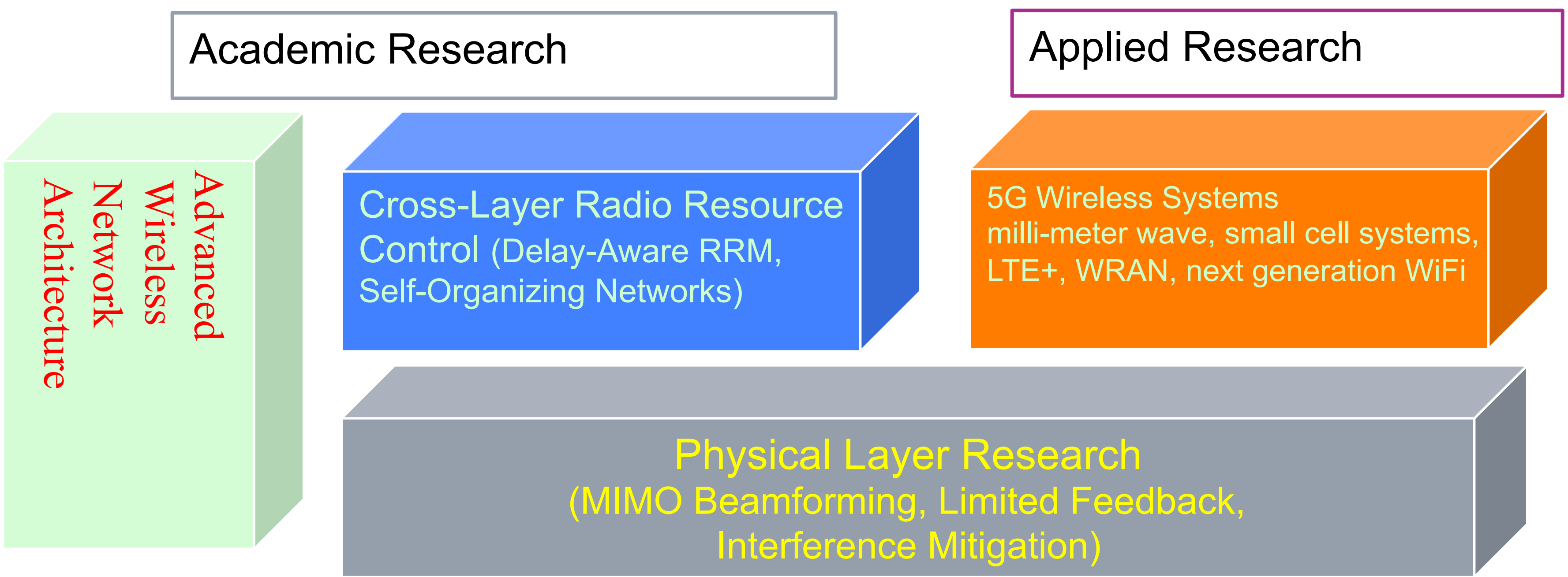
Hong Kong University of Science and Technology

Today's Lecture

- **Overview of Course**
- **Overview of Digital Communications**

Who am I?

B.Eng (1st Hons), Dept of EEE, University of Hong Kong (89-92)
System Engineer, HK Telecom (92-95) **Life was good!**
Croucher Foundation Scholarship and Sir Edward Youde Fellowship
Ph.D., Cambridge University (95-97) **Life was ….**
Research Scientist, Bell Labs, Lucent Technologies, New Jersey (98-04)
Joined Department of ECE, HKUST at 2004
Fellows of IEEE and HKIE; Chair Professor, Dept of ECE, HKUST;
Changjiang Chair Professor, Croucher Senior Research Fellow;
Director & Co-founder, Huawei-HKUST Joint Innovation Lab;
Technology Advisor for Qualcomm, Huawei, ASTRI, TCL, DJI…



Introduction

Digital Communications is the basic and key workhorse behind the Information Age.

This course is intended to discuss

- What is signal/information?
- How we represent signals / information?
- How to do wireless communications
- Overview and Evolution of Mobile Cellular Network

Intended Learning

Course Objectives	Details	Assessments
CO1	Recognize the key technological developments of digital communications and wireless systems	Multiple choice / short questions for students to demonstrate their recognition of the key technological developments of digital communications and wireless systems
CO2	Identify the fundamental principles related to digital communication technology	Problem sets / written exam questions for students to apply their knowledge of digital communication theory to solve simple engineering problems
CO3	Use Matlab to solve simple simulation problems in digital communications	Problem sets / written exam questions for students to apply their knowledge of digital communication theory to solve simple engineering problems
CO4	Able to comprehend technical specifications and understand how and why practical wireless systems are designed	Group term project (with project report) for students to apply their knowledge of digital communication theory to illustrate and document quantitative ways of performance evaluation in modern digital

Assessment Weighting

The final grade will be determined as a weighted combination of the results as following:

Homework (3 at 5%)	15%
Midterm	25%
Project	10%
Final Exam	50%

Homework

- It is very important that you do the homework yourself in order to understand the material of the course.
- Will benefit most from the homework if you attempt to do the problems <u>before</u> consulting your friends or TA.
- The homework should be <u>your own</u> and not a copy of your friends' solutions.
- Homework to be submitted to the TA by email.
- Detailed homework schedule will be announced in the web.

Textbook and References

Textbook

- R. E. Ziemer and R. L. Peterson, “Introduction to Digital Communication,” Prentice Hall, 2001.

Reference books

- R. E. Ziemer and W. H. Tranter, Principles of Communications: Systems, Modulation, and Noise, Houghton Mifflin, 4th Edition, 1995.
- John G. Proakis and M. Salehi, Communication Systems Engineering, Prentice Hall, 1994.
- Simon Haykin, Digital Communications, Wiley, 1988.
- Y.K. Kwok, V. Lau, Wireless Internet and Mobile Computing: Interoperability and Performance, John Wiley and Sons, 2007.

Course Notes

- Course notes are available from the canvas
- They will be posted at least 1 lecture in advance.
- Please read them before each lecture.
- I will also make some hand written comments on the white board and you should also note these in your copies of the notes.

Getting Help

- If you have trouble learning and following this course then it is important you seek help as early as possible.
- Tutorials will start NEXT week.
- Latest announcement and updates can be found in the course website.

Necessary Background

It is good for students taking this course to be familiar with the following topics:

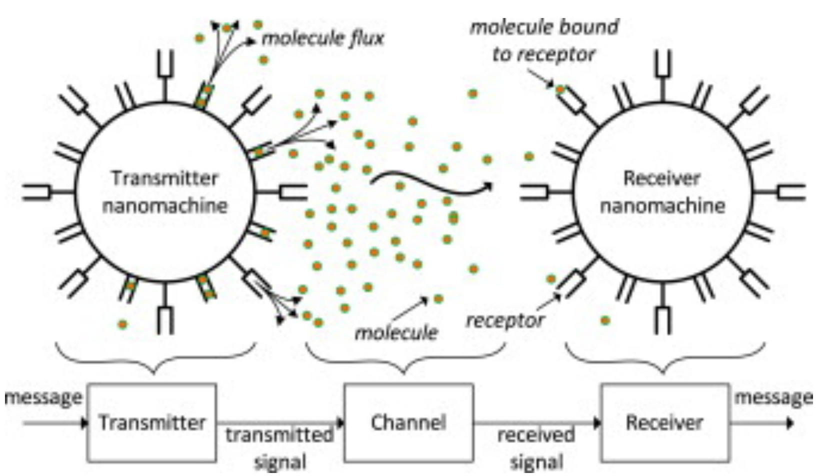
- Signal and Linear System Analysis (from ELEC3100)
    - Signal Models
    - Fourier Transform
    - Signals and Linear Systems
    - Sampling Theory
- Noise and Stochastic Processes (from ELEC3100)

Math as a Language

- Qualitative / Quantitative Approach
  - Mathematics may be used to make things precise
  - Mathematics is NOT the final goal
  - The final goal is to use the theory to explain
    - How things work in practice?
    - **Why** things work in practice?
- Math as a language
  - Presentation: Make insights precise
  - Comprehension: Read insights behind math equations

Communication

- What is "communication"?
  - Transfer of "information" from one location to another location
- Examples
  - Telecommunication
  - Short-range communication
  - Molecular communications



Why this is not a practical means of telecommunication?

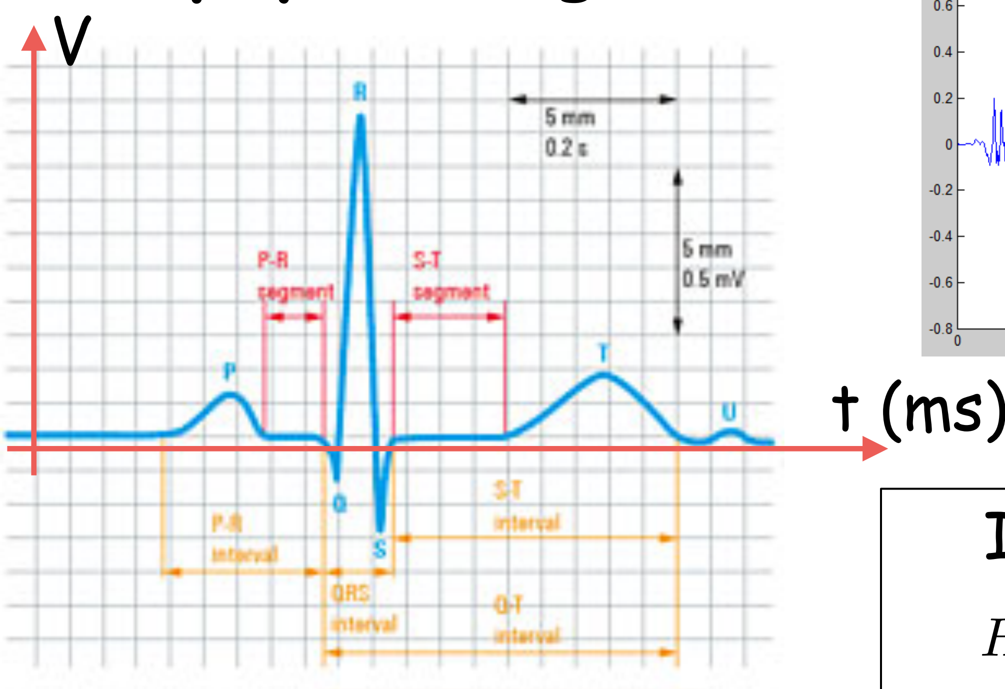
Attenuation occurs

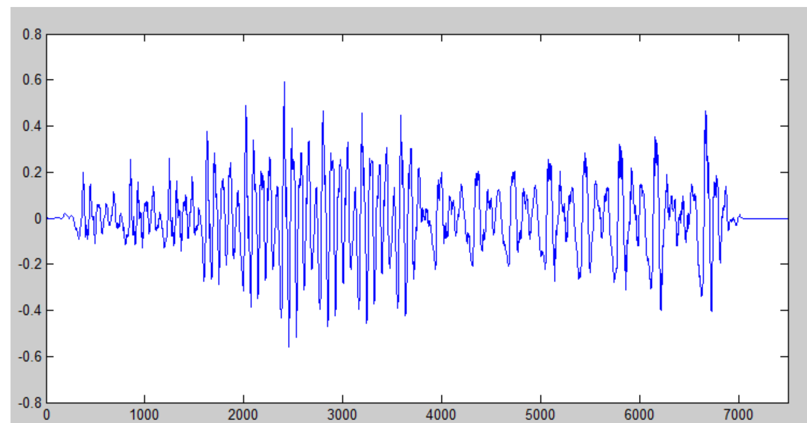
Passive system, power keep decaying to the surrounding

Part I - Basics of Signals

What is Information?

- Introduction to Communications
  - ⍰Information is represented by physical signals s(t)





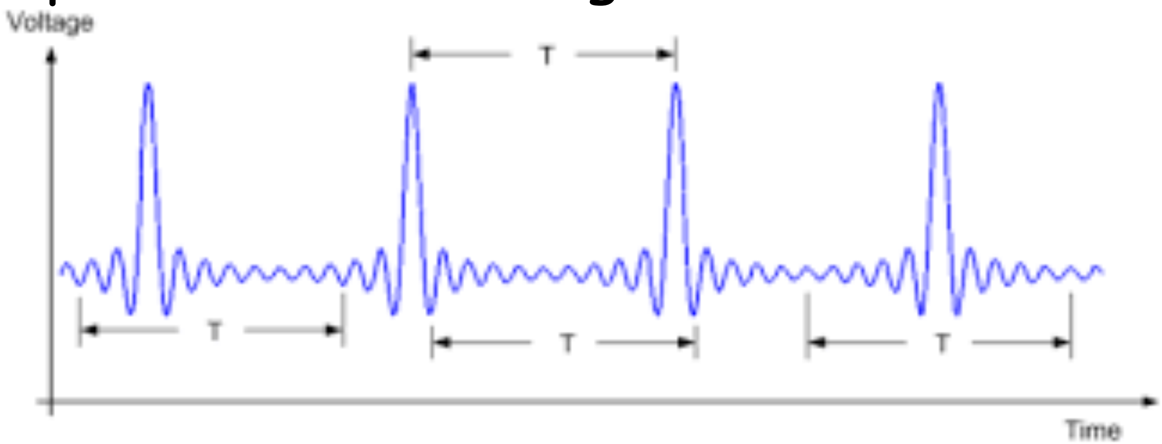
Information Entropy

$$H(X) = -\sum_X p(X) \log_2 p(X)$$

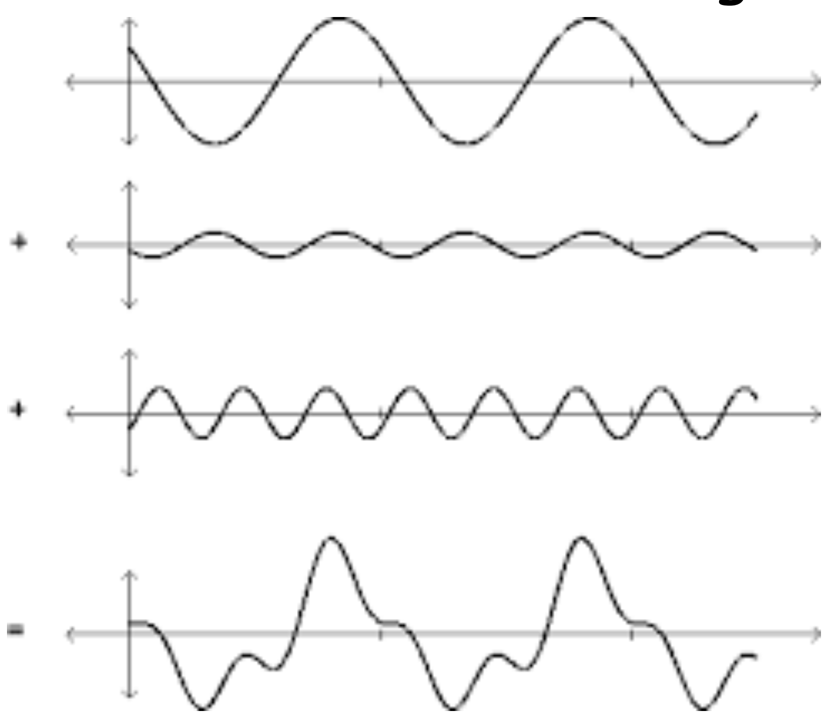
Representation of Signals -

- Time Domain Representation
  - Signal is represented by a time function s(t)
  - Waveform visually intuitive
  - If I want to describe a signal, I need to illustrate the entire waveform for $t \in [0, \infty)$
  - Periodic Signal:
    - Signals with a repeatable waveform called "cycle"
    - **Period T**: If there is a T>0 such that s(t+T) = s(t) for all t, then T is the period
    - **Frequency f** (# of cycles per second) f = 1/T Hz
-

Examples of Periodic Signals



Which one below has the highest frequency?

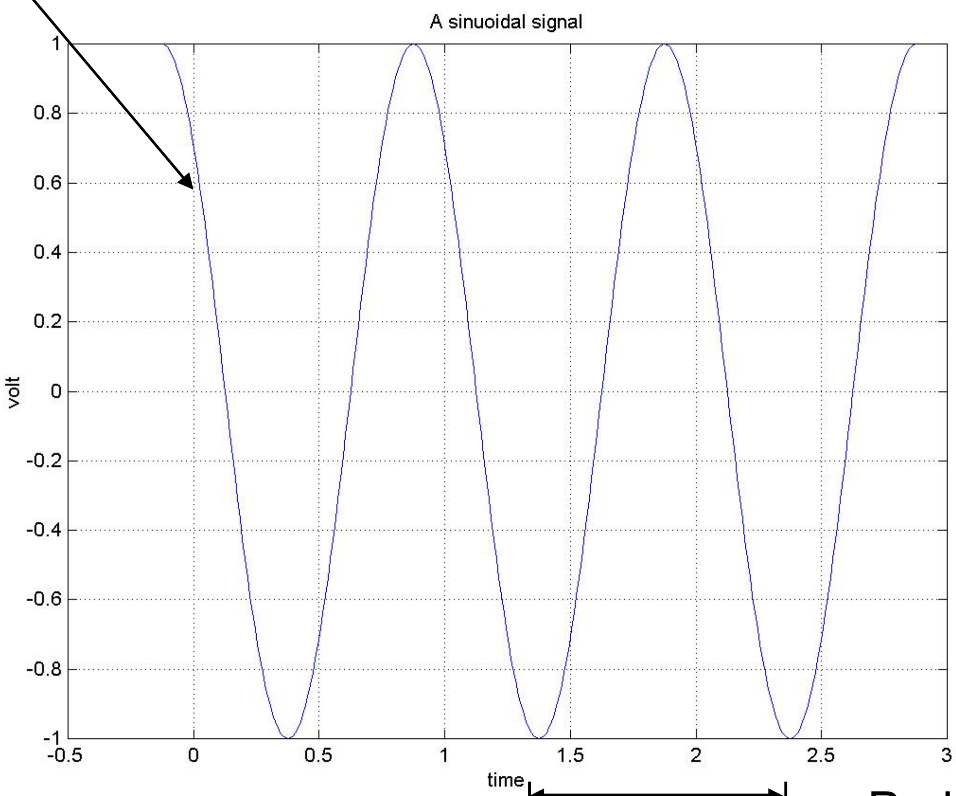


To describe a periodic signal, we just need to describe a cycle

Example - Sinusoidal Signals

Starting Phase = 30

$$s(t) = A\sin(\omega t + \phi) \quad T = \frac{2\pi}{\omega} \quad f = \frac{\omega}{2\pi}$$



Amplitude = A

Period = T

How to represent sinusoidal

To describe a sinusoidal signal, we just need to describe 3 numbers (Amplitude, frequency, phase)

$$s(t) = A\sin(\omega t + \phi)$$

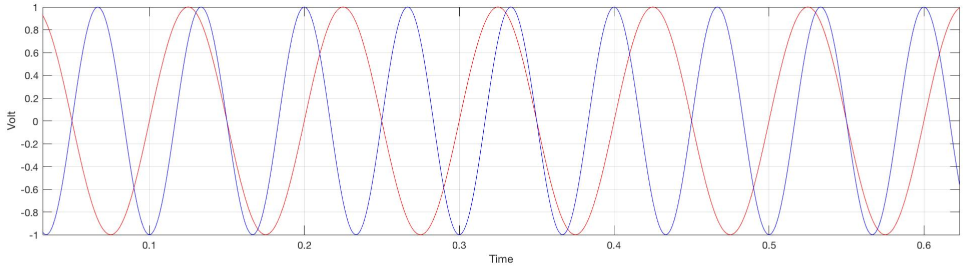
Sometimes, it is mathematically convenient to describe s(t) using complex sinusoidal:

$$\boxed{\begin{array}{l} s(t) = \Re\left[A\exp(j\phi)\exp(j\omega t)\right] = \Re\left[Z\exp(j\omega t)\right] \\ \text{where } Z = A\exp(j\phi) \text{ is the ''complex amplitude''} \end{array}}$$

Equivalently, we can fully characterise a sinusoidal signal using (Z, f).

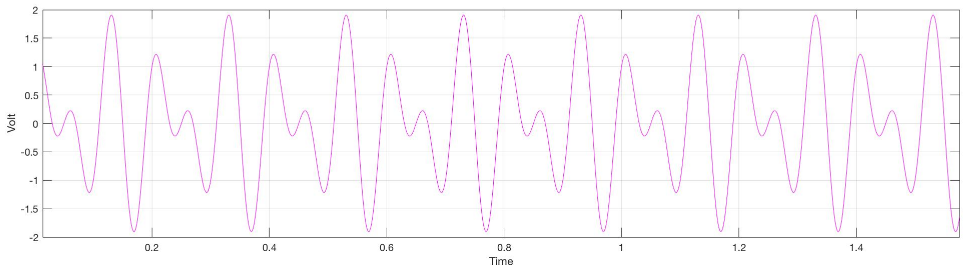
Discussion

- If $s_1(t)$ and $s_2(t)$ are periodic signals, is the following always periodic?
  - $s(t) = s_1(t) + s_2(t)$ no, let $\omega_1 = 2$ rad/s $\omega_2 = \sqrt{2}$ rad/s
  - $s(t) = s_1(t) \times s_2(t)$ no

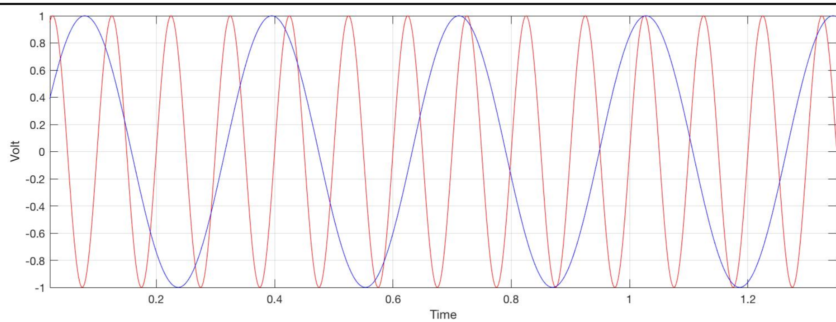


$$s_1(t) = \cos(2\pi(10)t)$$

$$s_2(t) = \cos(2\pi(15)t)$$

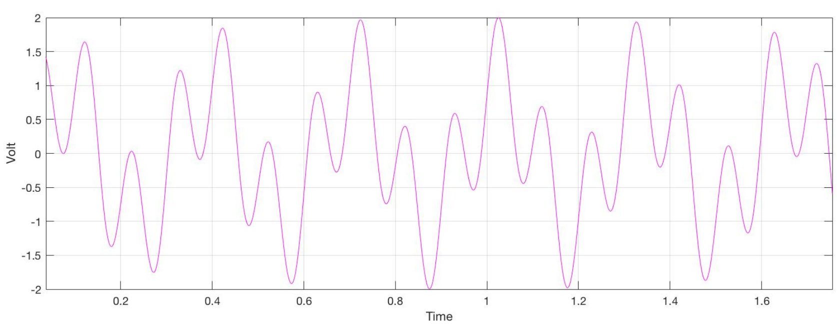


$$s_1(t) + s_2(t)$$



$$s_1(t) = \cos(2\pi(10)t)$$

$$s_2(t) = \cos(2\pi\sqrt{(10)}t)$$



$$s_1(t) + s_2(t)$$

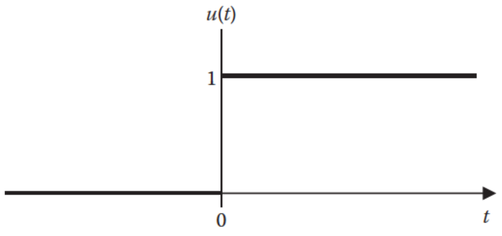
Aperiodic Signals

- Signals that does not repeat itself

1.4.1 Unit Step Function

The *unit step function* $u(t)$, also known as *Heaviside unit function*, is defined as

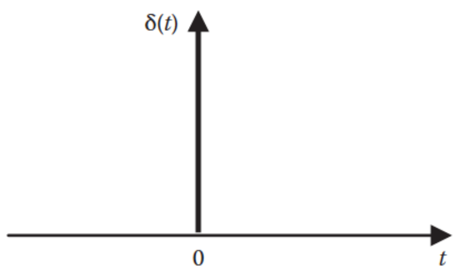
$$\boxed{u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}} \tag{1.19}$$



1.4.2 Unit Impulse Function

The derivative of the unit step function $u(t)$ is the **unit impulse function** $\delta(t)$, which we write as

$$\boxed{\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t \neq 0 \\ \text{undefined}, & t = 0 \end{cases}} \tag{1.23}$$

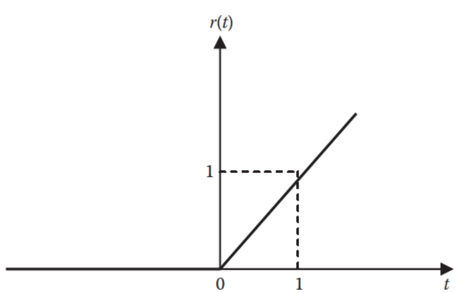


1.4.3 Unit Ramp Function

Integrating the unit step function $u(t)$ results in the **unit ramp function** $r(t)$; we write

$$r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda = tu(t) \tag{1.31}$$

$$\boxed{r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}}$$

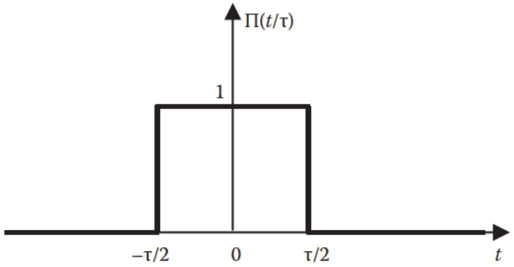


Aperiodic Signals

1.4.4 Rectangular Pulse Function

The **unit rectangular pulse function** is defined as

$$\Pi\left(\frac{t}{\tau}\right)=\begin{cases}1, & |t|<\tau/2\\ 0, & \text{otherwise}\end{cases}=\begin{cases}1, & -\tau/2<t<\tau/2\\ 0, & \text{otherwise}\end{cases}$$

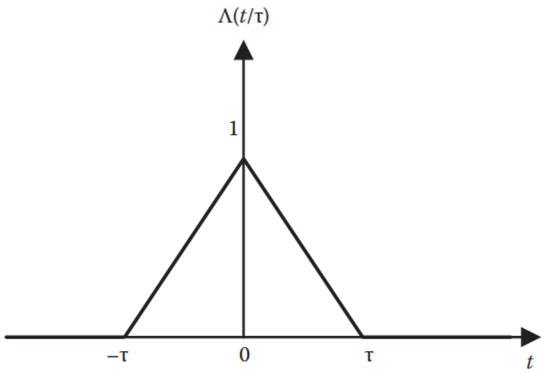


$$\Pi\left(\frac{t}{\tau}\right)=u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)$$

1.4.5 Triangular Pulse Function

The unit triangular function is defined as

$$\Lambda\left(\frac{t}{\tau}\right)=\begin{cases}1-\dfrac{|t|}{\tau}, & -\tau<t<\tau\\ 0, & \text{otherwise}\end{cases}=\begin{cases}1+\dfrac{t}{\tau}, & -\tau<t<0\\ 1-\dfrac{t}{\tau}, & 0<t<\tau\\ 0, & \text{otherwise}\end{cases}$$

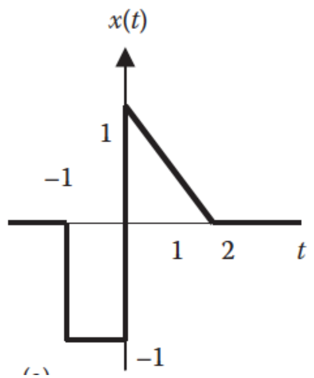


Basic Operations on Signals

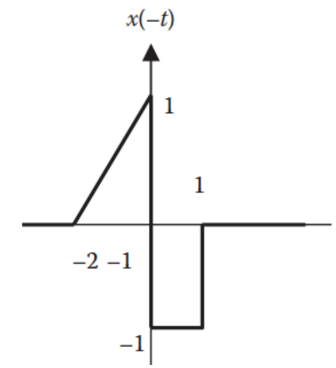
1.6.1 Time Reversal Given a signal $x(t)$, its time reversal is $x(-t)$.

$$x(t)=\begin{cases}-1, & -1<t<0\\ 1-t, & 0<t<2\\ 0, & \text{otherwise}\end{cases}$$

$$x(-t)=\begin{cases}-1, & -1<-t<0\\ 1-(-t), & 0<-t<2\\ 0, & \text{otherwise}\end{cases}=\begin{cases}-1, & 0<t<1\\ 1+t, & -2<t<0\\ 0, & \text{otherwise}\end{cases}$$



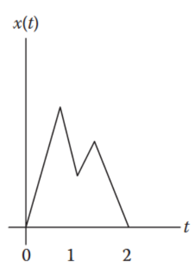
we obtain $x(-t)$ by replacing every t with $-t$

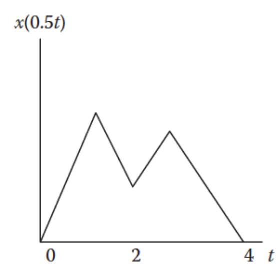


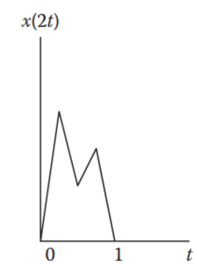
1.6.2 Time Scaling **Time scaling** involves the compression or expansion of a signal in time.

compressed if $|a| > 1$ or expanded if $|a| < 1$;

$x(t)$, $x(at)$.





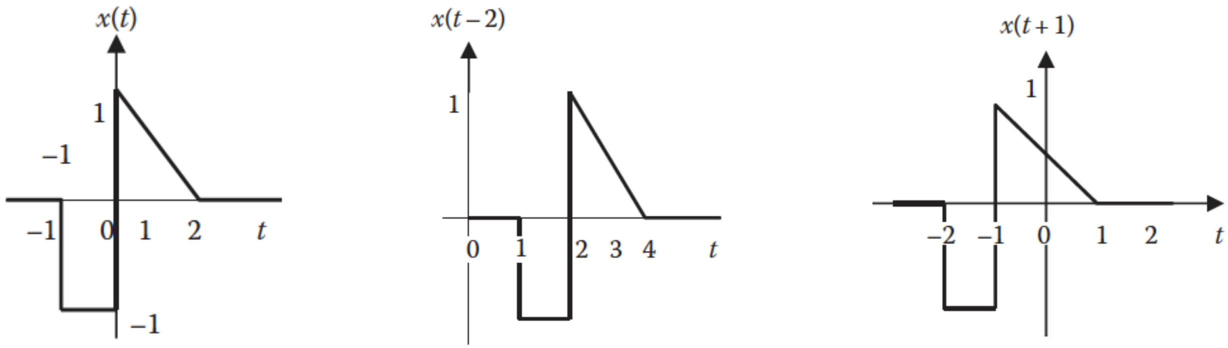


Basic Operations

1.6.3 Time Shifting $x(t) \longrightarrow x(t - t_o)$,

. If $t_o > 0$, then the signal $x(t - t_o)$ is *delayed*

When $t_o < 0$, the signal $x(t - t_o)$ is an *advanced* replica of $x(t)$.



$$x(at+b) = x\left(a\left(t+\frac{b}{a}\right)\right)$$ involves two steps:

1. Scale by factor a. If a is negative, reflect $x(t)$ about the vertical axis.
2. If b/a is negative, shift $x(t)$ to the right. If b/a is positive, shift $x(t)$ to the left.

Basic Operations

1.6.4 Amplitude Transformations $y(t) = Ax(t) + B$

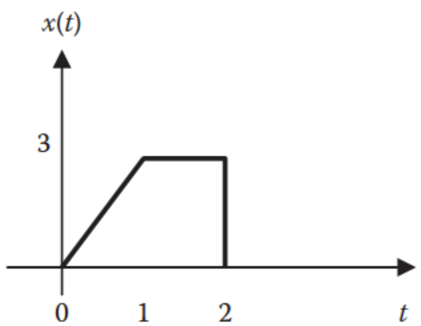
$$y(t) = -2x(t) + 4$$

We notice that $A = -2$ means amplitude reversal ($-x(t)$ implies reflection about the horizontal axis) and amplitude scaling ($|A| = 2$). Also, $B = 4$ shifts vertically the amplitude of the signal.

Example 1.9

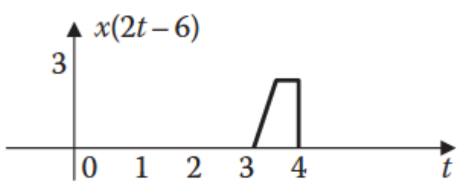
A continuous-time signal is shown in Figure 1.34. Sketch each of the following signals:

(a) $x(2t-6)$, (b) $x(t/2+1)$, (c) $y(t) = -1 + 2x(t)$



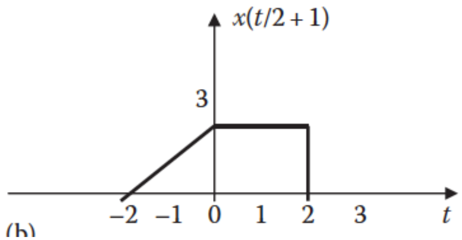
Solution

Method 1: We can write this as $x(2(t-3))$, indicating that $x(t)$ is compressed by a factor of 2 and shifted to the right by three units. Hence, $x(2t-6)$ is as sketched in Figure 1.35a.



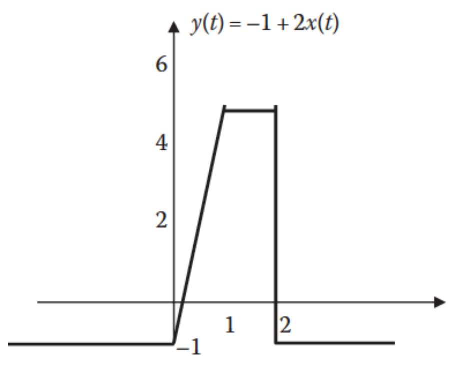
(a)

(b) *Method 1*: We write $x(t/2 + 1) = x(1/2(t + 2))$ indicating that $x(t)$ is expanded by a factor of 2 and advanced or shifted left by 2 units. The signal $x(t/2 + 1)$ is sketched in Figure 1.35b.



(c) This deals with amplitude amplification; there is no transformation of time t.

Method 1: The signal $x(t)$ is amplified by 2 and lowered by –1, as shown in Figure 1.35c, giving $-1 + 2x(t)$.



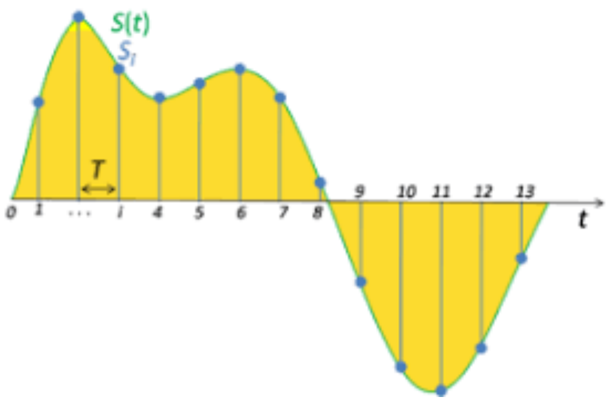
Analog vs Digital Signals

- Analog Signals ~ s(t) where t and s are both continuous quantities
  - e.g. t ~ time = Real Number, s(t) = Voltage = Real Number
- Digital Signals ~ s(t) where both t and s are discrete quantities
  - e.g. t ~ discrete time = integers or rational numbers
  - e.g. s(t) = quantised voltage = fixed point numbers

Analog to Digital Conversion (ADC)

Need to Discretize the x, y - axis

- (1) Discretization of Time (Sampling)



Design f_s, N

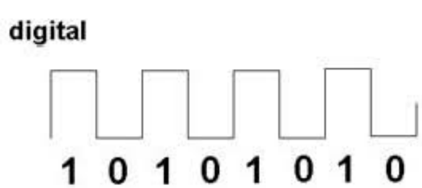
telephone. 8 bits
not good as CDs. 16 bits

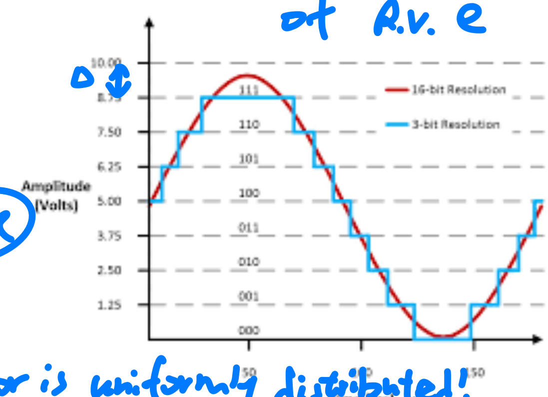
- (2) Discretization of s(t) (Quantization)
  - sampled values is quantised to N bit numbers
  - digital signals = {010111011.....}

$SNR_Q = \frac{P_s}{P_e}$

by variance of R.v. e

$|e| \le \frac{\Delta}{2}$





1 bit → +6 dB SNR

Assumptions: - input signal is sine wave
- quantization error is uniformly distributed!

Analog to Digital Conversion

TDMA:

↑ Can multiplex another waveform

- Discussion:
  - (1) Sampling: Is this process reversible?

Nyquist Sampling Theorem:

Yes, if you sample very fast

  - (2) Quantisation: Is this process reversible?

No :(

Many-to-one mapping!

$\sqrt{2} \rightarrow \boxed{Q(\,)} \rightarrow 1.4$

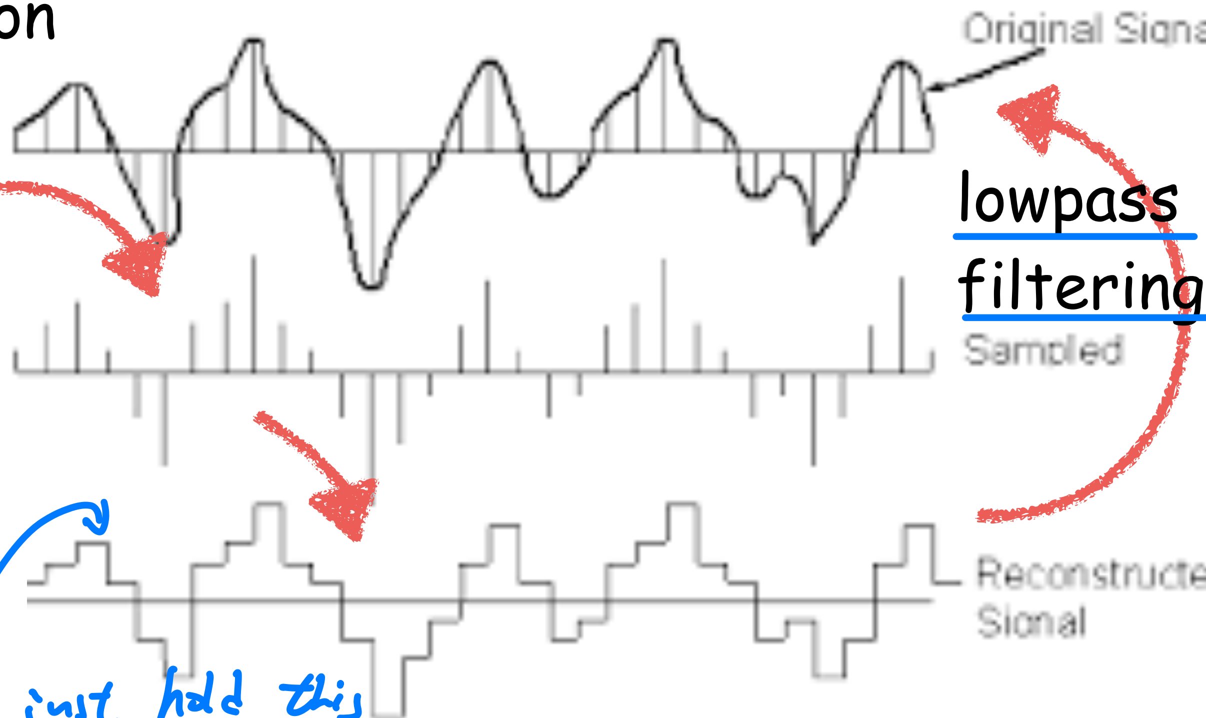
impossible to have infinite memory!

Difficult to design

Digital to Analog Conversion

Digital Information
(Bit streams)

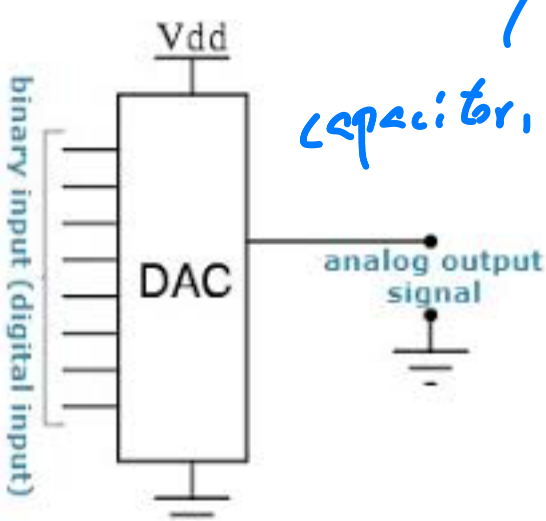
Digital Input Code				Analog Output Voltage
Bit 3	Bit 2	Bit 1	Bit 0	(V)
0	0	0	0	0.000
0	0	0	1	-0.625
0	0	1	0	-1.250
0	0	1	1	-1.875
0	1	0	0	-2.500
0	1	0	1	-3.125
0	1	1	0	-3.750
0	1	1	1	-4.375
1	0	0	0	-5.000
1	0	0	1	-5.625
1	0	1	0	-6.250
1	0	1	1	-6.875
1	1	0	0	-7.500
1	1	0	1	-8.125
1	1	1	0	-8.750
1	1	1	1	-9.375



lowpass
filtering

capacitor, just hold this value

only lost from quantization



Analog signals → ADC → digital signals

0101 . . .

DSP

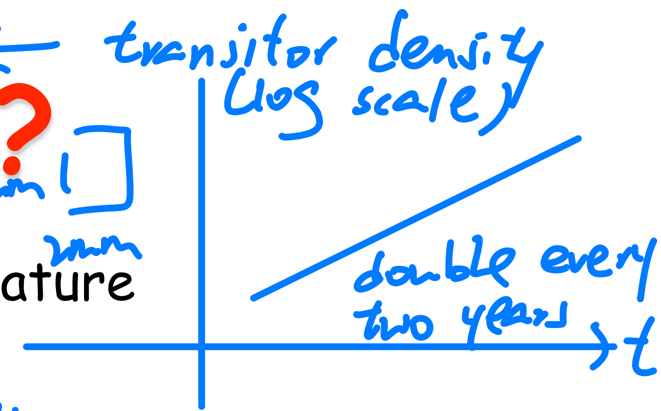
↓

DAC (reversible)

↓

analog

Why Digital?

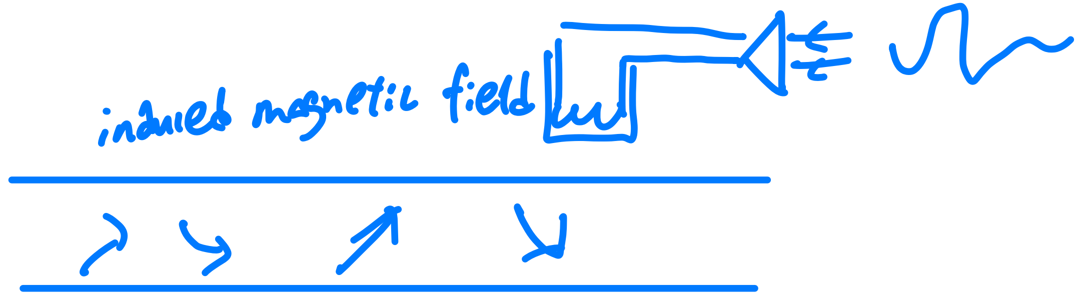


- Most of the physical signals are analog in nature
- Why bother to convert to digital?

  ☆ Perform benefit! ☆ Moore's law (important motivation)
- Robustness to Noise and Degradation!
- Analog Signals: The size can be enormous! 1. how to store a music! 2. (physical media will always introduce distortion of waveform)
  - Information is embedded in the waveform
  - Any distortion on the waveform —> distortion of the information
- Digital Signals:
  - Information is embedded in the "bits" behind the waveform.
  - Distortion on the waveform may not result in distortion of the bits

Analog

(Audio tape)



will oriented proportional to waveform

$\boxed{\text{CD}}$ 90's

MP3 .wav
lossless compression

sampling and quantization

PCM
① sampling $f_s = 44100\,\text{Hz}$
② Quantization $N = 16$ bits/sample

$R_s = 2 \times 44.1 \times 16 = 1411$ Mbps

2 independent channels for Hi-Fi

$$\text{size} = \frac{1.411\text{mb/s} \times 5 \times 60}{8 \text{ bytes}} = 53\text{MB} \sim 5\text{MB for MP3}$$

VCD ~ mp1 ~ must need compression!
57...

video

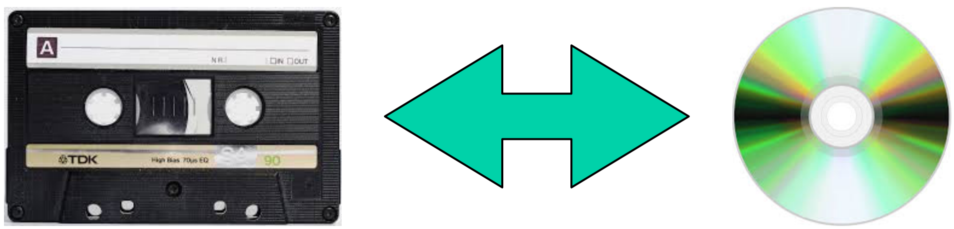
need to media!!!

DVD ~ mp2

mp4

Robustness of Digital

- Storage Applications
  - Audio Tape vs CD



- Telecommunication Applications
  - Long distance telephone has been very challenging
  - Resolved through digital transmission of signals

Long distance voice call over analog PSTN

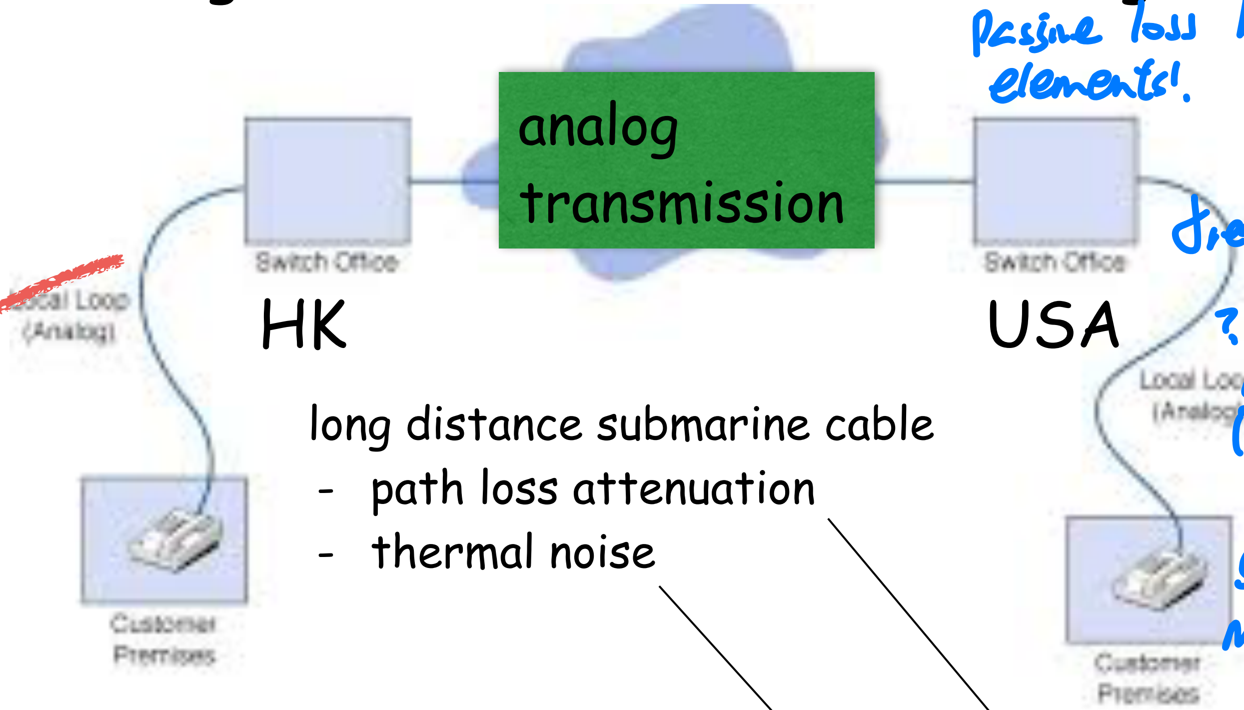
coaxial cable

attenuation? "ohmic loss"

passive loss by resistive elements!

d_{tx-Rx}

d incr $\uparrow$ $SNR = \frac{Sgx}{Noise}$

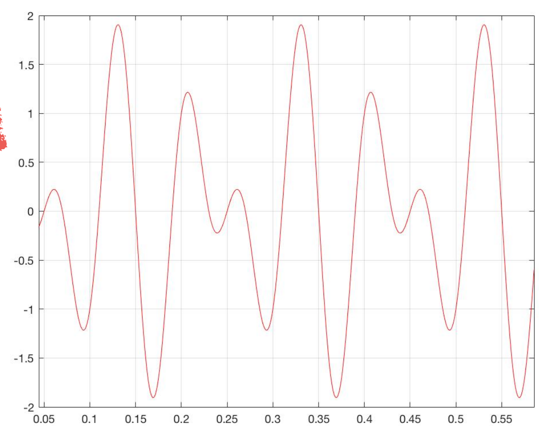


? Amplifier, but cannot solve! Because amplifier also 放大 noise, SNR 不變. 甚至會變低!

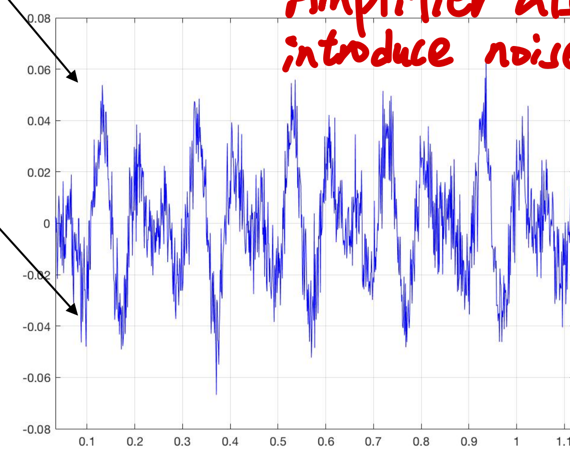
So it have a maximum range of transmission

Amplifier also introduce noise :(

long distance submarine cable
- path loss attenuation
- thermal noise

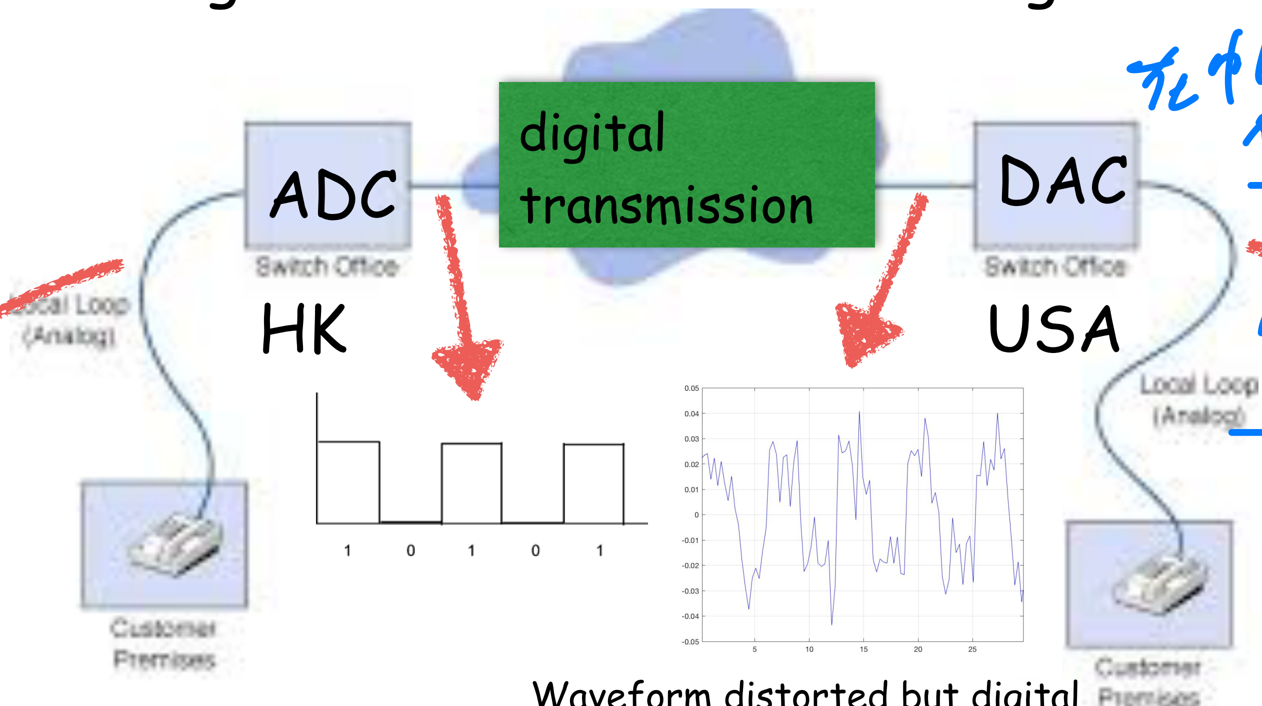


High Signal-to-Noise Ratio



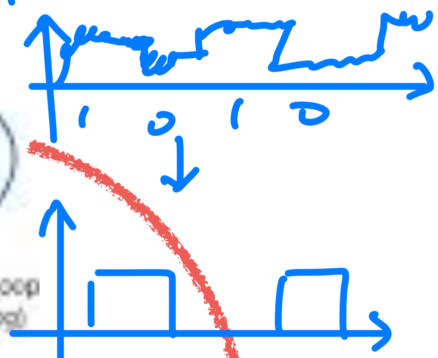
Poor Signal-to-Noise Ratio

Long distance voice call over digital PSTN

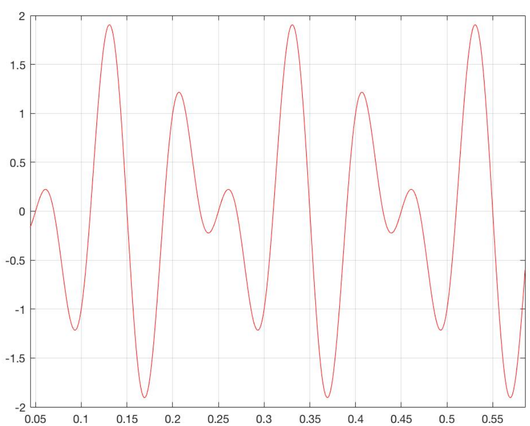


Waveform distorted but digital bits can still be recovered

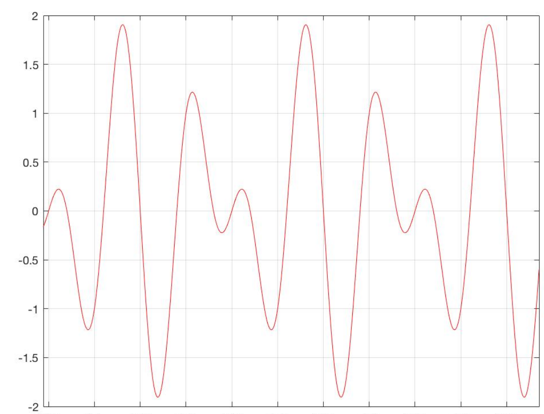
在中间设置 Repeater



Decode and format!



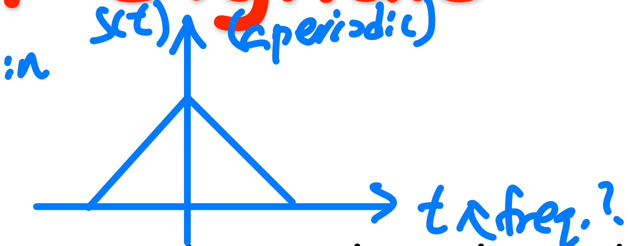
Good Signal-to-Noise Ratio



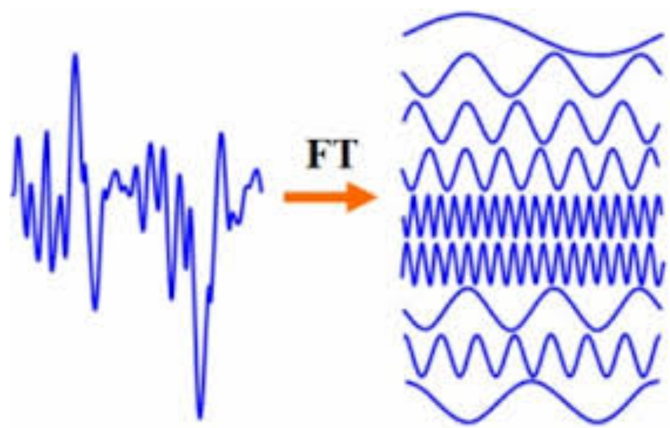
Good Signal-to-Noise Ratio

can describe frequency of this

Representation of Signals

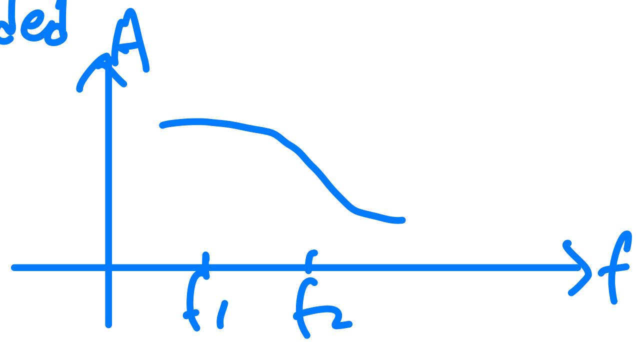


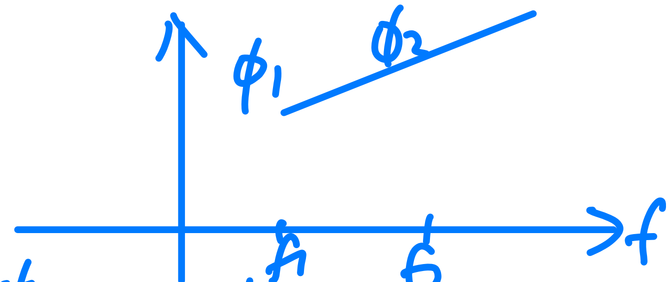
- Frequency Domain Representation
  - Frequency has been defined for period signals only. What do we mean by frequency for aperiodic signal?
  - Aperiodic signals can be decomposed into "sum" of sinusoidal signals



(amplitude spectrum) + (phase spectrum)

single-sided

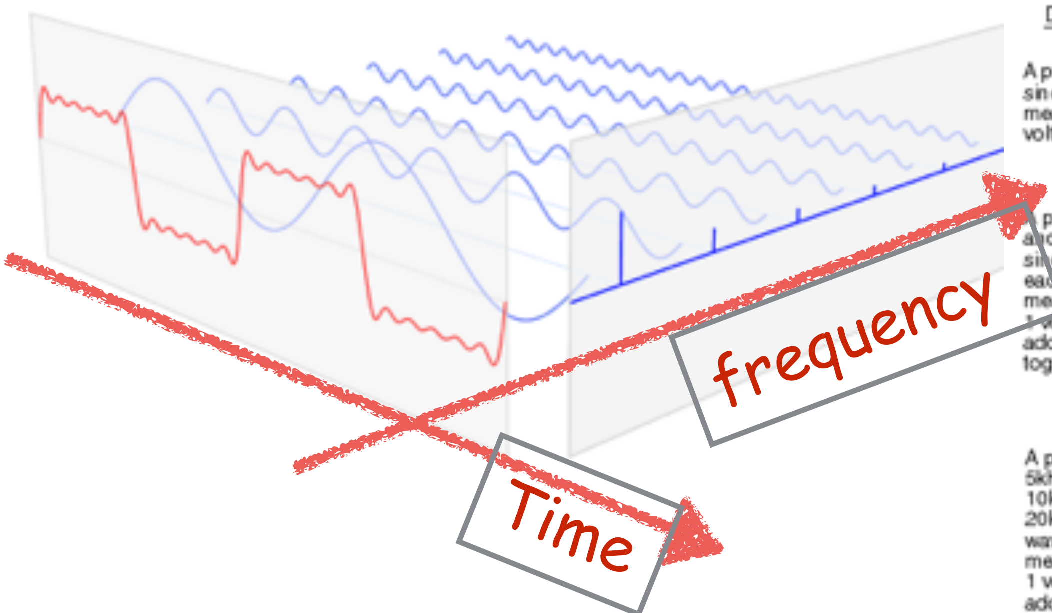


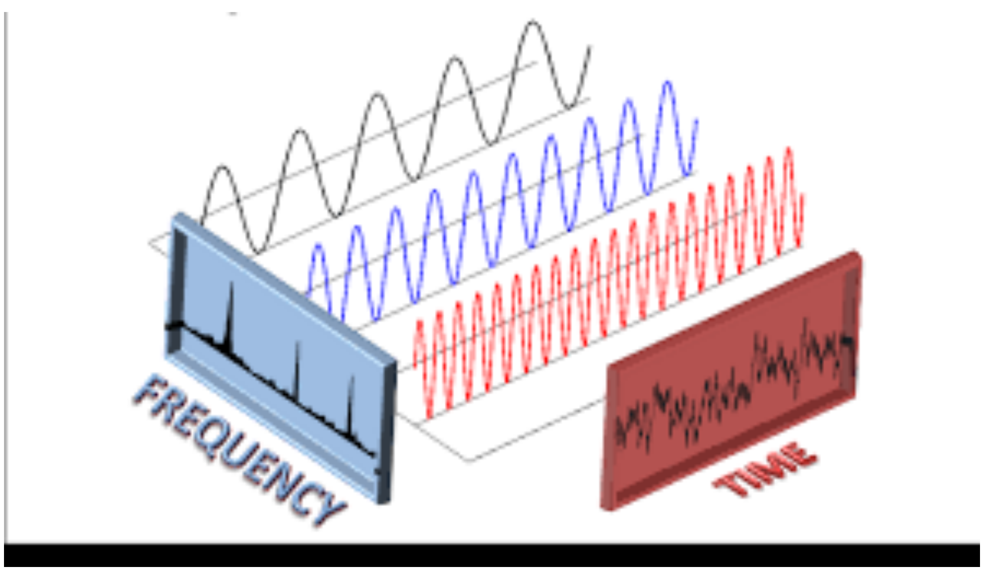


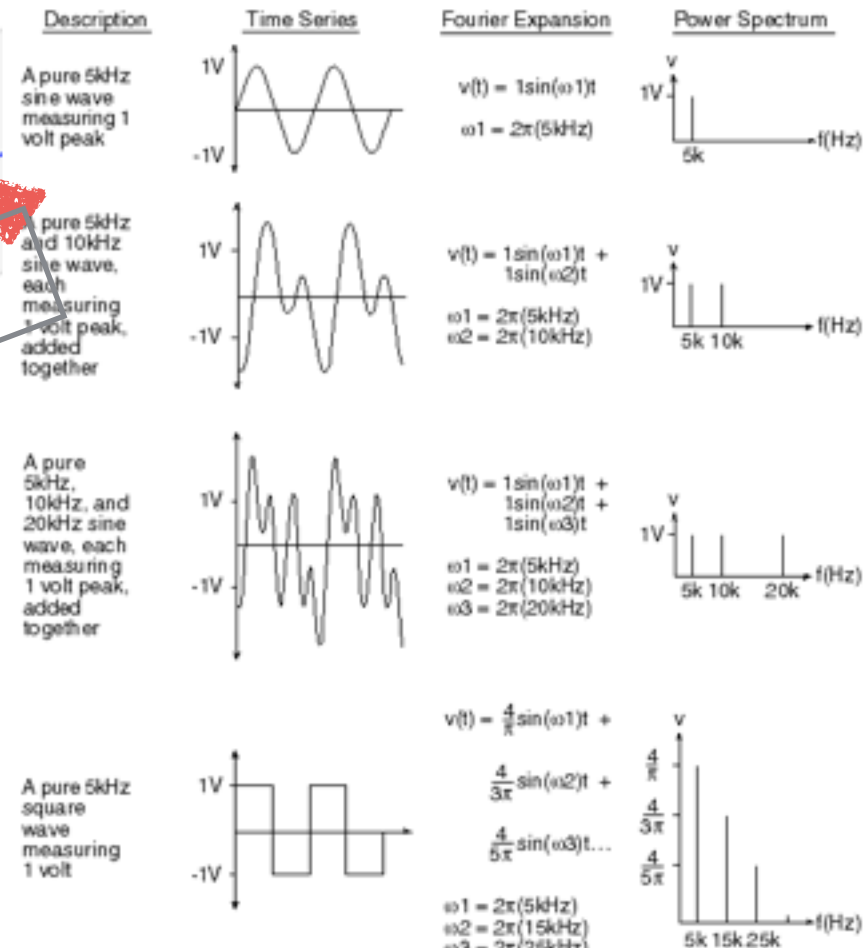
If the sinusoid is complex $\Rightarrow$ two sided spectrum

$$\tilde{x}_n(t) = Z_n e^{j2\pi f_n t}$$

$$\tilde{x}_n(t) = \frac{1}{2}\left(\tilde{x}_n(t) + \overline{\tilde{x}_n(t)}\right)$$


frequency
Time


FREQUENCY
TIME


Description
Time Series
Fourier Expansion
Power Spectrum
A pure 5kHz sine wave measuring 1 volt peak
1V
-1V
v(t) = 1sin(ω1)t
ω1 = 2π(5kHz)
v
1V
5k
f(Hz)
A pure 5kHz and 10kHz sine wave, each measuring 1 volt peak, added together
v(t) = 1sin(ω1)t + 1sin(ω2)t
ω1 = 2π(5kHz)
ω2 = 2π(10kHz)
5k 10k
A pure 5kHz, 10kHz, and 20kHz sine wave, each measuring 1 volt peak, added together
v(t) = 1sin(ω1)t + 1sin(ω2)t + 1sin(ω3)t
ω1 = 2π(5kHz)
ω2 = 2π(10kHz)
ω3 = 2π(20kHz)
5k 10k 20k
A pure 5kHz square wave measuring 1 volt
v(t) = 4/π sin(ω1)t + 4/3π sin(ω2)t + 4/5π sin(ω3)t...
ω1 = 2π(5kHz)
ω2 = 2π(15kHz)
ω3 = 2π(25kHz)...
4/π
4/3π
4/5π
5k 15k 25k

Fourier Transform

Time
Domain
s(t)

Frequency Domain
S(f)

Represent signals by means of the "waveform"

$$s(t) = \int_{-\infty}^{\infty} S(f) \exp(j 2\pi f t) df$$

"Uncountable Sum" over f

"Complex Amplitude" capturing amplitude & phase

"Complex Sinusoidal" at frequency f

Represent signals by means of the "list of frequency components"

- Frequency = f
- Amplitude = |S(f)|
- Phase = <S(f)

$$S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j 2\pi f t) dt$$

- Time domain & frequency domain representations are two sides of the same coin
- Physical parameters (e.g. Energy) computed from time and frequency domains are the same

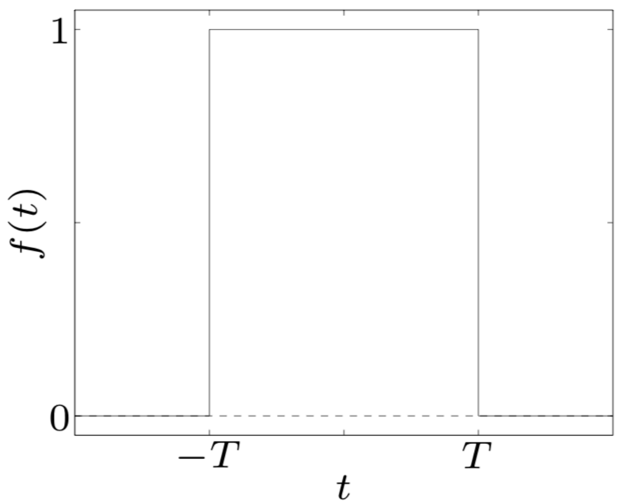
$$\int_{-\infty}^{\infty} |S(f)|^2 df = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

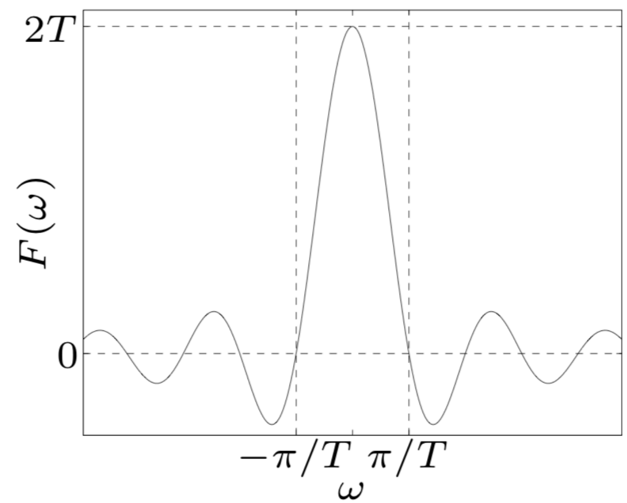
Fourier Transform

- Examples:

rectangular pulse: $f(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & |t| > T \end{cases}$

$$F(\omega) = \int_{-T}^{T} e^{-j\omega t}\, dt = \frac{-1}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) = \frac{2\sin \omega T}{\omega}$$



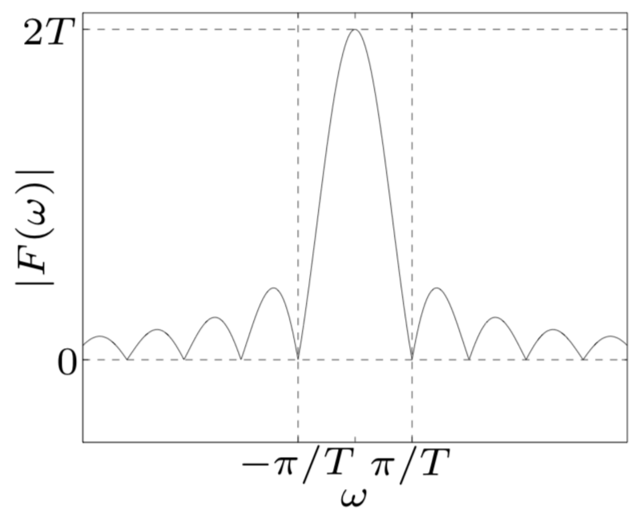


unit impulse: $f(t) = \delta(t)$

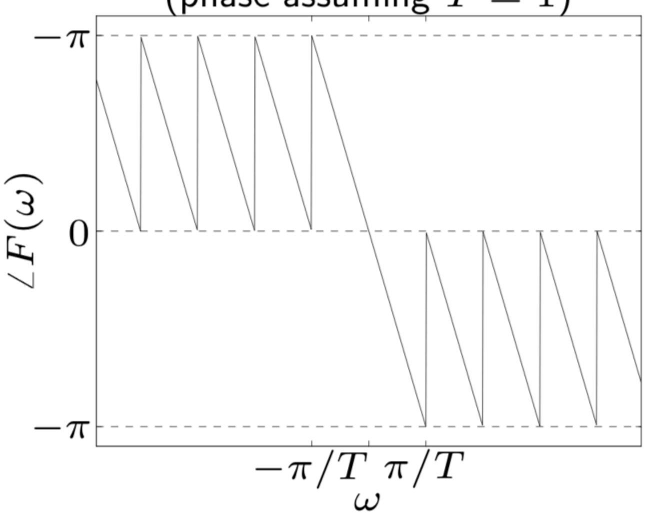
$$F(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = 1$$

shifted rectangular pulse: $f(t) = \begin{cases} 1 & 1-T \le t \le 1+T \\ 0 & t < 1-T \text{ or } t > 1+T \end{cases}$

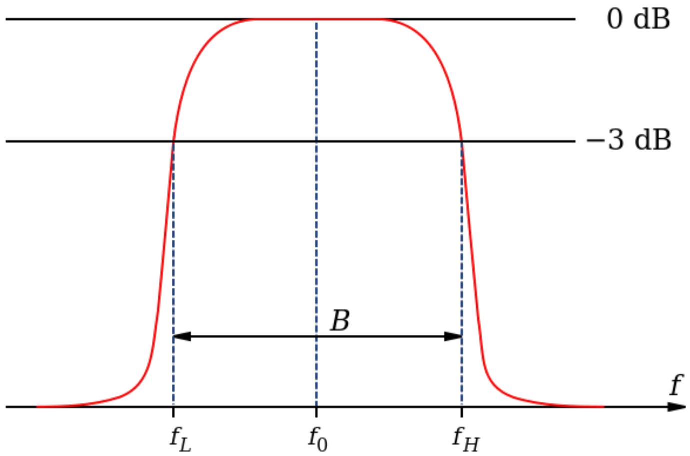
$$\begin{aligned} F(\omega) = \int_{1-T}^{1+T} e^{-j\omega t}\, dt &= \frac{-1}{j\omega}\left(e^{-j\omega(1+T)} - e^{-j\omega(1-T)}\right) \\ &= \frac{-e^{-j\omega}}{j\omega}\left(e^{-j\omega T} - e^{j\omega T}\right) \\ &= \frac{2\sin\omega T}{\omega} e^{-j\omega} \end{aligned}$$



(phase assuming $T = 1$)



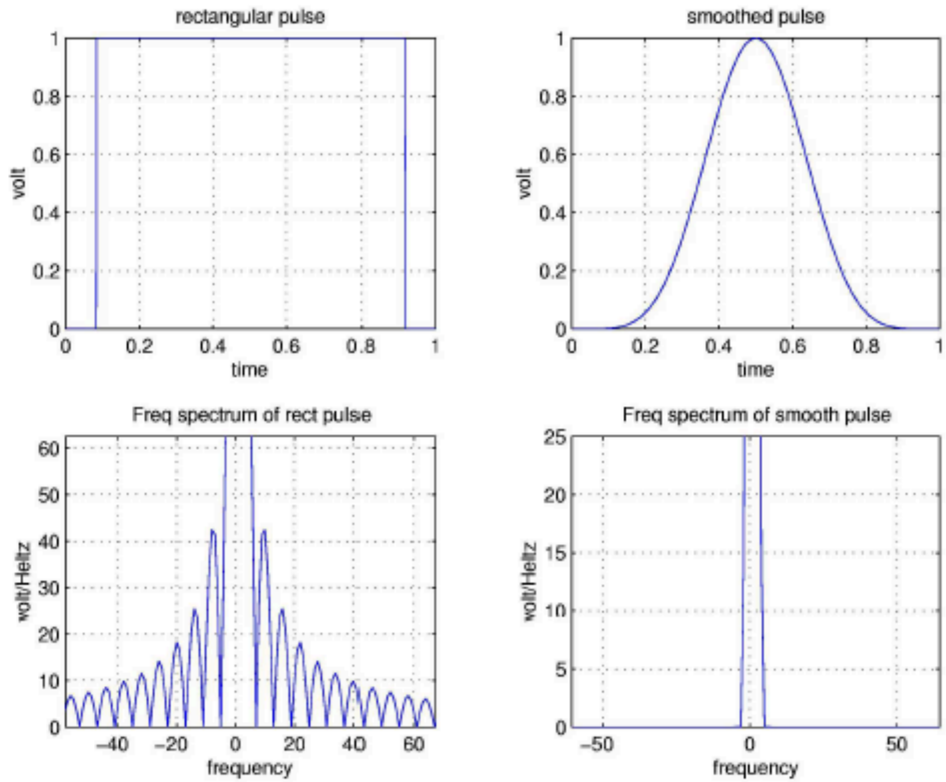
Bandwidth of signal = range of frequencies with significant energy



- Factors affecting Bandwidth
  - Pulse shape: sharp pulse → larger bandwidth
  - Pulse duration: small pulse → larger bandwidth

$$TW \geq \kappa$$

Uncertainty Principle



Properties of Fourier Transform

5.3.1 LINEARITY

$$\boxed{\mathcal{F}\left[a_1 x_1(t) + a_2 x_2(t)\right] = a_1 X_1(\omega) + a_2 X_2(\omega)}$$

for any a_1, a_2

5.3.2 TIME SCALING

If $\mathcal{F}(\omega) = [x(t)]$

$$\boxed{\mathcal{F}\left[x(at)\right] = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)}$$

a is a real constant.

5.3.3 TIME SHIFTING

$$\boxed{\mathcal{F}\left[x(t - t_0)\right] = e^{-j\omega t_0} X(\omega)}$$

5.3.4 FREQUENCY SHIFTING

$$\boxed{\mathcal{F}\left[x(t) e^{j\omega_0 t}\right] = X(\omega - \omega_0)}$$

this is a dual of the time shifting property

5.3.5 TIME DIFFERENTIATION

$$\boxed{\mathcal{F}\left[x'(t)\right] = j\omega X(\omega)}$$

5.3.6 FREQUENCY DIFFERENTIATION

$$\mathcal{F}\left[(-jt)^n x(t)\right] = \frac{d^n}{d\omega^n} X(\omega)$$

5.3.7 Time Integration

$$\boxed{\mathcal{F}\left[\int_{-\infty}^{t} x(t)\,dt\right] = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)}$$

Fourier Transform Pairs

$x(t)$	$X(\omega)$
$\delta(t)$	1
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\Pi(t/\tau)$	$\tau \sin c\left(\frac{\omega\tau}{2}\right)$
$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \sin c^2\left(\frac{\omega\tau}{2}\right)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

A signal $x(t)$ has a Fourier transform given by

$$X(\omega) = \frac{5(1+j\omega)}{8-\omega^2+6j\omega}$$

Without finding $x(t)$, find the Fourier transform of the following:

(a) $x(t-3)$
(b) $x(4t)$
(c) $e^{-j2t}x(t)$
(d) $x(-2t)$

Solution

We apply the relevant property for each case:

(a) $$\mathcal{F}[x(t-3)] = e^{-j\omega 3}X(\omega) = \frac{5(1+j\omega)e^{-j\omega 3}}{8-\omega^2+j6\omega}$$

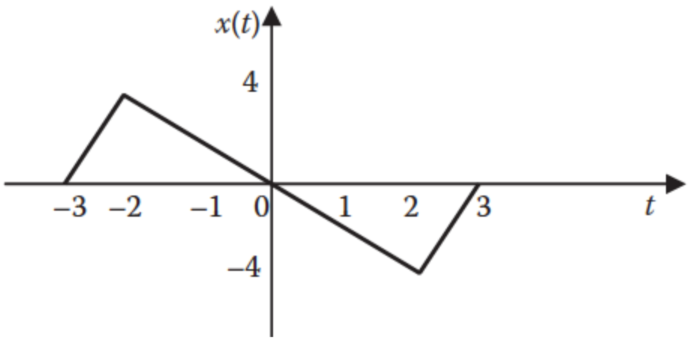
(b) $$\mathcal{F}[x(4t)] = \frac{1}{4}X\left(\frac{\omega}{4}\right) = \frac{\frac{5}{4}(1+j\omega/4)}{8-\omega^2/16+j6\omega/4} = \frac{5(4+j\omega)}{128-\omega^2+j24\omega}$$

(c) $$\mathcal{F}[e^{-j2t}x(t)] = X(\omega+2) = \frac{5\left[1+j(\omega+2)\right]}{8-(\omega+2)^2+6j(\omega+2)} = \frac{5(1+j\omega+j2)}{4-\omega^2-4\omega+6j\omega+j12}$$

(d) $$\mathcal{F}\left[x(-2t)\right] = \frac{1}{2}X\left(\frac{\omega}{-2}\right) = \frac{\frac{5}{2}(1-j\omega/2)}{8-\frac{\omega^2}{4}-\frac{6j\omega}{2}} = \frac{5(2-j\omega)}{32-\omega^2-12j\omega}$$

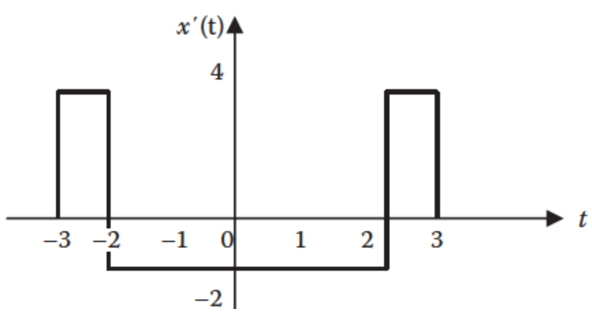
Example 5.5

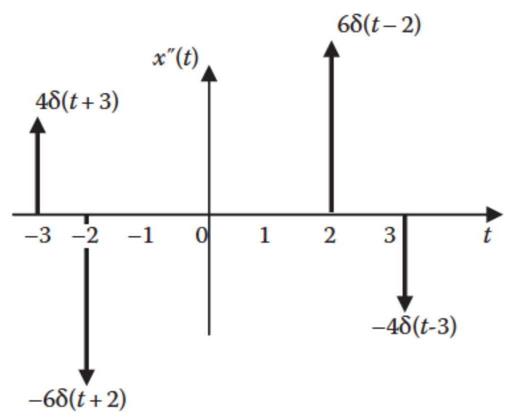
Determine the Fourier transform of the signal in Figure 5.6.



Solution

Although the Fourier transform of $x(t)$ can be found directly using Equation 5.6, it is much easier to find it using the derivative property. Taking the first derivative of $x(t)$ produces the signal in Figure 5.7a. Taking the second derivative gives us the signal in Figure 5.7b. From this, $x''(t) = 4\delta(t+3) - 6\delta(t+2) + 6\delta(t-2) - 4\delta(t-3)$





Taking the Fourier transform of each term, we obtain

$$(j\omega)^2 X(\omega) = 4e^{j3\omega} - 6e^{j2\omega} + 6e^{-j2\omega} - 4e^{-j3\omega}$$

$$-\omega^2 X(\omega) = 4\left(e^{j3\omega} - e^{-j3\omega}\right) + 6\left(e^{j2\omega} - e^{-j2\omega}\right)$$

$$= j8\sin 3\omega - j12\sin 2\omega$$

$$X(\omega) = \frac{j}{\omega^2}\left(12\sin 2\omega - 8\sin 3\omega\right)$$

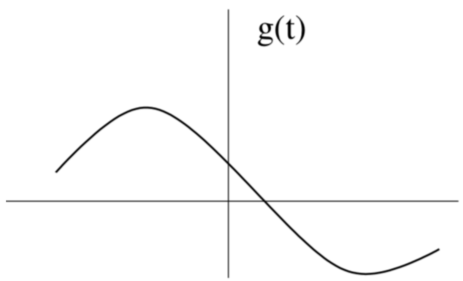
Frequency Domain Representation of Signals

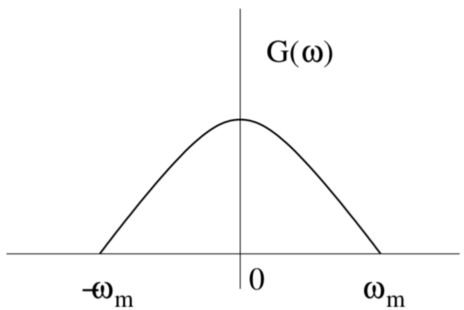
- why we are interested to represent signals in frequency domain?
- Application 1 - Efficient Storage of Signals
  - Most physical signals have long timespan in time domain. (e.g. waveform of a song may last for 6 minutes)
  - However, most of the physical signals are 'band-limited" ~ only have finite frequency span in frequency domain (e.g. bandwidth of music ~ 20kHz)
  - One needs to store significant frequency components —> compression of signals

Frequency Domain

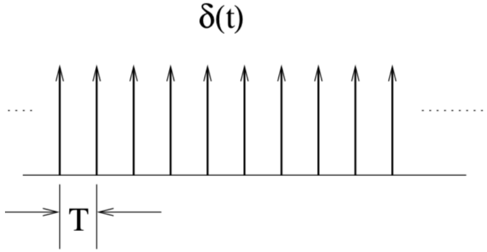
- Application 2: Signal Processing
  - e.g. Nyquist Sampling of signal g(t)

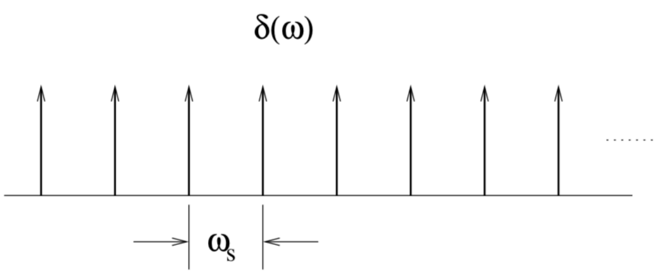
Let $g(t)$ be a bandlimited signal whose bandwidth is f_m





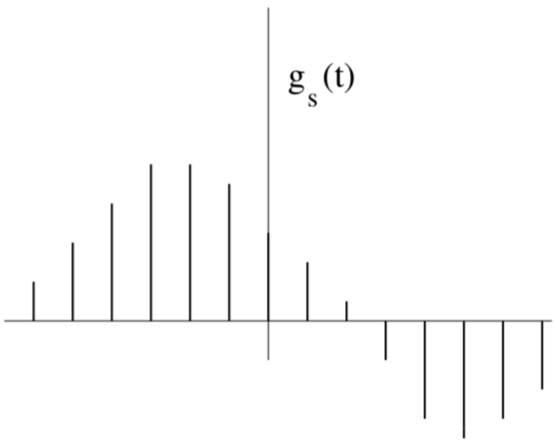
$\delta_T(t)$ is the sampling signal with $f_s = 1/T > 2f_m$.

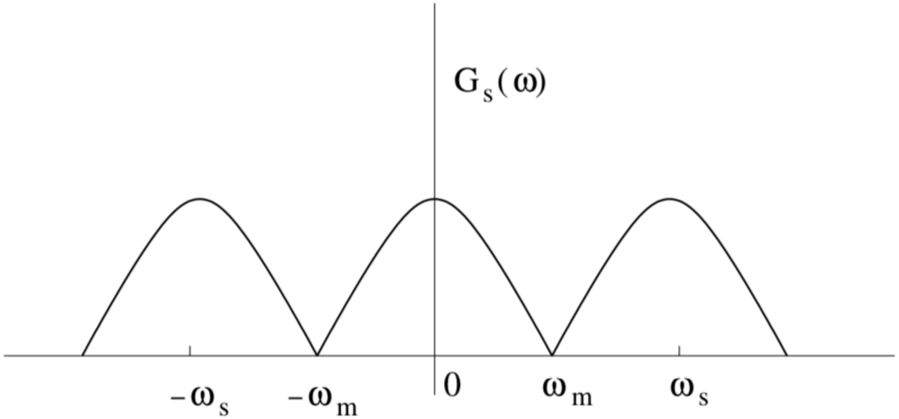




sampled signal $g_s(t) = g(t)\delta_T(t)$

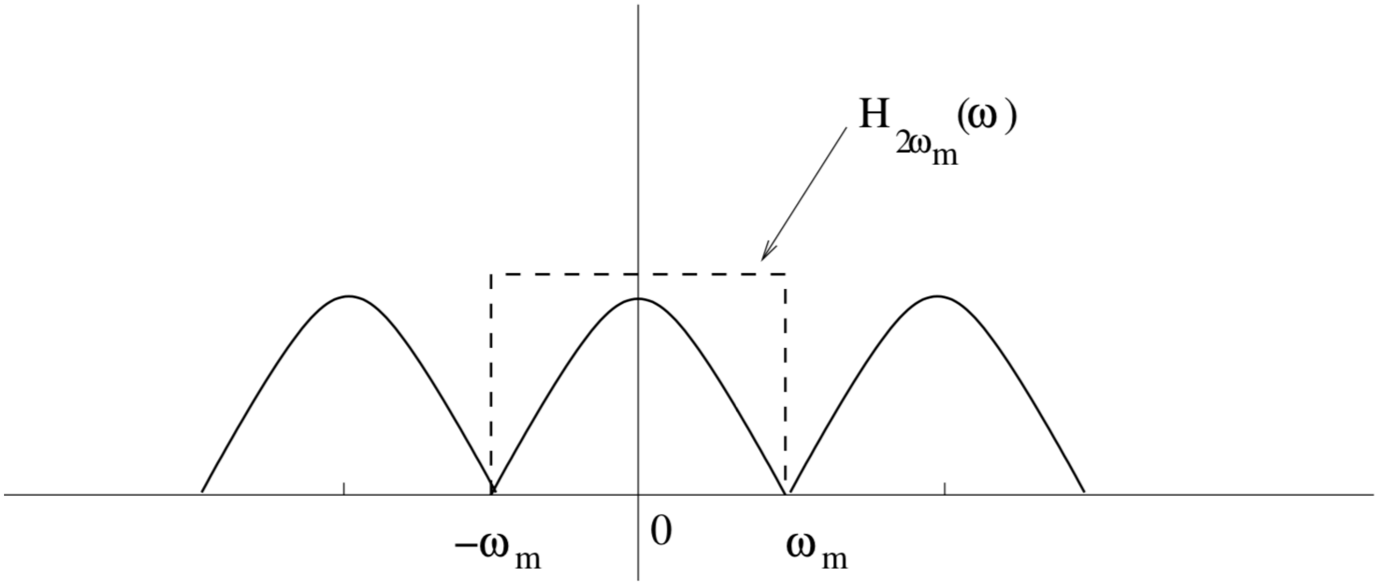
$$
\begin{aligned}
\mathcal{F}(g_s(t)) &= \mathcal{F}\left[g(t)\delta_T(t)\right] \\
&= \frac{1}{T}\sum_{n=-\infty}^{+\infty} G(\omega - n\omega_0)
\end{aligned}
$$



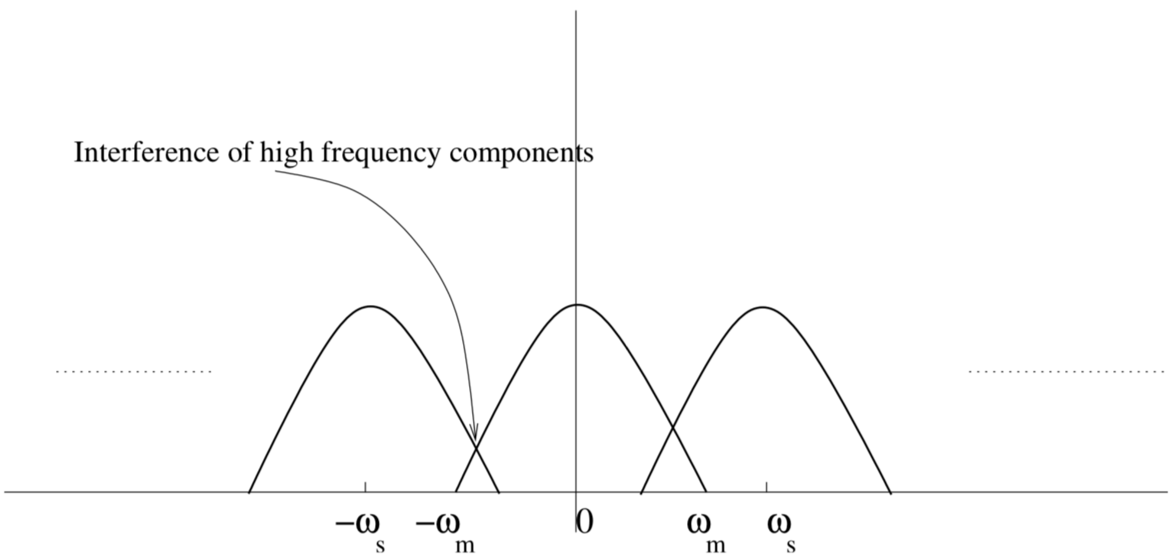


No overlapping (aliasing) of $G(f)$ if $f_s > f_m$

Recovery of sampled signal to the original signal g(t)



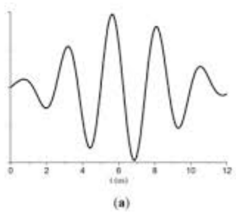
(Aliasing) inadequate sampling $\omega_s < 2\omega_m$.

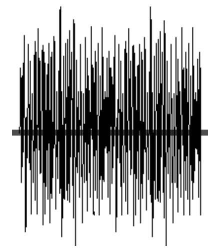


Frequency Domain Representation

thermal noise

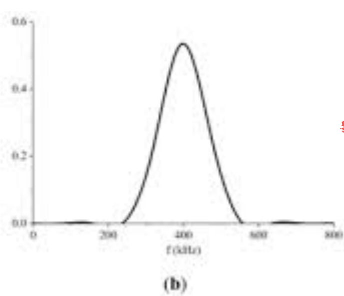
- Application 3: Telecommunication (noise filtering)
  - Signals are usually contaminated with noise (thermal noise).

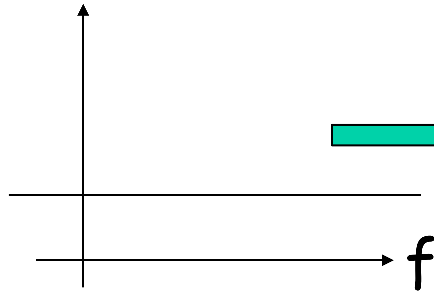




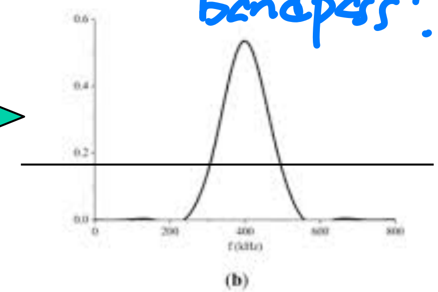
  - In time domain, noise added into signals cannot be separated
  - Yet, in frequency domain, signal and noise occupies different frequency spectrum ==> low pass filtering to "clean up" noisy signals

white noise





bandpass!

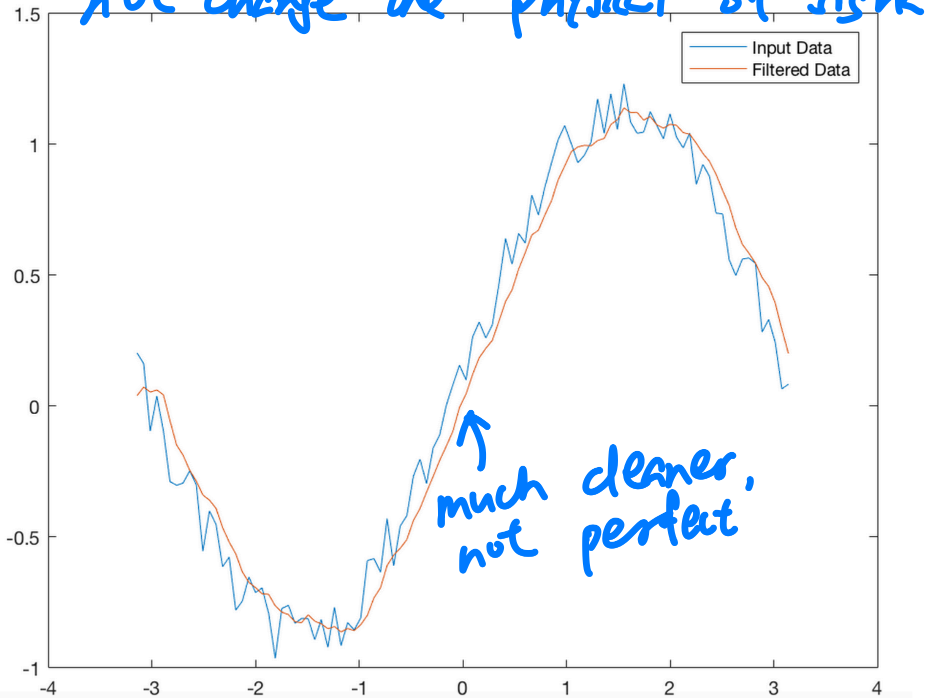


• Filtering of Noisy Signals

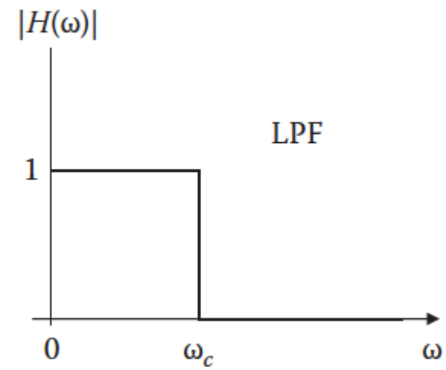
A **filter** is a device designed to pass signals with desired frequencies and block or attenuate others.

1. A *lowpass filter* (LPF) passes low frequencies and rejects high frequencies.
2. A *highpass filter* (HPF) passes high frequencies and rejects low frequencies.
3. A *bandpass filter* (BPF) passes frequencies within a frequency band and blocks or attenuates frequencies outside the band.
4. A *bandstop filter* (BSF) passes frequencies outside the frequency band and stops or attenuates frequencies within the band.

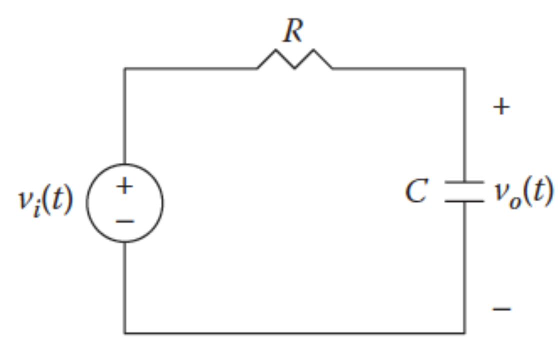
not change the physical of signal



much cleaner, not perfect



(a)



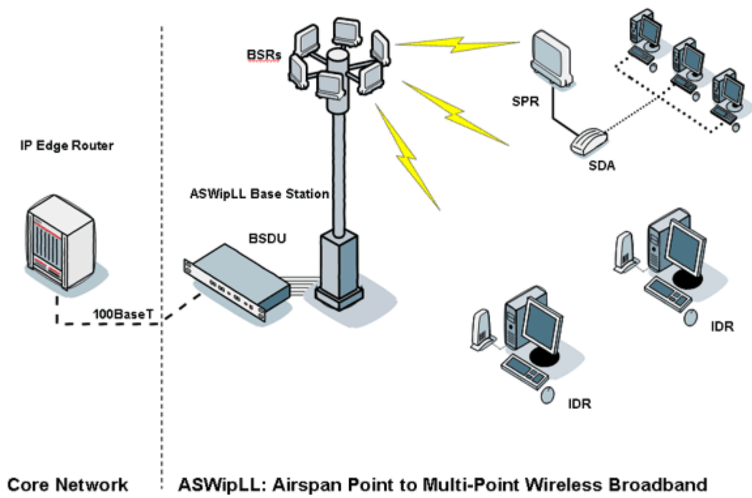
Wireless Communications and Cellular Networks

Why Wireless? ← most examples

convenient!

Pros: Ubiquitous Communications, Convenient, Mobile Communications, Cost Effective if rural area
[The dream of Tesla to go for Wireless] ⇒ very expensive

cost effectiveness



Challenges: Hostile Wireless Channels, Limited Spectrum, Limited Coverage, Limited Range depends on what I define!

Who invented Wireless Communications?

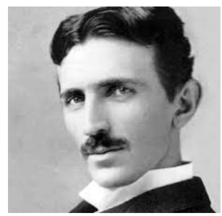
not engineer!

Dream: wireness!

Marconi

or

Tesla

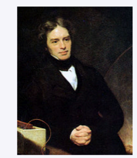


Wireless Telegraph

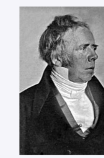
Physicists

Experiments and Theory

Joseph Henry



Michael Faraday



Hans Christian Ørsted

Mathematicians

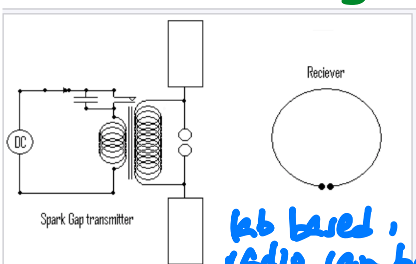
Maxwell and electromagnetic waves

James Clerk Maxwell

Oliver Heaviside

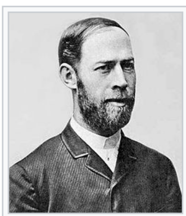
rigorously represent!

Engineers

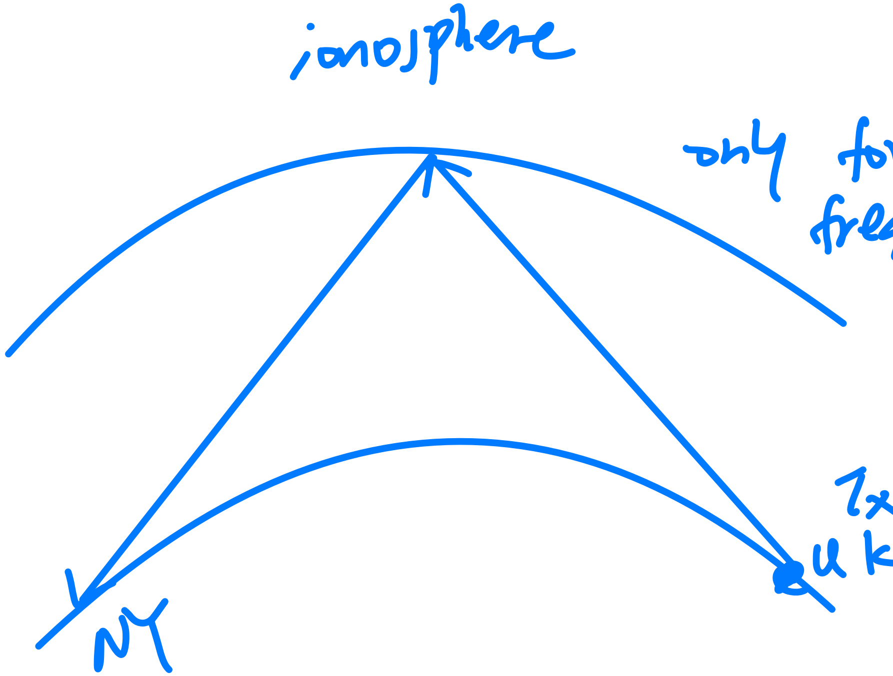


1887 experimental setup of Hertz's apparatus.

lab based, radio can be generated



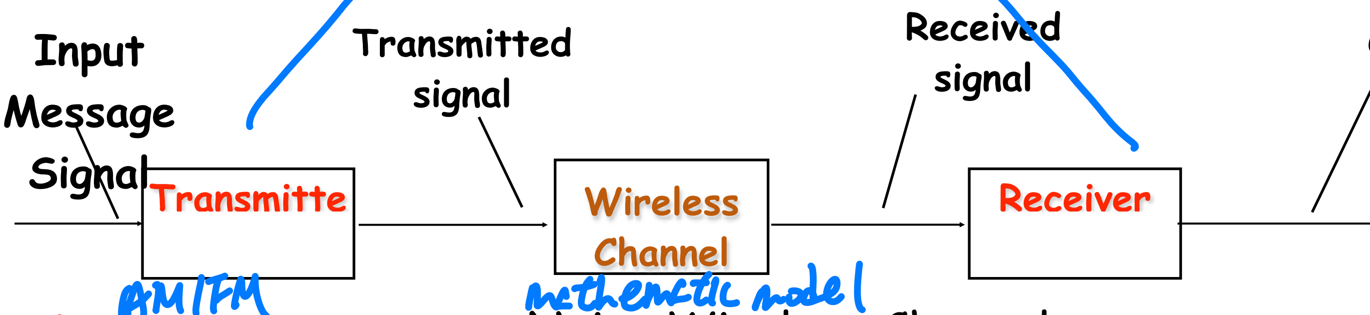
Heinrich Hertz



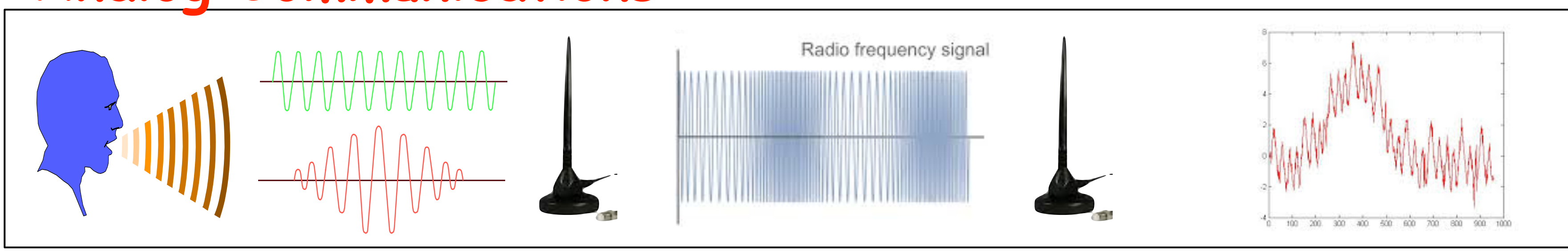
only for certain low frequency communication!

can design those!

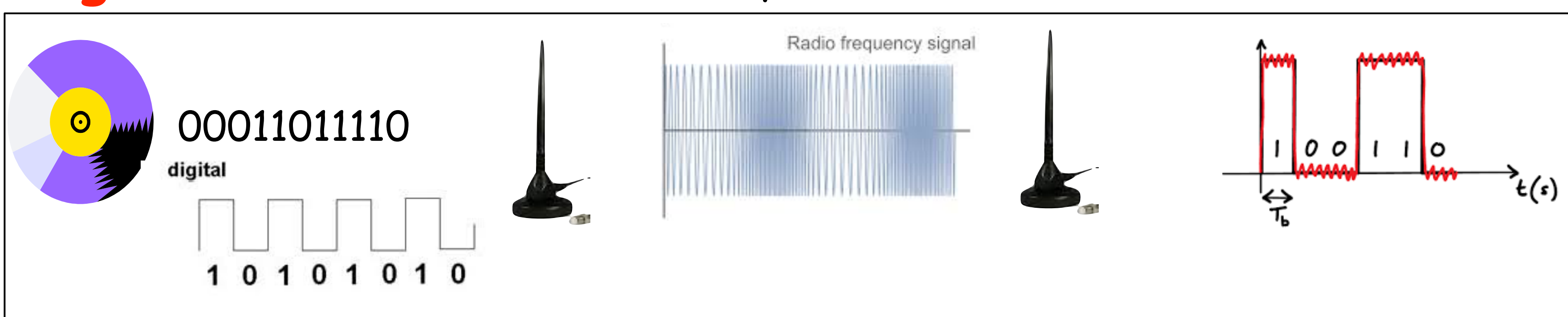
Wireless Communication System



Analog Communications Noisy Wireless Channels

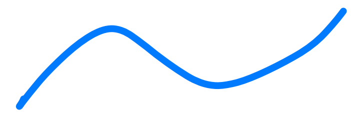


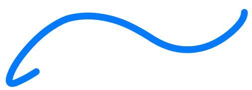
Digital Communications Noisy Wireless Channels



$b_1 b_2 \cdots b_n \rightarrow$ Tx $\xrightarrow{\theta s}$ ch $\rightarrow$ Rx $\rightarrow \hat{b}_1 \hat{b}_2 \cdots \hat{b}_{100}$

Analog bandpass signal





$s(t)$

$y(t) = s(t) + n(t)$

$SNR = \dfrac{P_s}{P_n}$ 越大越好

↓

Performance : $BER = \dfrac{\text{\# of bits corrupted}}{\text{\# of bits Tx}}$ performance metric!

越小越好

$\in [0, 1]$ 0和1一样的!

worst case: 0.5 (coin flipping receiver!)

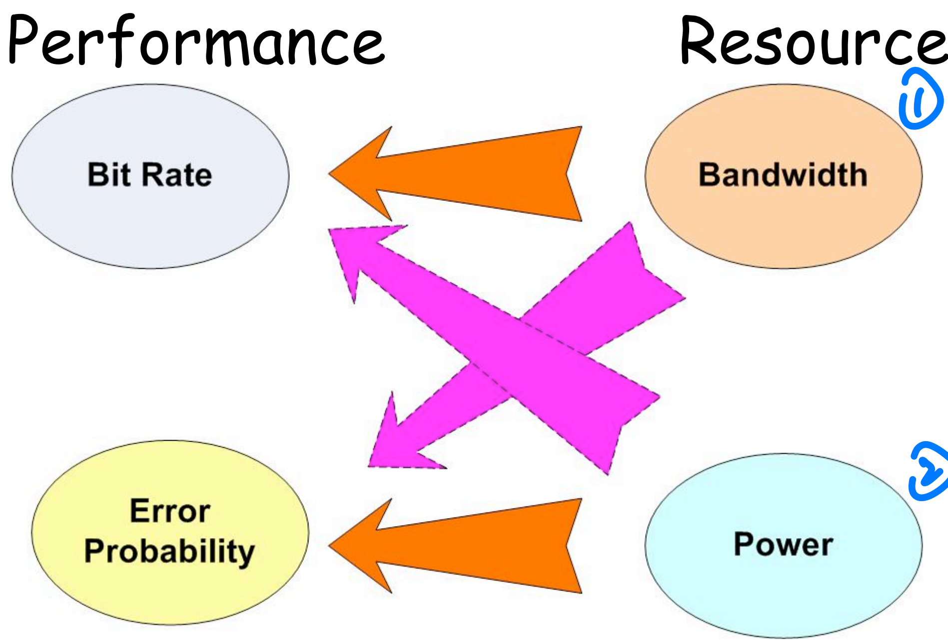
Bit rate: # of bits transmitted in 1 second!

↳ independent on how transmission speed!

What makes Communications Systems Challenging?

- There is no "universally optimal design" that fits for all situations.
- MUST know about the specific channel / situation before we could think about what design to use.
  - Example: **wireless systems requires a different design from an optical fibre.**
- **Complex Tradeoff between "Performance" and "Resource"**

4 tradeoffs



① Get license
↓
Bandwidth ↑,
much more expensive!

② Power mass,
不準太大!

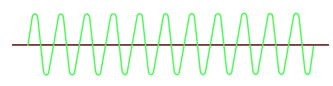
Wireless Channel

- Carries signals - could be a telephone wire, free space and often presents distorted signal to demodulator
- Effects include
  - Attenuation
  - Noise (e.g., additive Gaussian noise or AWGN.
  - Filtering
    - Channel can have a bandwidth that is small compared to the signal bandwidth (e.g. in a telephone channel).
    - Transmitted pulses will be changed in shape and smeared out in time causing Intersymbol interference or ISI.
  - Fading (Wireless Communications)
    - Signal amplitude can change in a random fashion
    - Fading is very important - studied in Cellular and wireless Personal Communications class
  - Time Variation
    - Time-varying channels cause signal fading. Also different components of the signal can be faded at different levels and this often causes random filtering of the signals (hence ISI).

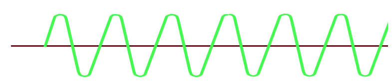
coaxial cable : low pass channel

Radio Waves often form part

- There are three basic concepts about radio channels that are important to understand
- Concept 1: The radio waves can have different frequencies



High frequency-
the waves vary
quickly

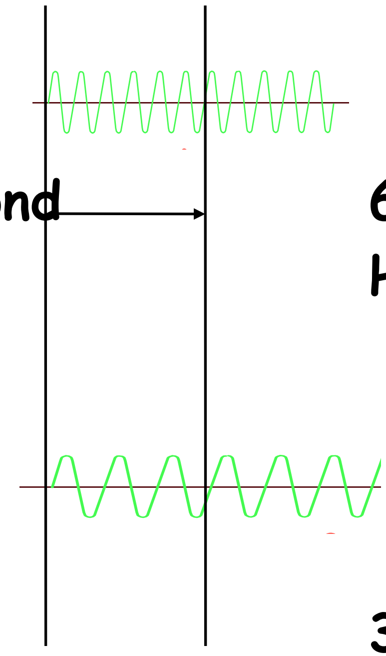


Low frequency-
the waves vary
slowly

Concept 1: Frequency

- The frequency of the waves is specified in cycles per second- or Hertz (after the inventor of the first antenna)

1 Second

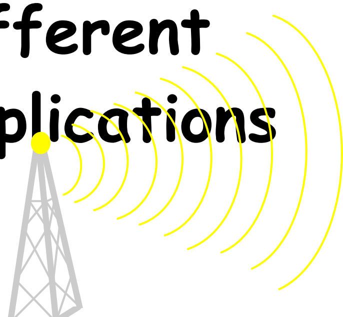


6 cycles/s or 6 Hz

3 cycles/s or 3 Hz

Radio Spectrum

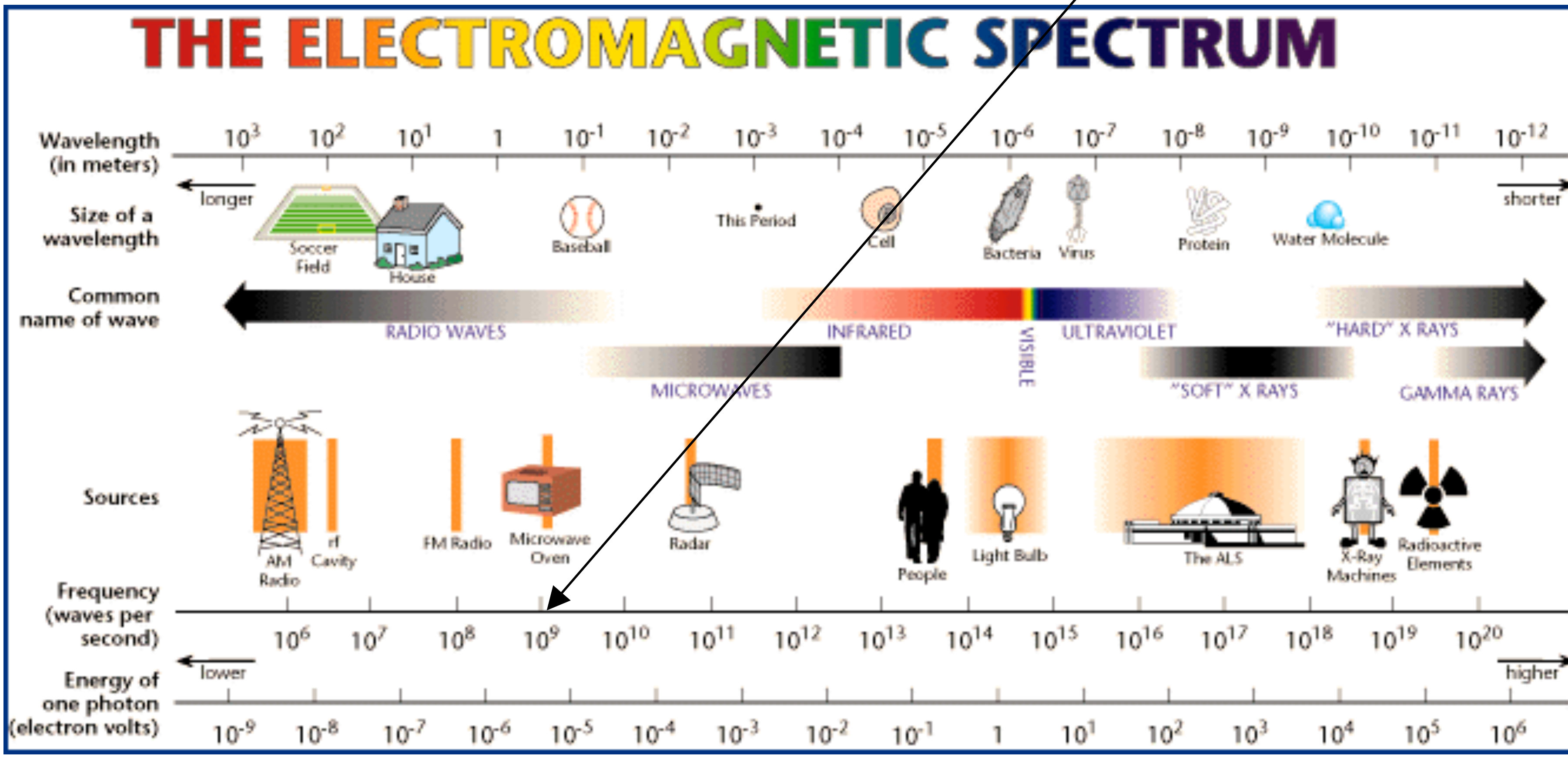
- **The set of all frequencies from 0Hz to infinity is known as the radio spectrum and is used for many different applications**



Frequency	Usage
30-300Hz ELF	
300-3kHz VF	
3k-30kHz VLF	
30k-300kHz LF	
300k-3MHz MF	Broadcast AM
3M-30MHz HF	
30M-300MHz VHF	Paging/TV/ Broadcast FM
300M-3GHz UHF	Mobile
3G-30GHz SHF	Satellite
30G-300GHz EHF	
	Remote control
	Camera
	Medicine

EM Spectrum

Mobile Phone, Wireless LANs, etc



Concept 2: Sharing and Regulation

- Concept 2: Radio waves travel or propagate through a common channel that everybody shares
- That is for a particular frequency only one person, user or company can use it- otherwise there will be interference and chaos!

Regulation of Radio Spectrum

- The government effectively owns the radio spectrum and regulates it
- In some cases the government sells the spectrum to a user or company
- The government of different countries must coordinate the regulation of the spectrum

Scam: start with # (from fake power station)

Regulation of Radio Spectrum

Wifi, many time slots are wasted
low spectrum efficiency

- In most countries this process has been performed by auction- the government will sell the spectrum to the highest bidder

GSM

2.4GHz 5.6GHz

- This is thought more efficient since it allows the spectrum to be allocated by the free market
- Most auctions now take place over the internet and may take several weeks!
- In the UK the 3G spectrum was auctioned for over HK$200 Billion! Absolutely incredible!
- In HK auctioning has also been performed

spectrum efficiency of wifi is non-desirable,
robust, just operate then good!

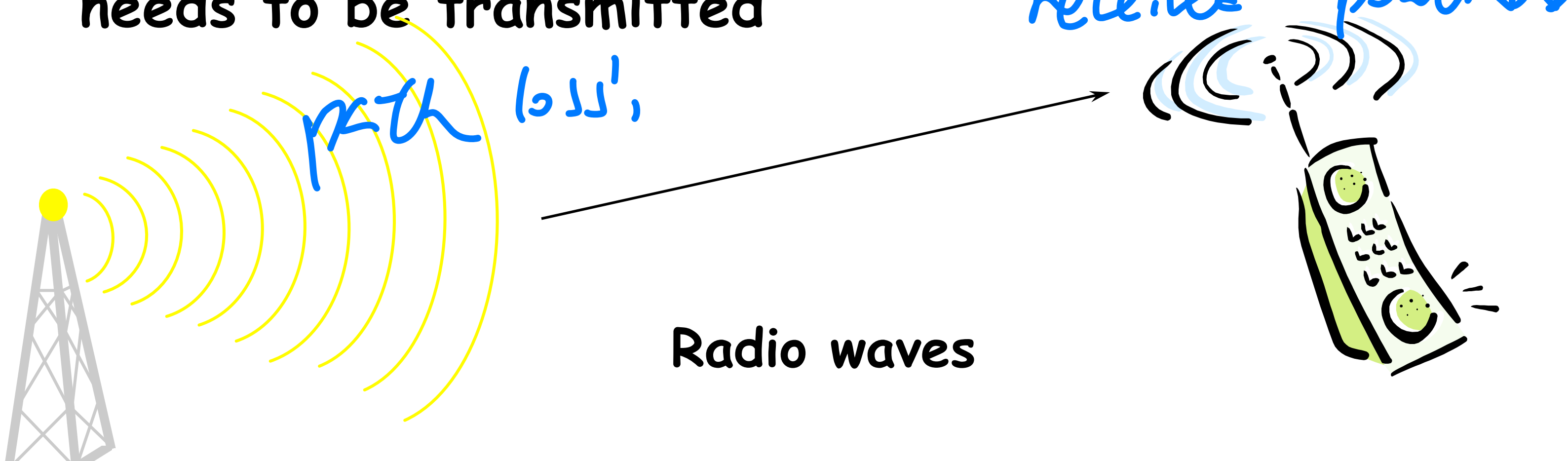
attenuation

Concept 3: Propagation

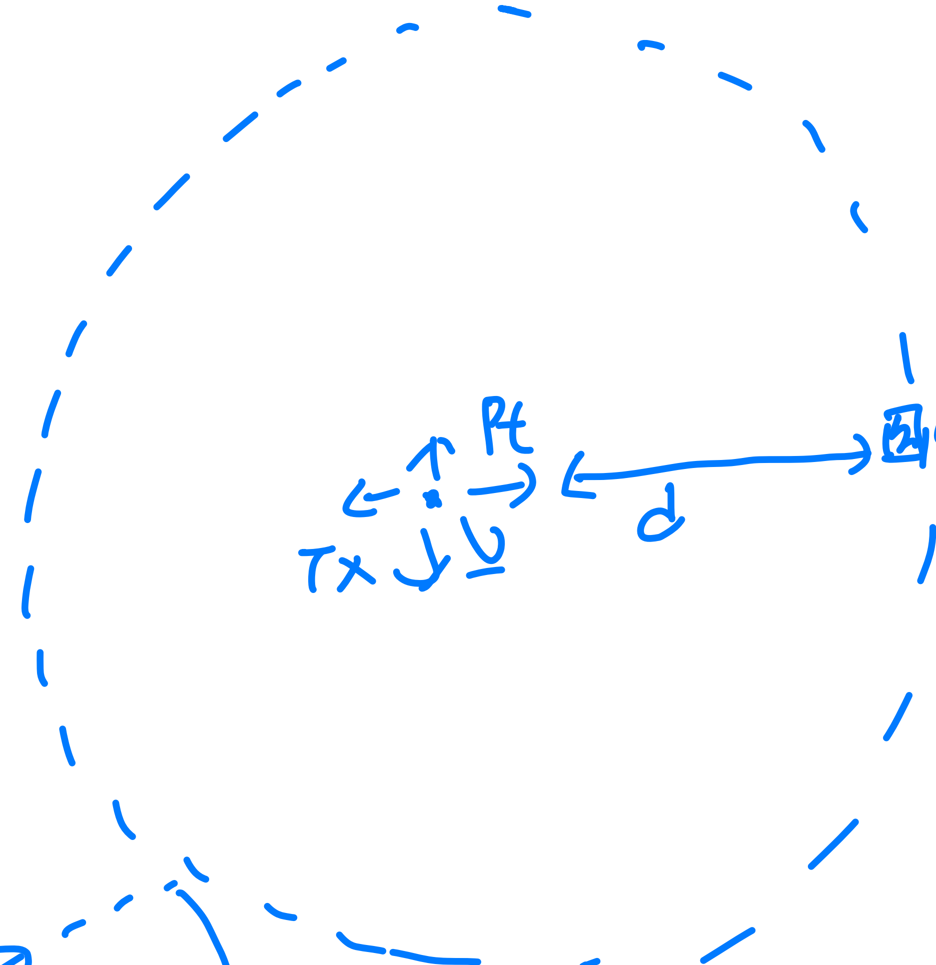
- As the radio waves travel from the transmitter to the receiver their strength decreases or attenuates

$P_r \propto \frac{1}{d^2}$

- Therefore the further the receiver is away from the transmitter the more power that needs to be transmitted



$P_r \propto \frac{1}{d^2}$

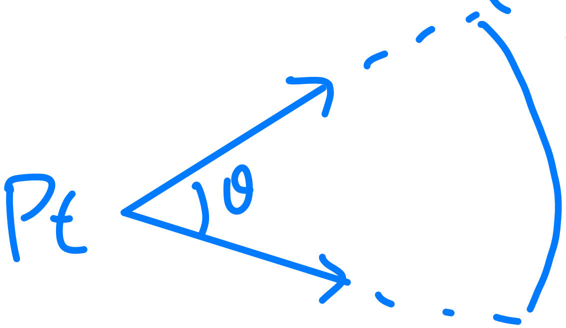


$$P_r = \frac{P_t}{4\pi d^2} A_e$$

$$\Downarrow$$

$$P_r \propto \frac{1}{d^2}$$

☆ $\boxed{S = O(d^2)}$



EM wave spread to bigger and bigger surface area!

still spread

⇓

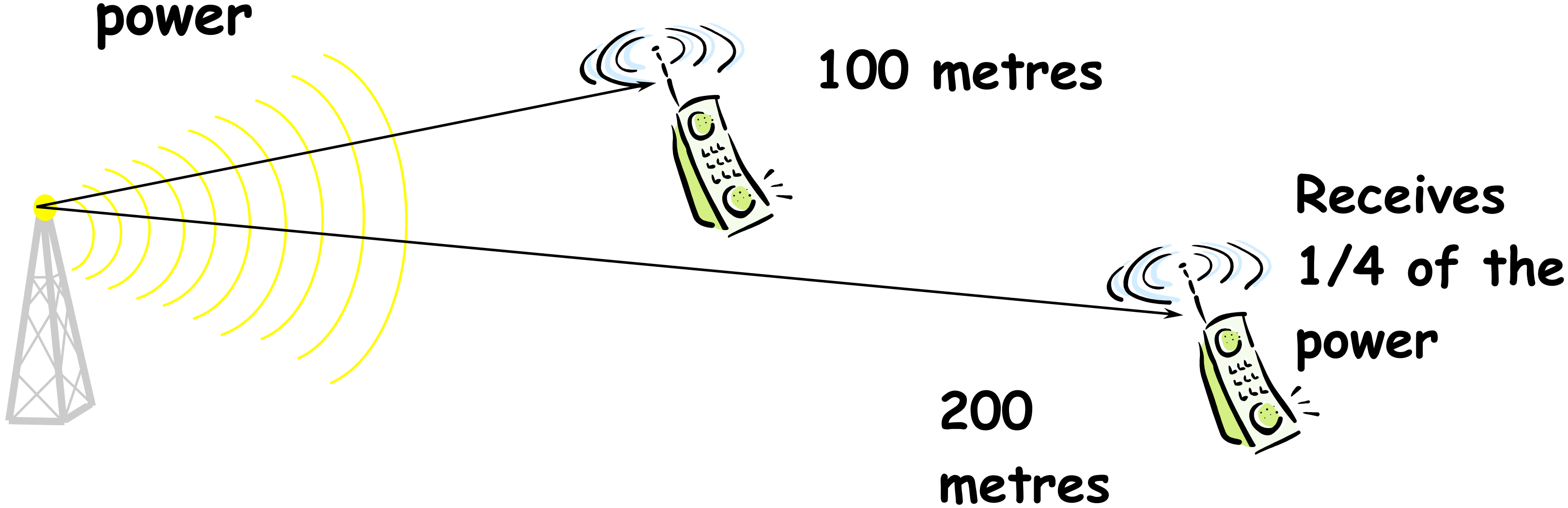
have inverse square law

$$P_r = \frac{P_t}{S} A_e \propto \frac{1}{d^2}$$

 ⇄ laser ⇒ $O(1)$

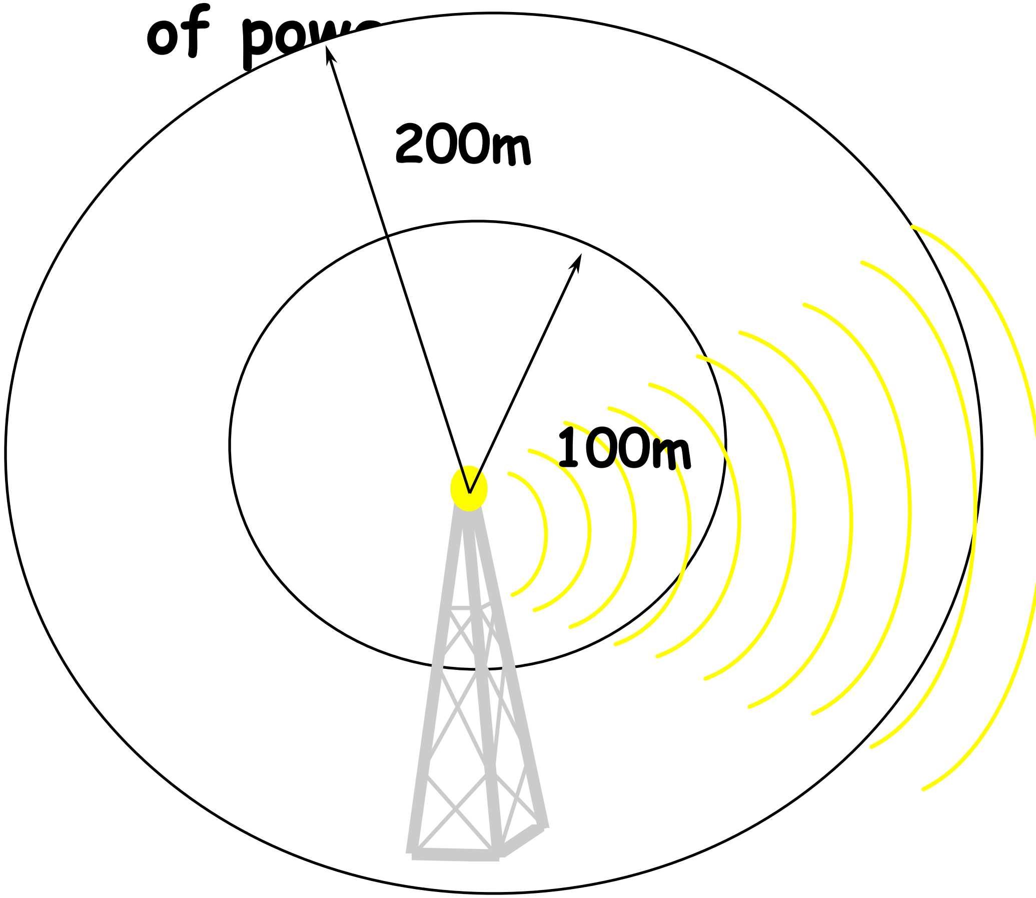
Propagation

- Calculating the attenuation is very difficult
- However it roughly obeys an inverse square distance law- that is every doubling of distance means the receiver receives 1/4 less power



Propagation

- The reason for this is simple conservation of pow

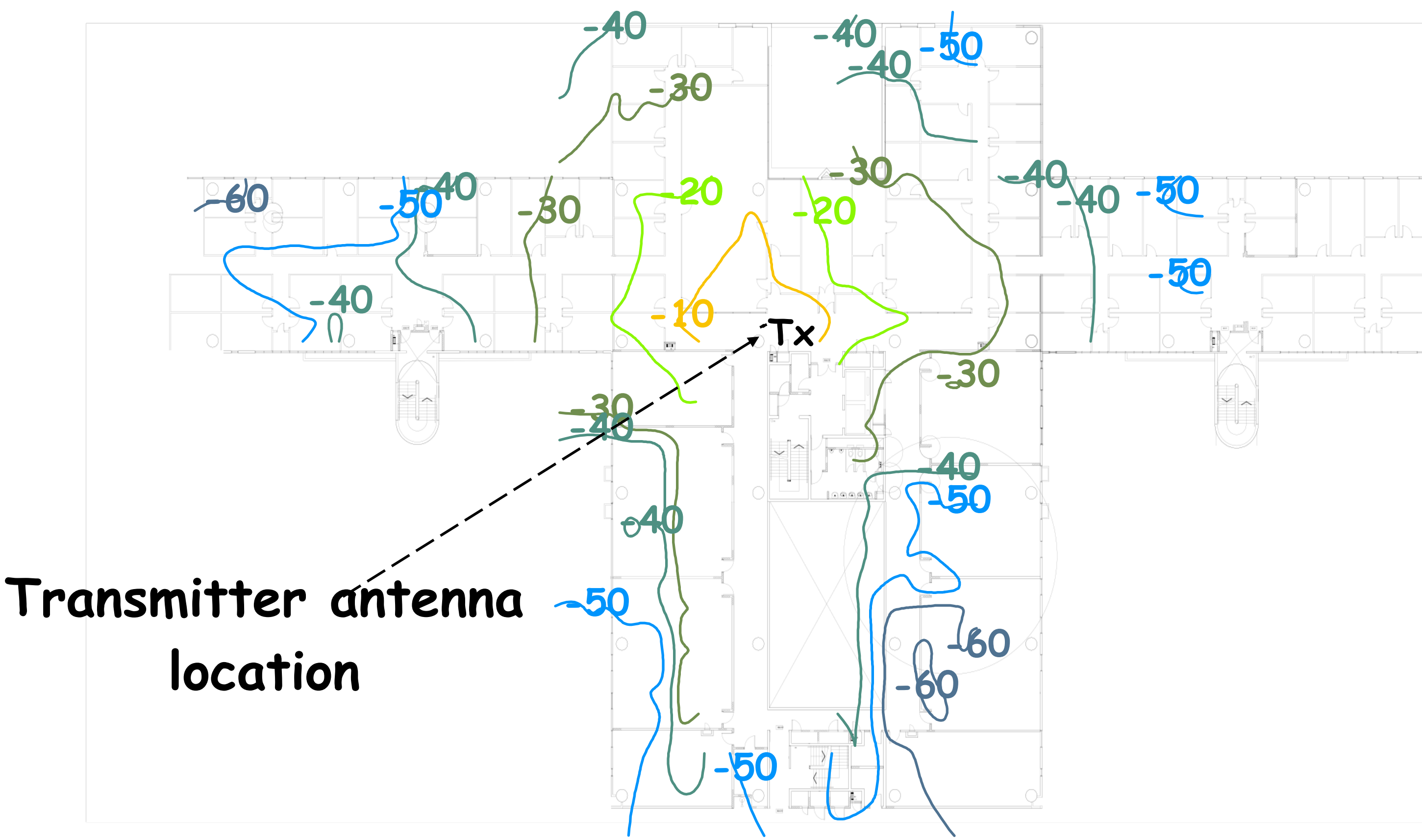


- Total power input must equal total power output
- Surface area of sphere is $4\pi r^2$
- Therefore as radius or distance increases total power on surface must decrease as inverse square distance

Propagation

不同 spectrum → very different path loss

- In more complicated situations it is not so straight forward however it roughly follows the general inverse distance square law



Path loss

Good

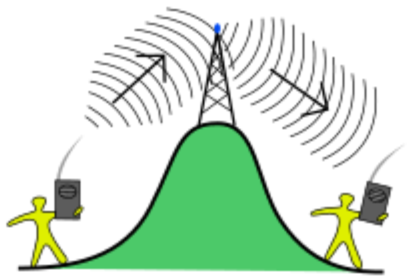
*** - Capacity (important!)

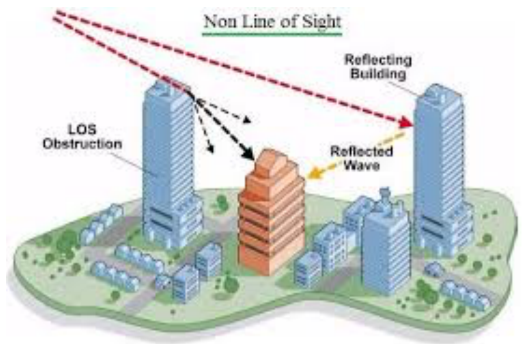
Bad

- Coverage limited

Propagation of Wireless

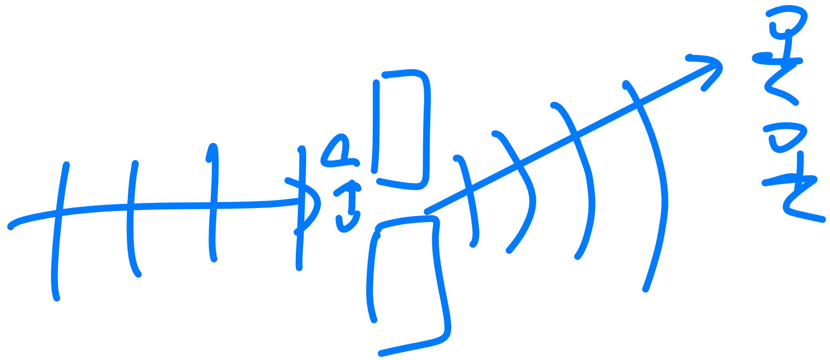
- Line-of-Sight Propagation (LOS)
  - You need to see the transmitter to receiver signals ?
  - Light rays, high frequency radio signals (28GHz, 60GHz)





- Non Line-of-Sight Propagation (NLOS)
  - Radio signals can bend upon obstacles
  - Diffraction, reflection, scattering
  - Effective for lower frequencies (e.g. FM radio, VHF, UHF, 2GHz)
  - Very important for wireless access applications

Diffraction



$a \approx \lambda$

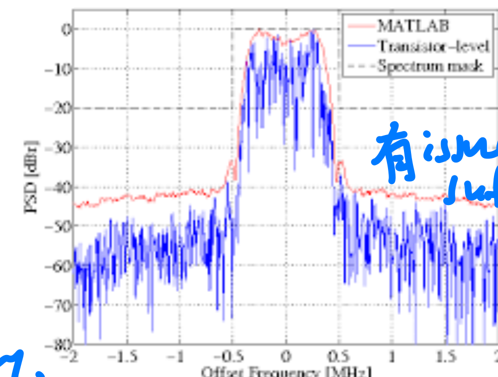
$f_c = 2\text{GHz}$ (very high demand!)
Auction \$\$

$f_c = 28\text{GHz}$ (low demand!)
spectrum density much lower! (不用这个!)

Wireless Spectrum

- All radio spectrum resources are regulated
- In general, a license is required to transmit signals on a particular spectrum
  - A spectrum license usually specifies
    - Carrier frequency (e.g. 2GHz for 3G, 2.4GHz for ISM)
    - Allowed bandwidth (e.g. 5MHz bandwidth about 2GHz carrier frequency) ☆ capacity!
    - Maximum transmission power mask

under that spectrum
for safety reason
determines your coverage



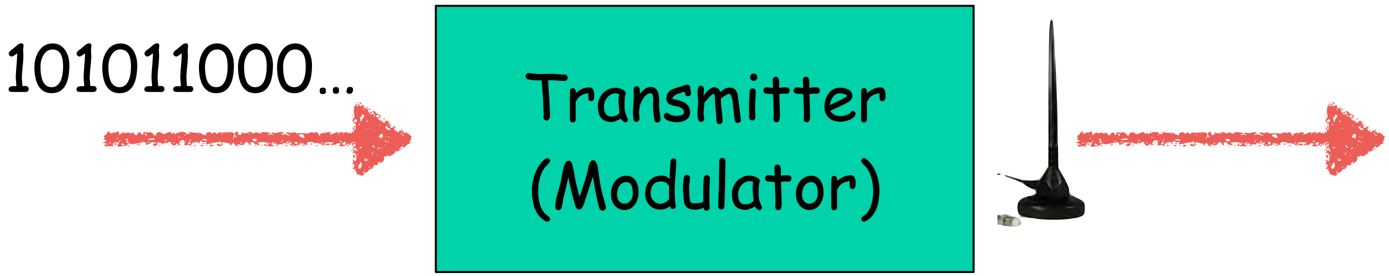
Output Spectrum comparison between MATLAB π

有issue, not sufficient to separate!

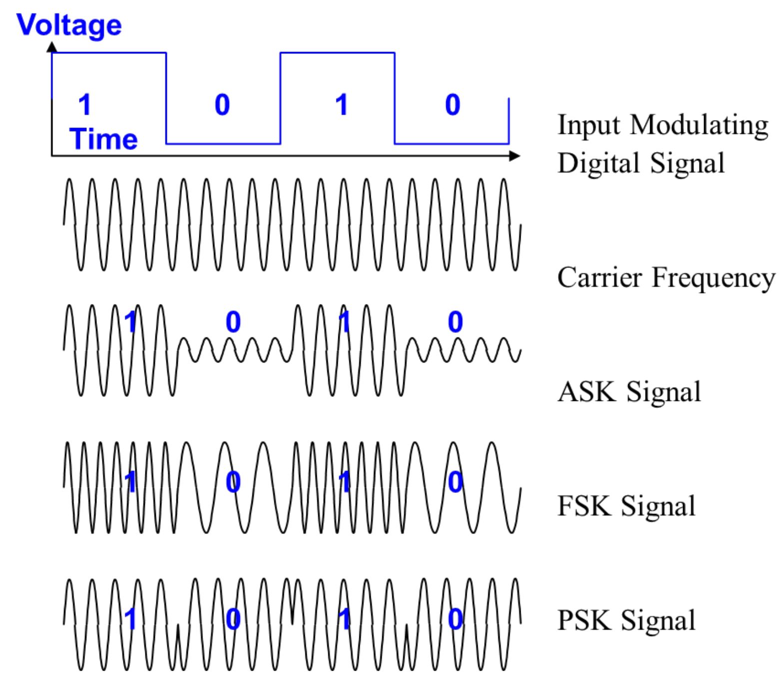
more large carrier frequency, more larger the difficult of modulation

- Some spectra are license-free
  - ISM Spectrum @ 2.4GHz and @5 GHz used for WiFi, bluetooth.
- Higher frequency spectra have more available bandwidth but unfavourable propagation (poor coverage)
- Lower frequency spectra have favourable propagation (good coverage) but

Transmitter (Modulator)



- In most cases, information signals cannot be directly transmitted over the wireless channel
  - Radiation efficient not good
  - frequency spectrum of information signal does not match that of the licensed spectrum
- Transmitter needs to find ways to "carry" information on the RF signals
- Basic parameters
  - {Symbol duration Ts + Signal Set S}
  - For binary modulator, bit rate = 1/Ts

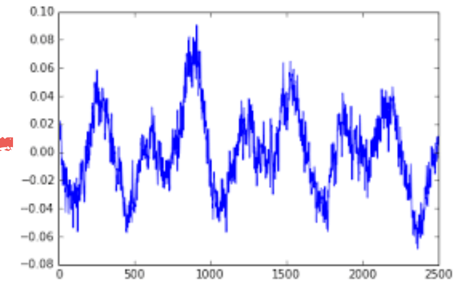


Receiver (Demodulator)

101011000…

recovered information bits

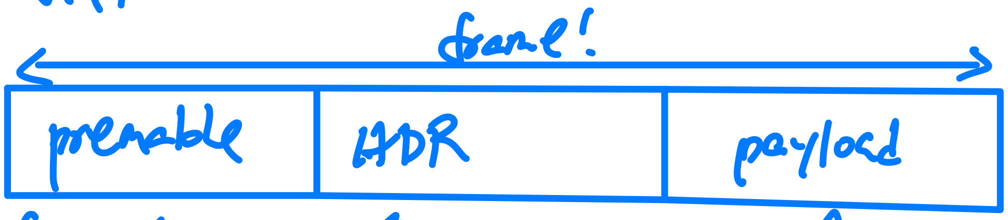
Receiver (Demodulator)



Noisy RF signal

- Receiver's task is to clean up the noisy received RF signal and recover the information bits from it.
- Although the waveform is distorted, as long as the information bits behind the waveform can be recovered, there is no loss of information.
- Performance metric
  - Bit Error Rate (BER) = # of bits corrupted / total # of bits transmitted
  - What is the worst case BER? 0.5
  - Frame Error Rate (FER) = # of frames corrupted / total # of frames transmitted
  - What is the worst case FER? 1, cannot flip the frame!

Wifi:



- for other purpose!

? modulation
? Bit rate

like passenger in mini bus