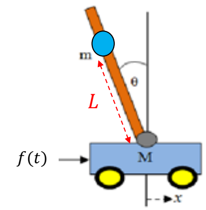
T13

Inverted Pendulum
Half-Power Frequency
Geometric Evaluation
Butterworth Filter
Block Diagram

1

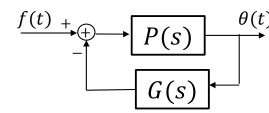
Inverted Pendulum



$$H(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0}}$$

$= 0$



Simple inverted pendulum

System function $P(s) = \dfrac{1/ML}{s^2 - \frac{g}{L}}$

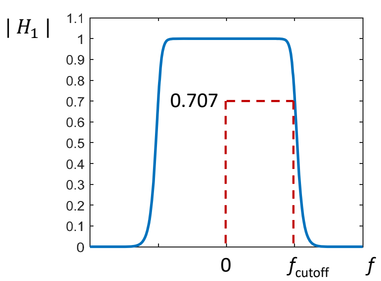
Inverted pendulum with feedback

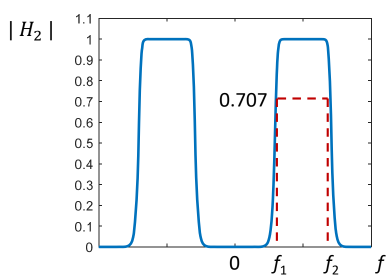
System function $H(s) = \dfrac{1/ML}{s^2 + K_2 s + \left(\frac{K_1}{ML} - \frac{g}{L}\right)}$

Question : What is the zeta parameter ?

2

Half-Power Frequency (i.e. Cutoff frequency)





Cutoff frequency : f_{cutoff}

Cutoff frequencies : f_1 , f_2

$$|H(j\omega)| = \frac{1}{\sqrt{2}}|H(j\omega)|_{max}$$

– voltage gain

$P \propto V^2$

3

4

Geometric Evaluation

$$H(s) = \frac{s - z_1}{(s - p_1)(s - p_2)}$$

$$|H(j\omega)| = \frac{|\overrightarrow{j\omega} - \overrightarrow{z_1}|}{|\overrightarrow{j\omega} - \overrightarrow{p_1}||\overrightarrow{j\omega} - \overrightarrow{p_2}|}$$

$\boldsymbol{\omega = \omega_1}$

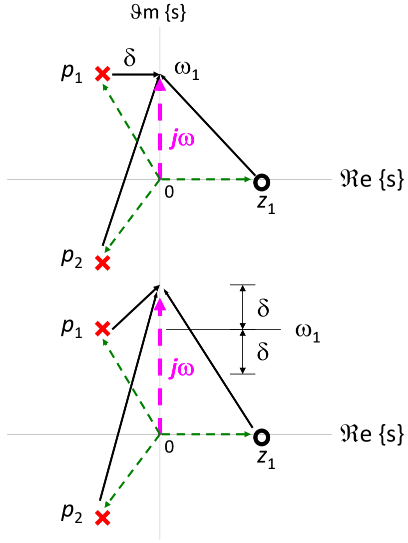
$$\left|\overrightarrow{j\omega} - \overrightarrow{p_1}\right| = Min = \delta$$

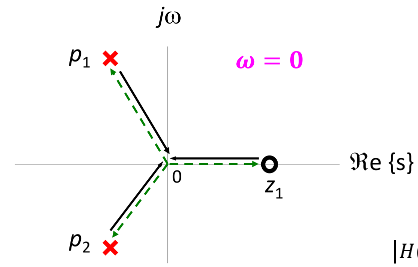
$$|H(j\omega)| = Max$$

$\boldsymbol{\omega = \omega_1 \pm \delta}$

$$\left|\overrightarrow{j\omega} - \overrightarrow{p_1}\right| = \sqrt{2}\,\delta$$

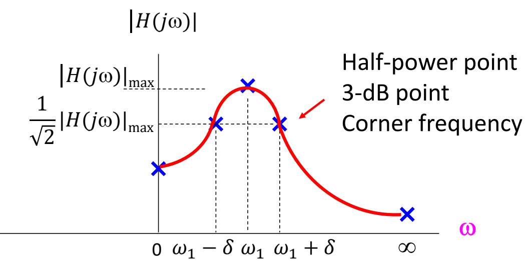
$$|H(j\omega)| = \frac{1}{\sqrt{2}}\,|H(j\omega)|_{max}$$





$\omega \to \infty$

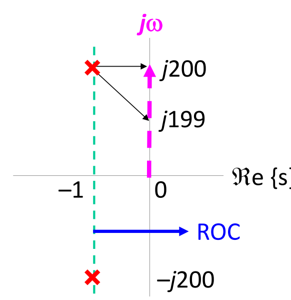
$$|H(j\omega)| \to 0$$



e.g. Given : $H(s) = \dfrac{4 \times 10^3}{(s + 1 + j200)(s + 1 - j200)}$ $\quad Re\{s\} > -1$

a) Plot poles and zeros
b) Stable ? Causal ? Oscillatory ?
c) Sketch the magnitude response
d) Maximum gain ? Half-power frequency ?
e) Type of this filter ?
f) Is it a real system ?

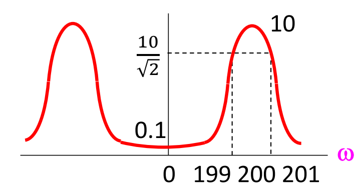
$$H(s) = \frac{4 \times 10^3}{(s + 1 + j200)(s + 1 - j200)} \qquad Re\{s\} > -1$$



$$|H(0)| \approx \left|\frac{4 \times 10^3}{(j200)(-j200)}\right| = 0.1$$

$$|H(j200)| \approx \left|\frac{4 \times 10^3}{(j400)(1)}\right| = 10$$

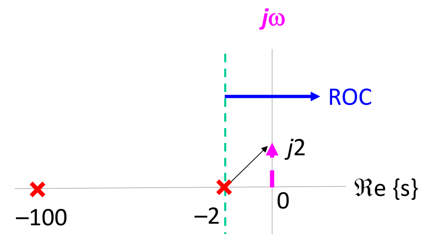
$$|H(j199)| \approx \left|\frac{4 \times 10^3}{(j400)(1 - j)}\right| = \frac{10}{\sqrt{2}}$$



e.g. Given : $H(s) = \dfrac{400}{(s+100)(s+2)}$ $\quad Re\{s\} > -2$

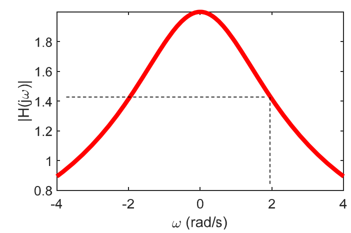
a) Plot poles and zeros
b) Stable ? Causal ? Oscillatory ?
c) Sketch the magnitude response
d) Maximum gain ? Half-power frequency ?
e) Type of this filter ?
f) Is it a real system ?

$$H(s) = \frac{400}{(s+100)(s+2)} \qquad Re\{s\} > -2$$



$$|H(0)| = \left|\frac{400}{(100)(2)}\right| = 2$$

$$|H(j2)| \approx \left|\frac{400}{(100)(j2+2)}\right| = \frac{2}{\sqrt{2}}$$



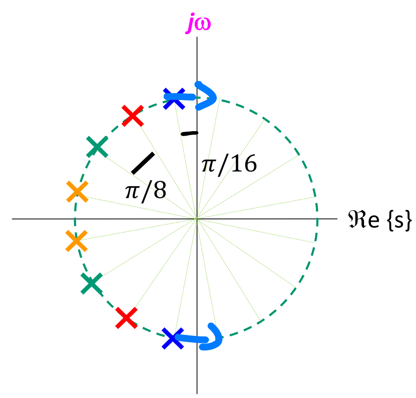
Butterworth Filter

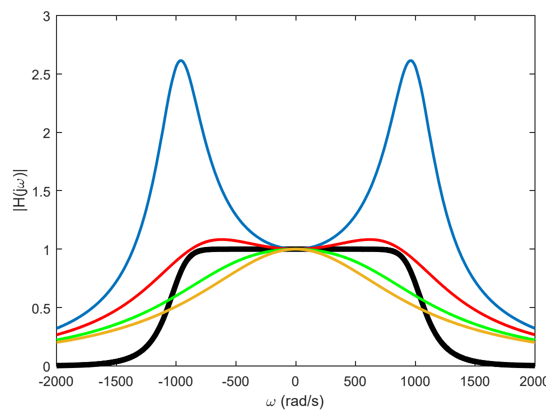
$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2N}}} = \frac{1}{\prod_{k=1}^{N}\left(1-\frac{s}{j\omega_c\, e^{j\frac{(2\pi k-\pi)}{2N}}}\right)}$$

e.g. Given : N = 8 and the cutoff frequency = 1000 rad/s
a) Sketch the pole locations.
b) Stable ? Causal ? Hence specify ROC.
c) Type of this filter ?

$$|H(j\omega)| = \frac{1}{\prod_{k=1}^{8}\left(1-\frac{s}{j1000\, e^{j\frac{(2\pi k-\pi)}{16}}}\right)} \qquad s = e^{j\frac{\pi}{2}}\,1000\, e^{j\frac{(2\pi k-\pi)}{16}}$$

$$|H(j\omega)| = |H_1(j\omega)|\,|H_2(j\omega)|\,|H_3(j\omega)|\,|H_4(j\omega)|$$

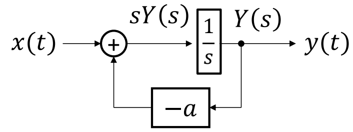




Block Diagram

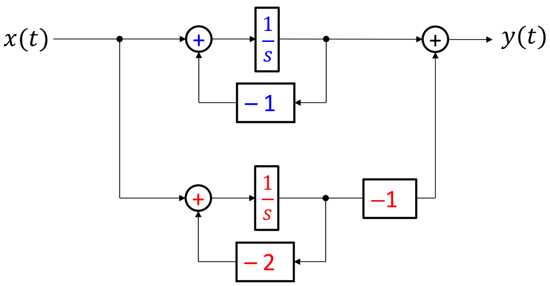
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+a} \qquad \frac{dy(t)}{dt} + ay(t) = x(t)$$

Integrator



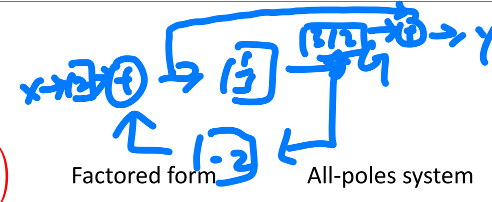
$$Y(s) = \frac{1}{s}[X(s) - aY(s)]$$

e.g. $H(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{(-1)}{s+2}$



Parallel form

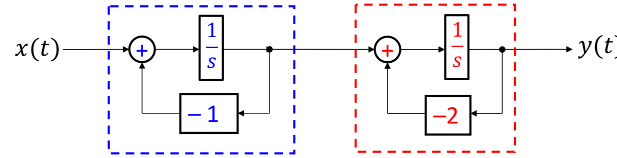
All-poles system



$$H(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)$$

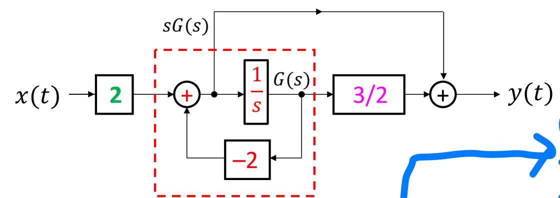
Factored form

All-poles system



$$H(s) = \left(\frac{2s+3}{s+2}\right) = 2\left(\frac{s+3/2}{s+2}\right)$$

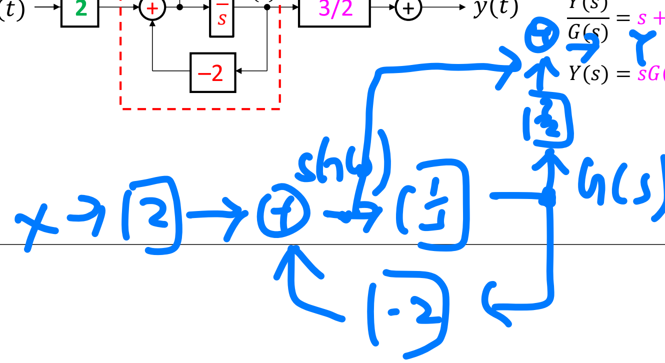
Single-zero-single-pole system



$$\frac{G(s)}{X(s)} = \frac{1}{s+2}$$

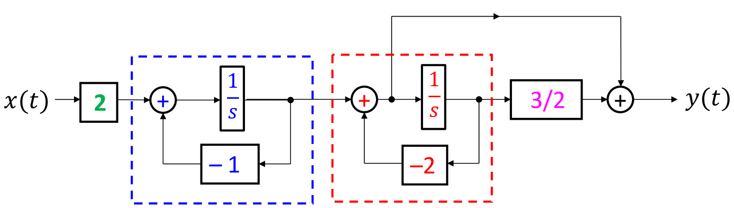
$$\frac{Y(s)}{G(s)} = s + \frac{3}{2}$$

$$Y(s) = sG(s) + \frac{3}{2}G(s)$$



$$H(s) = \frac{2s+3}{(s+1)(s+2)} = 2\left(\frac{s+3/2}{s+2}\right)\left(\frac{1}{s+1}\right)$$

Factored form



Cascaded form

$$H(s) = \frac{2s+3}{s^2+3s+2}$$

Direct Form

Question : Why to learn direct form ?

$$\frac{G(s)}{X(s)} = \frac{1}{s^2+3s+2}$$

All-poles system

$$\frac{Y(s)}{G(s)} = 2s + 3$$

All-zeros system

$$s^2G(s) = X(s) - 3sG(s) - 2G(s)$$