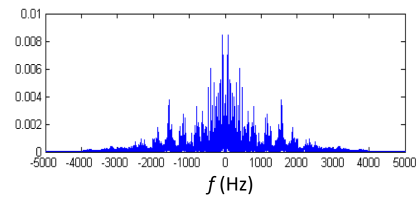
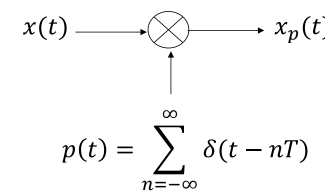
T09

From Continuous-time to Discrete-time
Interpolation
Sampling theorem
Aliasing
Sampling by gating
Anti-Aliasing Filter

From Continuous-Time to Discrete-Time

Frequency spectrum of real signal $x(t)$





Question :
- Band-limited signal ?
- Application of sampling ?

Question : Why is it important to know the sampling frequency ?

reverse know original frequency?

C/D conversion

e.g. $x(t) = \cos(2\pi t)$ $\rightarrow$ $x[n] = \cos\left(\frac{2\pi}{4} n\right) = \cos\left(\frac{\pi}{2} n\right)$

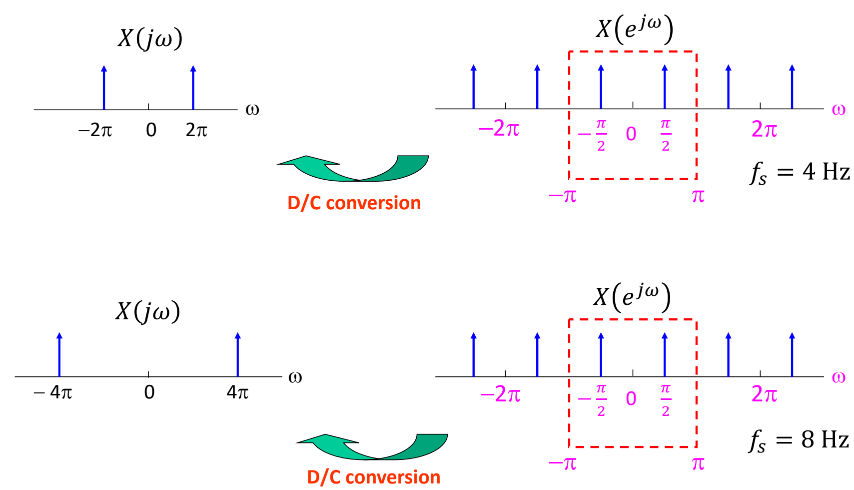
T= 1 sec $\quad$ $f_s = 4$ Hz

How to tell
the difference ?

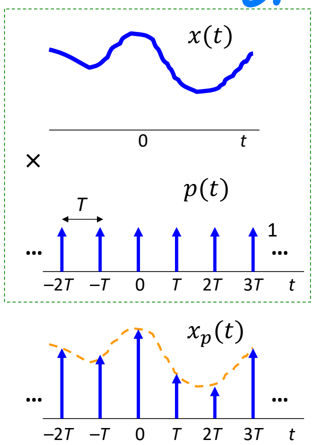
C/D conversion

e.g. $x(t) = \cos(4\pi t)$ $\rightarrow$ $x[n] = \cos\left(\frac{4\pi}{8} n\right) = \cos\left(\frac{\pi}{2} n\right)$

T= 0.5 sec $\quad$ $f_s = 8$ Hz

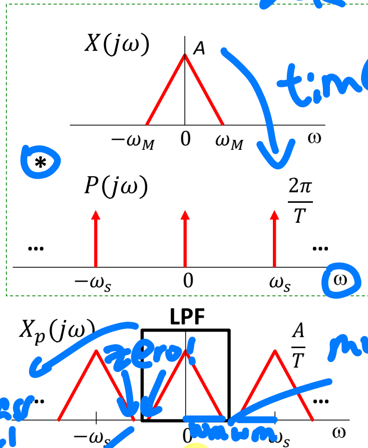


ensure ↙ 44.1Hz sampling
20k



F.T.

CTFT



ideal low pass filter!

must be bandlimited!

$\omega_s > 2\omega_m$

$$x_p(t) = x(t)\,p(t) = \sum_{n=-\infty}^{\infty} \boxed{x(nT)}\,\delta(t-nT)$$

$$X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

Time-domain

Frequency-domain

constant!

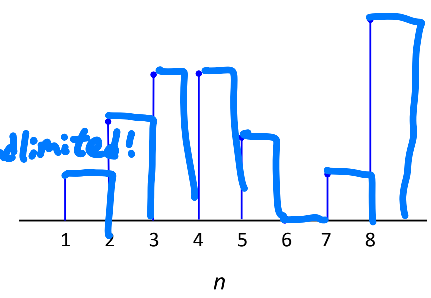
$\omega_s = 2\omega_m$, requirement!

40K, no ideal LPF

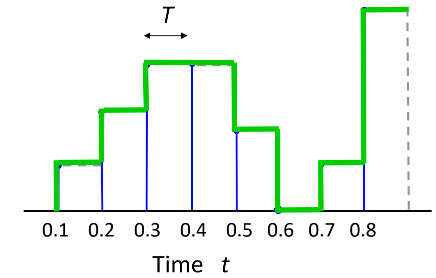
Interpolation

sinc function

$x[n]$ DT signal

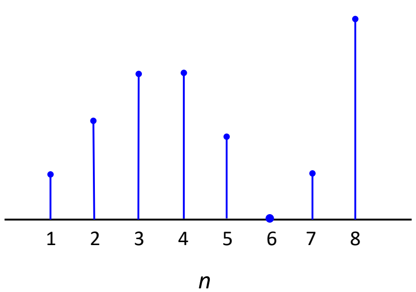


$x(t)$ CT signal **staircase approximation**

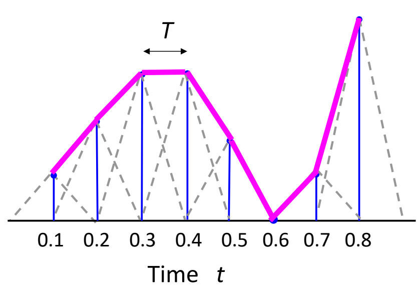


choose approprite fs

$x[n]$ DT signal



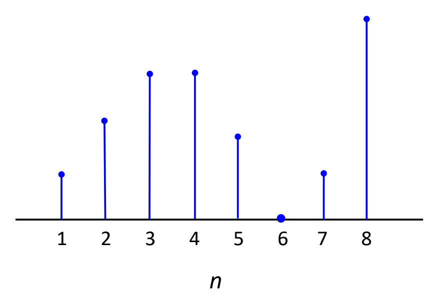
$x(t)$ CT signal **Matlab "plot"**



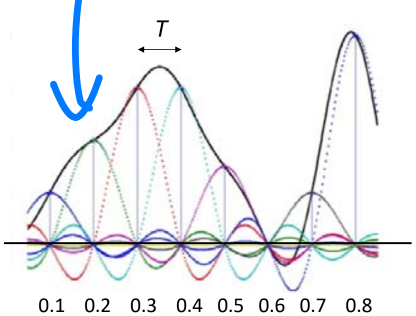
just perfect smooth curve

Question : What is the meaning of using ideal LPF for reconstruction ?

$x[n]$ DT signal



$x(t)$ CT signal **MATH sinc function**

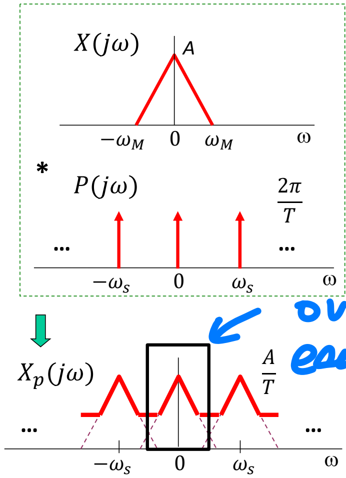


Sampling Theorem

The minimum sampling frequency

$\omega_s > 2\omega_M$

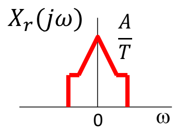
Nyquist rate : ?



overlap each other!

Aliasing

- Distortion
- $\omega_s < 2\omega_M$



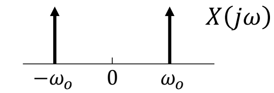
LPF

Aliasing

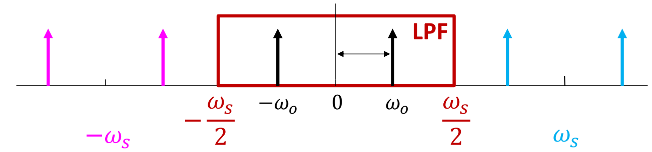
$\omega_s \downarrow$, time duration $\uparrow$

Question : Actually meaning?

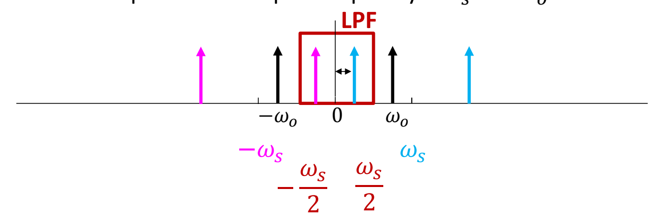
e.g. $x(t) = \cos(\omega_o t)$ $\xrightarrow{\text{F.T.}}$



a) What is the perceived output frequency if $\omega_s > 2\omega_o$?



b) What is the perceived output frequency if $\omega_s < 2\omega_o$?



Different CT signals with different actual signal frequencies shown below

2.2	3.8	4.2	5.8	– 1.8	(Hz)
4.4π	7.6π	8.4π	11.6π	– 3.6π	(rad/s)

CT

If the sampling frequency = 2 Hz

DT angular frequency = CT actual angular frequency (in rad/s) / sampling frequency (in Hz)

2π + 0.2π   4π – 0.2π   4π + 0.2π   6π – 0.2π   – 2π + 0.2π

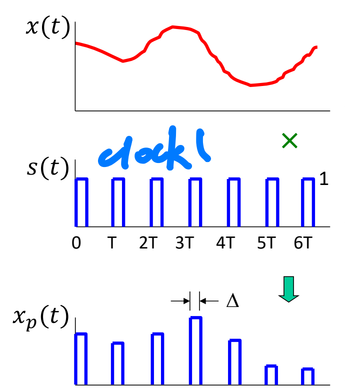
$$\cos(2\pi n + 0.2\pi n) = \cos(0.2\pi n)$$
$$\cos(4\pi n - 0.2\pi n) = \cos(0.2\pi n)$$
$$\cos(4\pi n + 0.2\pi n) = \cos(0.2\pi n)$$
$$\cos(6\pi n - 0.2\pi n) = \cos(0.2\pi n)$$
$$\cos(-2\pi n + 0.2\pi n) = \cos(0.2\pi n)$$

$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

wrong!

D/C conversion $\Rightarrow$ $\cos(0.4\pi t)$

Sampling by Gating



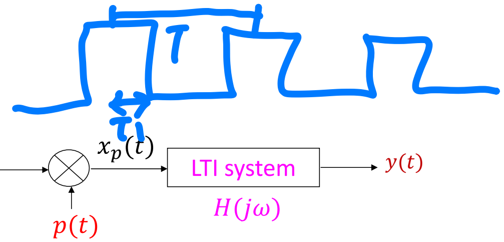
clock!

pulse train!

因为delta function 高度∞，不可model!

Question : The spectrum of the sampled signal $x_p(t)$?

e.g.



Bandlimited signal with unilateral bandwidth of $2\pi \times$ 10 kHz

Period square wave with $T_1 = 5\ \mu s$ and $T = 20\ \mu s$

modulation!

a) Sketch the spectrum of $x_p(t)$.

b) Specify this LTI system if $y(t) = 2\,x(t)\cos(1\times 10^5 \pi t)$.

c) Design a system to reconstruct $x(t)$ using $y(t)$.

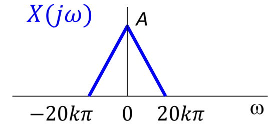
$$x_p(t) = x(t)\,p(t)$$

$$X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$\omega_s = \frac{2\pi}{20\times 10^{-6}} = 10^5\pi$$

$$Y(j\omega) = X_p(j\omega)\,H(j\omega)$$

?

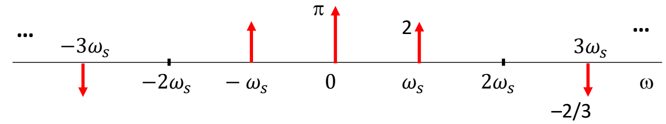


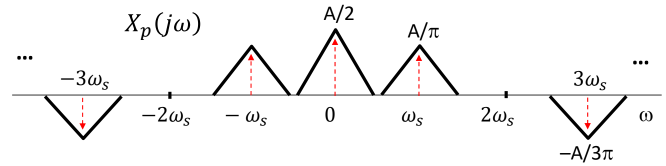
$$P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\sin\left(\frac{2\pi k T_1}{T}\right)}{k}\delta(\omega - k\omega_s)$$

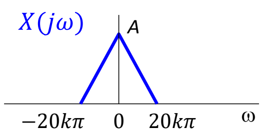
$$= \sum_{k=-\infty}^{\infty} \frac{2\sin\left(\frac{\pi k}{2}\right)}{k}\delta(\omega - k\omega_s)$$

$\frac{5}{20}$

Even function

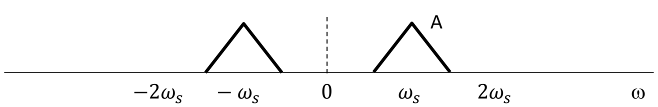


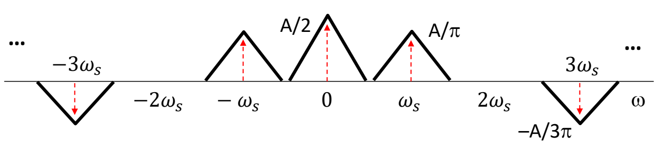




$$y(t) = 2\,x(t)\cos(1\times 10^5\pi t)$$

$$Y(j\omega) = X(j(\omega-\omega_s)) + X(j(\omega+\omega_s))$$





$H(j\omega)$

?

bandpass, gain $= \pi$!

Anti-aliasing Filter

quality

why?

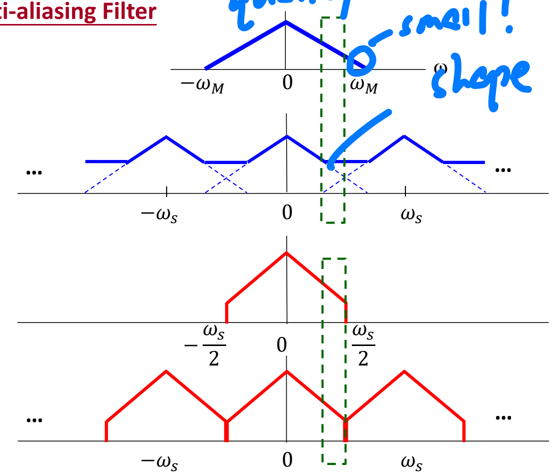
small!

shape

not the same

distortion!

How ?



minimize the spectrum!

$$W_m = \frac{W_s}{2}$$

Question : How to minimize aliasing (e.g. moire effect) if $\omega_s < 2\omega_M$?