Diversity Techniques for Combating Flat Fading

E $P_e(\gamma_b)$ on Rayleigh Flat Fading Channel

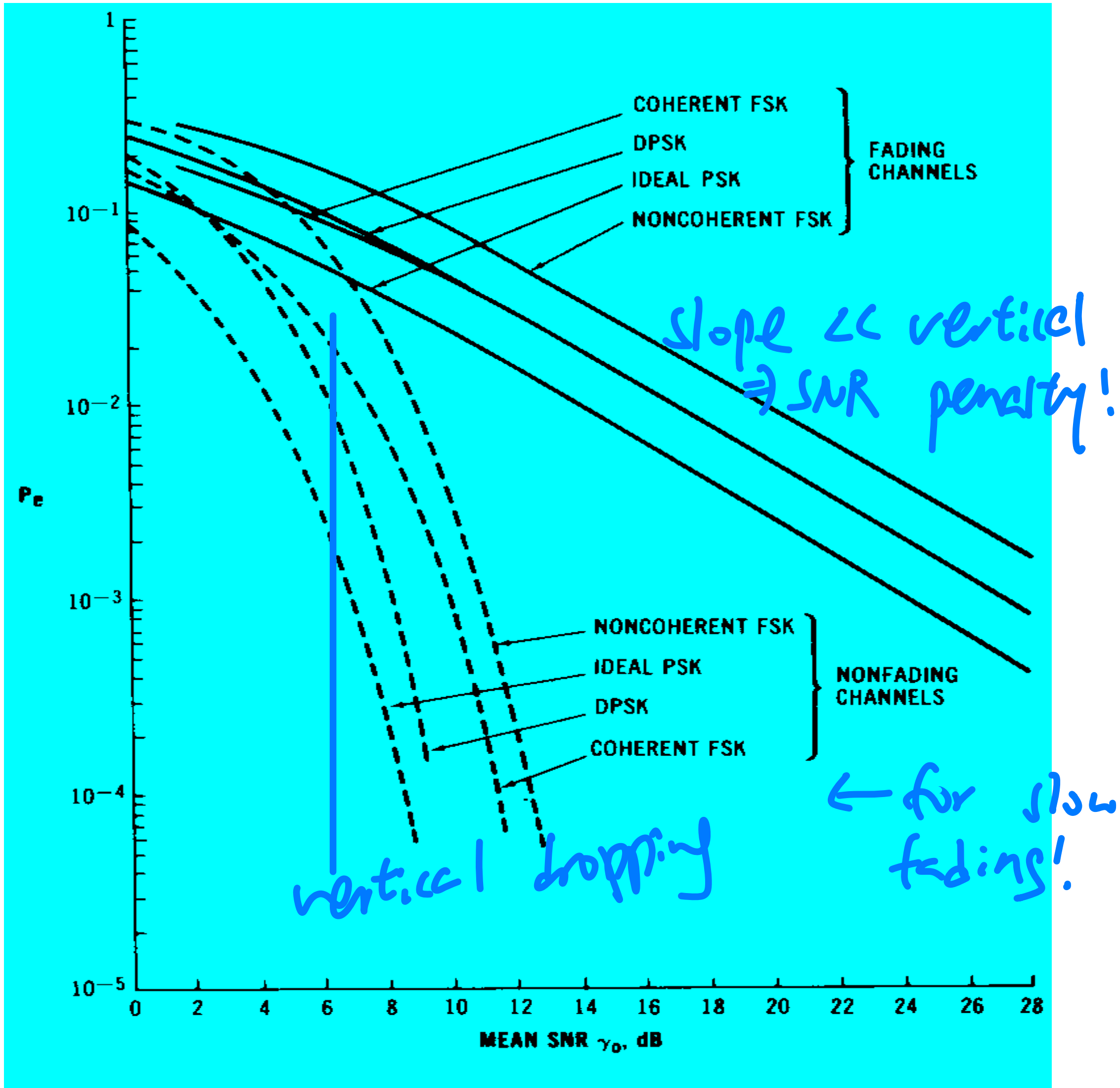


Figure 3.17. Probability of error for several systems in Rayleigh fading.

Remedies: Power Control

- Power Control
  - Control the ***transmit*** power such that the ***received*** SNR is equal to a *pre-designed value* to achieve a *desired* P_e
  - **Open loop**: use the Reverse-link measurement to control Forward-link transmit power or vice versa
    - effective on attenuation affecting both uplink and downlink identically (e.g. shadowing, blockage, and path loss)
    - not accurate in FDD: Rayleigh fading are different in uplink & downlink as they are in different frequency bands
  - **Closed loop**: measure the received signal strength or error rate and report it back to the transmitter for power control
    - Overhead (more bits to be transmitted) in a feedback channel for signal strength or error rate reporting
    - need to be fast enough for the time-varying fades

1. <u>closed loop</u> [slow mobility]

cannot work for high mobility

$y = \alpha s + n$

$\tilde{P}e \sim \frac{1}{4SNR}$

$s \to \boxed{Tx} \to [\frac{1}{\alpha}s] \to y = \alpha[\underbrace{\tfrac{1}{\alpha}s}_{\text{Tx symbol}}] + n$

$\uparrow$

$\hat{\alpha} \leftarrow$ from feedback

outdated

$y = s + n$

effect of α cancelled

$Pe \sim Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$

'1' ... '1'

preamble	HDR	payload

WiFi frame

$y_p = \alpha[1] + n$

$cN \to \infty$

$\hat{\alpha} = \frac{1}{N}\sum_{i=1}^{N} y_p(i) \to \alpha$

$\hat{\alpha}_{LS} = \arg\min \|\vec{y_p} - \alpha 1\|^2 \to$ sample mean

- Now receiver knows α, how transmitter know?
from feedback !

$$\hat{\alpha} \leftrightarrow (t = T_f)$$

want $\alpha(T_f) \approx \alpha(0)$
expect $\alpha(T_f)$ highly correlated with $\alpha(0)$

$T_f \ll T_c$ ← mobility

$T_c \sim 50$ ms
$T_c \sim 1$ms (vehicle)

Pros and Cons on Power Control

- Pros:
  - simple and easy to implement
  - open loop is effective for path loss and shadowing
  - closed loop can track the very slow fading with moderate overhead in feedback channel
  - ***If works well => fixed gain AWGN channel => easy modem design***
- Limitation or Cons:
  - Cannot track fast Rayleigh fading
  - Overhead in the feedback channel
  - Increase CCI (co-channel interference) when mobile is in the cell boundary

Is Power Control (PC) Optimal?

- PC sends more power in bad channel conditions and less power in good channel condition. ***Make sense?!***
- In good channel conditions, need less energy/bit (E_b) while in bad channel conditions, need a higher E_b
- Optimal scheme sends more bits and more power (but small energy/bit) in good channel conditions and fewer bits and less power (though large energy/bit) in bad channel conditions --- just the opposite of PC
- Why does current digital cellular system use PC? (Ans: need 2-way realtime communications)
- How about wireless LAN or other non-realtime traffic?

Diversity

- **Diversity**:
  - Get **multiple** **independent** observations of the information
- ***Q1) Where are these multiple independent observations coming from?***
  - time diversity with interleaving
  - frequency diversity using frequency hopping
  - multi-path diversity
  - antenna diversity or spatial diversity

send same symbols multiple time

through — time → if separation ↑↑
— frequency → ⇒create diversity!

Open loop

(No knowledge of α at Tx)

Uncertainty on "α"

(Diversity order $= L$)

↳ $\boxed{\text{Diversity}}$ — multiple observations about 's' — independent "α"

$P_e \sim \left(\frac{1}{SNR}\right)^L$ - like job application, submit multiple

$\log P_e \sim L \log\left(\frac{1}{SNR}\right)$ - assumption: uncertainty between α and $T\alpha$, independent!

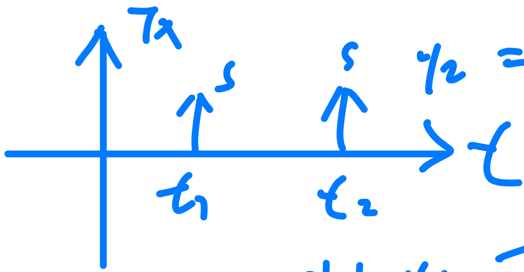
↳ linear slope

modulation is not diversity, just decision for one!

(Q1) Where are these diversity observations from?

(Q2) How to make use of ...

Q1:



$y_2 = \alpha_2 s + n$

multiple observation

$y_1 = \alpha_1 s + n$

$\eta = \frac{R_b}{W} \downarrow 50\%$

$R_b \leftarrow R_b \div 2$!

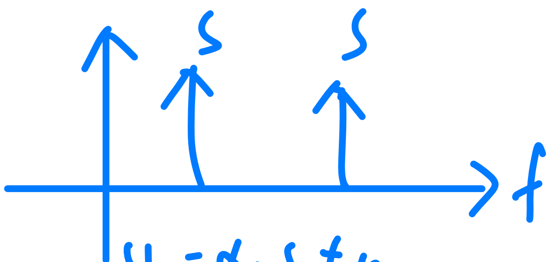
α_1, α_2 are independent?

$\boxed{\Delta t > T_c} \Rightarrow$ uncorrelated

$\propto \frac{1}{\delta\tau}$

α_1, α_2 are jointly Gaussian

Freq. Diversity



$y_1 = \alpha_1 s + n_1$

$y_2 = \alpha_2 s + n_2$

$\eta = \frac{R_b}{W} \downarrow 50\%$

$W \leftarrow W \times 2$

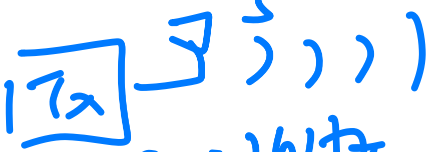
$\Delta f > B_c \propto \frac{1}{\delta\tau}$

B_c, T_c all from channel environment

Heavy penalty!

Spatial Diversity

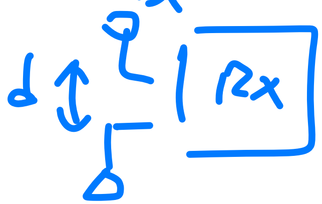
Rx



$f_c = 2\text{GHz}$

$\lambda = 15\text{ cm}$

Tx

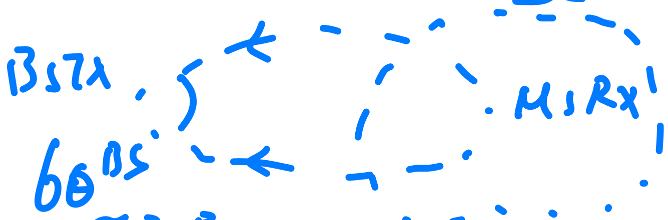


$y_1 = \alpha_1 s + n_1$

$y_2 = \alpha_2 s + n_1$

spacial spreation : d should be large

$\Delta d > D_c \propto \frac{1}{\sigma_\theta}$ angle spread



$\sigma_\theta^{BS} \approx 30^\circ$

$\sigma_\theta^{MS} \approx 2\pi$

$D_c^{BS} = \underline{10\lambda}$ — wavelength at carrier

$D_c^{MS} = \frac{\lambda}{2}$

$\boxed{\text{standardization}}$

- 不, no need buy Tx, Rx from same company!
- promote inter-operability!

① $\eta = 100\%$

② full diversity (diversity = 2)

③ Transparent to Tx

$\boxed{\text{3GPP}}$

each company only have one vote!

chip set

equipment venders!

operators (CMHK, HKTelecom)

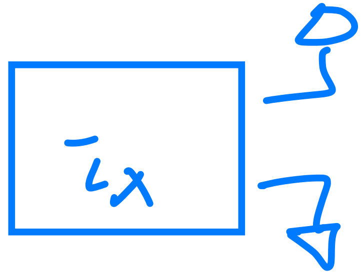
:(no room for complete performance between companies standard for Tx, transmit format!

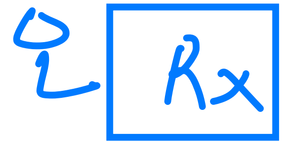
not for Rx !!!

不用改 Tx !!!

small company → good!

Tx Diversity





· 100% ↑

· full diversity

$\delta d > D_c \approx 10\lambda$

att. \ time	①	②
①	S	0
②	0	S

$y_1 = \alpha_1 S + n_1, \quad y_2 = \alpha_2 S + n_2$

full div.
50% ↑

Space time coding. Tx standard!!!

att. \ time	1	2
1	s_1	$-s_2^*$
2	s_2	s_1^*

$$y_1 = \alpha_1 s_1 + \alpha_2 s_2 + n_1$$

$$y_2 = \alpha_1(-s_2^*) + \alpha_2(s_1^*) + n_2$$

$$y_2^* = -\alpha_1^* s_2 + \alpha_2^* s_1 + n_2^*$$

$$\begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^* & -\alpha_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix}$$

observation vector !

unknown data

$\Rightarrow$ full diversity (2) (use 2 times)

$\Rightarrow$ 100% η

conjugate operation is non-linear

how about both Tx and Rx have multiple antennas?

multiple input, multiple output

$\eta = 200\%$!!!

$y_1 = \alpha_1 s + n_1 \rightarrow$

$y_2 = \alpha_2 s + n_2 \rightarrow$ $\boxed{Rx\ ?}$ $\xrightarrow{r}$ $\boxed{\text{Demod}}$ $\rightarrow \hat{s}$

$\vdots$

$y_L = \alpha_L s + n_L$

simple metric !

(will prove is optimal!)

$y \rightarrow \boxed{\text{Demod}} \rightarrow \hat{s}$

(1) <u>selection combining</u>

γ_{sel} = good ⇐ bad

$y_1 = \alpha_1 s + n_1$

reliability

if good term have more power ⇒ reliable ↑↑ ⇒ if α ↑

$|\alpha_1|$ is deep fade!!! $\Rightarrow$ reliable $\downarrow$

$|\alpha| \propto$ Raleigh distribution

$|\alpha_1|$ is large $\Rightarrow$ reliable

$|\alpha_1|$ is small $\Rightarrow$ not reliable!

(1) Selection combining

$\gamma_{rel} = y_{i^*}$ where $i^* = \arg\max\{|\alpha_1|^2, \cdots, |\alpha_L|^2\}$

$$P_e \sim \left(\frac{1}{4SNR}\right)^L \quad \text{fail } L \text{ times}$$

if randomly pick one, without diversity!

discard all other observations!!!

Embrace all the observations together!

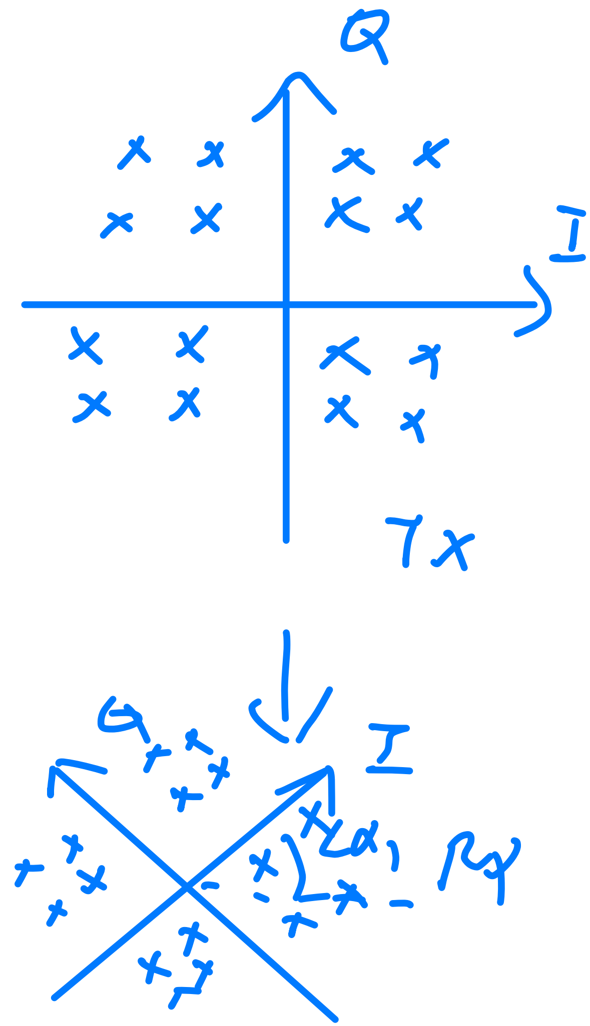
(2) Equal Gain Combining

$y_1 = \alpha_1 s + n_1$

$= |\alpha_1| e^{j\angle\alpha_1} s + n_1$

$r_{EGC} = e^{-j\angle\alpha_1} y_1 + e^{-j\angle\alpha_2} y_2$

Derotation!...



(3) Maximal ratio combining

if $|\alpha_2|$ is small 不该加!

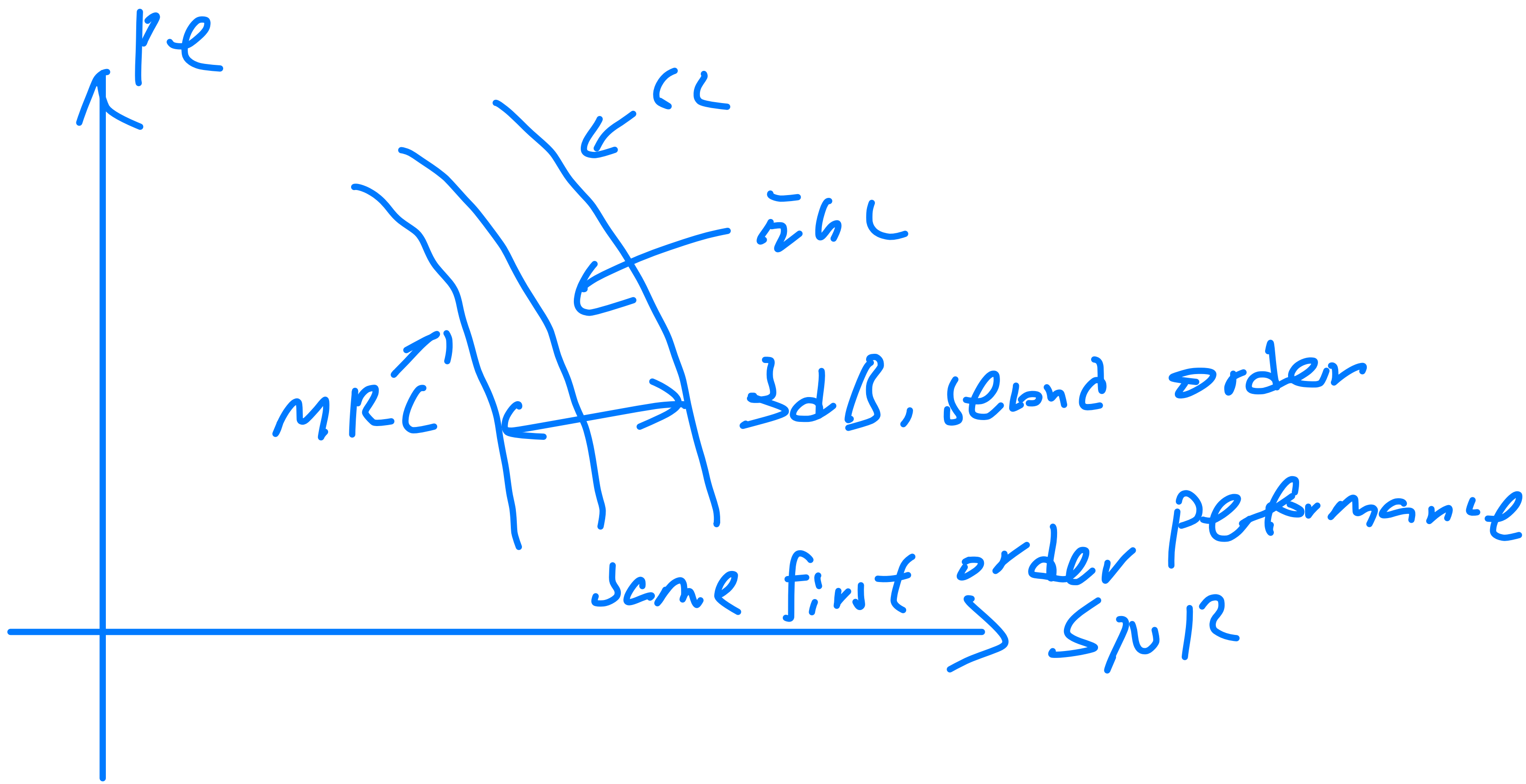
$$\Rightarrow r_{EGC} = e^{-j\angle\alpha_1} y_1 + \cancel{e^{-j\angle\alpha_2} y_2} + \cdots$$

$$r_{MRC} = |\alpha_1| e^{-j\angle\alpha_1} y_1 + |\alpha_2| e^{-j\angle\alpha_2} y_2 + \cdots$$

$$= \alpha_1^* y_1 + \alpha_2^* + \cdots +$$

$\angle\alpha$ $\angle y$

Optimal way for combining!!!



Depends on operating region!

$$\hat{s} = \arg\max_{s \in \{16QAM\}} \log p(y_1 y_2 \mid s ; \alpha_1 \alpha_2)$$

ML

observations hypothesis

conditional to my data!

$$p(y_1 y_2 \mid s ; \alpha_1, \alpha_2) \sim K \exp\left(-\frac{(y_1 - \alpha_1 s)^2}{2\sigma^2}\right) \exp\left(-\frac{(y_2 - \alpha_2 s)^2}{2\sigma^2}\right)$$

$$\sim K \exp -\left[\frac{(y_1 - \alpha_1 s)^2 + (y_2 - \alpha_2 s)^2}{2\sigma^2}\right]$$

$$\log p(\;) \sim \text{const} - \left[\frac{(y_1 - \alpha_1 s)^2 + (y_2 - \alpha_2 s)^2}{2\sigma^2}\right]$$

$$\hat{s} = \underset{s \in 16QAM}{\operatorname{argmin}} \left[|y_1 - \alpha_1 s|^2 + |y_2 - \alpha_2 s|^2 \right]$$

$$|z|^2 = z z^*$$

$$\hat{s} = \underset{s}{\operatorname{argmin}} \Big[|\alpha_1|^2 |s|^2 + |\alpha_2|^2 |s|^2 - (y_1 \alpha_1^*) s^* - (y_1 \alpha_1)^* s - (y_2 \alpha_2^*) s^* - (y_2 \alpha_2)^* s \Big]$$

$$r = \alpha_1^* y_1 + \alpha_2^* y_2$$

$$= \arg\min_s \left[(|\alpha_1|^2 + |\alpha_2|^2)|s|^2 - (r s^* + r^* s) \right]$$

sufficient statistics with respect to detection of s!

MLC metric!!!

$\Downarrow$

optimal !!!

(channel situation) from pilots / prembles

Recap: 3 ways to combine

(1) diversity observations:

$$r_{sel} = y_i - i^* = \arg\max \{ |\alpha_1|^2, |\alpha_2|^2 \cdots \}$$

select most reliable one!

(2) Equal gain combining

$$r_{EGC} = e^{-j\angle\alpha_1} y_1 + \cdots$$

(3) Maximal Ratio Combining (MRC) $r_{MRC} = \alpha_1^* y_1 + \alpha_2^* y_2 + \cdots$

same Pe slope, (3) will be the best

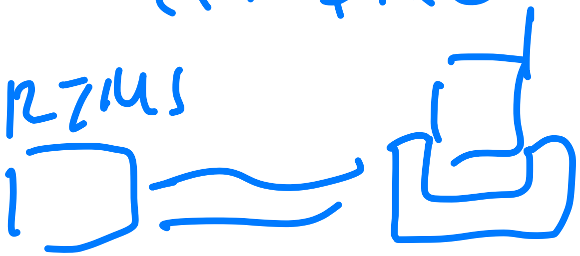
小灵通

(2G → 3G)

GSM → $$$ ↓ ⇒ profit margin ↓

cordless phone → $ (PHS)

(for fixed line network)

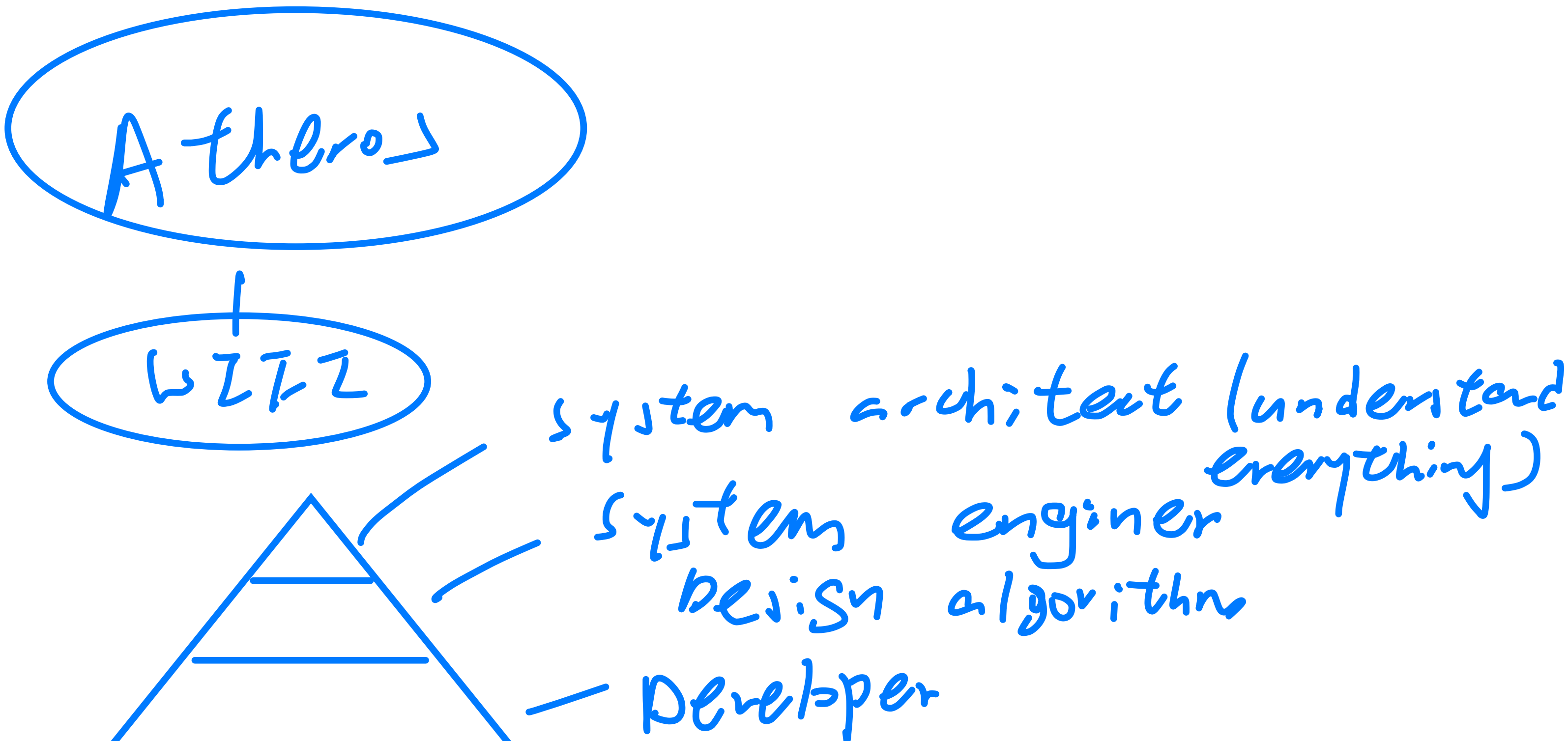


$f = 2.4\,GHz$

no need license!

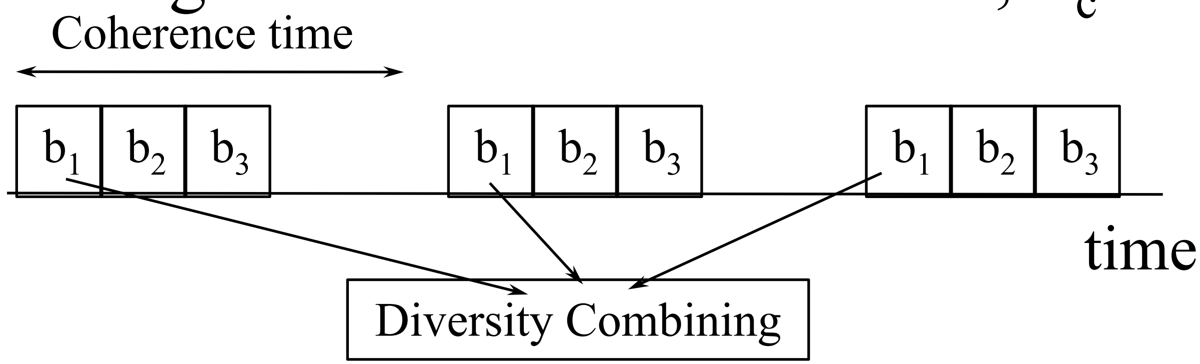
→ need think new business model!

WIFI

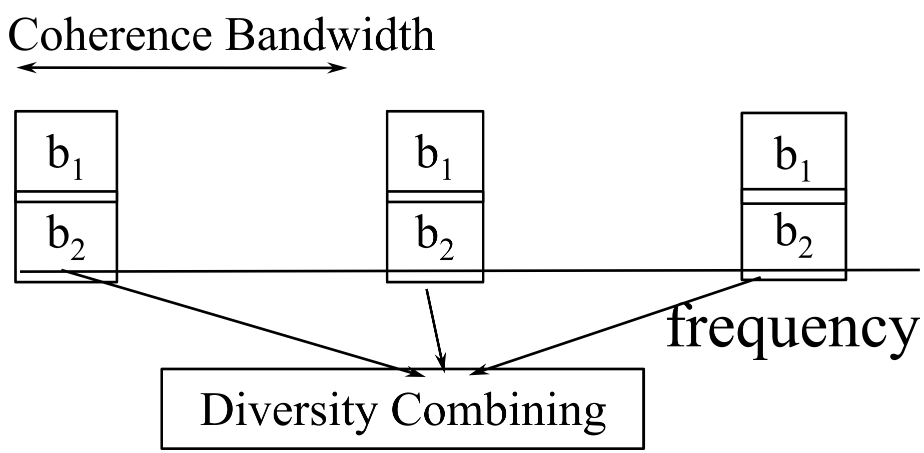

Atheros
WIFI
system architect (understand everything)
system engineer
Design algorithm
Developer

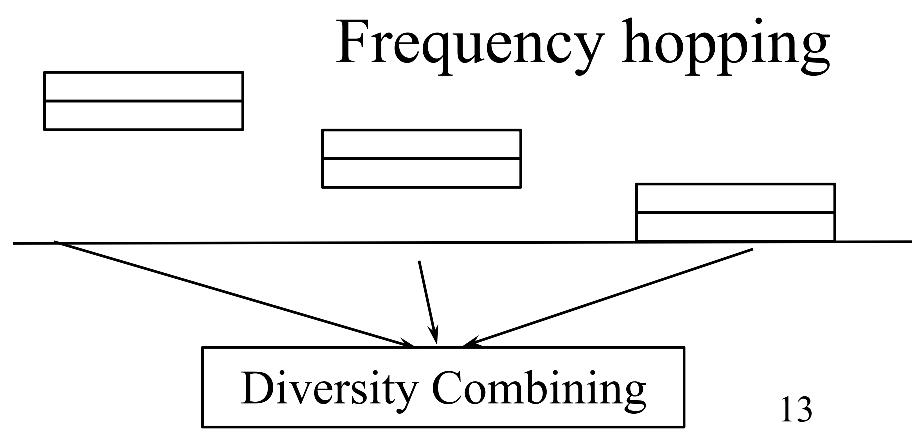
Time or Frequency Diversity

- Time diversity --- interleave the repeated bits over a duration longer than the coherence time, T_c



- Frequency diversity --- send signal in multiple frequency locations separated by more than B_c





Diversity Combining Methods

- ***Q2) How to utilize these multiple independent observations?***:
  - Selection Combining: pick one of the r_i based on their signal strengths (**need to know the relative signal strength**)
    - low bit SNR only when both are low
    - $P(r_1 \ldots r_L < c) = (1-e^{-c/\Gamma})^L$ (assuming that r_i are independent and exponentially distributed)
  - Equal-gain Combining: $r = r_1 + r_2 + \ldots + r_L$
    - better than selection combining when all SNRs are low
    - worst than selection combining when one has a much larger SNR than the others
  - Optimal combining: $r = a_1 r_1 + \ldots + a_L r_L$ where a_i are proportional to the square root of the SNRs for r_i.
    - Optimal ***if the SNRs can be estimated correctly***
- ***Objective: Lower the variations of SNR***

Diversity Gain

fading

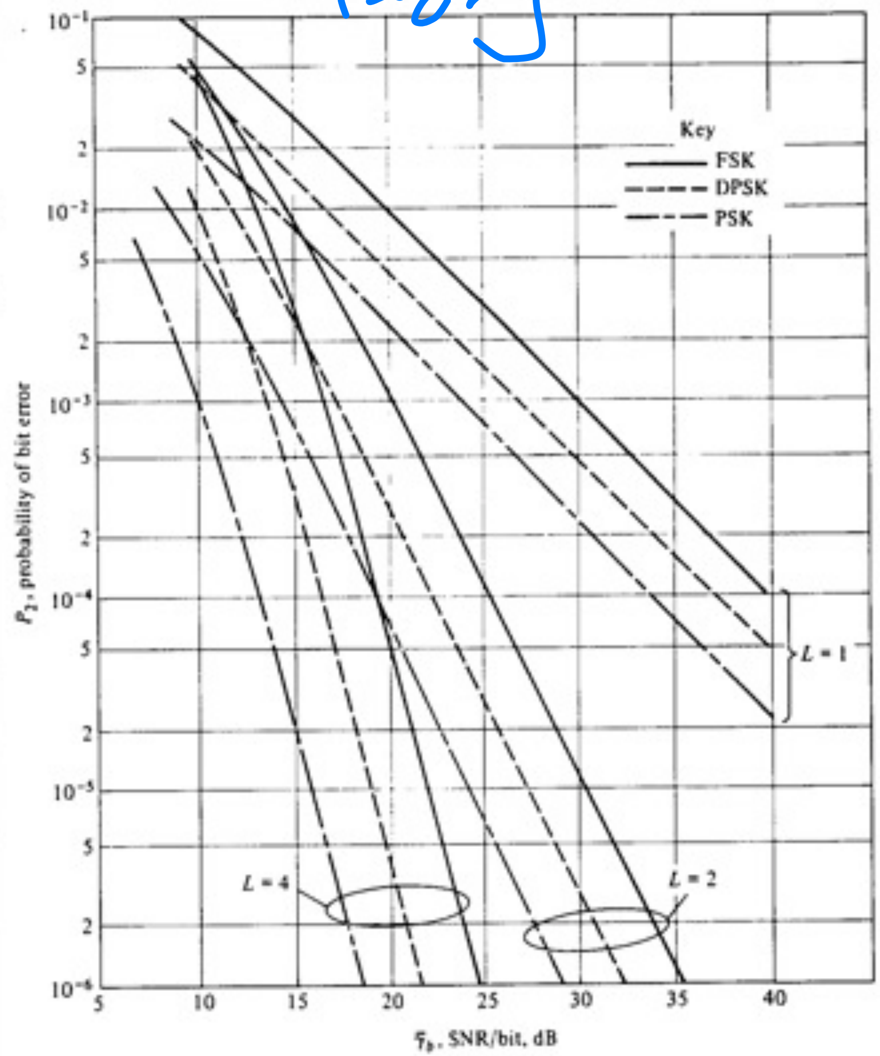


FIGURE 7.4.2
Performance of binary signals with diversity.

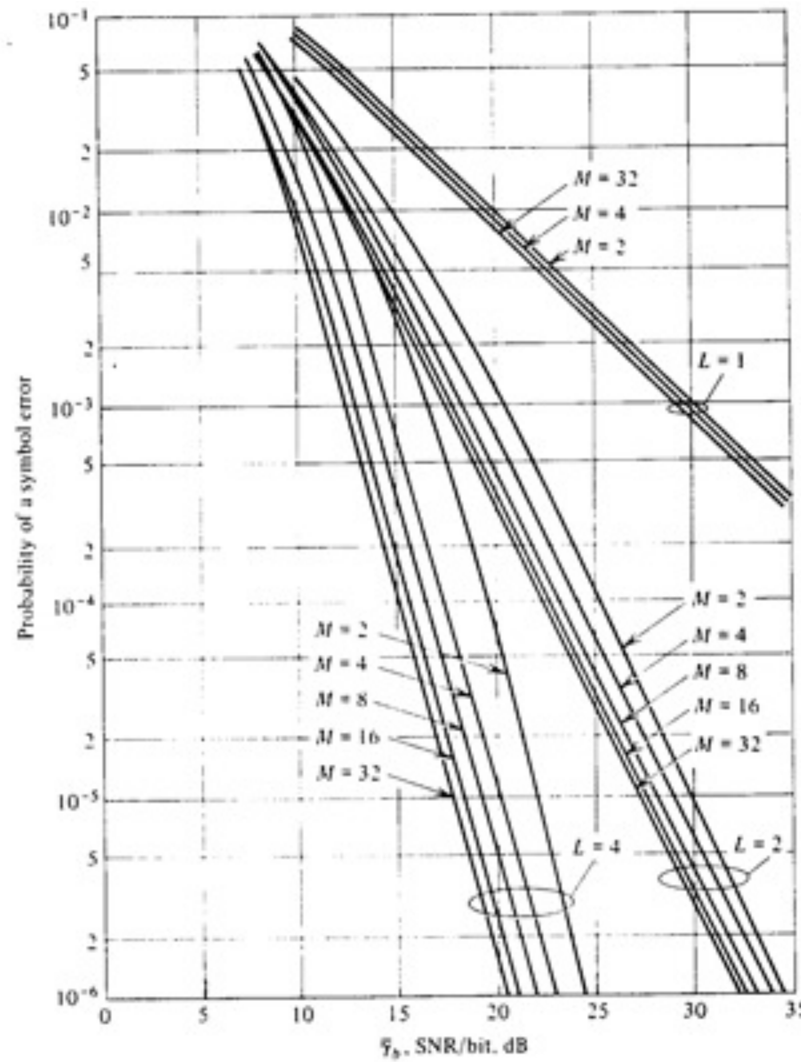
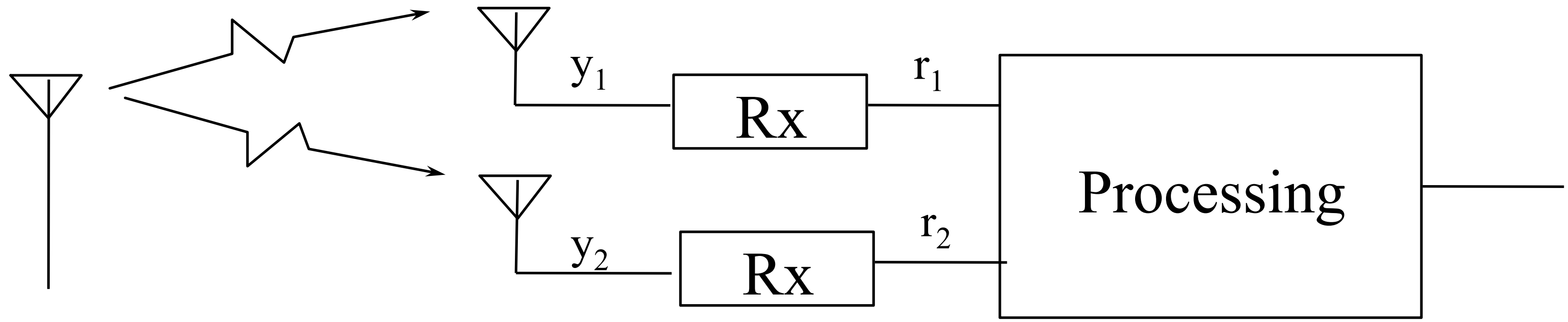


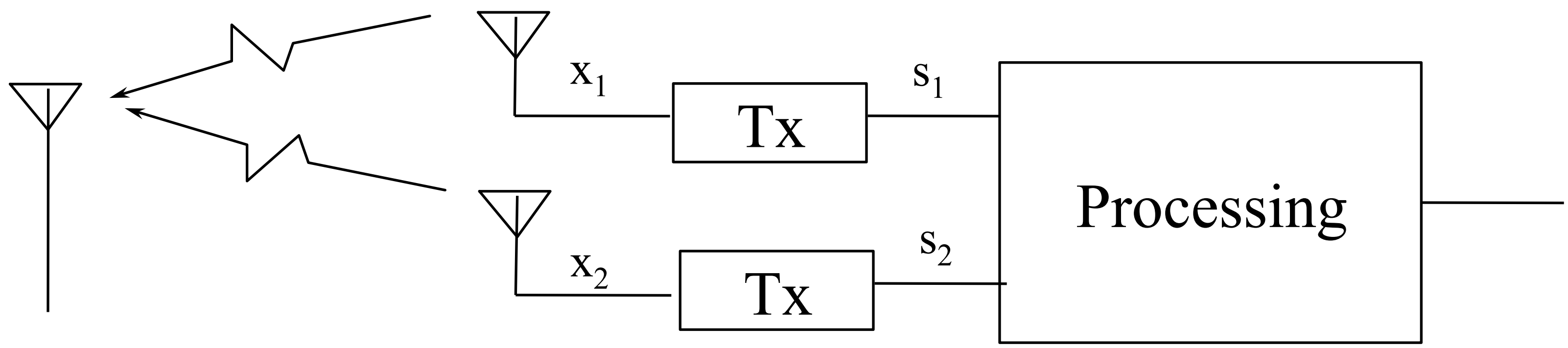
FIGURE 7.7.5
Performance of orthogonal signaling with M and L as parameters.

Antenna Diversity

- **Antenna Diversity**
  - Received Antenna diversity



  - Transmit Antenna diversity (Only CDMA System or with multiple receive antennas as in MIMO systems)



Time, Freq, & Antenna Diversity

- Time or freq diversity
  - requires bandwidth expansion by a factor of L for L-th order diversity --- **Tradeoff between BW and diversity**
  - for fixed **energy per information bit E_b**, energy in each repeated bits is only 1/N of the E_b; hence, only lower the SNR variations, but does not change the average SNR
  - has complete control over the multiple representations, (e.g. can use coding instead of just repeating the same bits.)
- Antenna Diversity
  - no bandwidth expansion
  - receiver antenna diversity --- no control over the multiple representations, always the same TX signal
  - transmit antenna diversity --- not straight-forward

Tradeoff in Diversity

- Time and Frequency Diversity
  - Bandwidth Vs diversity gain
  - By repeating the symbols L times (or L-order diversity),
    - bandwidth is increased by L times
    - Variance of the channel gain is lowered by L times
- Antenna Diversity
  - Antenna components Vs diversity gain
  - Using L antennas (L-order diversity)
    - hardware increases by L times
    - Variance of the channel gain is lowered by L times
- ***Any better ways? Yes, Error Correcting Code!***