

T07

CT Fourier transform for periodic signal

Relation between CTFS and CTFT

Tables 4.1 and 4.2

Duality property

DT Fourier series to DT Fourier transform

DT Fourier transform for aperiodic signal

DT Fourier transform for periodic signal

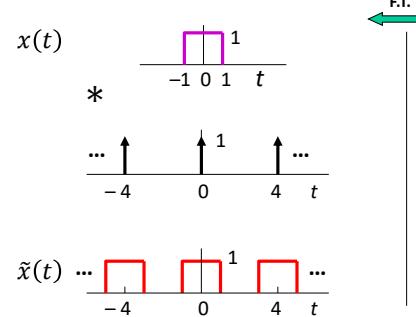
Tables 5.1 and 5.2

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CT Fourier Transform for Periodic Signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{k=-\infty}^{\infty} x(t - kT) \quad \text{Poisson sum}$$

e.g.

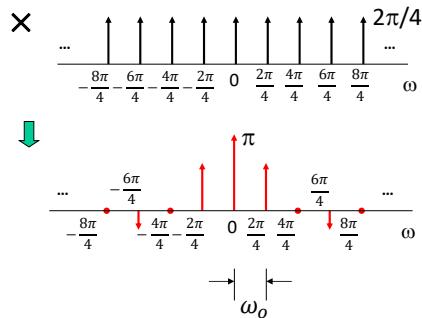
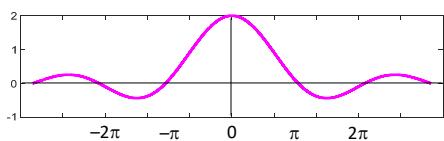


$$X(j\omega) = \frac{2 \sin(\omega)}{\omega}$$

$$\frac{2\pi}{4} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{4}\right) \quad T = 4$$

$$X(j\omega) \left[\frac{2\pi}{4} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{4}\right) \right]$$

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Question : Difference on F.T. between aperiodic and periodic ?

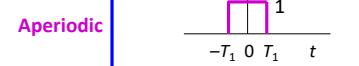
Relation of CTFS and CTFT

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$



$$a_k = \frac{1}{T} \int_0^T \tilde{x}(t) e^{-jk\omega_0 t} dt \quad (1)$$

$$x(t) = \begin{cases} \tilde{x}(t) & -\frac{T}{2} \leq t < T/2 \\ 0 & \text{otherwise} \end{cases}$$



$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

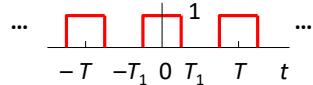
$$= \int_{-T/2}^{T/2} \tilde{x}(t) e^{-j\omega t} dt$$

(2)

$$a_k = \frac{1}{T} X(j\omega)|_{\omega=k\omega_0}$$

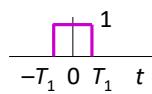
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e.g. Periodic square wave and Aperiodic square pulse



$$a_k = \frac{2 \sin(k\omega_o T_1)}{Tk\omega_o}$$

$$= \frac{2 \sin(k\omega_o T_1)}{Tk \left(\frac{2\pi}{T}\right)} \\ = \frac{\sin(k\omega_o T_1)}{k\pi}$$



$$X(j\omega) = \frac{2 \sin(\omega T_1)}{\omega}$$

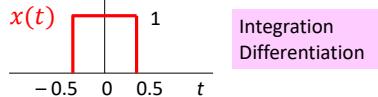
$$a_k \Leftrightarrow \frac{1}{T} X(j\omega)|_{\omega=k\omega_o} = \frac{2 \sin(k\omega_o T_1)}{k \left(\frac{2\pi}{T}\right)}$$

Question : Difference between its FS and its FT for a periodic signal ?

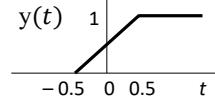
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e.g. Given :

$$y(t) = \begin{cases} 0 & t < -0.5 \\ t + 0.5 & -0.5 \leq t < 0.5 \\ 1 & 0.5 \leq t \end{cases}$$



Obtain the F.T. of $y(t)$



$$x(t) = \begin{cases} 1 & |t| < 0.5 \\ 0 & otherwise \end{cases}$$

\longleftrightarrow

$$X(j\omega) = \frac{2 \sin\left(\frac{\omega}{2}\right)}{\omega}$$

$$X(j0) = ?$$

$$y(t) = \int_{-\infty}^t x(t) dt$$

\longleftrightarrow

$$Y(j\omega) = \frac{1}{j\omega} X(j\omega) + \pi X(0) \delta(\omega)$$

$$\frac{d}{dt} y(t) = x(t)$$

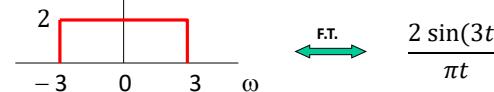
$$X(0) = ?$$

Tables 4.1 and 4.2

e.g. Given : $|X(j\omega)| = 2 u(\omega + 3) - u(\omega - 3)]$

Obtain $x(t)$

$$\angle X(j\omega) = -2\omega + \pi$$



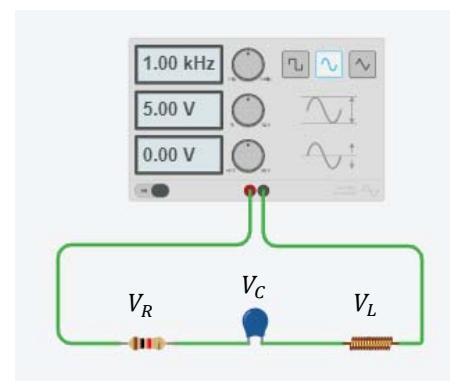
Linearity & Time shifting

$$X(j\omega) = \begin{cases} 2 e^{j(-2\omega + \pi)} & |\omega| < 3 \\ 0 & otherwise \end{cases} = (-1) e^{-j2\omega} \begin{cases} 2 & |\omega| < 3 \\ 0 & otherwise \end{cases}$$

$$x(t) = \frac{2 \sin(3(t-2))}{\pi(t-2)} = \frac{2 \sin(3t)}{\pi t} * \delta(t-2)$$

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e.g.



$$v_R(t) = i(t) R$$

$$V_R(j\omega) = I(j\omega) R$$

$$R = \frac{V}{I}$$

$$v_C(t) = \frac{1}{C} \int i(t) dt$$

$$V_C(j\omega) = \frac{1}{C j\omega} I(j\omega)$$

$$v_L(t) = L \frac{d}{dt} i(t)$$

$$V_L(j\omega) = L j\omega I(j\omega)$$

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e.g. Find $x(t)$ for the following LTI system.

$$x(t) \xrightarrow{H(j\omega) = \frac{1}{3+j\omega}} y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

Convolution

$$y(t) = x(t) * h(t) \quad \xleftarrow{\text{F.T.}} \quad Y(j\omega) = X(j\omega)H(j\omega)$$

$$\begin{aligned} X(j\omega) &= \frac{Y(j\omega)}{H(j\omega)} \\ &= \frac{1}{3+j\omega} - \frac{1}{4+j\omega} \\ &= \left(\frac{1}{3+j\omega} - \frac{1}{4+j\omega} \right) (3+j\omega) \\ &= \frac{1}{4+j\omega} \end{aligned}$$

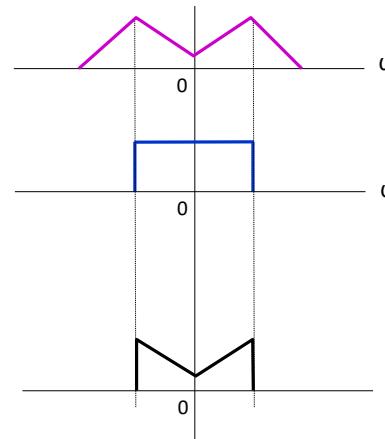
Inverse F.T. $\Rightarrow x(t) = e^{-4t}u(t)$

Question : Is it a causal system ?

$$h(t) = ?$$

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Input spectrum



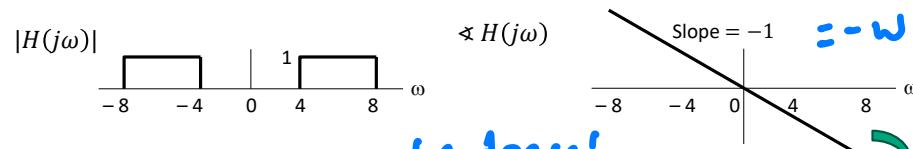
Frequency response

Sketch the output spectrum

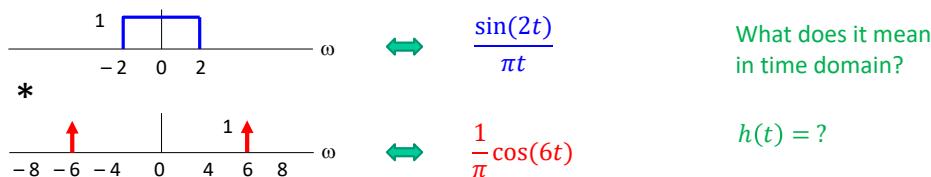
$$Y(j\omega) = X(j\omega)H(j\omega)$$

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e.g. The magnitude response and phase response of an LTI system are given below.



- a) What is the type of this filter?
b) Find the impulse response $h(t)$.



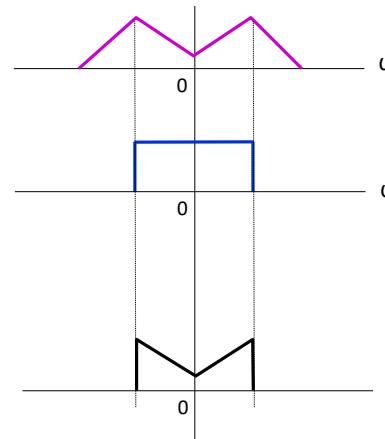
$$(2\pi) \frac{\sin(2t)}{\pi t} \frac{1}{\pi} \cos(6t) = \frac{2 \sin(2t)}{\pi t} \cos(6t)$$

$$H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)}$$

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$$[e^{-j\omega}] \rightarrow \delta(t-1)$$

e.g. Given :

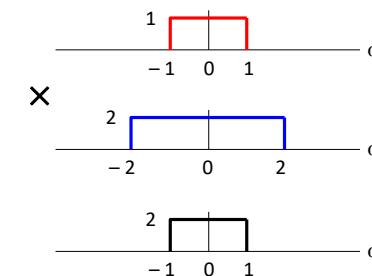


$$\text{e.g. Given : } x(t) = \left(\frac{\sin(t)}{\pi t} \right) * \left(\frac{2 \sin(2t)}{\pi t} \right)$$

a) Sketch the F.T. of $x(t)$ Convolution

b) Find the energy of $x(t)$ Parseval's Relation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$



$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\begin{aligned} \text{Energy} &= \frac{2 \times 2}{2\pi} \\ &= \frac{4}{\pi} \end{aligned}$$

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Duality Property

$$\begin{array}{ccc} x(t) & \longleftrightarrow & X(j\omega) \\ y(t) = X(j\omega)|_{\omega=t} & & Y(j\omega) = 2\pi x(t)|_{t=-\omega} \\ \text{Proof: } Y(j\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} X(jt)e^{-j\omega t} dt & & (1) \end{array}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$x(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(-jt) e^{j\omega t} dt \quad (2)$$

$$(1) \rightarrow (2) \quad \Rightarrow \quad Y(j\omega) = 2\pi x(-\omega) = 2\pi x(t)|_{t=-\omega}$$

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?

$$\begin{array}{ccc} x(t) & \longleftrightarrow & X(j\omega) \\ y(t) = X(j\omega)|_{\omega=t} & & Y(j\omega) = 2\pi x(t)|_{t=-\omega} \end{array}$$

$$\text{e.g. Given: } te^{-|t|} \stackrel{\text{F.T.}}{\Leftrightarrow} \frac{-j4\omega}{(1+\omega^2)^2}$$

$$\text{Obtain the FT of } \frac{4t}{(1+t^2)^2}$$

$$\begin{aligned} F\left\{\frac{4t}{(1+t^2)^2}\right\} &= \frac{2\pi(-\omega)e^{-|-\omega|}}{-j} \\ &= -j2\pi\omega e^{-|\omega|} \end{aligned}$$

$$Y(j\omega) = \int_{-\infty}^{\infty} \frac{4t}{(1+t^2)^2} e^{-j\omega t} dt$$

$$\begin{array}{ccc} te^{-|t|} & \stackrel{\frac{-j4\omega}{(1+\omega^2)^2}}{\longleftrightarrow} & \frac{-j4t}{(1+t^2)^2} \\ -j4t & \longleftrightarrow & 2\pi(-\omega)e^{-|\omega|} \end{array}$$

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DT Fourier series to DT Fourier transform
DT Fourier transform for aperiodic signal
DT Fourier transform for periodic signal
Tables 5.1 and 5.2

DT Fourier Series

$$\omega_o = \frac{2\pi}{N} \quad N = \text{finite}$$

$$\omega = k\omega_o \quad \omega = \text{finite}$$

$$x[n] = \sum_{k=<N>} \mathbf{a}_k e^{jk\frac{2\pi}{N}n}$$

$$\mathbf{a}_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\frac{2\pi}{N}n}$$

Periodic Signals

DT Fourier Transform

$$\omega_o = \frac{2\pi}{N} \quad N \rightarrow -\infty \text{ to } \infty$$

$$k\omega_o \rightarrow \omega$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Aperiodic Signals

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Question : What is the difference between CTFT and DTFT ?

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Table 5.1 and 5.2

5.1

Section	Property	Aperiodic Signal	Fourier Transform
5.3.2	Llinearity	$x[n] + y[n]$	$X(j\omega) + Y(j\omega)$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(j\omega)$
5.3.4	Frequency Shifting	$e^{jn_0\omega} x[n]$	$X(j(\omega - n_0))$
5.3.5	Conjugation	$x^*[n]$	$X^*(j\omega)$
5.3.6	Time Reversal	$x[-n]$	$X^*(-j\omega)$
5.3.7	Time Expansion	$x[n] \quad \quad n = \text{multiple of } k$	$X(j\omega/k)$
5.4	Convolution	$x[n] * y[n]$	$X(j\omega)Y(j\omega)$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega')Y(j\omega - \omega') d\omega'$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(j\omega)$
5.3.5	Accumulation	$\sum_{k=0}^{n-1} x[k]$	$\frac{1}{1 - e^{-j\omega}} X(j\omega)$
5.3.8	Differentiation in Frequency	$x[n]$	$j \frac{dX(j\omega)}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$X(j\omega) = X^*(j\omega)$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real and even	$ X(j\omega) = X^*(j\omega) $
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$\delta(X(j\omega)) = -\delta(X(-j\omega))$
5.3.4	Even/Odd Decomposition of Real Signals	$x[n] = \Re{x[n]} + j\Im{x[n]}$	$\delta(X(j\omega)) = -\delta(X(-j\omega))$
5.3.9	Parseval's Relation	$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$

4.1

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
4.3.1	Linearity	$a(t) + b(t)$	$X(j\omega) + Y(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.3	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.4	Conjugation	$x^*(t)$	$X^*(j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X^*(-j\omega)$
4.3.5	Time and Frequency Scale	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega')Y(j\omega - \omega') d\omega'$
4.3.4	Differentiation in Time	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} Y(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$x(t)$	$\frac{1}{j\omega} X(j\omega) - \frac{1}{\omega} Y(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$X(j\omega) = X^*(-j\omega)$ $\delta(x(X(j\omega))) = \delta(x(X(-j\omega)))$ $\delta(x(X(j\omega))) = -\delta(x(X(-j\omega)))$ $ X(j\omega) = X(-j\omega) $
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$ X(j\omega) = X(-j\omega) $ $X(j\omega) = X(-j\omega)$
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$ X(j\omega) = X(-j\omega) $ $X(j\omega) = -X^*(-j\omega)$
4.3.3	Even/Odd Decomposition for Real Signals	$x(t) = f_{\text{even}}(t) + f_{\text{odd}}(t)$	$X(j\omega)$ purely imaginary and odd
4.3.7	Parseval's Relation for Aperiodic Signals	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$

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5.2

2π-periodic, like T.S. coefficients!

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

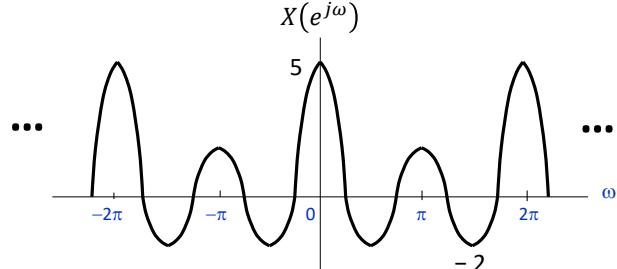
Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\delta[n]$	$\sum_{k=-\infty}^{\infty} a_k e^{j\omega k}$	$a_k = 1$
$\cos(\omega_0 n)$	$2\pi \sum_{k=-\infty}^{\infty} \delta(k - \frac{\omega_0}{2\pi})$	$a_k = 0, \text{ otherwise}$
$\sin(\omega_0 n)$	$j \sum_{k=-\infty}^{\infty} [\delta(k - \omega_0) - \delta(k + \omega_0)]$	$a_k = 0, \text{ otherwise}$
$\sin(\omega_0 n)$	$\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_k = -a_{-k} = \frac{1}{2}, \text{ otherwise}$
$\delta(t)$	$2\pi \delta(\omega)$	$a_0 = 1, a_k = 0, k \neq 0$ (this is the Fourier series representation for any choice of $T = T'$)

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=0}^{N-1} a_k e^{j\omega k}$	$2\pi \sum_{k=0}^{N-1} a_k (\delta(\omega - k\omega_0))$	$a_k = 1$
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_k = 0, \text{ otherwise}$
$\cos(\omega_0 t)$	$a_0 + \frac{1}{2} \sum_{k=1}^{\infty} [a_k \delta(\omega - k\omega_0) + a_{-k} \delta(\omega + k\omega_0)]$	$a_0 = a_{-1} = \frac{1}{2}, a_k = 0, \text{ otherwise}$
$\sin(\omega_0 t)$	$\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_k = -a_{-k} = \frac{1}{2}, a_0 = 0, \text{ otherwise}$
$\delta(t)$	$2\pi \delta(\omega)$	$a_0 = 1, a_k = 0, k \neq 0$

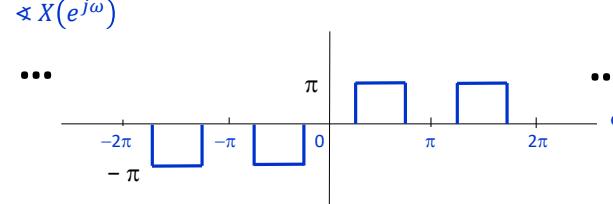
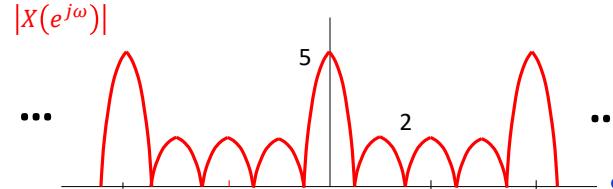
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DT Fourier Transform for Aperiodic Signal

$$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\varphi X(e^{j\omega})}$$

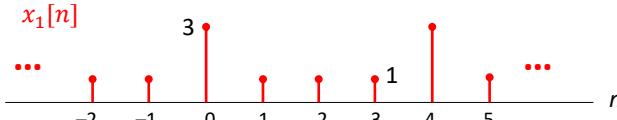
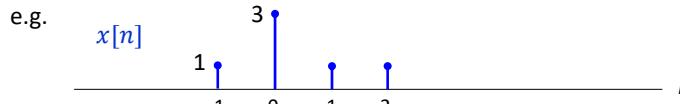
e.g. Magnitude spectrum? Phase spectrum? Is $x[n]$ real?

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DT Fourier Transform for Periodic Signal



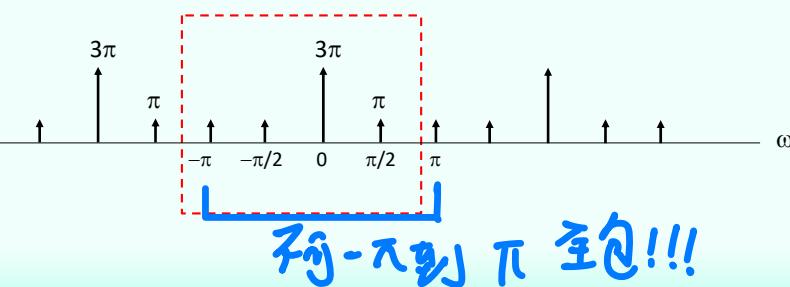
- a) Represent $x_1[n]$ in terms of $x[n]$.
- b) Find the Fourier transform of $x[n]$.
- c) Find the Fourier transform of $x_1[n]$.
- d) Find the FS of $x_1[n]$.

$$\begin{aligned} x_1[n] &= x[n] * \sum_{k=-\infty}^{\infty} \delta[n - 4k] \\ &= \sum_{k=-\infty}^{\infty} x[n - 4k] \quad N = 4 \end{aligned}$$

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$$X_1(e^{j\omega}) = \frac{\pi}{2} \sum_{k=-\infty}^{\infty} \left(3 + 2 \cos\left(\frac{\pi k}{2}\right) + e^{-j\pi k} \right) \delta\left(\omega - \frac{\pi k}{2}\right)$$

for $-\pi \leq \omega < \pi$

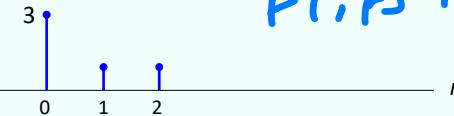


Question : Why is the frequency component at π excluded ?

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F_T, f_S is just 2π !

Apply LTI!



$$x[n] = \delta[n+1] + 3\delta[n] + \delta[n-1] + \delta[n-2]$$

$$X(e^{j\omega}) = e^{j\omega} + 3 + e^{-j\omega} + e^{-j2\omega} = 3 + 2\cos(\omega) + e^{-j2\omega}$$

$$X_1(e^{j\omega}) = X(e^{j\omega}) \frac{2\pi}{4} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{4}\right) \quad \omega \rightarrow \omega - \frac{\pi k}{2}$$

$$= \sum_{k=-\infty}^{\infty} \frac{\pi}{2} \left(3 + 2 \cos\left(\frac{\pi k}{2}\right) + e^{-j\pi k} \right) \delta\left(\omega - \frac{\pi k}{2}\right)$$

$$a_k = \frac{1}{2\pi} X_1(e^{j\omega}) = \frac{1}{4} \left(3 + 2 \cos\left(\frac{\pi k}{2}\right) + e^{-j\pi k} \right)$$

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A DT LTI system has frequency response as shown below.

$$H(e^{j\omega}) = \begin{cases} 4e^{-j\omega} & |\omega| < \frac{2\pi}{3} \\ 0 & \frac{2\pi}{3} < |\omega| < \pi \end{cases} \quad \text{for } |\omega| \leq \pi \quad H(e^{j\omega}) \text{ is } 2\pi\text{-periodic}$$

e) What is the type of this filter ? *low pass!*

If $x_1[n]$ is applied to the above system,

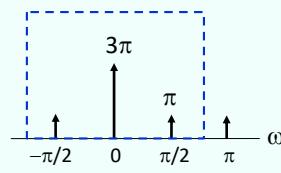
f) Find the Fourier transform of the output.

g) Write down the expression of the output as the sum of real cosine signal(s).

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$$H(e^{j\omega}) = \begin{cases} 4e^{-j\omega} & |\omega| < \frac{2\pi}{3} \\ 0 & \frac{2\pi}{3} < |\omega| < \pi \end{cases} \quad \text{for } |\omega| \leq \pi$$

$$X_1(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \frac{\pi}{2} \left(3 + 2 \cos\left(\frac{\pi k}{2}\right) + e^{-j\pi k} \right) \delta\left(\omega - \frac{\pi k}{2}\right)$$



$$Y_1(e^{j\omega}) = X_1(e^{j\omega}) H(e^{j\omega})$$

$$= 12\pi\delta(\omega) + 4e^{-j\frac{\pi}{2}}\pi\delta\left(\omega - \frac{\pi}{2}\right) + 4e^{j\frac{\pi}{2}}\pi\delta\left(\omega + \frac{\pi}{2}\right) \quad \text{for } |\omega| \leq \pi$$

$$y_1[n] = 6 + 4 \cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right) = 6 + 4 \sin\left(\frac{\pi}{2}n\right)$$

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$$H(e^{j\omega}) = \frac{\frac{4}{5}e^{-j\omega}}{1 - \left(\frac{4}{5}\right)e^{-j\omega}} = \frac{\frac{4}{5}e^{-j\omega}}{1 - \left(\frac{4}{5}\right)e^{-j\omega}} \cdot \frac{1}{1 - \left(\frac{4}{5}\right)e^{-j\omega}}$$

$$\left(\frac{4}{5}\right)^n u[n] \leftrightarrow \frac{1}{1 - \left(\frac{4}{5}\right)e^{-j\omega}}$$

$$h[n] = \frac{4}{5} \left(\frac{4}{5}\right)^{n-1} u[n-1]$$

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e.g. A causal and stable LTI system S has the property that

$$x[n] = \left(\frac{4}{5}\right)^n u[n] \xrightarrow{\text{System S}} y[n] = n \left(\frac{4}{5}\right)^n u[n] = x[n] * h[n]$$

Determine the frequency response for the system S.

Differentiation in frequency

$$X(e^{j\omega}) = \frac{1}{1 - \left(\frac{4}{5}\right)e^{-j\omega}}$$

$$Y(e^{j\omega}) = j \frac{d}{d\omega} \left[\frac{1}{1 - \left(\frac{4}{5}\right)e^{-j\omega}} \right]$$

Convolution

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{4}{5}e^{-j\omega}}{1 - \left(\frac{4}{5}\right)e^{-j\omega}} = \frac{\frac{4}{5}e^{-j\omega}}{\left[1 - \left(\frac{4}{5}\right)e^{-j\omega}\right]^2}$$

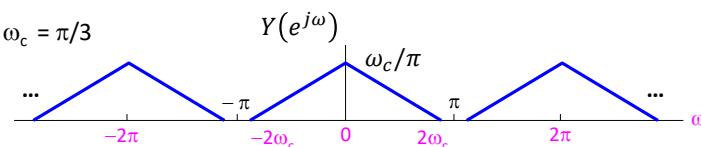
Question : What is the impulse response ? Is it a causal system ?

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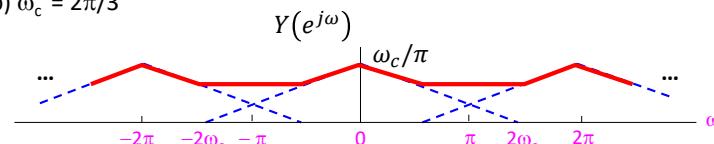
e.g. Given : $y[n] = \left(\frac{\sin(\omega_c n)}{\pi n}\right)^2$ where $0 < \omega_c < \pi$. Multiplication

Sketch the spectrum for the following cases.

a) $\omega_c = \pi/3$

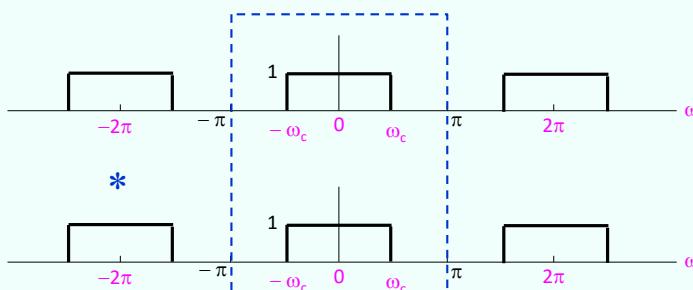


b) $\omega_c = 2\pi/3$



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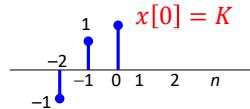
$$y[n] = \left(\frac{\sin(\omega_c n)}{\pi n} \right)^2 = \frac{\sin(\omega_c n)}{\pi n} \times \frac{\sin(\omega_c n)}{\pi n}$$



$$\frac{1}{2\pi} (1)(1)(2\omega_c) = \frac{\omega_c}{\pi}$$

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According (1), (2) and (3) :

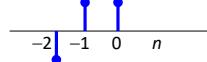


According to (4) and (2) :

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 3 \quad \text{Parseval's Relation}$$

$$(-1)^2 + (1)^2 + (K)^2 = 3$$

$$K = 1$$



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e.g. The following four facts are given about a real signal $x[n]$ with Fourier transform. Determine $x[n]$.

- (1) $x[n] = 0$ for $n > 0$ (3) $\text{Im}\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)$
 (2) $x[0] > 0$ (4) $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$

According to (3) :

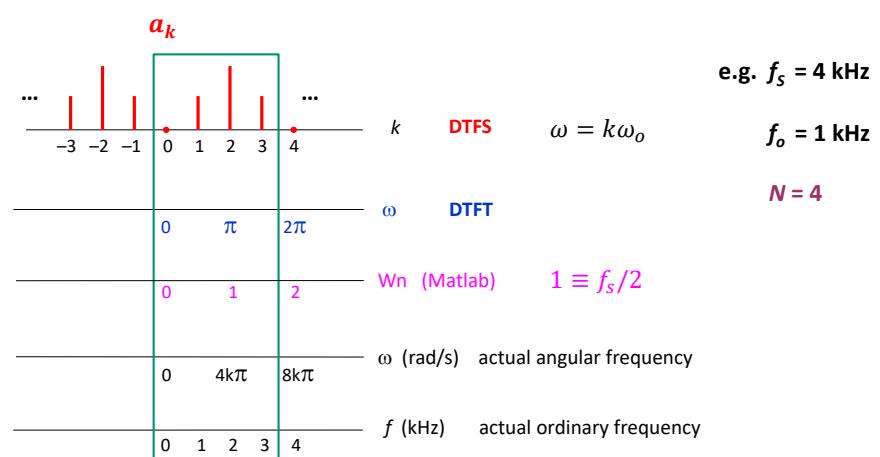
Real & odd $j \text{Im}\{X(e^{j\omega})\} = j \sin(\omega) - j \sin(2\omega)$

$$\frac{x[n] - x[-n]}{2} = F^{-1}\{j \sin(\omega) - j \sin(2\omega)\}$$

$$\frac{x[n] - x[-n]}{2} = F^{-1}\left\{\frac{1}{2}(e^{j\omega} - e^{-j\omega}) - \frac{1}{2}(e^{j2\omega} - e^{-j2\omega})\right\}$$

$$x[n] - x[-n] = \delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]$$

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