T05

Properties (Table 3.1)   **Frequency domain**

Fourier analysis for DT signals and systems
Difference between CTFS and DTFS
Difference between H(jω) and H(e^{jω})

1

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e.g. Find the F.S. coefficients for all values of k.

$$a_k \quad \boxed{e^{jk\frac{2\pi}{T}t}}$$
Basis function

$$x(t) = 2 + 4\cos\left(2\pi t + \frac{\pi}{3}\right) + 8\sin\left(6\pi t - \frac{\pi}{4}\right)$$

$$= 2 + 2\,e^{j\left(2\pi t+\frac{\pi}{3}\right)} + 2\,e^{-j\left(2\pi t+\frac{\pi}{3}\right)} + \frac{4}{j}\,e^{j\left(6\pi t-\frac{\pi}{4}\right)} + \left(-\frac{4}{j}\right)e^{-j\left(6\pi t-\frac{\pi}{4}\right)}$$

$$= 2 + 2\,e^{j\frac{\pi}{3}}\,e^{j2\pi t} + 2\,e^{-j\frac{\pi}{3}}\,e^{-j2\pi t} + \frac{4}{j}\,e^{-j\frac{\pi}{4}}\,e^{j6\pi t} + \left(-\frac{4}{j}\right)e^{j\frac{\pi}{4}}\,e^{-j6\pi t}$$

$$a_1 = 2\,e^{j\frac{\pi}{3}} \qquad\qquad a_3 = 4\,e^{-j\frac{\pi}{2}}\,e^{-j\frac{\pi}{4}} = 4\,e^{-j\frac{3\pi}{4}} \qquad\qquad \frac{1}{j} = -j = e^{-j\frac{\pi}{2}}$$

$$a_{-1} = a_1^* = 2\,e^{-j\frac{\pi}{3}} \qquad a_{-3} = a_3^* = 4\,e^{j\frac{3\pi}{4}} \qquad a_0 = 2$$

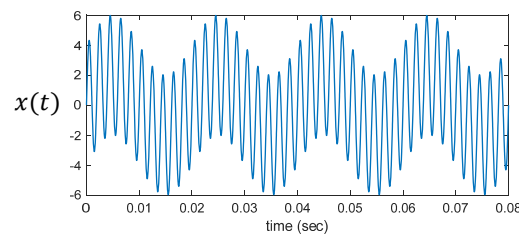⬆ conjugate symmetry ⬆ $a_k = 0$ otherwise

Question : Why to decompose a signal ?

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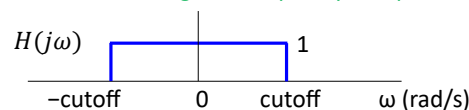e.g. A real periodic signal $x(t)$ is plotted below according to a given data file.

$$\omega_o = \frac{2\pi}{T} \qquad a_k \quad e^{jk\frac{2\pi}{T}t}$$

Question : Is $x(t)$ a real signal ?

$$x(t) = a_{-10}e^{j(-10)\frac{2\pi}{T}t} + a_{-1}e^{j(-1)\frac{2\pi}{T}t} + a_1 e^{j\frac{2\pi}{T}t} + a_{10}e^{j(10)\frac{2\pi}{T}t} \qquad \text{Decomposition}$$
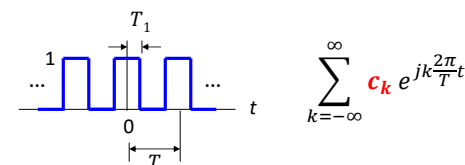
Question : Design a system to remove the highest frequency component ?

Question : cutoff ?
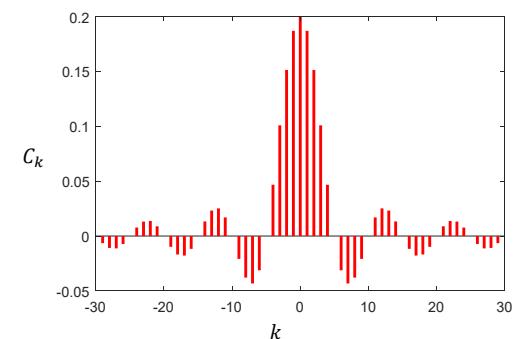

$H(j\omega)$ ... 1
$-$cutoff   0   cutoff   ω (rad/s)

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e.g.   For example, $T = 5$ $T_1 = 0.5$

$$\sum_{k=-\infty}^{\infty} c_k\, e^{jk\frac{2\pi}{T}t}$$

C_k

$$c_k = \frac{\sin(k\omega_o T_1)}{k\pi} = \frac{\omega_o T_1}{\pi}\,\text{sinc}\left(\frac{k\omega_o T_1}{\pi}\right)$$

$$c_0 = \frac{k\omega_o T_1}{k\pi} = \frac{\left(\frac{2\pi}{T}\right)T_1}{\pi} = (1)\frac{2T_1}{T} \qquad \text{for } k = 0$$
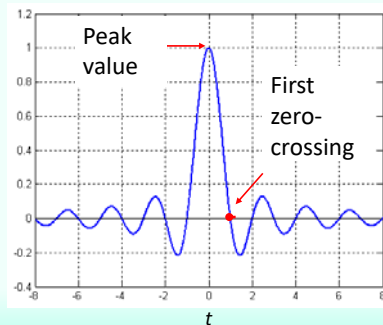
Question : Magnitude and phase of c_k ?

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Slide 5

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\lim_{t \to 0} \frac{\sin(\pi t)}{\pi t} = \frac{\pi t}{\pi t} = 1$$

Peak value

First zero-crossing

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} = 0$$

$$\sin(\pi t) = 0$$

$$t = 1$$

Slide 6

e.g. The input and the frequency response of an LTI system is given.

$$x(t) = \sin(2\pi t) + \frac{1}{2}\cos(4\pi t) \qquad H(j\omega) = 4\,e^{-j\frac{\omega}{8}}$$

a) What is the fundamental angular frequency ω_o ?

b) Is the input a real signal ?

c) Plot the F.S. coefficient of $x(t)$.

d) Find the eigenvalue for each eigenfunction.

e) Represent the output $y(t)$ as a sum of real sinusoids.

f) Is the system a real system ?

g) What does the system do?

h) Suggest a corresponding impulse response.

Slide 7

$$x(t) = \frac{1}{j2}\,e^{j(2\pi t)} - \frac{1}{j2}\,e^{-j(2\pi t)} + \frac{1}{4}\,e^{j(4\pi t)} + \frac{1}{4}e^{-j(4\pi t)}$$

$$= \frac{1}{j2}\,e^{j(1)(2\pi t)} - \frac{1}{j2}\,e^{j(-1)(2\pi t)} + \frac{1}{4}\,e^{j(2)(2\pi t)} + \frac{1}{4}e^{j(-2)(2\pi t)}$$

$k = 1 \qquad k = -1 \qquad k = 2 \qquad k = -2$

$$\sum_{k=-\infty}^{\infty} a_k\, e^{jk\frac{2\pi}{T}t}$$

$$a_1 = \frac{1}{j2} = \frac{1}{2}e^{-j\frac{\pi}{2}} \qquad a_{-1} = a_1^* = \frac{1}{2}e^{j\frac{\pi}{2}}$$

$$a_2 = \frac{1}{4} \qquad a_{-2} = a_2^* = \frac{1}{4}$$

$$a_k = 0 \qquad otherwise$$

Slide 8

Magnitude

$|a_k|$

1/2

1/4

Even symmetry

$-2 \quad -1 \quad 0 \quad 1 \quad 2 \qquad k$

Phase

$\sphericalangle\, a_k$

$\pi/2$

Odd symmetry

$-2 \quad -1 \quad 0 \quad 1 \quad 2 \qquad k$

$-\pi/2$
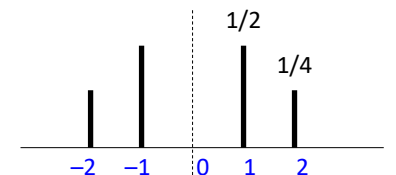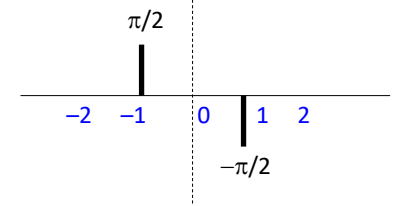
Question : Why to show the phase in the range of $-\pi$ and π ?

$$H(j\omega) = 4\, e^{-j\frac{\omega}{8}} \qquad H(j2\pi) = 4\, e^{-j\frac{2\pi}{8}} \qquad \text{Conjugate symmetry} \qquad H(-j2\pi) = 4\, e^{j\frac{2\pi}{8}}$$

$$H(j4\pi) = 4\, e^{-j\frac{4\pi}{8}} \quad\Longleftrightarrow\quad H(-j4\pi) = 4\, e^{j\frac{4\pi}{8}}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k\, H(jk\omega_o)\, e^{jk\omega_o t}$$

$$\boxed{x(t) = \sin(2\pi t) + \frac{1}{2}\cos(4\pi t)}$$

$$= \frac{1}{j2} 4\, e^{-j\frac{2\pi}{8}} e^{j(1)(2\pi t)} - \frac{1}{j2} 4\, e^{j\frac{2\pi}{8}} e^{j(-1)(2\pi t)} + \frac{1}{4} 4\, e^{-j\frac{4\pi}{8}} e^{j(2)(2\pi t)} + \frac{1}{4} 4\, e^{j\frac{4\pi}{8}} e^{j(-2)(2\pi t)}$$
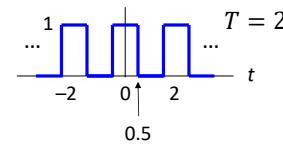
$$= 4\sin\left(2\pi t - \frac{2\pi}{8}\right) + 2\cos\left(4\pi t - \frac{4\pi}{8}\right)$$

$$h(t) = \; ?$$

$$= \boxed{4\sin\left(2\pi\left(t - \frac{1}{8}\right)\right) + 2\cos\left(4\pi\left(t - \frac{1}{8}\right)\right)}$$
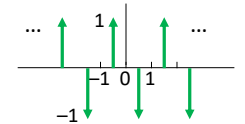
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Properties (Table 3.1)

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$T = 2$

$$s(t) \quad\Longleftrightarrow\quad c_k = \frac{\sin(k\omega_o T_1)}{k\pi} = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

e.g. Find the F.S. of $y(t)$.

$$y(t) = \frac{d}{dt} s(t) \quad\Longleftrightarrow\quad b_k = jk\omega_o c_k = jk\pi c_k = j\sin\left(\frac{k\pi}{2}\right)$$
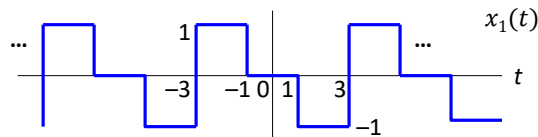
Differentiation

$$a_k = \frac{1}{2}$$

$$y(t) = x(t+0.5) - x(t-0.5) \quad\Longleftrightarrow\quad b_k = a_k e^{-jk\omega_o(-0.5)} - a_k e^{-jk\omega_o(0.5)}$$

Time-shifting
Linearity

$$= j2 a_k \sin\left(\frac{k\pi}{2}\right)$$

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e.g. What is the value of a_0 for each of the following signals ?

$x_1(t)$

$$a_0 = \; ?$$

$x_2(t)$
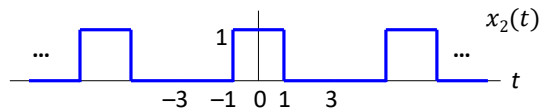
$$a_0 = \; ?$$

e.g. Does each of the following signals have valid F.S. coefficients ?

$$y_1(t) = \int_{-\infty}^{t} x_1(t)\, dt \qquad y_2(t) = \int_{-\infty}^{t} x_2(t)\, dt$$

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e.g. Given : $\quad x(t) = 2\, e^{j\left(2\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(4\pi t - \frac{\pi}{6}\right)}$

a) Sketch the F.S. coefficient of $x(t)$.

b) Sketch the F.S. coefficient of $y(t) = x^*(t)$.

c) Sketch the F.S. coefficient of $y(t) = x(-t)$.

d) Sketch the F.S. coefficient of $y(t) = x(2t)$.

e) Sketch the F.S. coefficient of $y(t) = e^{j2\pi t} x(t)$.

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$$x(t) = 2\, e^{j\left(2\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(4\pi t - \frac{\pi}{6}\right)} \qquad \omega_o = 2\pi \qquad T = 1$$

a) F.S. coefficient of $x(t)$

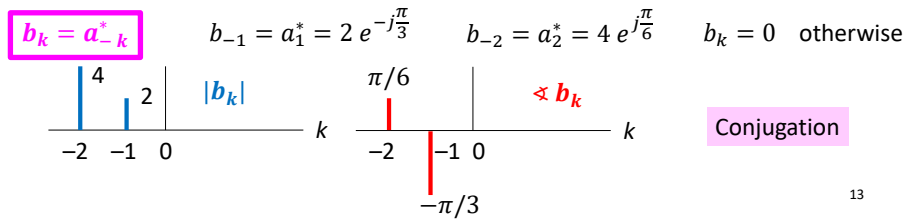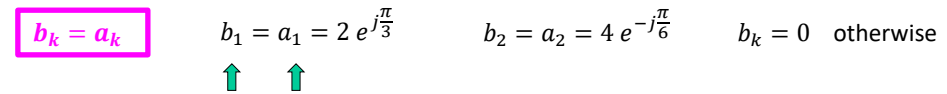$$a_1 = 2\, e^{j\frac{\pi}{3}} \qquad\qquad a_2 = 4\, e^{-j\frac{\pi}{6}} \qquad a_k = 0 \quad \text{otherwise}$$

$|a_k|$: 2 at $k=1$, 4 at $k=2$

$\sphericalangle\, a_k$: $\pi/3$ at $k=1$, $-\pi/6$ at $k=2$

b) $y(t) = x^*(t)$

$$\boxed{b_k = a^*_{-k}} \qquad b_{-1} = a_1^* = 2\, e^{-j\frac{\pi}{3}} \qquad b_{-2} = a_2^* = 4\, e^{j\frac{\pi}{6}} \qquad b_k = 0 \quad \text{otherwise}$$

$|b_k|$: 4 at $k=-2$, 2 at $k=-1$

$\sphericalangle\, b_k$: $\pi/6$ at $k=-2$, $-\pi/3$ at $k=-1$

Conjugation

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c) $y(t) = x(-t)$

$$\boxed{b_k = a_{-k}} \qquad b_{-1} = a_1 = 2\, e^{j\frac{\pi}{3}} \qquad b_{-2} = a_2 = 4\, e^{-j\frac{\pi}{6}} \qquad b_k = 0 \quad \text{otherwise}$$

$|b_k|$: 4 at $k=-2$, 2 at $k=-1$

$\sphericalangle\, b_k$: $\pi/3$ at $k=-1$, $-\pi/6$ at $k=-2$

Time reversal

d) $y(t) = x(2t)$

$$\boxed{b_k = a_k} \qquad b_1 = a_1 = 2\, e^{j\frac{\pi}{3}} \qquad b_2 = a_2 = 4\, e^{-j\frac{\pi}{6}} \qquad b_k = 0 \quad \text{otherwise}$$
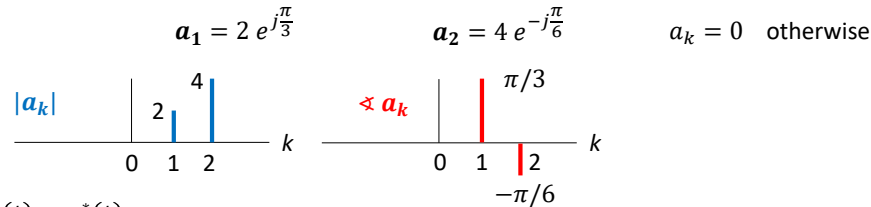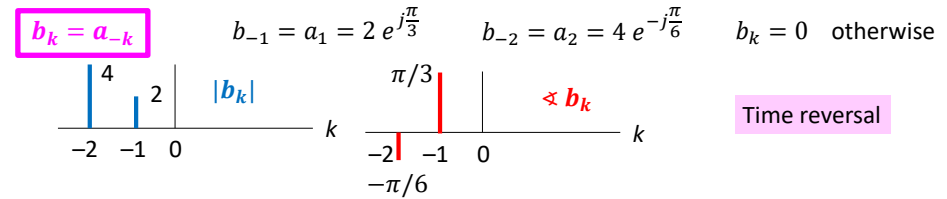
Question : Do they have the same meaning when $k = 1$?

Time scaling

$$x(2t) = 2\, e^{j\left(4\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(8\pi t - \frac{\pi}{6}\right)}$$

$$x(t) = 2\, e^{j\left(2\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(4\pi t - \frac{\pi}{6}\right)}$$

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e) $y(t) = e^{j2\pi t} x(t) = 2\, e^{j\left(4\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(6\pi t - \frac{\pi}{6}\right)} \qquad \boxed{\omega_o = 2\pi}$

$M = 1$

$$\boxed{b_k = a_{k-M}} \qquad b_2 = a_1 = 2\, e^{j\frac{\pi}{3}} \qquad b_3 = a_2 = 4\, e^{-j\frac{\pi}{6}} \qquad b_k = 0 \quad \text{otherwise}$$

$|b_k|$: 2 at $k=2$, 4 at $k=3$

$\sphericalangle\, b_k$: $\pi/3$ at $k=2$, $-\pi/6$ at $k=3$

Frequency shifting
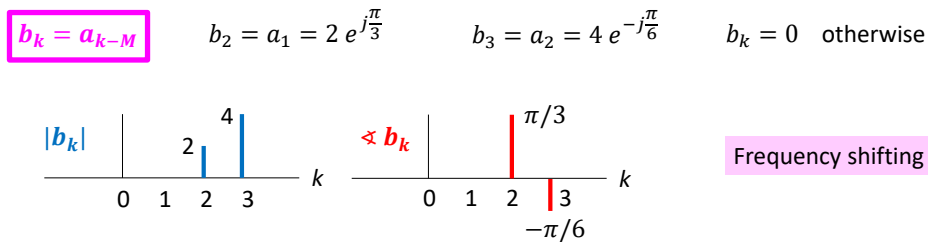
$$x(t) = 2\, e^{j\left(2\pi t + \frac{\pi}{3}\right)} + 4\, e^{j\left(4\pi t - \frac{\pi}{6}\right)}$$

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e.g. Multiplication $\qquad z(t) = x(t)\, y(t) \quad \Longrightarrow \quad \boxed{c_k = \sum_{l=-\infty}^{\infty} a_l\, b_{k-l} = a_k * b_k}$

$$y(t) = \cos\left(\frac{2\pi}{10} t\right) \qquad \times$$

$x(t)$

FS

FS

b_k : 0.5 at $k=-1$ and $k=1$

a_k : peaks, 0.5, $1/\pi$, $-1/3\pi$

$$\frac{2\pi}{10} \quad 2.5$$

$$a_k = \frac{\sin(k\omega_o T_1)}{k\pi} = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$
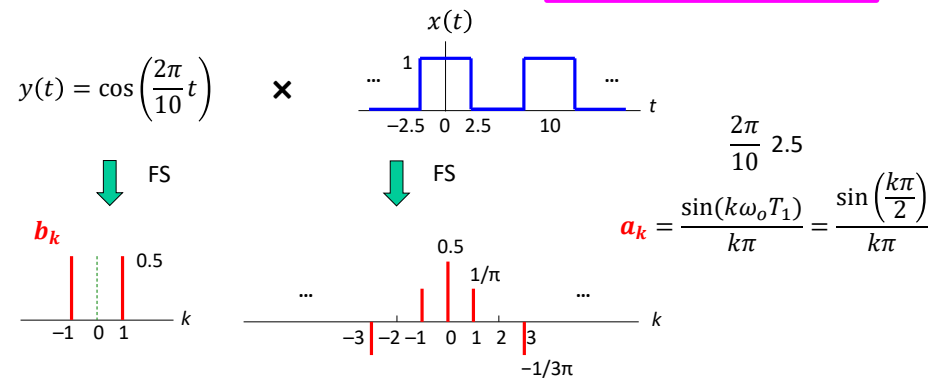
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Method 1 : By convolution sum

e.g. $c_{-2} = 0.5\left(\dfrac{1}{\pi}\right) + 0.5\left(-\dfrac{1}{3\pi}\right)$

c_k

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b_k $*$ a_k

$0.5\,a_{k-1}$

$0.5\,a_{k+1}$

$c_k = a_k * b_k$
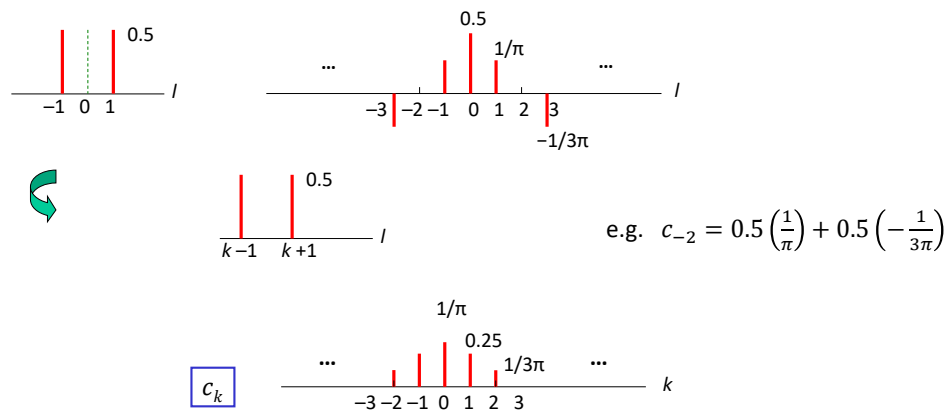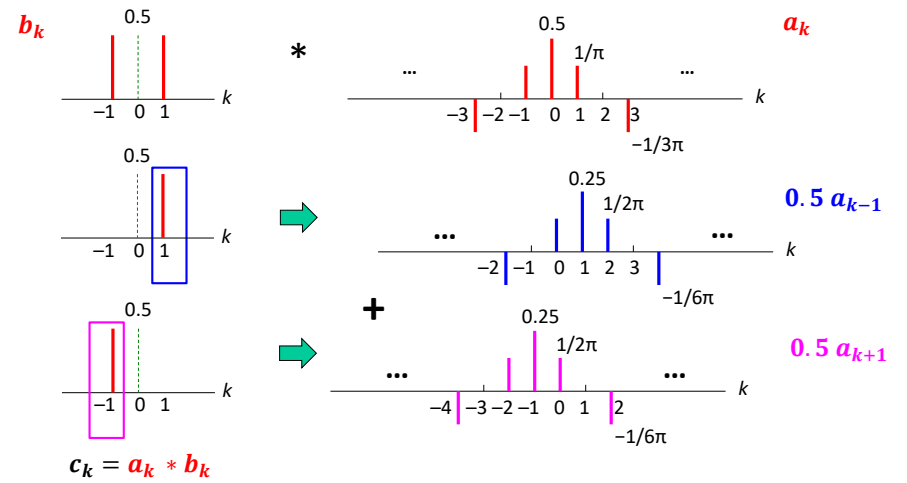
Method 2 : By frequency shifting & Linearity

$x(t)\cos\left(\dfrac{2\pi}{10}t\right)$

$\dfrac{1}{2}e^{j\left(\frac{2\pi}{10}t\right)}x(t) + \dfrac{1}{2}e^{-j\left(\frac{2\pi}{10}t\right)}x(t)$

$c_k = \dfrac{a_{k-1} + a_{k+1}}{2}$

e.g. $c_{-2} = \dfrac{a_{-3} + a_{-1}}{2}$

a_{k-1}

a_{k+1}

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k	- 4	- 3	- 2	- 1	0	1	2	3	4
a_k	0	- 1/3π	0	1/π	0.5	1/π	0	- 1/3π	0
a_{k-1}	1/5π	0	- 1/3π	0	1/π	0.5	1/π	0	- 1/3π
a_{k+1}	- 1/3π	0	1/π	0.5	1/π	0	- 1/3π	0	1/5π
c_k	- 1/15π	0	1/3π	0.25	1/π	0.25	1/3π	0	- 1/15π

$c_k = \dfrac{a_{k-1} + a_{k+1}}{2}$

e.g. $x(t) = 1 + 4\cos(2\pi t) + 6\sin(4\pi t)$

a) Find the F.S. coefficients.

$$x(t) = 1 + 2\,e^{j2\pi t} + 2\,e^{-j2\pi t} + \frac{3}{j}\,e^{j4\pi t} - \frac{3}{j}\,e^{-j4\pi t}$$

$$a_0 = 1 \qquad a_1 = 2 \qquad a_{-1} = 2 \qquad a_2 = 3e^{-j\frac{\pi}{2}} \qquad a_{-2} = 3e^{j\frac{\pi}{2}}$$

b) Find the total average power.   Parseval's relation

$$P = \sum_{k=-\infty}^{\infty} |a_k|^2 = (1)^2 + 2\,(2^2) + 2(3)^2 = 27 \qquad\qquad P = \frac{1}{T}\int_0^T |x(t)|^2\,dt$$
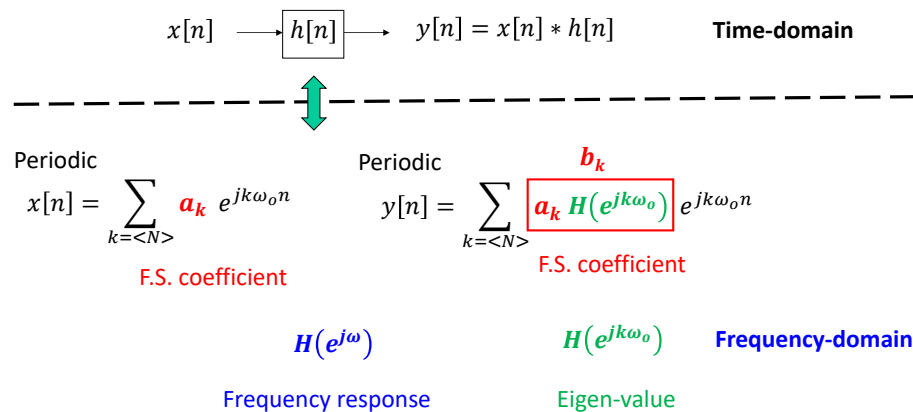
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Fourier analysis for DT signals and systems
Difference between CTFS and DTFS
Difference between H(j⍵) and H(e^{jω})

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Fourier Analysis for DT Signals and LTI Systems

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n] \qquad \textbf{Time-domain}$$

Periodic

$$x[n] = \sum_{k=<N>} a_k\,e^{jk\omega_o n}$$

F.S. coefficient

Periodic

$$y[n] = \sum_{k=<N>} \overset{b_k}{\boxed{a_k\,H(e^{jk\omega_o})}}\,e^{jk\omega_o n}$$

F.S. coefficient

$$H(e^{j\omega}) \qquad\qquad H(e^{jk\omega_o}) \qquad \textbf{Frequency-domain}$$

Frequency response   Eigen-value

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Difference between CTFS and DTFS

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\,e^{jk\frac{2\pi}{T}t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} \overset{b_k}{\boxed{a_k\,H(jk\omega_o)}}\,e^{jk\omega_o t}$$

$$a_k = \frac{1}{T}\int_0^T x(t)\,e^{-jk\frac{2\pi}{T}t}\,dt$$

$$\boxed{\text{Infinite number of } a_k}$$

$$x[n] = \sum_{k=<N>} a_k\,e^{jk\frac{2\pi}{N}n}$$

$$y[n] = \sum_{k=<N>} \overset{b_k}{\boxed{a_k\,H(e^{jk\omega_o})}}\,e^{jk\frac{2\pi}{N}n}$$

$$a_k = \frac{1}{N}\sum_{n=<N>} x[n]\,e^{-jk\frac{2\pi}{N}n} \qquad \text{Analysis equation}$$

$$\boxed{\text{Only } N \text{ number of } a_k}$$

$$a_k = a_{k+N}$$

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$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\frac{2\pi}{T}t} \qquad\qquad x[n] = \sum_{k=<N>} a_k\, e^{jk\frac{2\pi}{N}n}$$

a_0

$a_1\, e^{j(1)\frac{2\pi}{T}t}$

$a_2\, e^{j(2)\frac{2\pi}{T}t}$

$a_3\, e^{j(3)\frac{2\pi}{T}t}$

$a_4\, e^{j(4)\frac{2\pi}{T}t}$

$a_5\, e^{j(5)\frac{2\pi}{T}t}$

$\vdots$
$\vdots$
$\vdots$

e.g. N = 3

$a_0\, e^{j(0)\frac{2\pi}{N}n}$

$a_1\, e^{j(1)\frac{2\pi}{N}n}$

$a_2\, e^{j(2)\frac{2\pi}{N}n}$

$a_3\, e^{j(3)\frac{2\pi}{N}n} = a_0\, e^{j(0)\frac{2\pi}{N}n}$

$a_4\, e^{j(4)\frac{2\pi}{N}n} = a_1\, e^{j(1)\frac{2\pi}{N}n}$

$a_5\, e^{j(5)\frac{2\pi}{N}n} = a_2\, e^{j(2)\frac{2\pi}{N}n}$
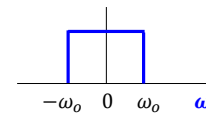
$\vdots$
$\vdots$
$\vdots$

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The difference between $H(j\omega)$ and $H(e^{j\omega})$

CTFS

$$y(t) = \sum_{k=-\infty}^{\infty} \boxed{a_k\, H(jk\omega_o)}^{b_k}\, e^{jk\omega_o t}$$

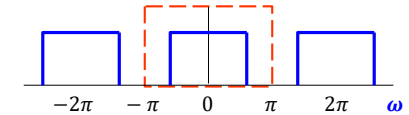a_k and b_k are not periodic

$H(j\omega)$ is not periodic

$-\omega_o \quad 0 \quad \omega_o \qquad \omega$

Actual frequency

DTFS

$$y[n] = \sum_{k=<N>} \boxed{a_k\, H(e^{jk\omega_o})}^{b_k}\, e^{jk\frac{2\pi}{N}n}$$

a_k and b_k are periodic

$H(e^{j\omega})$ is periodic

$-2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi \qquad \omega$

Normalized frequency

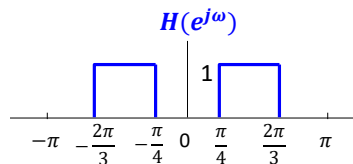Question : What is the highest frequency ?

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e.g. Given the frequency response ($-\pi \le \omega \le \pi$) of a filter and the following inputs :

$$x_1[n] = (-1)^n \qquad\qquad x_2[n] = 1 + \sin\left(\frac{3\pi}{2}n + \frac{\pi}{4}\right)$$

$$y_1[n] = \;?\qquad\qquad\qquad y_2[n] = \;?$$

$H(e^{j\omega})$

$-\pi \quad -\frac{2\pi}{3} \quad -\frac{\pi}{4} \quad 0 \quad \frac{\pi}{4} \quad \frac{2\pi}{3} \quad \pi$

Determine the output for each of the above inputs.

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e.g. Given :
$$x[n] = 1 + \cos\left(\frac{2\pi}{4}n\right) + 2\cos(\pi n) + \sin\left(\frac{5\pi}{2}n\right)$$

$$y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right) \qquad \omega_o = ? \qquad N = ?$$

Find the eigenvalues for k = 0, 1, 2 and 3.

$$x[n] = 1 + \frac{1}{2}e^{j\left(\frac{2\pi}{4}n\right)} + \frac{1}{2}e^{-j\left(\frac{2\pi}{4}n\right)} + 2e^{j\left(\frac{2\pi}{2}n\right)} + \frac{1}{j2}e^{j\left(\frac{5\pi}{2}n\right)} - \frac{1}{j2}e^{-j\left(\frac{5\pi}{2}n\right)}$$

$$= 1 + \frac{1}{2}e^{j\left(\frac{\pi}{2}n\right)} + \frac{1}{2}e^{-j\left(\frac{\pi}{2}n\right)} + 2\,e^{j\left(\frac{2\pi}{2}n\right)} - \frac{j}{2}e^{j\left(\frac{\pi}{2}n\right)} + \frac{j}{2}e^{-j\left(\frac{\pi}{2}n\right)}$$

$$= 1 + \frac{1}{2}(1-j)\,e^{j\left(\frac{\pi}{2}n\right)} + \frac{1}{2}(1+j)\,e^{-j\left(\frac{\pi}{2}n\right)} + 2\,e^{j\left(\frac{2\pi}{2}n\right)} \qquad\qquad a_k\, e^{jk\frac{2\pi}{N}n}$$

k = 0 $\qquad\qquad$ k = 1 $\qquad\qquad$ k = −1 $\qquad$ k = 2

Question : a_3 ?

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$$y[n] = \frac{1}{2} e^{j\left(\frac{5\pi}{2}n+\frac{\pi}{4}\right)} + \frac{1}{2} e^{-j\left(\frac{5\pi}{2}n+\frac{\pi}{4}\right)}$$

$$= \frac{1}{2} e^{j\frac{\pi}{4}} \, e^{j\left(\frac{\pi}{2}n\right)} + \frac{1}{2} e^{-j\frac{\pi}{4}} \, e^{-j\left(\frac{\pi}{2}n\right)} \qquad\qquad b_k \, e^{jk\frac{2\pi}{N}n}$$

$k = 0$	$a_0 = 1$	$H(e^{j0}) = 0$	$b_0 = 0$
$k = 1$	$a_1 = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}}$	$H\left(e^{j\frac{\pi}{2}}\right) = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{2}}$	$b_1 = \frac{1}{2} e^{j\frac{\pi}{4}}$
$k = 2$	$a_2 = 2$	$H(e^{j\pi}) = 0$	$b_2 = 0$
$k = 3$	$a_3 = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}$	$H\left(e^{j\frac{3\pi}{2}}\right) = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}}$	$b_3 = \frac{1}{2} e^{-j\frac{\pi}{4}}$

$$a_k \, H\left(e^{jk\omega_o}\right) = b_k$$

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e.g. Given the following periodic signal $x[n]$:
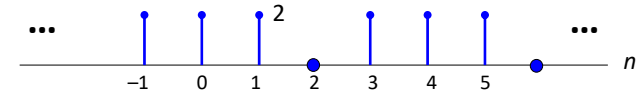
a) Find the fundamental frequency.
b) Find the FS coefficients.
c) Express $x[n]$ as a sum of complex sinusoids.
d) Express $x[n]$ as a sum of real sinusoids.

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$$N = 4$$

$$\omega_o = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] \, e^{-jk\frac{2\pi}{N}n}$$
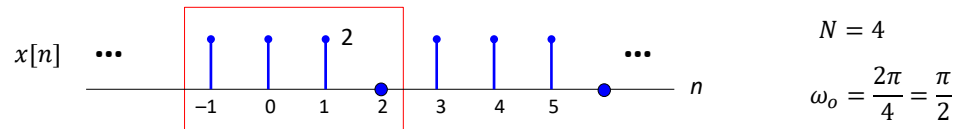
$$= \frac{1}{4} \sum_{n=-1}^{1} 2 \, e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{2}{4} \left(1 + e^{-jk\frac{2\pi}{4}} + e^{jk\frac{2\pi}{4}}\right)$$

$$= \frac{1}{2} \left(1 + 2\cos\left(\frac{k\pi}{2}\right)\right)$$

$$a_0 = \frac{3}{2}$$

$$a_1 = \frac{1}{2}$$

$$a_2 = -\frac{1}{2}$$

$$a_3 = a_{-1} = a_1^*$$

$$x[n] = \sum_{k=<N>} a_k \, e^{jk\frac{2\pi}{N}n}$$

$$x[n] = \frac{3}{2} + \frac{1}{2} e^{j\frac{\pi}{2}n} - \frac{1}{2} e^{j\pi n} + \frac{1}{2} e^{-j\frac{\pi}{2}n}$$
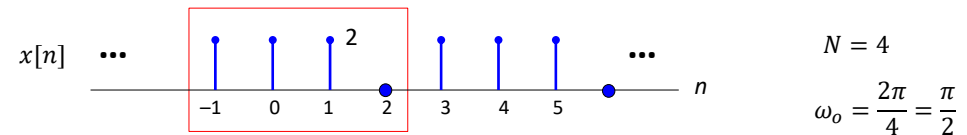
$$x[n] = \frac{3}{2} + \frac{1}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) - \frac{1}{2} e^{j\pi n}$$

$$x[n] = \frac{3}{2} + \cos\left(\frac{\pi}{2}n\right) - \frac{1}{2}\cos(\pi n)$$

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Slide 32

$$N = 4$$

$$\omega_o = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_k = (2)\frac{1}{N}\frac{\sin\left(\frac{2\pi k}{N}\left(N_1+\frac{1}{2}\right)\right)}{\sin\left(\frac{\pi k}{N}\right)}$$

$$= \frac{1}{2}\frac{\sin\left(\frac{3k\pi}{4}\right)}{\sin\left(\frac{\pi k}{4}\right)}$$

$$a_0 = \frac{3}{2}$$

$$a_1 = \frac{1}{2}\frac{\sin\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)} = \frac{1}{2}$$

$$a_2 = \frac{1}{2}\frac{\sin\left(\frac{6\pi}{4}\right)}{\sin\left(\frac{2\pi}{4}\right)} = -\frac{1}{2}$$

$$a_3 = a_{-1} = a_1^*$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

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