MATH2351: Introduction to Differential Equations
2024-25 Fall Midterm Exam
Question Paper
Duration: 80 minutes
Total Points: 100

Question 1: Find the real-valued equilibrium solutions of the following differential equation: *[4 points]*

$$y' = (2 + y^2)(y^2 - 2\pi y)$$

Question 2: Given the following differential equation:

$$y' = 7y - 6$$

(a) Find the equilibrium solution of the differential equation. *[4 points]*
(b) Find the non-equilibrium solution of the differential equation using the method of Calculus. *[4 points]*
(c) Find the general solution expression for ALL solutions and specify the range of the parameter in your general solution expression corresponding to the equilibrium solution found in part (a) and the non-equilibrium solution found in part (b). *[2 points]*

Question 3: Find a fundamental set of solutions for the following differential equation: *[10 points]*

$$y'' + 8y' + 16y = 0$$

Question 4: Solve the following initial value problem: *[12 points]*

$$y'' - 4y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

Question 5: Find the general solution of the following differential equation: *[20 points]*

$$y'' - 2y' - 3y = 2e^{3t} - \sin(2t) + t$$

Question 6: Solve the following initial value problem: *[12 points]*

$$ty' - 2y - 7t^2 = 0, \quad y(-1) = 2$$

Question 7: Given the following differential equation:

$$\frac{dy}{dx} = \frac{y\cos(x)}{3y^4 + 5}$$

(a) Find all the solutions of the differential equation. *[10 points]*
(b) Find the particular solution of the differential equation that satisfies the initial condition $y(0) = 1$. *[3 points]*
(c) Find the particular solution of the differential equation that satisfies the initial condition $y(0) = 0$. *[2 points]*

Question 8: Given a solution $y_1(x) = \frac{1}{x}$ of the following differential equation:

$$y'' + \frac{3}{x}y' + \frac{1}{x^2}y = 0 \quad \text{for} \quad x > 0$$

(a) Find the second linearly independent solution $y_2(x)$ of the differential equation such that $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of this equation for $x > 0$ by applying the method of reduction of order. *[12 points]*
(b) Compute the Wronskian of $y_1(x)$ and $y_2(x)$ and verify that the solutions $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of the differential equation for $x > 0$. *[3 points]*
(c) Find the general solution of the differential equation for $x > 0$. *[2 points]*

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