Correlation of signals

$r_{xy}[l]=\sum_{n=-\infty}^{\infty} x[n]\,y[n-l]$

$\rho_{xy}[l]=\frac{r_{xy}[l]}{\sqrt{r_{xx}[0]\,r_{yy}[0]}}$

Convolution

$x[n]*h[n]=\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

DFT: $X[k]=\sum_{n=0}^{N-1} x[n]\,W_N^{kn}$, $W_N^{kn}=e^{-j\frac{2\pi}{N}}$

$$\vec{X}=D_N\vec{x},\quad D_N=\begin{pmatrix} 1 & 1 & 1 & \cdots & 1\\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)}\\ 1 & W_N^2 & W_N^4 & \cdots & \\ \vdots & \vdots & & \ddots & \\ 1 & W_N^{(N-1)} & & & W_N^{(N-1)^2}\end{pmatrix}$$

$$\vec{x}=\frac{1}{N}D_N^*\vec{X},\quad D_N^*=\begin{pmatrix} 1 & 1 & 1 & \cdots & 1\\ 1 & W_N^{-1} & W_N^{-2} & \cdots & \\ \vdots & & & \ddots & \\ 1 & W_N^{-(N-1)} & & & W_N^{-(N-1)^2}\end{pmatrix}$$

Circular convolution

$$\begin{bmatrix} y_c[0]\\ y_c[1]\\ \vdots\\ y_c[N-1]\end{bmatrix}=\begin{bmatrix} h[0] & h[N-1] & \cdots & h[1]\\ h[1] & h[0] & & h[2]\\ \vdots & \vdots & \ddots & h[3]\\ h[N-1] & h[N-2] & \cdots & h[0]\end{bmatrix}\begin{bmatrix} g[0]\\ g[1]\\ \vdots\\ g[N-1]\end{bmatrix}$$

FIR filter Design.

Integral square error $=\sum_{n=-M}^{M}(h_t[n]-h_d[n])$ ← this should make to be 0 $+\sum_{n=-\infty}^{-M-1}(h_t[n]-h_d[n])+\sum_{n=M+1}^{\infty}(h_t[n]-h_d[n])$

Transpose block diagram

1. Reverse all the path, amplifiers
2. Adder to node, node to Adder
3. Swap Input output

PCM: 分成不同 levels

$L=2^n$, $n=\lceil \log_2 L\rceil$

L = levels, n = ? bits per level

$f_s \geq 2B$, B = Bandwidth.

Bit rate $=nf_s$

e = quantization error $=[x(t)-x_q(t)]$

$\overline{e^2}=E(e^2)=\int_{-\Delta/2}^{\Delta/2}\frac{e^2}{\Delta}\,de=\frac{\Delta^2}{12}$

Average power

sine wave: $\frac{L^2\Delta^2}{8}$, $SNR=\frac{3}{2}L^2$ (better)

uniform signal: $\frac{L^2\Delta^2}{12}$, $SNR=L^2$

Entropy: $H(x)=E(-\log_2 P)$

- Defines lowest bit/symbol

LZW Algorithm:

Example: 101101010000010

Dictonary D

Index	Entry	Index	Entry
0	ϕ	5	010
1	1	6	00
2	0	7	10
3	11		
4	01		

W ϕ ϕ 1 0 01 0 1

B 1 0 1 1 0 0 0

Pairs ← corresponding index in D

(0,1) (0 0) (1,1) (2,1) (4,0) (2,0) (1,0)

Encoding: 0001 0000 0011 0101 1000 0100 0010

Comparison of different encoding methods

- Shannon-Fano's, Huffman, Arithemic need know the distribution
- LZW no need.
- Huffman allows decoding in middle, faster as it can be conducted in parallel
- Arithemic coding: coding rate closer to Entropy, no need to store symbols in Advance

Channel Capacity: Max. C for reliable data

$C\leq B\log_2(1+\frac{S}{N})$

(7, 4) hamming code

Generating Matrix

$$G=(I_k\,|\,P)=\left(\begin{array}{cccc|ccc} 1&0&0&0&1&0&1\\ 0&1&0&0&1&1&1\\ 0&0&1&0&1&1&0\\ 0&0&0&1&0&1&1\end{array}\right)$$

NOT UNIQUE!

$$H=(P^T\,|\,I_{n-k})=\begin{pmatrix}1&1&1&0&1&0&0\\ 0&1&1&1&0&1&0\\ 1&1&0&1&0&0&1\end{pmatrix}$$

Encode as $c=uG$

$E=Hc^T=\vec{0}$, if not $\vec{0}$, then must have error!

Sydrome matrix:

$s=rH^T=(c+e)H^T$
$=eH^T$

Use sydrom decoding to detect error!

WSGN noise:

$n(t)$: $PSD=\mathcal{F}(\frac{N_0}{2}\delta(\tau))=\frac{N_0}{2}$

WSS: mean = constant, $R_x(t_1,t_2)$ depends only on time difference t_1-t_2

Nyquist noise theorem

$\overline{v^2}=4kTBR$ ← Resistance (B: Bandwidth)

Integrate and dump

$s(t)=\begin{cases}A & 0\leq t<T \quad '1'\\ -A & 0\leq t<T \quad '0'\end{cases}$

$v=\int_0^T (s(t)+n(t))\,dt$

$=\begin{cases}AT+N & \text{if '1' sent}\\ -AT+N & \text{if '0' sent}\end{cases}$

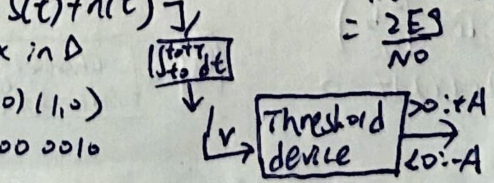
$VAR(N)=E(N^2)=E[(\int_0^T n(t)\,dt)^2]$

$=\frac{N_0T}{2}\equiv\sigma^2$

$P_e=Q\sqrt{\frac{A^2T^2}{\sigma^2}}=Q\sqrt{\frac{2E_b}{N_0}}$

As $\frac{E_b}{N_0}=SNR$, should maximize $\frac{E_b}{N_0}$

Block diagram:



More general case: $p(0)\neq p(1)$

$P_e=p_0Q(\frac{V_D-s_0(T)}{\sigma})+p_1Q(\frac{s_1(T)-V_D}{\sigma})$

Optimal Threshold:

$V_D=\frac{N_0}{4A}\ln(\frac{1-p_0}{p_0})$

if 0: A 0: −A

But in Ch 7.1−7.2, Let

$p_0=p_1=0.5$

$V_{D,opt}=\frac{s_{00}(T)+s_{01}(T)}{2}$

$P_e=Q\sqrt{\frac{(s_{01}(T)-s_{00}(T))^2}{4\sigma^2}}$

Let $\zeta=\frac{s_{01}(T)-s_{00}(T)}{\sigma}$

⇓

$P_e=Q\sqrt{\frac{\zeta^2}{4}}$

$x(t)\xrightarrow{WSS}\boxed{h(t),\ H(f)}\rightarrow y(t)$

WSS

$R_{xy}(\tau)=\int_{-\infty}^{\infty}h(s)R_x(\tau-s)\,ds$
$=h(\tau)*R_x(\tau)$

$R_{yx}(\tau)=h(-\tau)*R_x(\tau)$

$S_{yx}(f)=H^*(f)S_x(f)$

$\therefore S_y(f)=|H(f)|^2S_x(f)$

$S_w(f)=\frac{N_0}{2}|H(f)|^2$

Hence,

$\sigma^2=E(w^2(T))=\int_{-\infty}^{\infty}S_w(f)\,df$
$=\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df$

Need maximize SNR

$\zeta^2=\frac{|\int_{-\infty}^{\infty}H(f)G(f)e^{j2\pi fT}\,df|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df}$

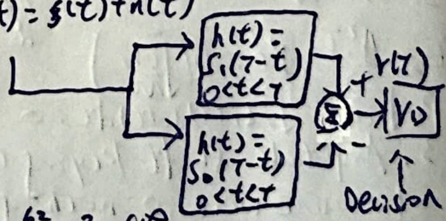
Hence, $\zeta^2=\frac{2}{N_0}\int_{-\infty}^{\infty}|G(f)|^2\,df$

$H_{opt}(f)=kG^*(f)e^{-j2\pi fT}$

$h_{opt}(t)=kg(T-t)$, which $g(t)=s_{01}(t)-s_{00}(t)$

Block diagram:

$y(t)=s(t)+n(t)$



$\zeta^2=\frac{2}{N_0}\int_{-\infty}^{\infty}|G(f)|^2\,df$
$=\frac{2}{N_0}\int_{-\infty}^{\infty}g^2(t)\,d(t)$
$=\frac{2E_s}{N_0}$

As last, $P_e=Q(\sqrt{\frac{E_s}{2N_0}})$

Digital Modulation

Reason: Because unity gain in only higher frequency, otherwise huge attenuation

- Analog Modulation

1. Linear Modulation

carrier : $A_c\cos(2\pi f_c t)$

Modulated signal $= x_c(t) = A_c m(t)\cos(2\pi f_c t)$

2. Angle Modulation

$x_c(t) = A_c\cos(w_c t + \phi(t))$

$\psi(t) = w_c t + \phi(t)$,

it is the instantaneous phase of $x_c(t)$

$w_c(t) = \frac{d\psi(t)}{dt} = w_c + \frac{d\phi(t)}{dt}$

$\psi(t) = w_c t + \phi(t)$

$w(t) = w_c + \frac{d\phi(t)}{dt}$

$\phi(t)$: phase deviation

$\frac{d\phi(t)}{dt}$: frequency deviation

2.1 PM: $\phi(t) = k_p m(t)$

k_p: phase deviation constant

2.2 FM:

carrier: $A_c\cos(2\pi f_c t)$

$x_c(t) = A_c\cos\left(w_c t + 2\pi f_d\int_0^t m(\tau)d\tau\right)$

Single tone:

$x_c(t) = A_c\cos[w_c t + \beta\sin w_m t]$

☆☆☆☆☆

1. TDM, 2. FDM

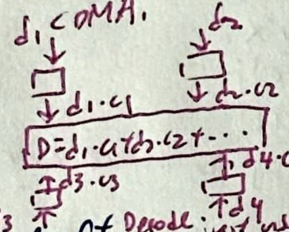
$T_s \geq \frac{1}{2B}$, 越小越好

FDM: $f_2 - f_1 \geq w_1 + w_2$

☆可以 TDM+FDM 混合

TDMA, FDMA, SDMA

CDMA.



Decode: just use dot product, as c_1, c_2, c_3, c_4 are orthogonal.

- Digital Modulation

1. Antipodal Signaling

$s_1(t) = A$

$s_0(t) = -A$

$E_b = A^2T$, $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t) = -A^2T$

2. Non-Return to Zero

$s_1(t) = A$

$s_0(t) = -A$

$E_b = \frac{A^2T}{2}$, $P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t)dt = 0$

3. Amplitude Shift Keying (ASK)

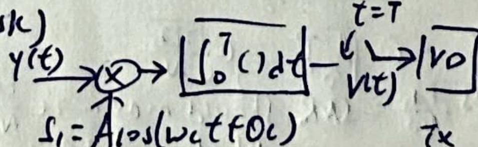
- 1: cosine wave, 0: nothing

$s_1(t) = A\cos(w_c t + \theta_c)$

$s_0(t) = 0$

$E_b = \frac{A^2T}{4}$, $P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

$\rho_{10}\sqrt{E_1E_0} = \int_0^T s_1(t)s_0(t)dt = 0$



题外话:

LZW over huffman

1. exploits dependencies (correlation) between symbols.
2. 不需要 distributions
3. Simple algorithm, perform quickly.
4. Achieves entropy.

As Huffman only do so if probabilities $= 2^n$

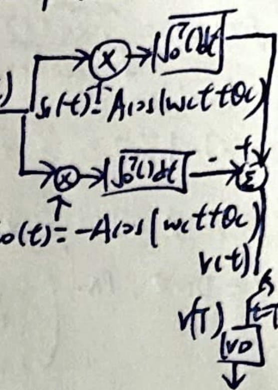
4. Phase shift keying (PSK/BPSK)

$s_0(t) = -A\cos(w_c t + \theta_c)$ (180° phrase shift)

$s_1(t) = A\cos(w_c t + \theta_c)$

$E_b = \frac{A^2T}{2}$, $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

$\rho_{10}\sqrt{E_1E_0} = -\frac{A^2T}{2}$



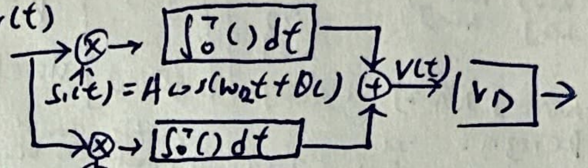
5. Frequency shift keying (FSK)

$s_0(t) = A\cos(w_1 t + \theta_c)$

$s_1(t) = A\cos(w_2 t + \theta_c)$

$f_2 > f_1$, $\Delta f \triangleq f_2 - f_1 = \frac{n}{T} = nR$

$E_g = \frac{A^2T}{2}$, $P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$



- Trade off between orthogonal and BER

Wireless Propagation

1. Reflection 2. Diffraction 3. Scattering

- Channel Effects

1. Noise 2. Attenuation 3. Fading, random change of Ampitude 4. Freq/time selectivity.

Signal strength $\propto \frac{1}{\text{distance}}$

Rayleigh Fading

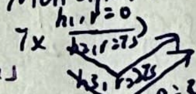
$y = hx + n$, $|h| \sim$ Rayleigh distribution

$p_\alpha(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)$, $E(\alpha) =$ av. power: $2\sigma^2$

- Flat Fading, very good. effect $\simeq \delta(t)$

- Freq. selective fading, like window, will cause ISI

Multipath channel and OFDM



Rx $y_k = \sum_{i=1}^{P} h_i x_{k-i}$ (FIR)

$P = 3$

$x[n] \to h[n] \to y[n]$, how to find $x[n]$?

use DFT: $X[k] = \frac{Y[k]}{H[k]}$, how to convert lin. convolution to circular convolution?

Use Cyclic Prefix (CP)

Tx: $\begin{bmatrix} x[1] \\ x[2] \\ x[3] \end{bmatrix} \xrightarrow{CP} \begin{bmatrix} x[2] \\ x[3] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$ } $P-1 = 2$ CPs $\begin{bmatrix} y[1] \\ y[2] \\ \vdots \end{bmatrix}$ $\begin{bmatrix} h_1 & 0 & 0 & 0 & 0 \\ h_2 & h_1 & 0 & 0 & 0 \\ h_3 & h_2 & h_1 & 0 & 0 \\ 0 & h_3 & h_2 & h_1 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 \end{bmatrix}$

At Rx, discard CP. 頭尾不要!

$\begin{pmatrix} h_3 & h_2 & h_1 & 0 & 0 \\ 0 & h_3 & h_2 & h_1 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 \end{pmatrix}\begin{pmatrix} x[2] \\ x[3] \\ x[1] \\ x[2] \\ x[3] \end{pmatrix} = x[n] \circledP h[n]$