

### T03

Representation of DT signals  
DT Impulse response  
Convolution sum for DT LTI system

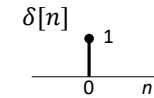
Representation of CT signals  
CT Impulse response  
Convolution integral for CT LTI system

Properties of LTI system  
Characterizing LTI system by Impulse Response

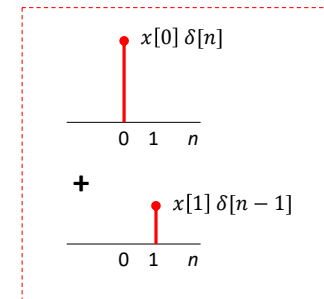
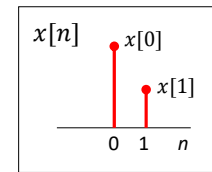
1

### Representation of DT signals

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$$

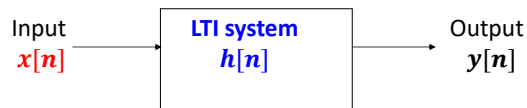


Difference between  $x[n]$  and  $x[k]$ ?



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### Impulse Response (DT)



e.g.  $x[n]$   $h[n] = \delta[n]$   $y[n] = ?$   $x[n]$

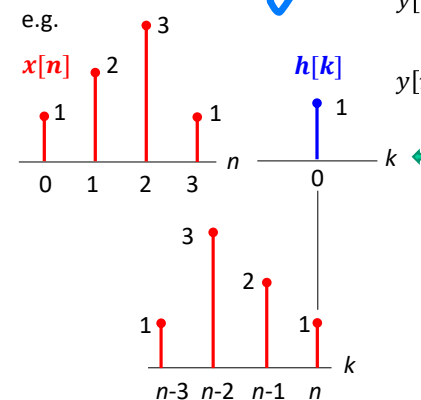
e.g.  $x[n] = \delta[n]$   $h[n]$   $y[n] = ?$   $h[n]$

*impulse response*

Question : How to relate input, output and LTI system together ?

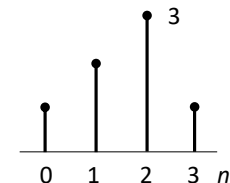
3

### Convolution Sum (DT)



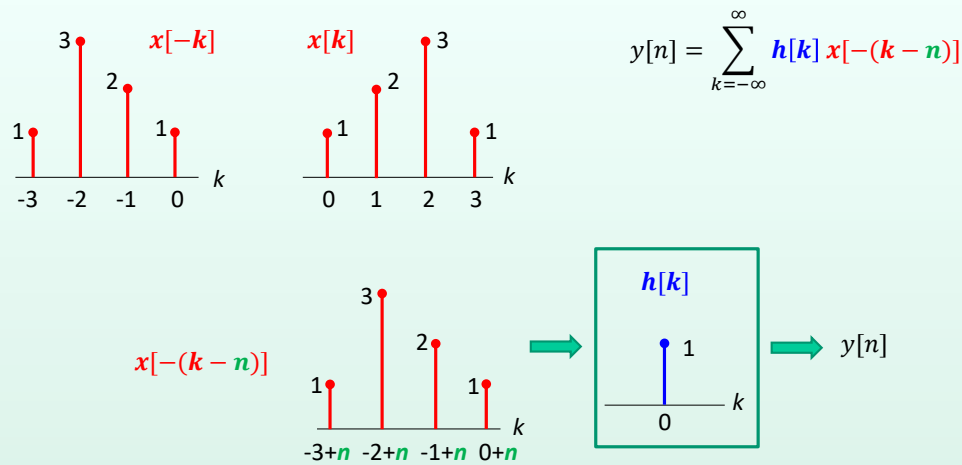
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] x[-(k-n)]$$

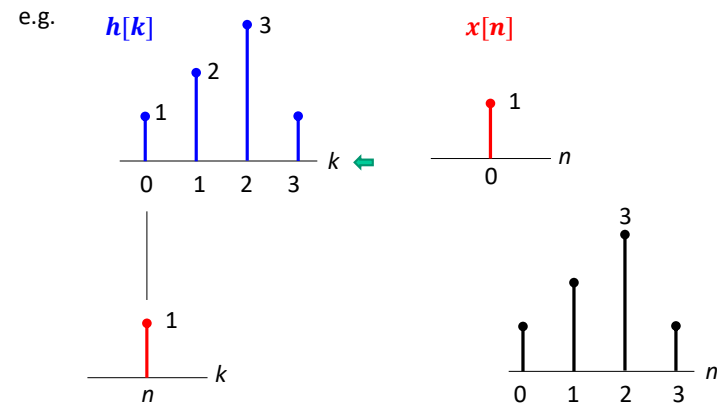


Question : What is the output  $y[n]$  ?  
How to physically implement this system ?

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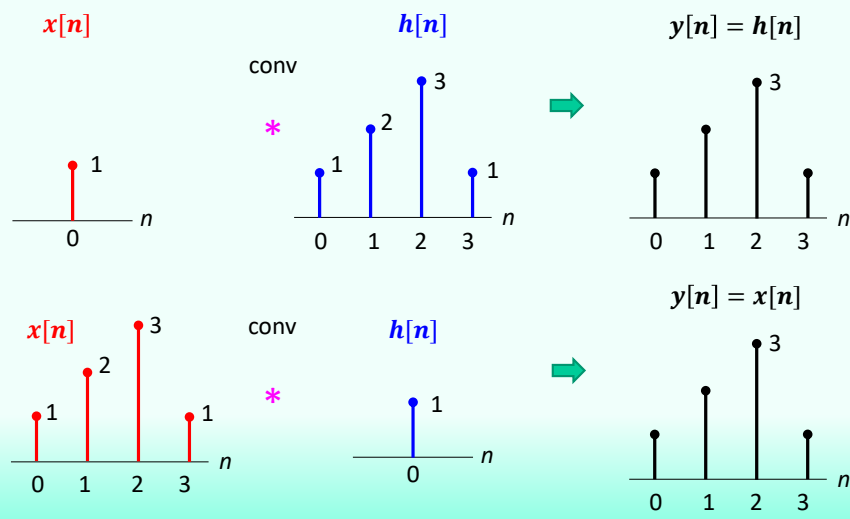
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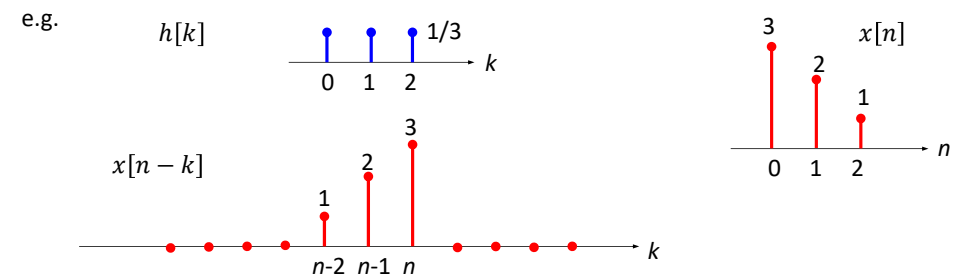
$$y[n] = ?$$

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Time reversal!



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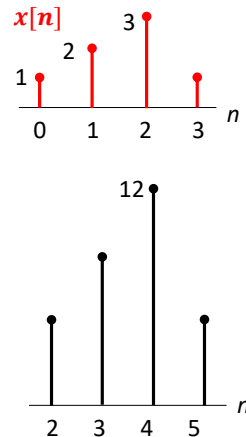
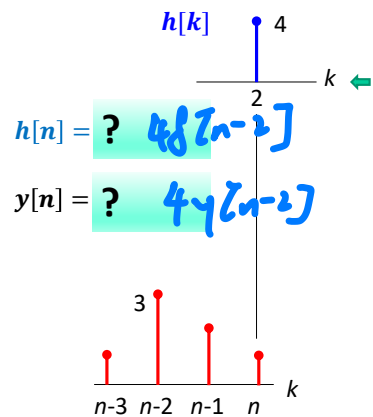
$$y[2] = (1)\frac{1}{3} + (2)\frac{1}{3} + (3)\frac{1}{3} \Rightarrow \text{Average} = \frac{1}{3}(1 + 2 + 3)$$

$$y[2] = x[2-0]h[0] + x[2-1]h[1] + x[2-2]h[2] = x[2]h[0] + x[1]h[1] + x[0]h[2]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k] h[k] = x[n-0]h[0] + x[n-1]h[1] + x[n-2]h[2]$$

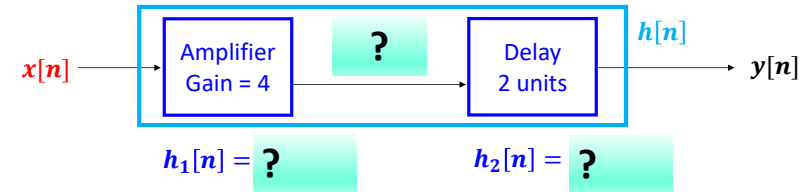
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e.g. Sketch the output.



Question : What is the expression of  $h[n]$  ?  
What is the input-output relationship ?

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Question :

- What is the input-output relationship ?
- What is the impulse response for each block ?
- What is the overall impulse response ?

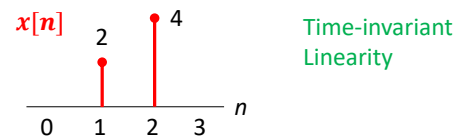
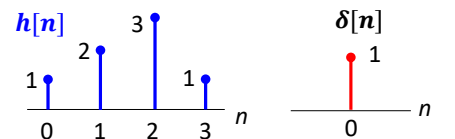
$y[n] = ?$

$h[n] = ?$

- What is the impulse response for each of the following systems ?  
Do-nothing , Echo , Integrator , Windowed smoother ... etc

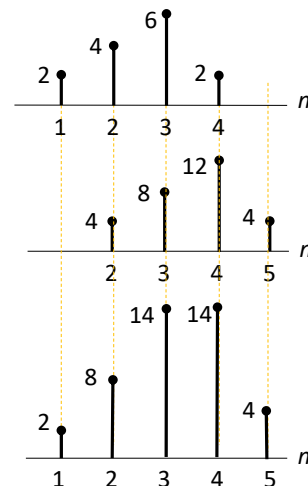
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e.g. Represent  $y[n]$  in terms of  $h[n]$ . Sketch the output.



$$x[n] = 2\delta[n-1] + 4\delta[n-2]$$

$y[n] = ?$



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Representation of CT signals  
CT Impulse response  
Convolution integral for CT LTI system

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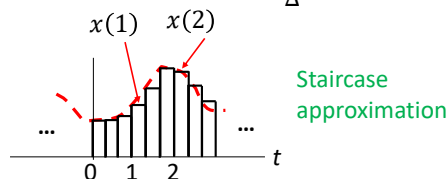
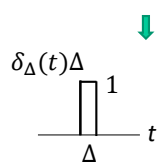
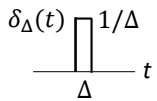
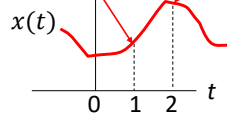
### Representation of CT signals

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

Difference between  $x(t)$  and  $x(\tau)$  ?

$$x(1) = \int_{-\infty}^{\infty} x(1) \delta(1 - \tau) d\tau$$

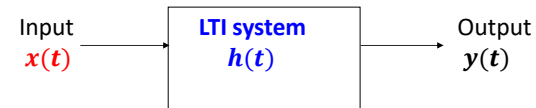
$$x(2) = \int_{-\infty}^{\infty} x(2) \delta(2 - \tau) d\tau$$



$$x(t) = \sum_{k=-\infty}^{\infty} [x(\Delta k) \delta_{\Delta}(t - \Delta k) \Delta]$$

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### Impulse Response (CT)



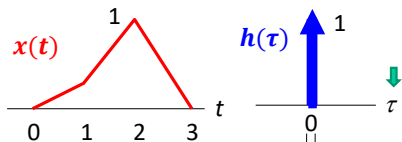
e.g.  $x(t)$   $h(t) = \delta(t)$   $y(t) = ?$

e.g.  $x(t) = \delta(t)$   $h(t)$   $y(t) = ?$

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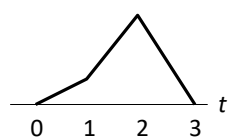
### Convolution Integral (CT)

e.g.



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau$$

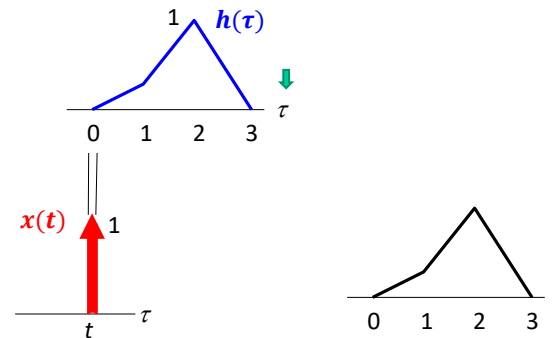
$$= \int_{-\infty}^{\infty} h(\tau) x(-(\tau - t)) d\tau$$



Question : What is the output ?  $y(t) = ?$

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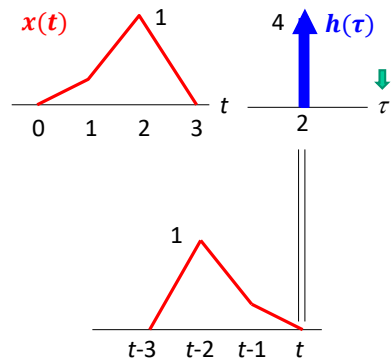
e.g.



Question : What is the output ?  $y(t) = ?$

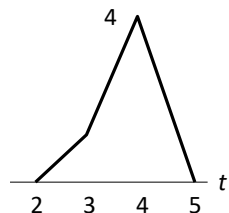
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e.g.



$$h(t) = ?$$

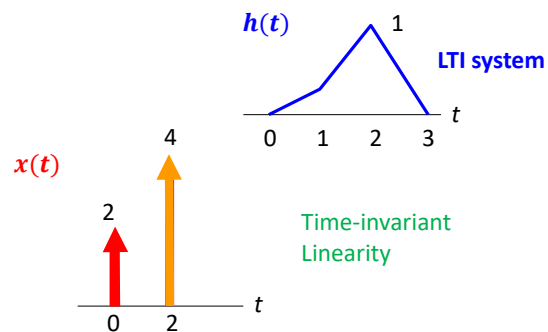
$$y(t) = ?$$



Question : What is the expression of  $h(t)$  ?  
What is the output ?

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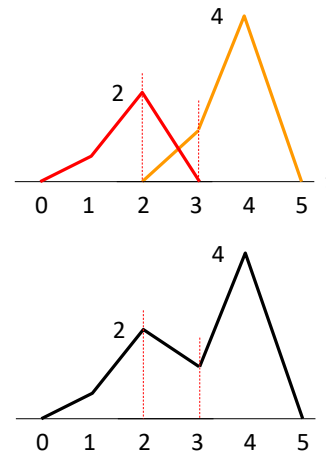
e.g. Represent  $y(t)$  in terms of  $h(t)$ . Sketch the output.



Time-invariant  
Linearity

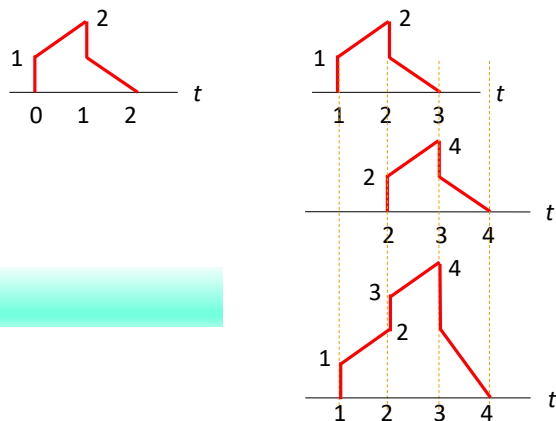
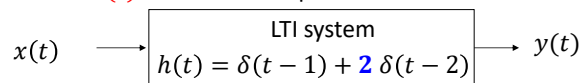
$$x(t) = 2\delta(t) + 4\delta(t-2)$$

$$y(t) = ?$$



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e.g. Represent  $y(t)$  in terms of  $x(t)$ . Sketch the output.

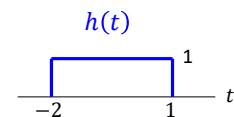


$$y(t) = ?$$

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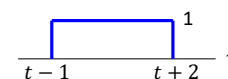
e.g.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau = \int_{-2}^1 x(t-\tau) d\tau$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{t-1}^{t+2} x(\tau) d\tau$$

$$h(t-\tau) = h(-(t-\tau))$$



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Properties of LTI system  
Characterizing LTI system by Impulse Response

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Properties of LTI System

**Commutative**  $x(t) * h(t) = h(t) * x(t)$   
 $x[n] * h[n] = h[n] * x[n]$

**Distributive**  $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$   
 $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

**Associative**  $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$   
 $(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$

$x(t) * \delta(t) = x(t)$

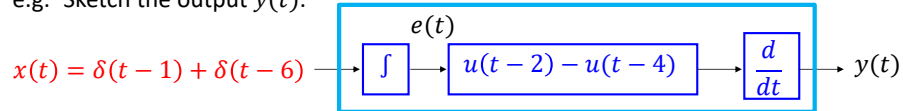
$x[n] * \delta[n] = x[n]$

$x(t) * \delta(t - t_o) = x(t - t_o)$

$x[n] * \delta[n - n_o] = x[n - n_o]$

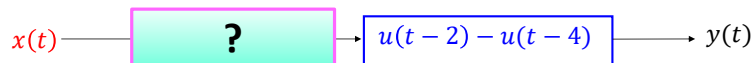
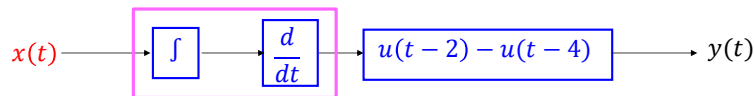
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e.g. Sketch the output  $y(t)$ .

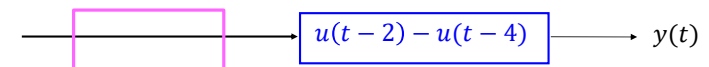


$$e(t) = \int_{-\infty}^t x(\tau) d\tau = u(t - 1) + u(t - 6)$$

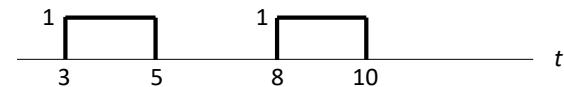
$$y(t) = \frac{d}{dt} \{ e(t) * [u(t - 2) - u(t - 4)] \}$$



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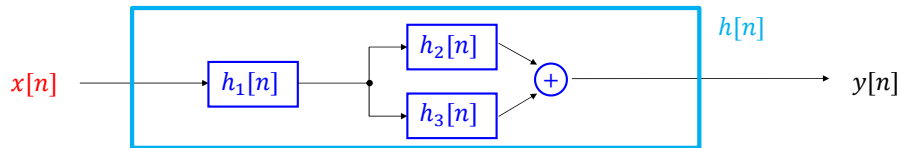
$x(t) = \delta(t - 1) + \delta(t - 6)$



$$y(t) = u(t - 3) - u(t - 5) + u(t - 8) - u(t - 10)$$

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e.g. Consider the following block diagram



a) Write down the overall impulse response  $h[n]$ .

$$h[n] = h_1[n] * (h_2[n] + h_3[n]) = \boxed{h_1[n] * h_2[n]} + \boxed{h_1[n] * h_3[n]}$$

b) If  $y[n] = h_1[n] + \frac{1}{2}h_1[n - 2]$  when  $x[n] = \delta[n]$ , suggest  $h_2[n]$  and  $h_3[n]$ .

$$y[n] = x[n] * h[n] = h[n] = \boxed{h_1[n]} + \boxed{\frac{1}{2}h_1[n - 2]}$$

$$h_2[n] = \boxed{?} \quad h_3[n] = \boxed{?}$$

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## Characterizing of LTI System by Impulse Response

System Properties  $\leftarrow$  By looking at Input/Output Relation

$$x(t) \quad y(t) \quad x[n] \quad y[n]$$

Memoryless , Causality , Stability , Time-invariant , Linearity, Invertibility

System Properties  $\leftarrow$  By looking at  $h(t)$  and  $h[n]$

Memoryless  $h(t) = 0$  for  $t \neq 0$  ;  $h[n] = 0$  for  $n \neq 0$

Causality  $h(t) = 0$  for  $t < 0$  ;  $h[n] = 0$  for  $n < 0$

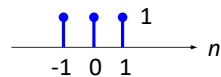
Stability  $\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$  ;  $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

Question : How about time-invariant and linearity ?

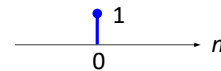
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Memoryless ? Causal ? Stable ?

e.g.  $h[n] = \delta[n - 1] + \delta[n] + \delta[n + 1]$



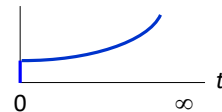
e.g.  $h[n] = \delta[n]$



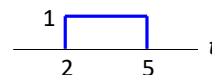
e.g.  $h[n] = \sum_{k=1}^{\infty} \delta[n - 10k]$



e.g.  $h(t) = e^t u(t)$



e.g.  $h(t) = u(t - 2) - u(t - 5)$



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