ISI Mitigation Techniques: Spread Spectrum and OFDM

3G!

4G!

Lecture 11

2 problem interference

Pe irreducible error floor

mixture of echos!

Spread Spectrum Systems

- In the last chapter, we talked about solution to issues due to "flat-fading channels" - diversity
- What about "frequency-selective fading channels"?
  - Interference from Multiple path (ISI)
  - Irreducible Error Floor
  - Can this be solved using "Diversity Techniques"?
- Spread spectrum communication has been devised as one way to overcome these problems
  - overcome narrowband interference
  - overcome the ISI in the multipath fading channel

Spread Spectrum Systems

- To be classified as spread spectrum the transmitted signal bandwidth is much larger than the information bit rate/bandwidth and is independent of the bit rate/ bandwidth

Definition:

For Spread-spectrum SS (modulation)

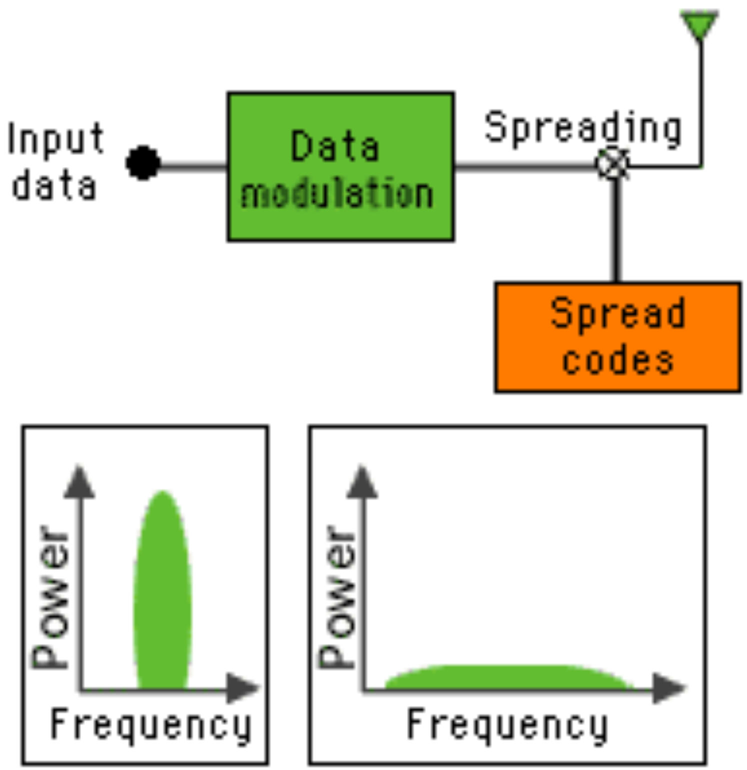
$$\boldsymbol{B_s} >> \boldsymbol{B_m} \quad \boldsymbol{B_s} \text{ is independent of } \boldsymbol{B_m}$$

where

$\boldsymbol{B_s}$ = Transmitted (signal) Bandwidth

$\boldsymbol{B_m}$ = Message Bandwidth

DSSS Communications



- The input data is modulated in the normal way using BPSK for example and forms the first output with spectrum as shown
- This signal is then multiplied by the much higher frequency spreading sequence to form the final transmitted signal with spectrum as shown

chip sequence!

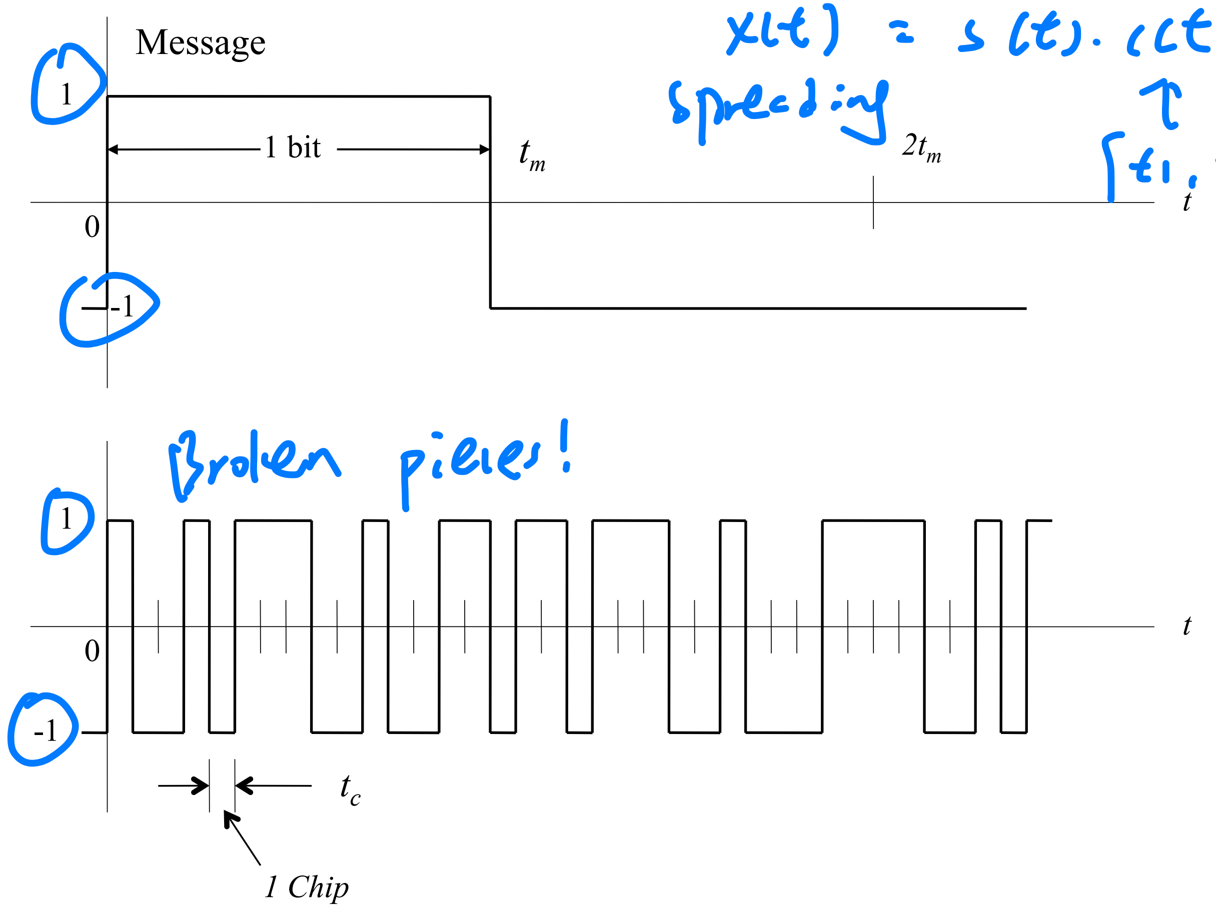
Data and spreading sequence

$x(t) = s(t) \cdot c(t)$

spreading

$\{+1, -1\}$

Broken pieces!



Relation between the code sequence and the binary message

DSSS Communication System

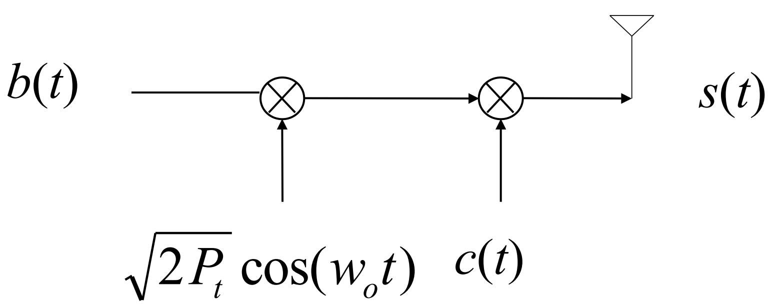
$b(t) = \pm 1$ Binary Message

$c(t) = \pm 1$ PN code (signal)

$P_t =$ Transmitted Power

$t_m =$ Bit period

$t_c =$ Chip period



DSSS Transmitter

Spread Spectrum Systems

- The standard BPSK output can be written as

T_s

$$S_d(t) = \sqrt{2P_t}\, b(t) \cos(\omega_0 t)$$

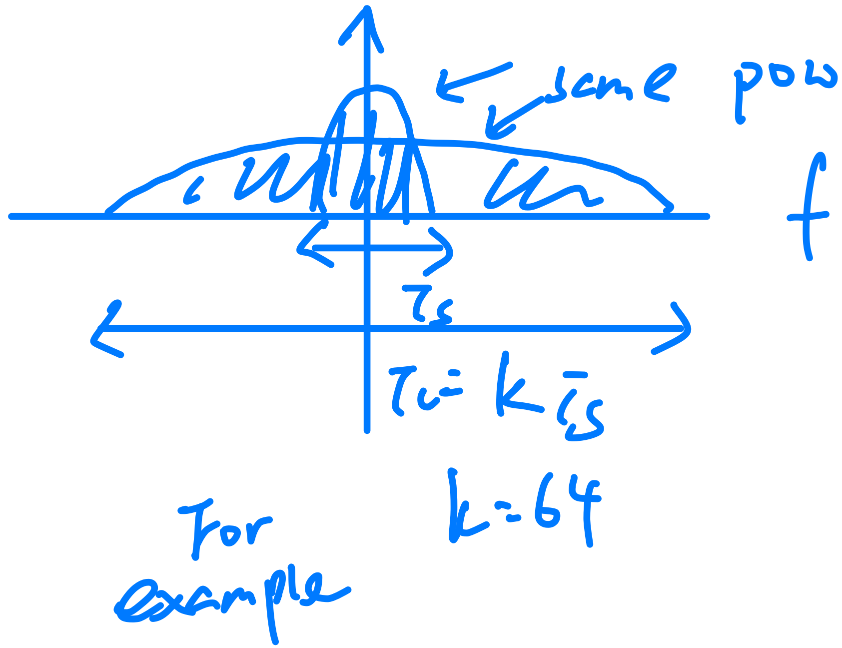
- After spreading the final output is

$$S(t) = \sqrt{2P_t}\, b(t) c(t) \cos(\omega_0 t)$$

$$E_s = \int_0^{T_s} s^2(t)\, dt \qquad = \frac{1}{T_s} \int_0^{T_s} b^2(t) c^2(t)\, dt$$

$$P_s = \frac{1}{T_s} \int_0^{T_s} s^2(t)\, dt = \frac{1}{T_s} \int_0^{T_s} b^2(t)\, dt$$

$SF = 64 \quad T_s = 64 \times T_c$

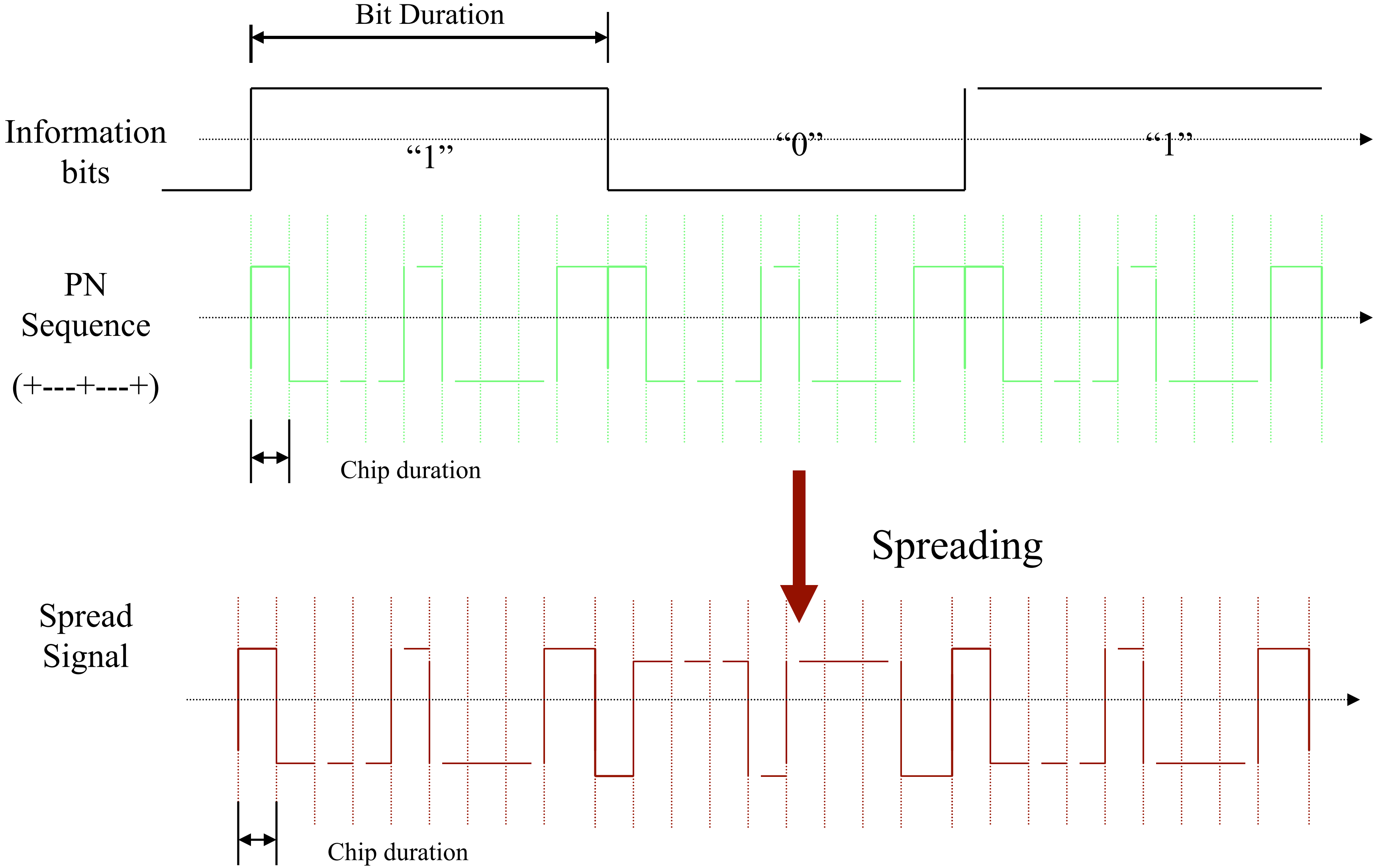


Despreading:

Rx need $c(t)$!

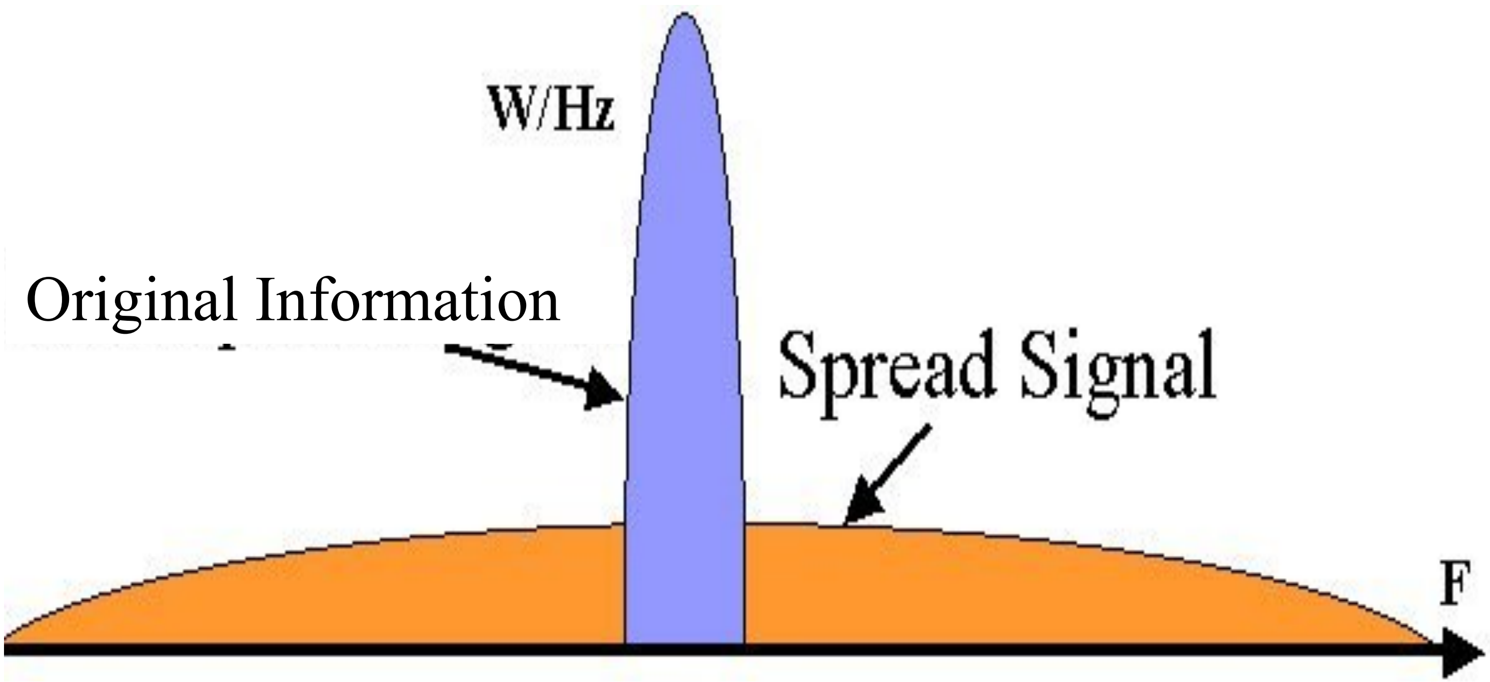
Also need to know despreading time!

DS Spreading – Time Domain Operation



$$\{x_n\} = \{d_m c_n\}$$ where m-symbol index, n=chip index

DS – Spreading – Frequency Domain



(bandpass BW)

$$W_{non-spread} = \frac{1}{T_s}, W_{spread} = \frac{1}{T_c}$$

Receiver

- The signal s(t) can then be demodulated by first multiplying by c(t) (known as de-spreading) so the product c(t)c(t) becomes unity and disappears
- The resulting signal can then be demodulated in the usual way
- Note that the data modulation does not have to be BPSK
- It is common for both the spreading and data modulation to be of the same kind
- Also note that the order of spreading and modulating can be reversed

Successful Despreading:
(1) Know the **code** of spreading
(2) Despread at the right **timing**

$y(t) = s(t) = b(t)c(t)$

$w(t) = \overline{y(t)\, c(t)}$)

Despread!

Take average by

$= \frac{1}{64} \sum_{i=1} y(i)\, c(i)$

$= \overline{[b(t)\, c(t)] \cdot c(t)}$

$= b(t)\, \overline{c^2(t)}$

$= b(t)$

$\Rightarrow$ same!

Must know spreading sequence!!!

Also the time!!!

否则 $\overline{c^2(t)}$

$= \overline{c(t)\, c_1(t)}$

OR

$= \overline{c(t)\, c(t-\tau)}$

$\Rightarrow$ GG!

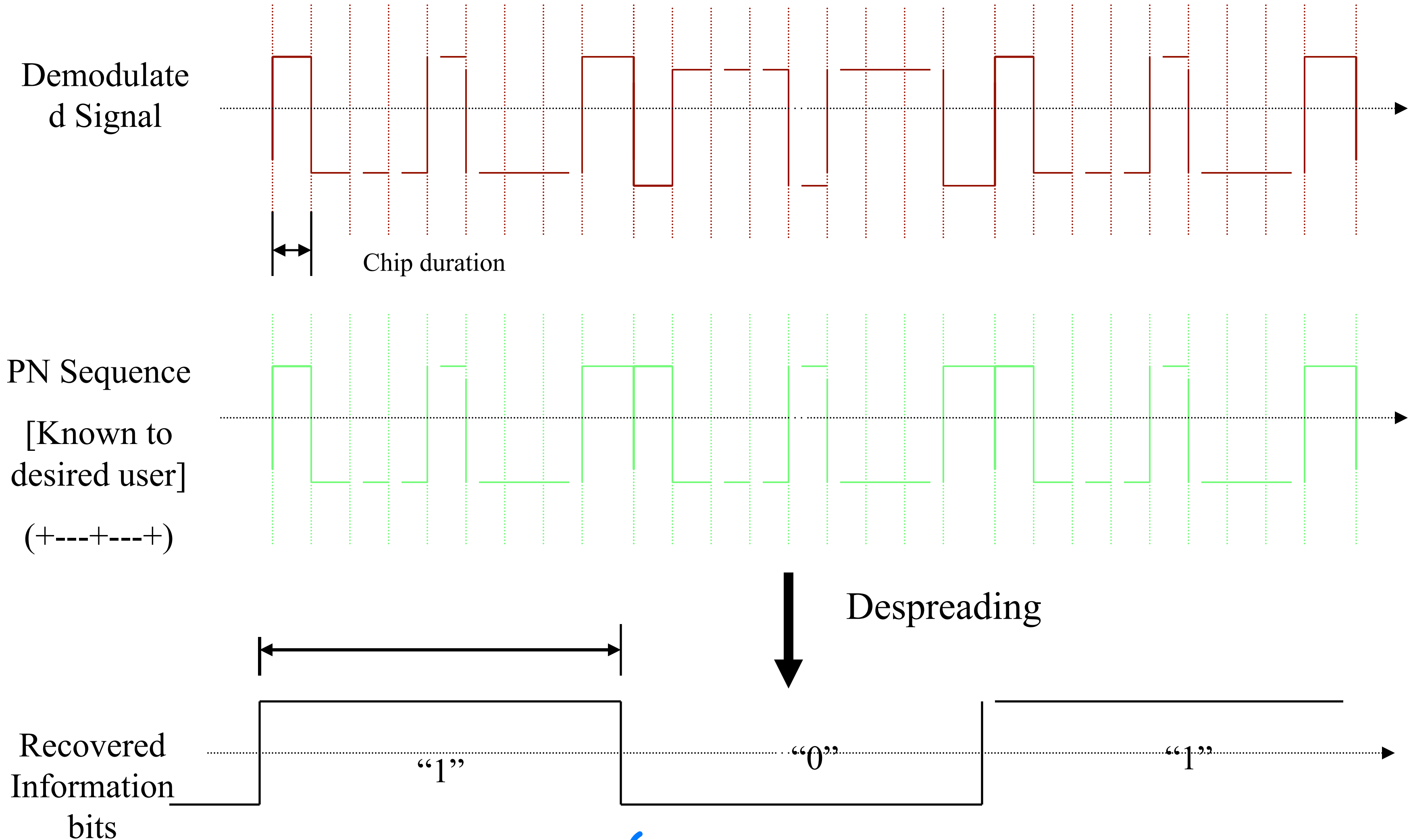
Pay a cost

$w_{tx} = 64 \times w_s$

Advantage: ?

Bit rate not decreased!

DS Despreading – Time Domain



Re-construct to original signal!

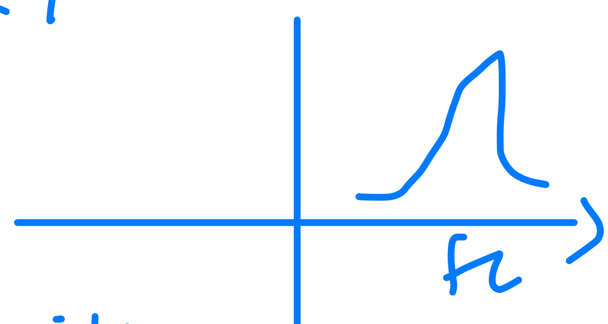
DS Spreading / Despreading

ISI from yourself → multiple user interference CDMA

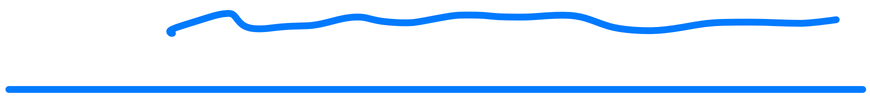
- Function of Spread Spectrum
  - **<u>Interference Suppression</u>**
  - Low Probability of Interception
  - Has no effect on channel noise

War, your 敌人 use spectrum analyzer

⇒



⇒ easily to jam



← white noise

thermal noise

Peak will be reduced by k times!!!

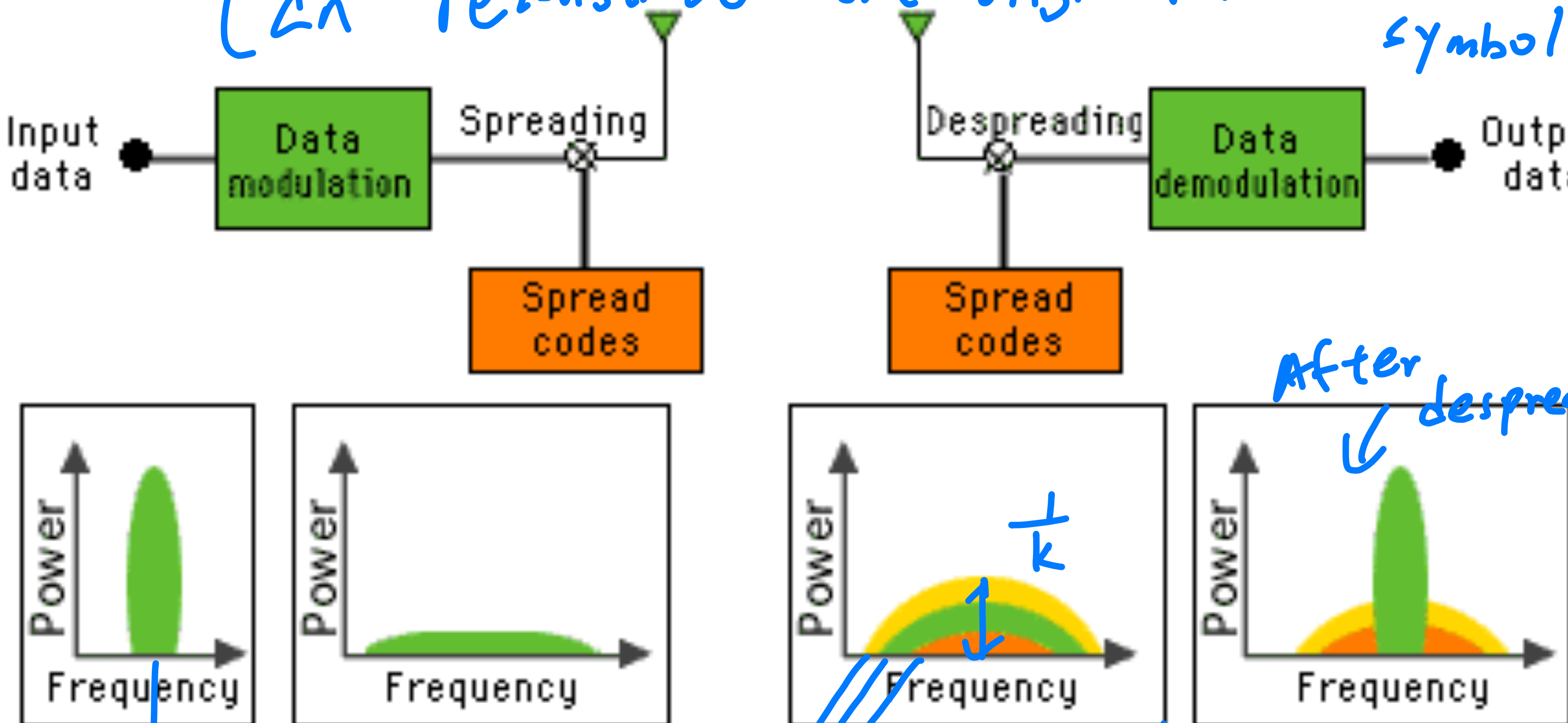
⇒ extra protection!

不知 engage communication!

illustrate in freq. domain!

DS/CDMA Communications

Can reconstruct the original modulation symbol!



After despreading

$\frac{1}{k}$

$\frac{1}{T_s}$

different codes!

$$SINR = \frac{P_{Good}}{P_{Bad}} = \frac{1}{2} < 0\,dB$$

Tradeoff

Interference suppression vs Bandwidth

Frequency selective

ISI $\Rightarrow$ irreducible error floor

Requirement of PN Codes

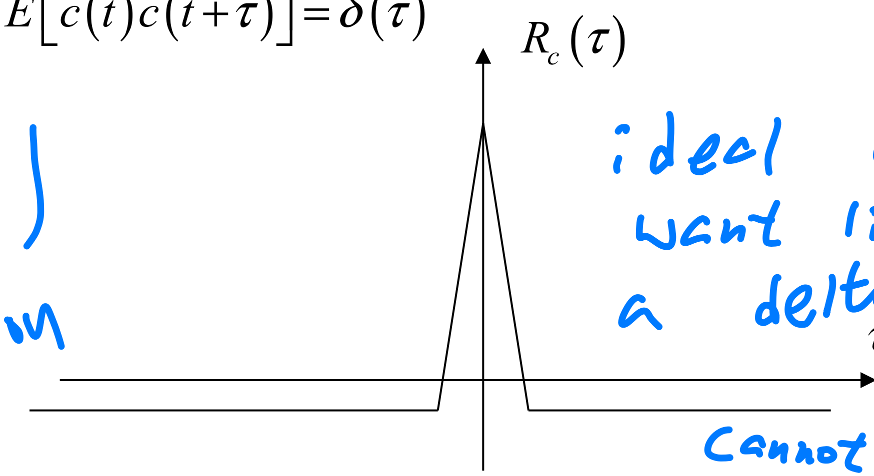
- Correlation
  - A statistical measure to compare the "similarity" between two random sequences.
- Auto-correlation
  - Compare how similar a random sequence is with respect to a time-shifted version of itself.

$$R_c(\tau)=E\left[c(t)c(t+\tau)\right]$$

- Good PN Codes
  - Those with small autocorrelation at non-zero offsets
  - Ideal case $\quad R_c(\tau)=E\left[c(t)c(t+\tau)\right]=\delta(\tau)$
  - Why??

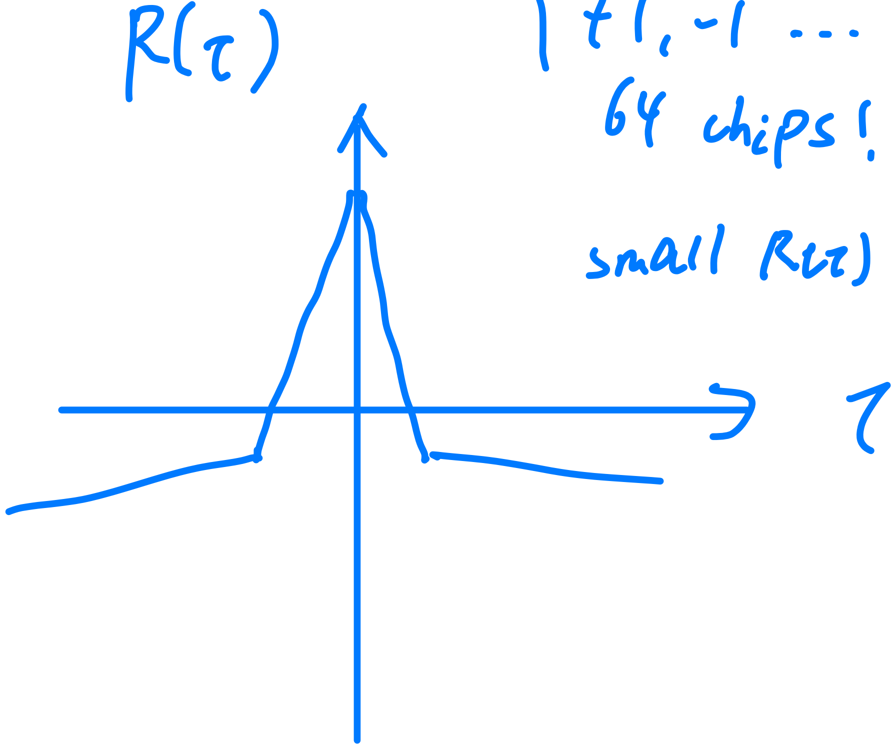
$c(t) \in \{1, -1\}$

Autocorrelation



ideal case
want like
a delta function

cannot predict
the future!



$c(t) \in [+1, -1]$

$[+1, -1, +1, -1 \ldots]$

64 chips!

small $R(\tau) \sim O\left(\frac{1}{\sqrt{64}}\right)$

$= O\left(\frac{1}{8}\right)$

ISI Mitigation using DS-SS in frequency selective fading channels

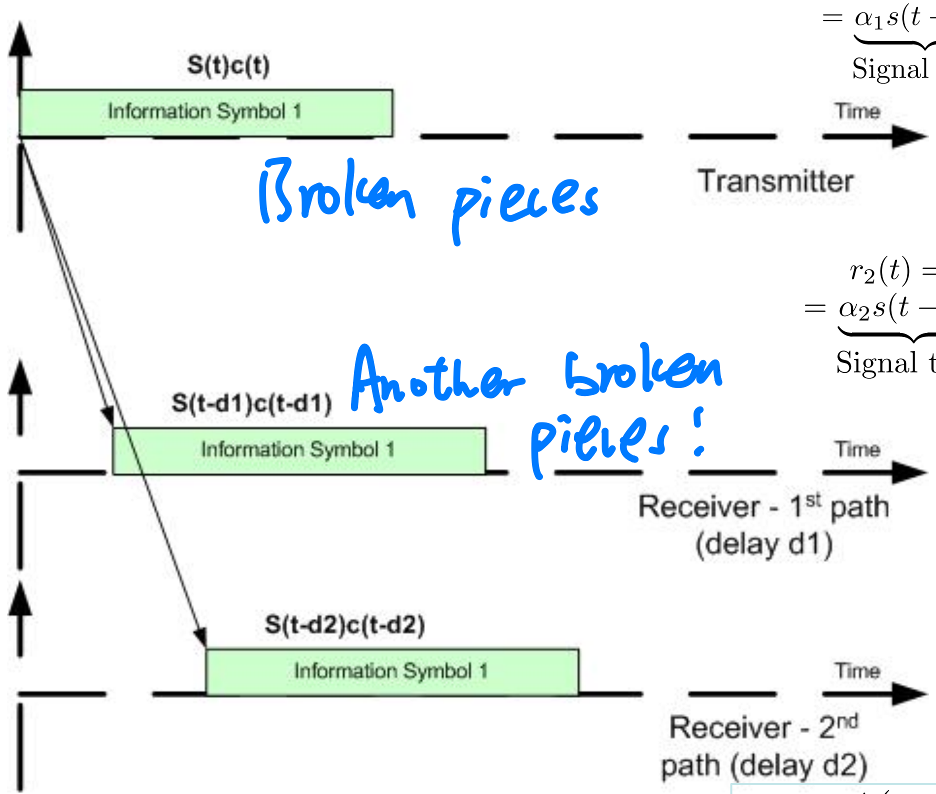
- Due to spread spectrum, the transmission BW is large.
- Usually, DSSS signal will experience frequency selective fading channels (with quite a lot of resolvable echos)
- There will be severe ISI → very high error floor
- How does the DSSS deal with the ISI??

Principle of RAKE Receiver

- The total number of resolvable multipath is $L = \left\lfloor W/B_c \right\rfloor$
- The received signal is composed of the superposition of two multipath components at different delays.
- To recover the original information bit, we despread the received signal with two delay-synchronized PN sequences. Each PN sequence is synchronized with respect to the corresponding path delay.
- Each "finger" will produce one observation on the received information bit.

Example of RAKE Receiver Processing

$$r_1(t) = Despread[y(t), c(t-d_1)]$$
$$= \underbrace{\alpha_1 s(t-d_1)}_{\text{Signal term}} + \underbrace{\alpha_2 s(t-d_2) < c^*(t-d_1)c(t-d_2) >}_{\text{ISI due to path 2}} + \underbrace{< c^*(t-d_1)n(t) >}_{\text{noise}}$$



Broken pieces

Another broken pieces!

$$r_2(t) = Despread[y(t), c(t-d_2)]$$
$$= \underbrace{\alpha_2 s(t-d_2)}_{\text{Signal term}} + \underbrace{\alpha_1 s(t-d_1) < c^*(t-d_2)c(t-d_1) >}_{\text{ISI due to path 1}} + \underbrace{< c^*(t-d_2)n(t) >}_{\text{noise}}$$

-ISI suppression due to small autocorrelations (e.g. for iid sequence)

$$\boxed{\begin{aligned} &< c_1^*(t-d_1)c_1(t-d_2) >= \frac{1}{N}\sum_t c_1^*(t-d_1)c_1(t-d_2) \\ &\leq \sqrt{\frac{2\log\log N}{N}} \text{ almost surely, } N = \frac{T_s}{T_c} \end{aligned}}$$

$$y(t) = \underbrace{\alpha_1 s(t-d_1)c(t-d_1)}_{\text{path 1}} + \underbrace{\alpha_2 s(t-d_2)c(t-d_2)}_{\text{path 2}} + \underbrace{n(t)}_{\text{noise}}$$

ISI!

-Diversity due to frequency selective fading

① know $c(t)$ ✓

② know the timing ✓

② ISI (bad)
still broken pieces

$$w_1 = \alpha_1 \underbrace{\overline{y(t)\, c(t-d_1)}}_{①} + \alpha_2 \frac{s(t-d_2)\cdot}{c(t-d_2)\, c(t-d_1)} + n_{oise}$$

$$\| $$

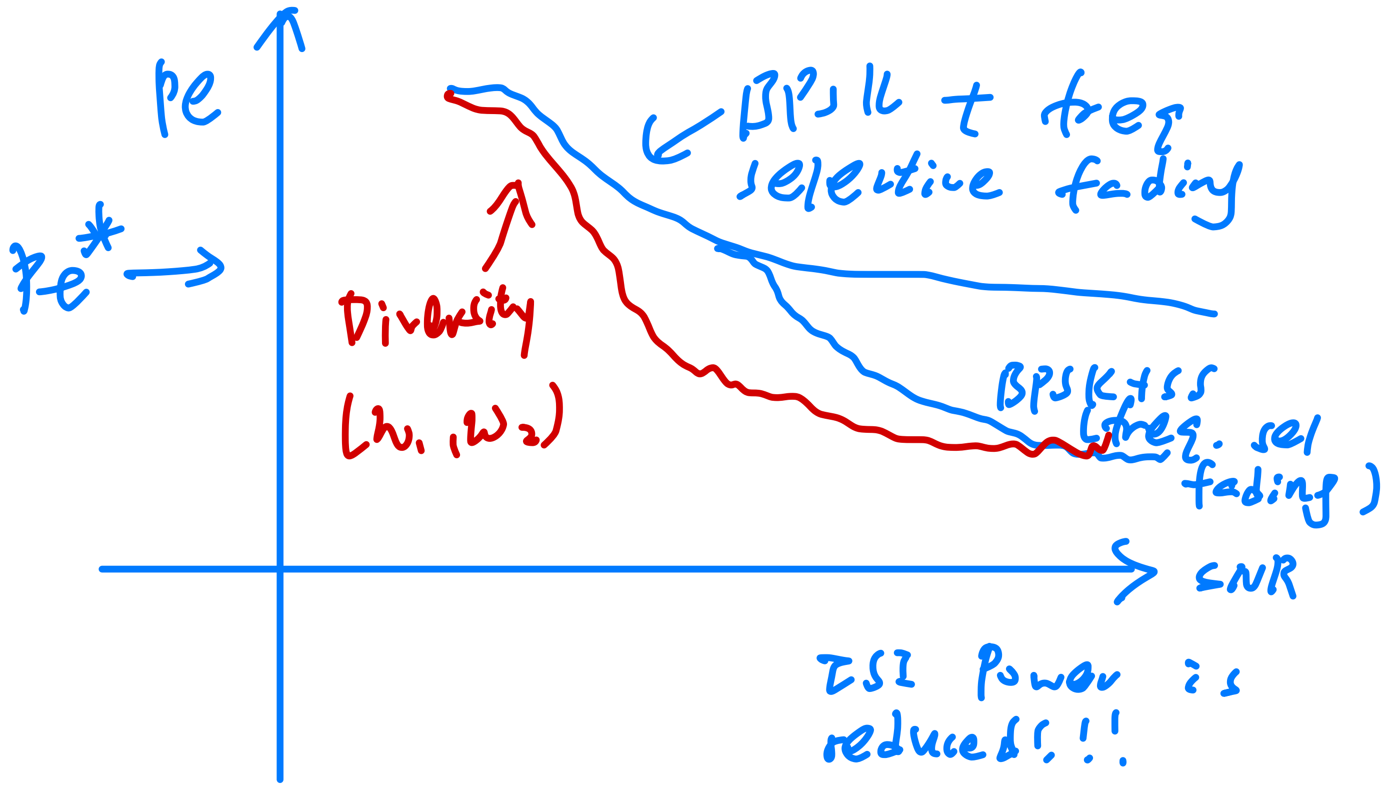
$$R_c(d_1 - d_2)$$

non-zero delay ⇒ small!

$$\sim O\left(\frac{1}{\sqrt{SF}}\right)$$

SF: spreading constant

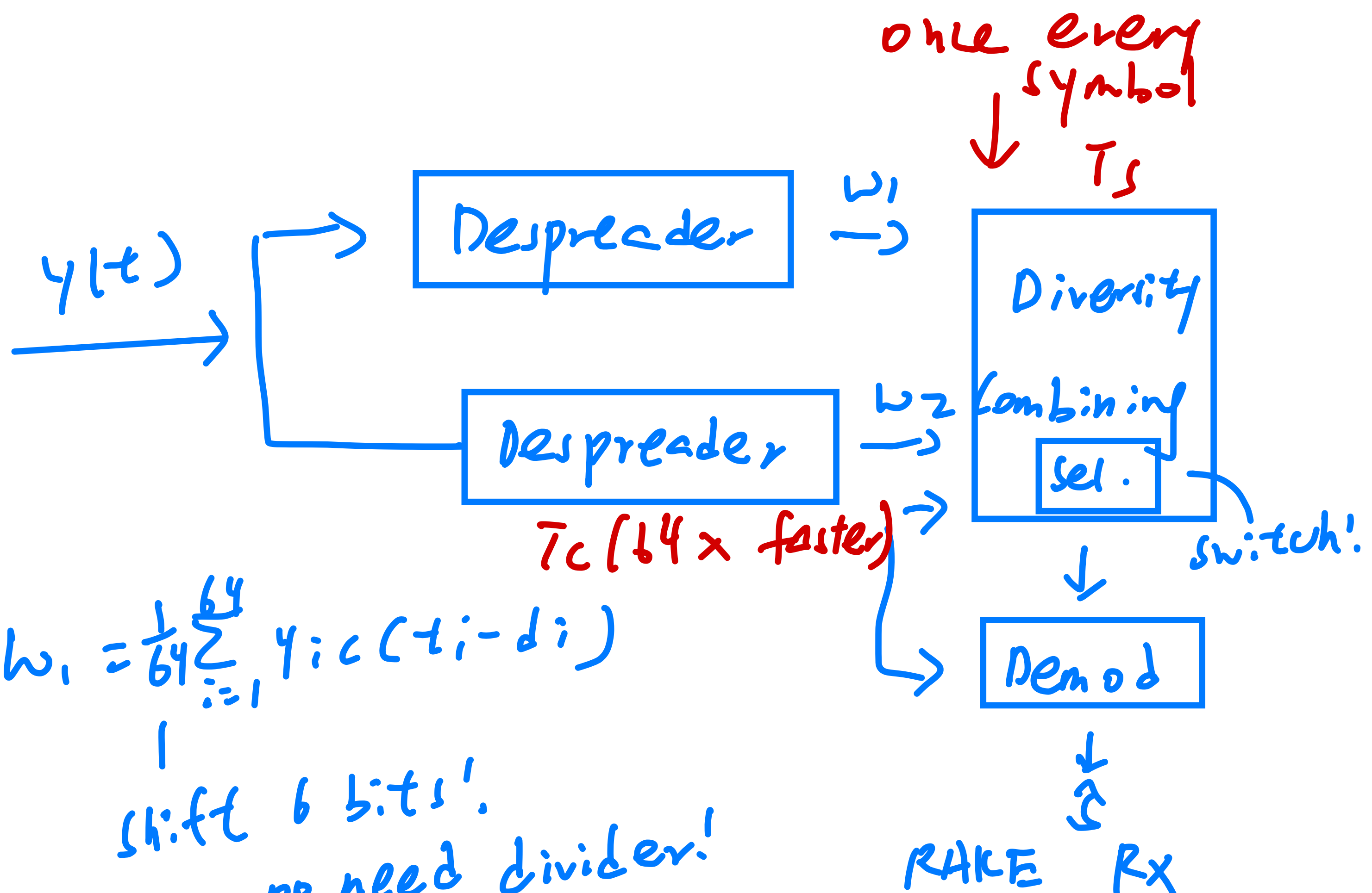
$$(\text{ISI power})_{eff} = \frac{1}{SF}(\text{Actual ISI power})$$



ISI Power is reduced!!!

$w_2 = \ldots$

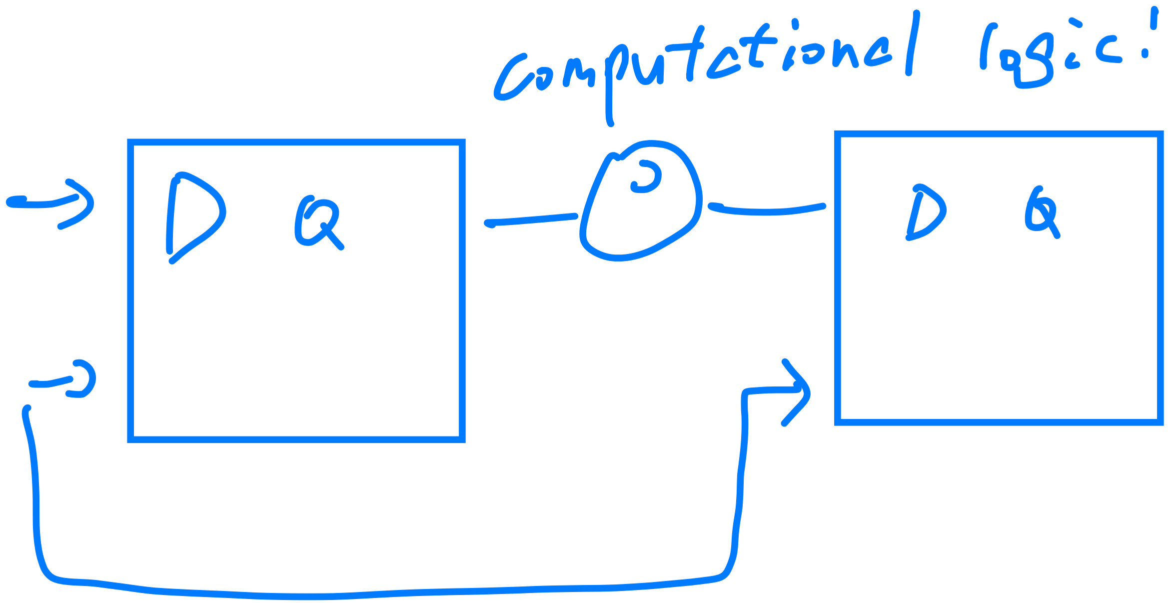
$r_{MRC} = \alpha_1^* w_1 + \alpha_2^* w_2$



$$w_1 = \frac{1}{64} \sum_{i=1}^{64} y_i \, c(t_i - d_i)$$

shift 6 bits!

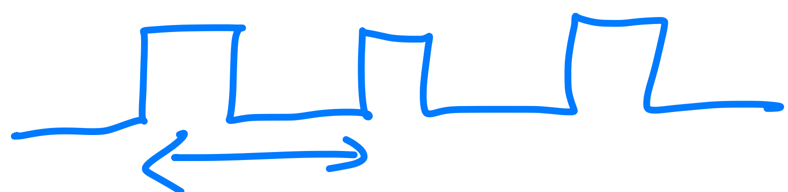
no need divider!



for i=1 to 64
w_i += $y_i\, c(t_i - d)$;

“Explicit” signaling

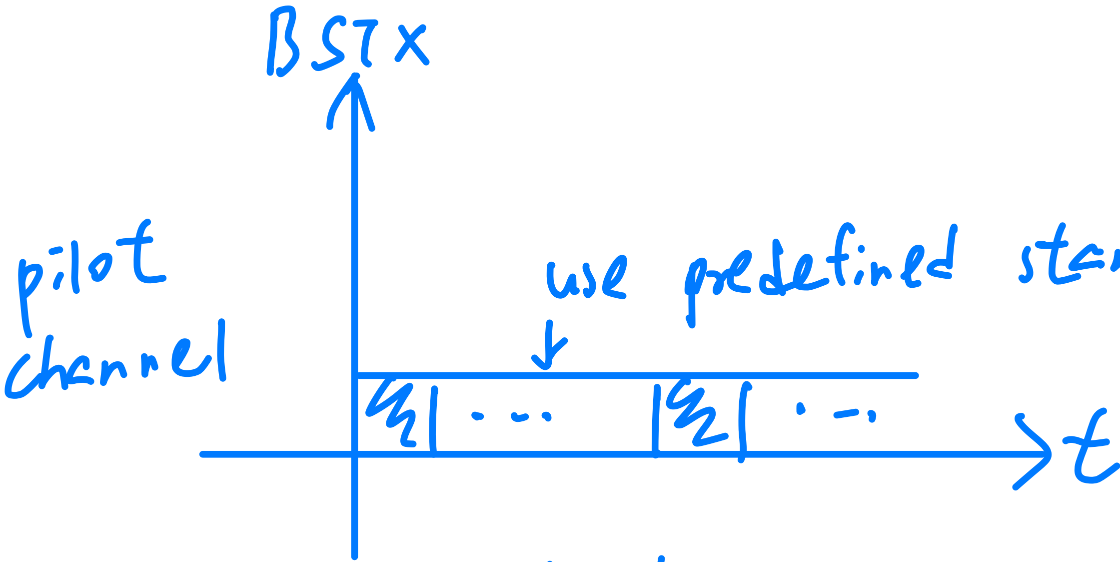
不应 multipler!
因为 $c(t_i - d) = \pm 1$
+1, same
−1, 2's complement!



$T_{ck} > T$

strecher

$\Rightarrow$ keeps finding correct spread timing!

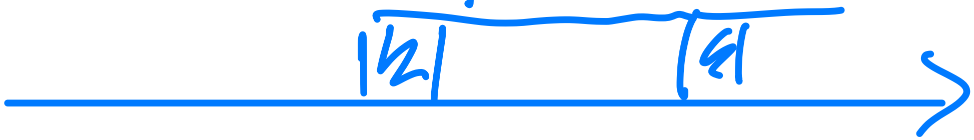


if know first chip time

$\Rightarrow$ know all other!

①

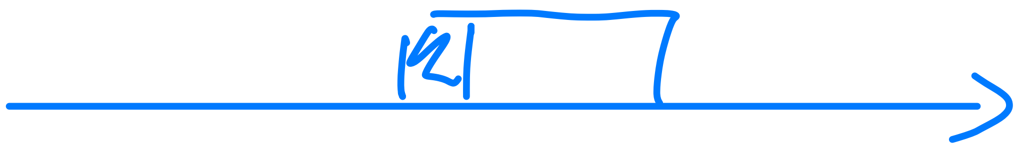
$\alpha_1 p(t-d_1)$



$+$

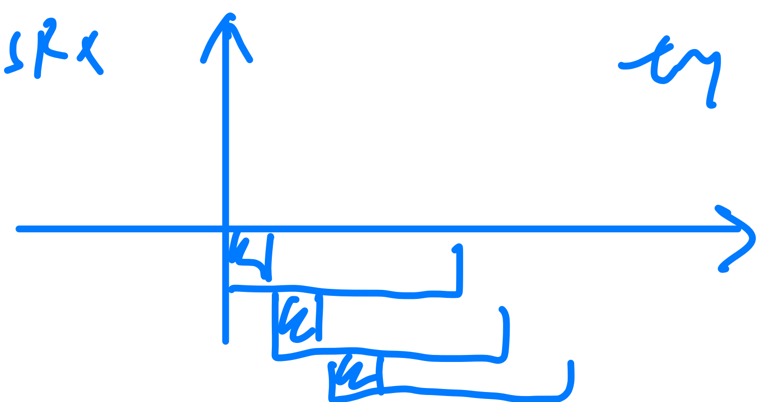
②

$\alpha_2 p(t-d_2)$

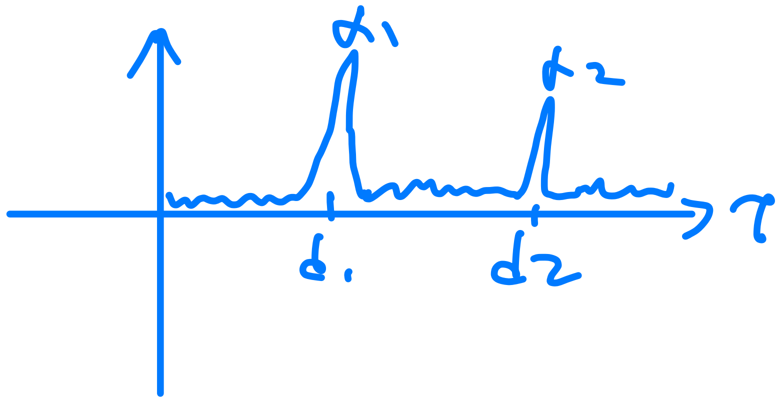


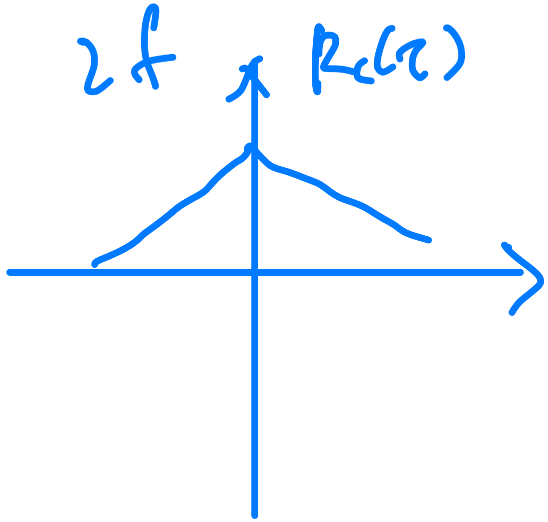
Exhaustive search

MSRx

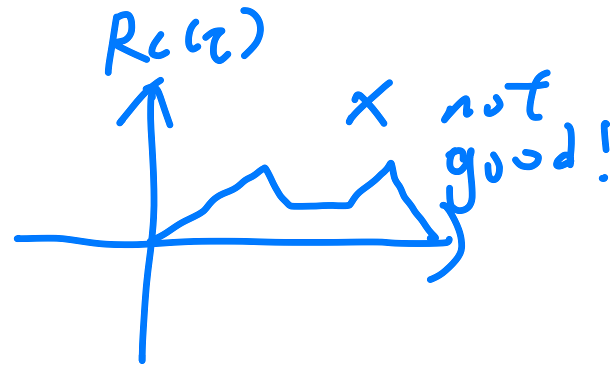


try all dispreading timing ⇒ try all autocorrelation!!!





$\Rightarrow$



RAKER: $O(L)$

Question if only have one despreader, but 2 peaks

Possible to get both interference suppression and diversity advantage? Yes!

only 1 despreader, 2 diversity

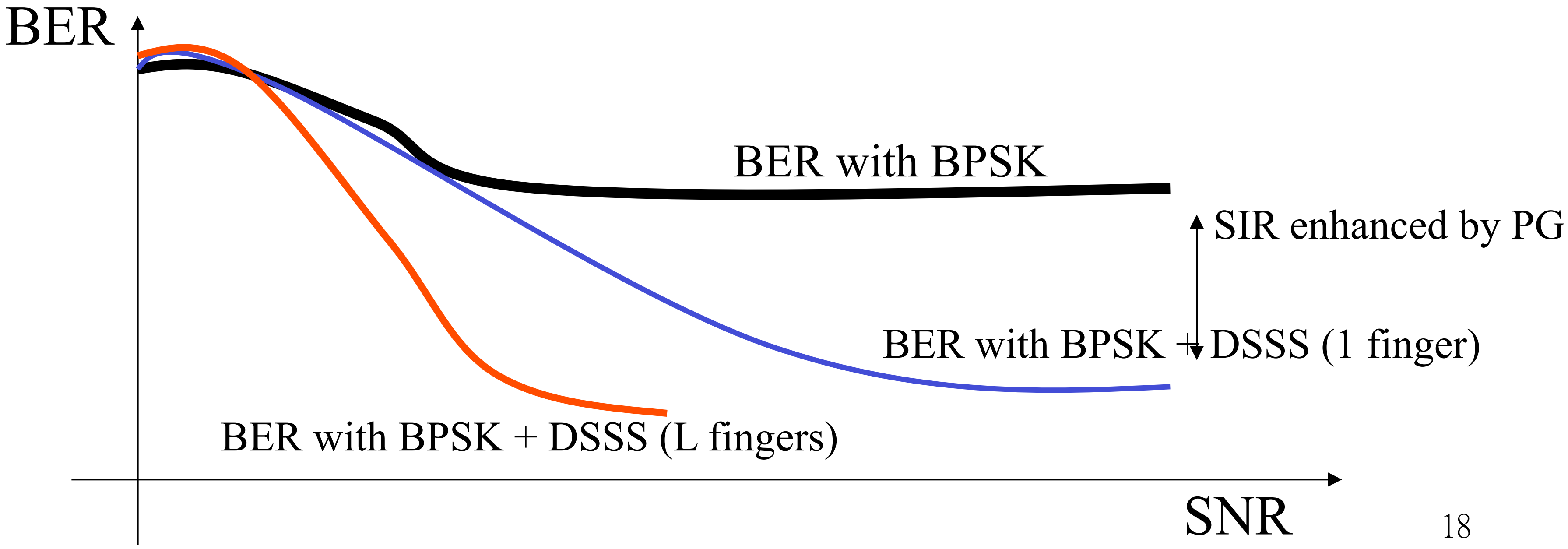
$$r_{sel} = [w_1, w_2]$$

But w_1, w_2 are not free

$$\Rightarrow O(L) \sim O(1)$$

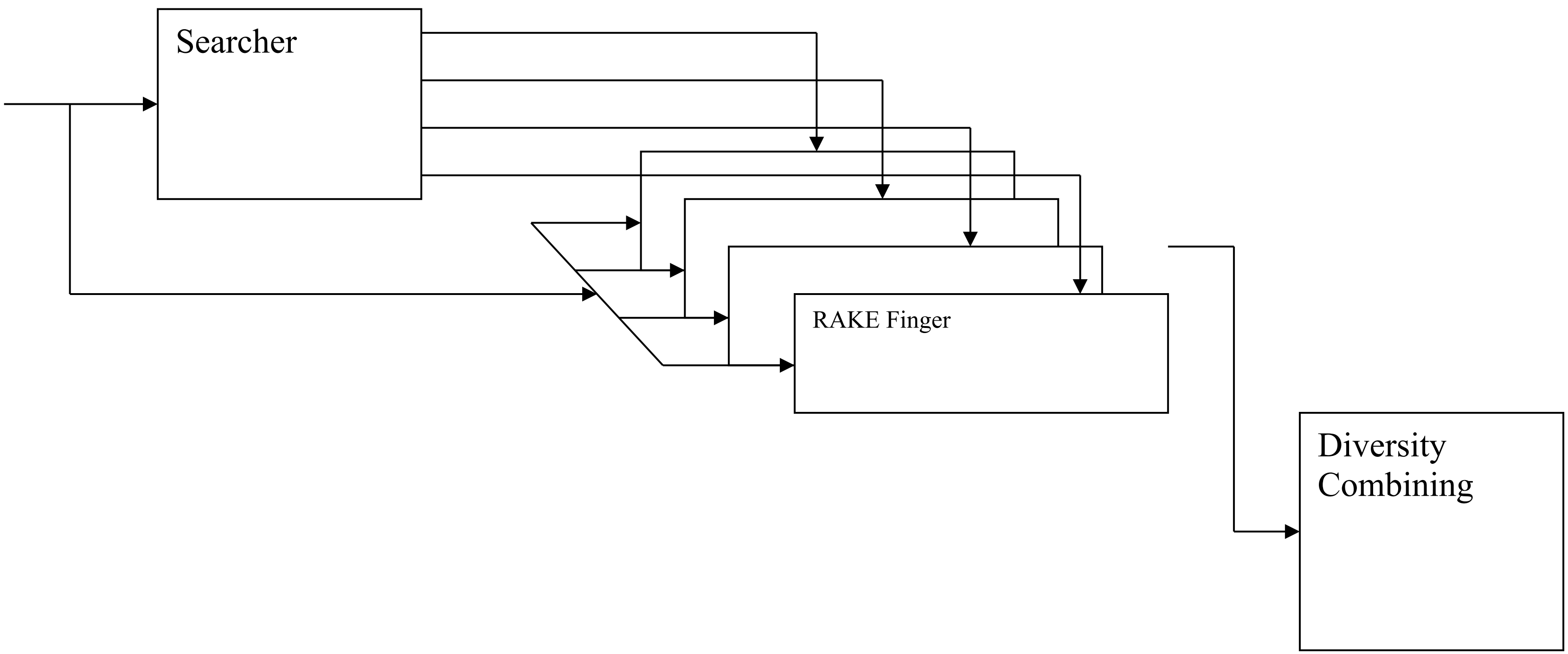
Performance of RAKE Receiver

- Due to SS, it can suppress ISI (by a discount factor "PG").
- Hence, the effective SINR with DS is enhanced by the processing gain. → BER floor is lowered.
- Moreover, the metrics from different fingers represent "independent observations" on the same information → achieve diversity → BER slope is enhanced.



Principle of RAKE Receiver

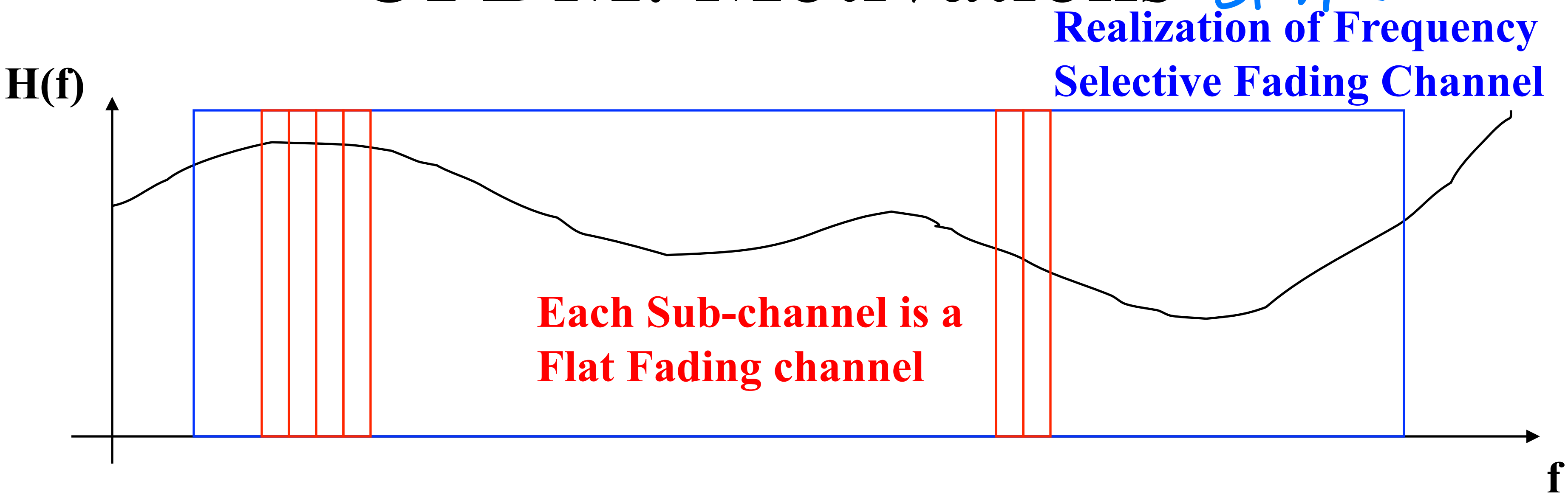
- If we have L fingers to capture observations from the L resolvable multipath, we have a L-order diversity system



Out of syllabus

OFDM: Motivations

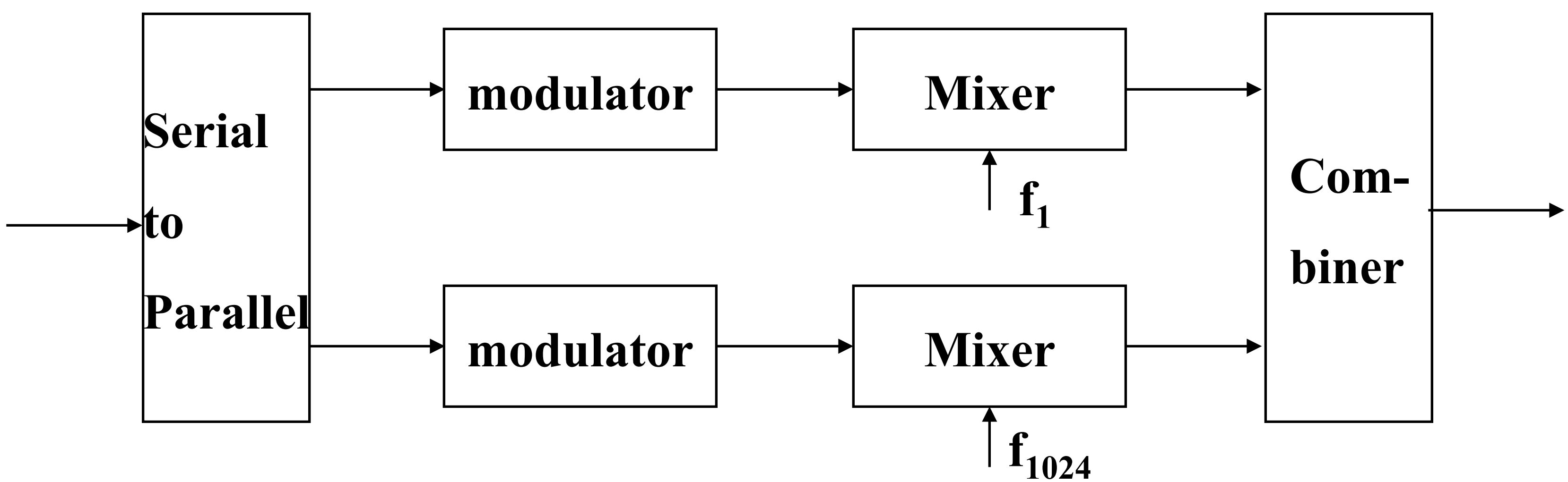
4u! Skip!



- Treat a Wideband FS fading channel as Multiple Narrowband Flat fading channels
- Change in TX so that RX does not suffer from ISI
- Use FEC with codeword span across all sub-channels achieve Frequency Diversity, but with no ISI problem

OFDM : Motivations

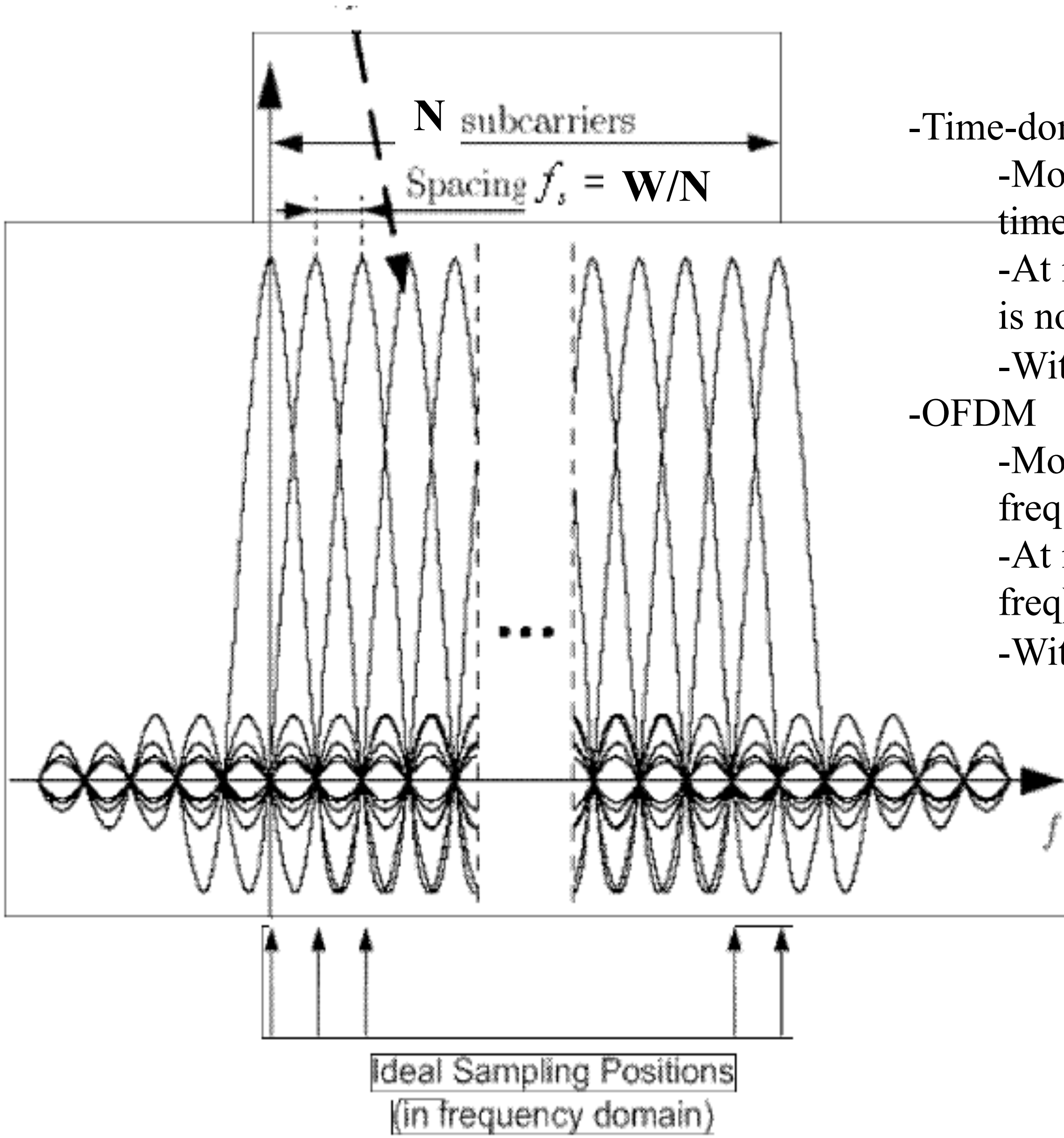
- Motivation:
  - Split a frequency selective fading channel into multiple, say N=1024, narrowband flat fading sub-channels
  - Send the bits over these sub-channels in parallel



OFDM: Motivation

- Problems:
  - Multiple transmitter front ends (mixer, modulator, etc)
  - require guard bands
- Solutions:
  - Do all these in digital domain using a wide baseband signal
  - Use DFT (discrete Fourier transform) to create the baseband equivalent of the transmit signal and then up-covert it to the center frequency using one front end
  - As DFT is an orthogonal transformation, no guard band is needed

OFDM Transmission



- Time-domain modulation
    - Modulation Pulse overlaps in time
    - At ideal sampling position, there is no ISI
    - With timing offset → ISI
- OFDM
    - Modulation pulse overlaps in freq
    - At ideal sampling position (in freq), there is no ICI
    - With frequency offset → ICI

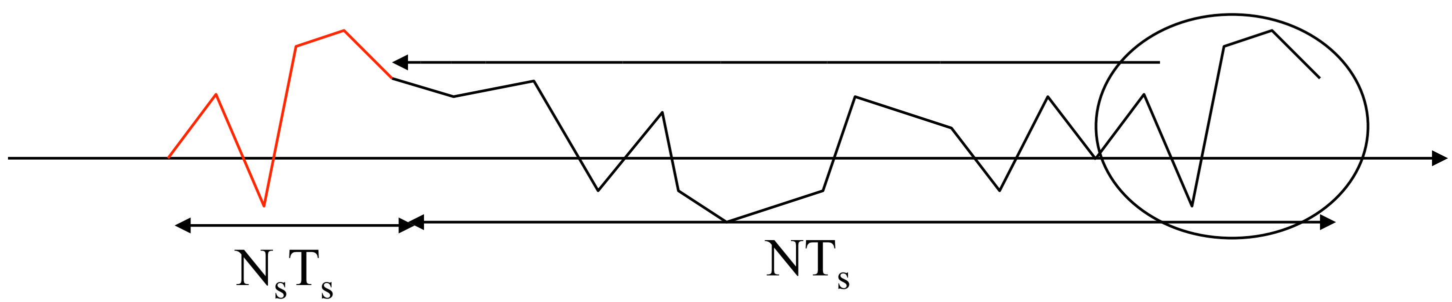
Cyclic Prefix

- If we just take the last N points out of the N $+N_s$ points to do the FFT at the receiver,
- Then $$y_n = x_n \otimes h_n$$

where $\otimes$ denotes circular convolution

and $Y_k = H_k X_k$

where Y_k, X_k, and H_k are the DFT of y_n, x_n, and $h_{n,}$ resp



Advantages of OFDM

- With cyclic prefix, we can eliminate ISI completely
- Provide frequency diversity
  - Forward error correcting code such as convolutional code with interleaver is needed as some sub-carriers will be in deep fade
- Potential
  - If the transmitter knows the channel conditions
    - can select only the good sub-carriers to transmit or transmit different numbers of bits based on the sub-carriers' gains → Power water-filling in frequency domain.
- If the transmitter knows the channel, OFDM with bit allocation is better than the best equalizer (e.g. MLSE)

Disadvantages of OFDM

- Overheads
  - Cyclic Prefix: can be reduced by increasing N
  - Power to transmit cyclic prefix: can be lower by increasing N
- Implementation issues
  - sensitivity to frequency offsets
    - especially when N is large and sub-carrier spacing is small
  - require highly linear power amp
    - high peak-to-average-power (PAP) ratio, especially when N is large
  - Poor Adjacent band rejection ~ 20/30dB only.
    - Q: In wireless LAN (802.11g), the AP can receive and decode packets on an adjacent channel. Why?
- Typical value for N is 2^7 to 2^{11}