

T03

Representation of DT signals

DT Impulse response

Convolution sum for DT LTI system

Representation of CT signals

CT Impulse response

Convolution integral for CT LTI system

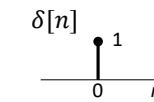
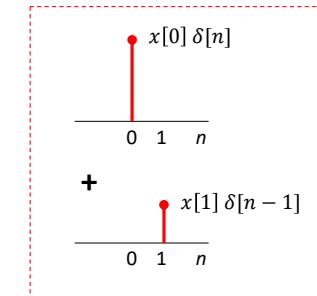
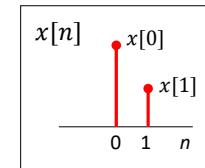
Properties of LTI system

Characterizing LTI system by Impulse Response

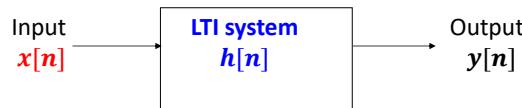
1

Representation of DT signals

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k]$$

Difference between $x[n]$ and $x[k]$?

2

Impulse Response (DT)e.g. $x[n]$

$$h[n] = \delta[n]$$

$$y[n] = ? \quad x[n]$$

e.g. $x[n] = \delta[n]$

$$h[n]$$

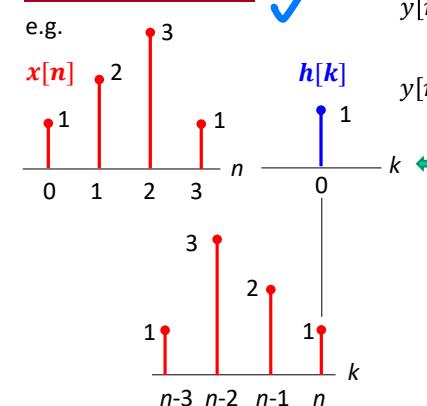
$$y[n] = ? \quad \text{impulse response}$$

Question : How to relate input, output and LTI system together ?

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Convolution Sum (DT)

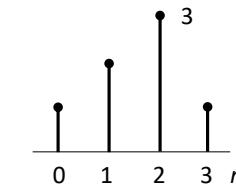
e.g.



$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

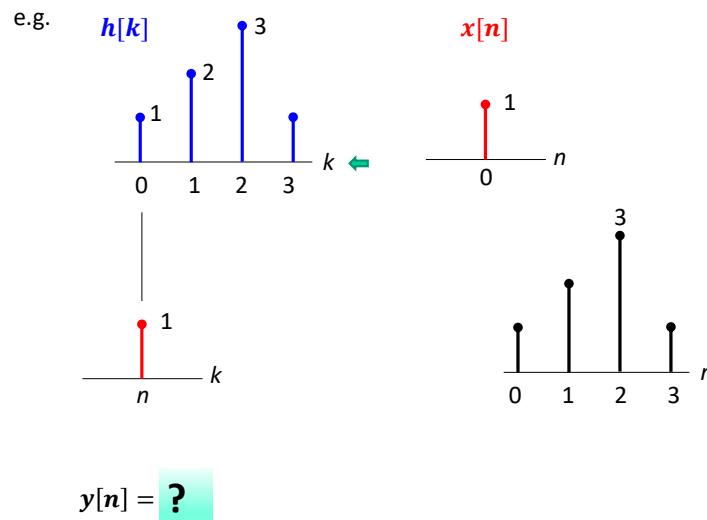
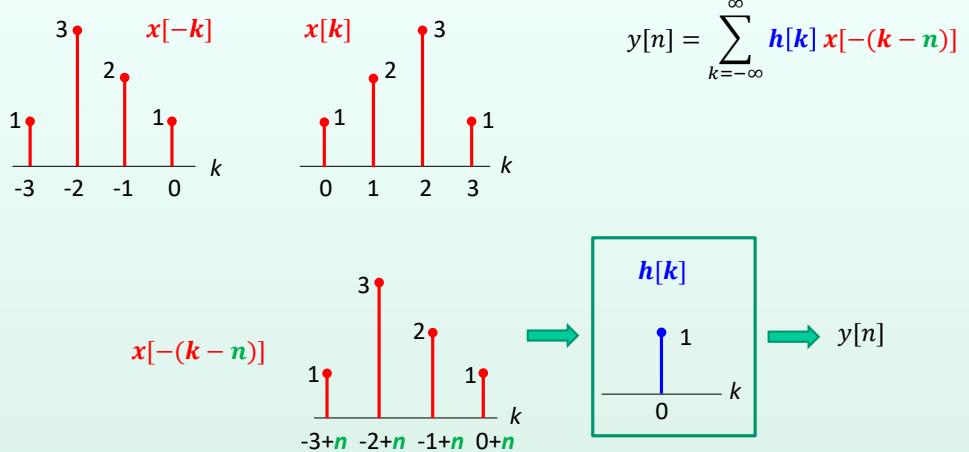
$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k] x[-(k-n)]$$

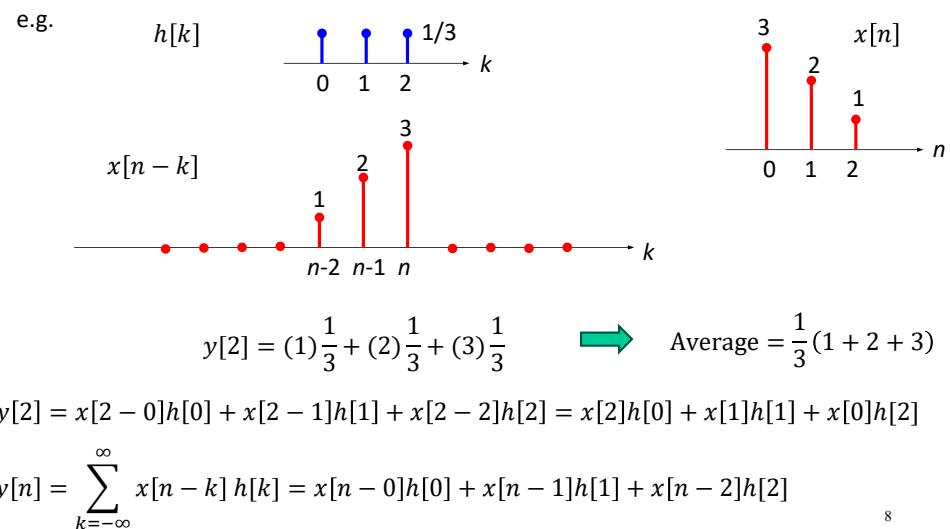
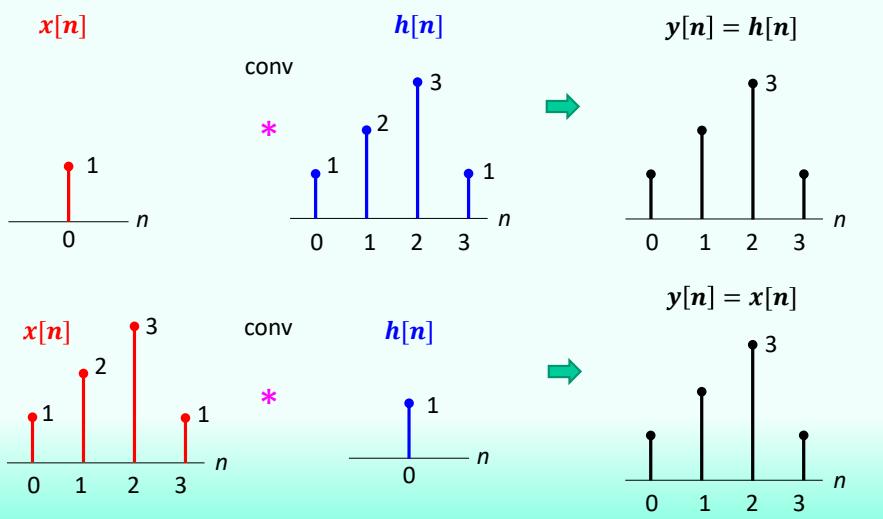
Question : What is the output $y[n]$?

How to physically implement this system ?

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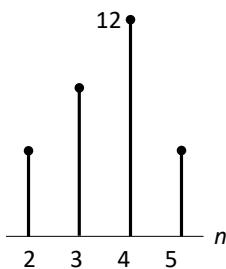
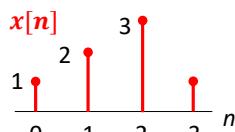
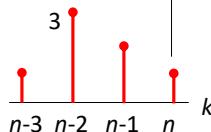
Time reversal!



e.g. Sketch the output.



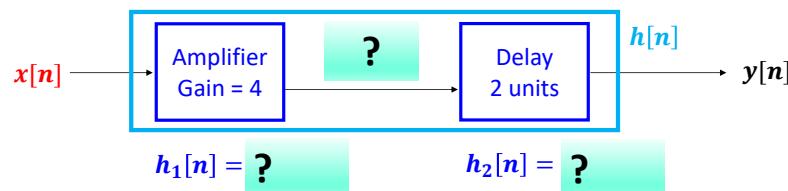
$$y[n] = ? \quad 4q[n-2]$$



Question : What is the expression of $h[n]$?

What is the input-output relationship ?

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Question :

- What is the input-output relationship ?

$$y[n] = ?$$

- What is the impulse response for each block ?

- What is the overall impulse response ?

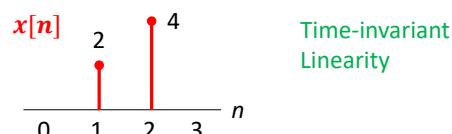
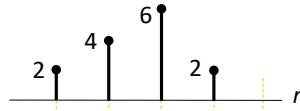
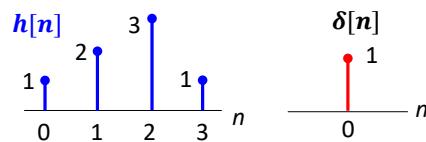
$$h[n] = ?$$

- What is the impulse response for each of the following systems ?

Do-nothing , Echo , Integrator , Windowed smoother ... etc

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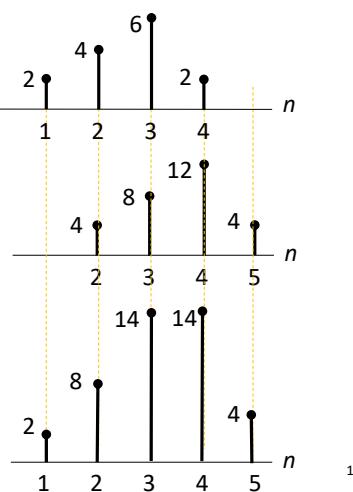
e.g. Represent $y[n]$ in terms of $h[n]$. Sketch the output.



Time-invariant Linearity

$$x[n] = 2\delta[n-1] + 4\delta[n-2]$$

$$y[n] = ?$$



Representation of CT signals
CT Impulse response
Convolution integral for CT LTI system

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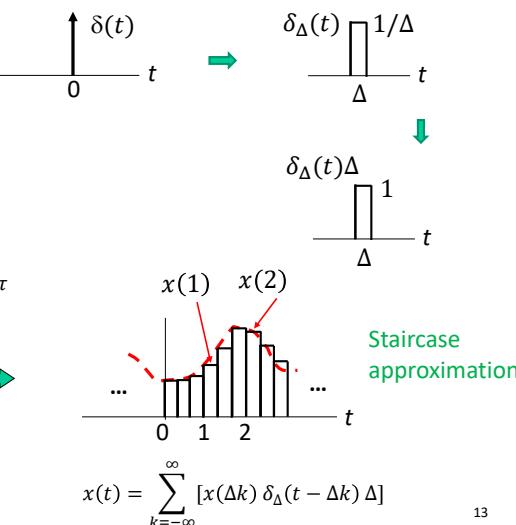
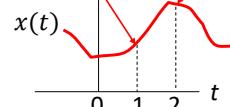
Representation of CT signals

$$x(t) = \int_{-\infty}^{\infty} x(\tau) [\delta(t) - \delta(\tau)] d\tau$$

Difference between $x(t)$ and $x(\tau)$?

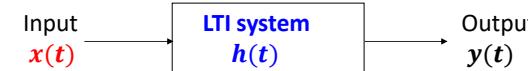
$$x(1) = \int_{-\infty}^{\infty} x(1) \delta(1 - \tau) d\tau$$

$$x(2) = \int_{-\infty}^{\infty} x(2) \delta(2 - \tau) d\tau$$



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Impulse Response (CT)



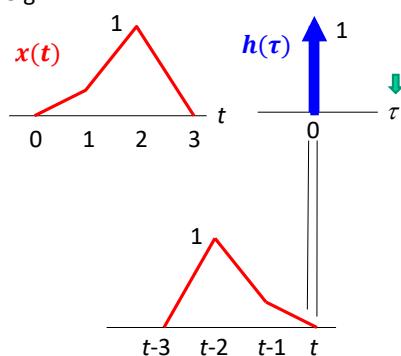
e.g. $x(t)$ $h(t) = \delta(t)$ $y(t) = ?$

e.g. $x(t) = \delta(t)$ $h(t)$ $y(t) = ?$

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Convolution Integral (CT)

e.g.



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(-(t - \tau)) d\tau$$

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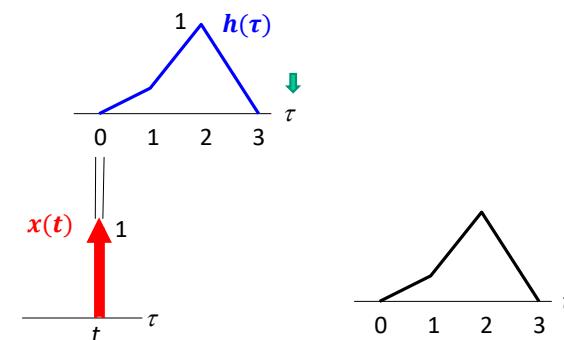
Question : What is the output ?

$$y(t) = ?$$

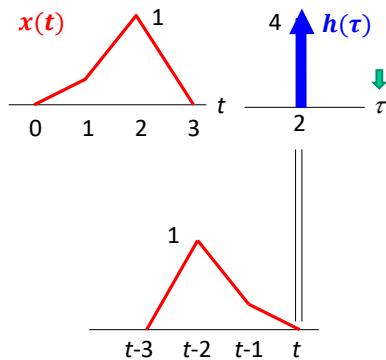
Question : What is the output ?

$$y(t) = ?$$

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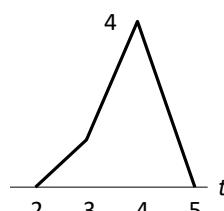


e.g.



$$h(t) = ?$$

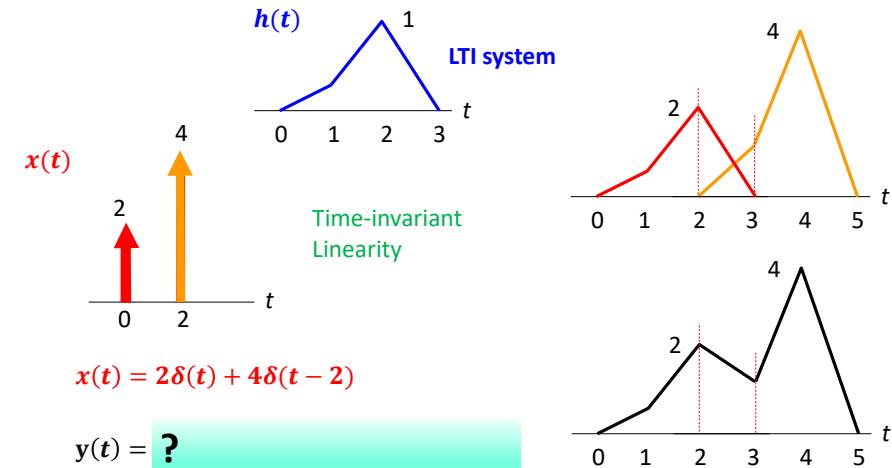
$$y(t) = ?$$



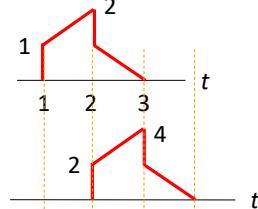
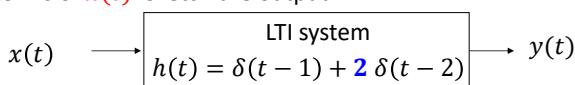
Question : What is the expression of $h(t)$?
What is the output ?

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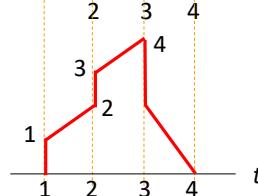
e.g. Represent $y(t)$ in terms of $h(t)$. Sketch the output.



e.g. Represent $y(t)$ in terms of $x(t)$. Sketch the output.



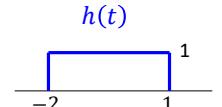
$$y(t) = ?$$



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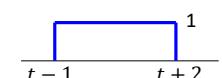
e.g.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(t) d\tau = \int_{-2}^{1} x(t-\tau) d\tau$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{t-1}^{t+2} x(\tau) d\tau$$

$$h(t-\tau) = h(-(t-\tau))$$



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Properties of LTI system

Characterizing LTI system by Impulse Response

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Properties of LTI System

Commutative $x(t) * h(t) = h(t) * x(t)$

$$x[n] * h[n] = h[n] * x[n]$$

Distributive $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

Associative $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

$$x(t) * \delta(t) = x(t)$$

$$x[n] * \delta[n] = x[n]$$

$$x(t) * \delta(t - t_o) = x(t - t_o)$$

$$x[n] * \delta[n - n_o] = x[n - n_o]$$

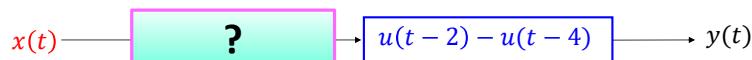
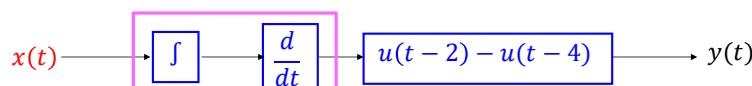
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e.g. Sketch the output $y(t)$.

$$x(t) = \delta(t - 1) + \delta(t - 6) \rightarrow e(t) \xrightarrow{\int} u(t - 2) - u(t - 4) \xrightarrow{\frac{d}{dt}} y(t)$$

$$e(t) = \int_{-\infty}^t x(\tau) d\tau = u(t - 1) + u(t - 6)$$

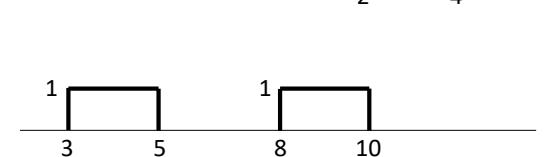
$$y(t) = \frac{d}{dt} \{ e(t) * [u(t - 2) - u(t - 4)] \}$$



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$$x(t) = \delta(t - 1) + \delta(t - 6) \rightarrow u(t - 2) - u(t - 4) \rightarrow y(t)$$

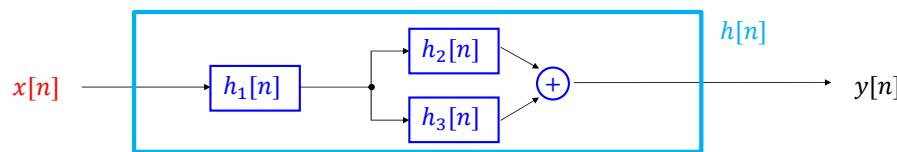
$$y(t) = u(t - 3) - u(t - 5) + u(t - 8) - u(t - 10)$$



$$y(t) = u(t - 3) - u(t - 5) + u(t - 8) - u(t - 10)$$

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e.g. Consider the following block diagram



a) Write down the overall impulse response $h[n]$.

$$h[n] = h_1[n] * (h_2[n] + h_3[n]) = \boxed{h_1[n] * h_2[n]} + \boxed{h_1[n] * h_3[n]}$$

b) If $y[n] = h_1[n] + \frac{1}{2}h_1[n - 2]$ when $x[n] = \delta[n]$, suggest $h_2[n]$ and $h_3[n]$.

$$y[n] = x[n] * h[n] = h[n] = \boxed{h_1[n]} + \boxed{\frac{1}{2}h_1[n - 2]}$$

$$h_2[n] = \boxed{?} \quad h_3[n] = \boxed{?}$$

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Characterizing of LTI System by Impulse Response

System Properties By looking at Input/Output Relation

$$\mathbf{x(t)} \quad \mathbf{y(t)} \quad \mathbf{x[n]} \quad \mathbf{y[n]}$$

Memoryless , Causality , Stability , Time-invariant , Linearity, Invertibility

System Properties By looking at $\mathbf{h(t)}$ and $\mathbf{h[n]}$

Memoryless $\mathbf{h(t)} = 0$ for $t \neq 0$; $\mathbf{h[n]} = 0$ for $n \neq 0$

Causality $\mathbf{h(t)} = 0$ for $t < 0$; $\mathbf{h[n]} = 0$ for $n < 0$

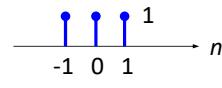
Stability $\int_{-\infty}^{\infty} |\mathbf{h}(\tau)| d\tau < \infty$; $\sum_{k=-\infty}^{\infty} |\mathbf{h}[k]| < \infty$

Question : How about time-invariant and linearity ?

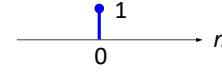
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Memoryless ? Causal ? Stable ?

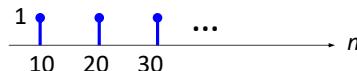
e.g. $h[n] = \delta[n - 1] + \delta[n] + \delta[n + 1]$



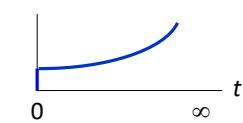
e.g. $h[n] = \delta[n]$



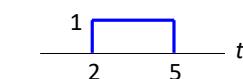
e.g. $h[n] = \sum_{k=1}^{\infty} \delta[n - 10k]$



e.g. $h(t) = e^t u(t)$



e.g. $h(t) = u(t - 2) - u(t - 5)$



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