Mon. Not. R. Astron. Soc. **000**, 1–11 (2011) Printed 13 January 2011 (MN LaTeX style file v2.2)

Galactic Dynamics

C. P. L. Berry[1⋆] and J. R. Gair[1]
[1] *Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge, CB3 0HA*

13 January 2011

ABSTRACT
I thought I would try out the MNRAS LaTeXstyle.

Key words: cake – biscuits.

1 EVENT RATES

1.1 The density function

We wish to calculate the probability that there is an encounter between a compact object on an orbital trajectory described by eccentricity e and periapse radius r_p and the supermassive black hole at the galactic centre. We begin by following the work of Bahcall & Wolf (1976, 1977) and assuming that the density function within the galactic core is just a function of the orbital energy; the number of stars is given by

$$N = \int \mathrm{d}^3 r \int \mathrm{d}^3 v f(\mathcal{E}). \tag{1}$$

We will define the energy per unit mass of the orbit as

$$\mathcal{E} = \frac{v^2}{2} - \frac{GM_\bullet}{r} \tag{2}$$

where $M_\bullet$ is the mass of the supermassive black hole. Close to the centre of the galactic core dynamics will be dominated by the influence of the black hole as it is significantly more massive than the surrounding stars. We will define the radius of influence for the black hole as

$$r_c = \frac{GM_\bullet}{\sigma_\star^2} \tag{3}$$

where σ^2 is the line-of-sight velocity dispersion. We will assume that the mass of stars enclosed within the radius is greater than the black hole mass, which is much greater than the mass of a typical star $M_\star$ (Bahcall & Wolf 1976). We will define a reference number density from the enclosed mass as

$$m_\star(r_c) = \frac{4\pi r_c^3}{3} n_\star M_\star. \tag{4}$$

Within the the core, the density function can be calculated using the approximation of Fokker-Planck formalism. The population of bound ($\mathcal{E} < 0$) stars is evolved numerically until a steady state is reached: the unbound ($\mathcal{E} > 0$) stars form a reservoir with an assumed Maxwellian distribution. Denoting a species of star by its mass M:

$$f_M(\mathcal{E}) = \frac{C_M n_\star}{(2\pi\sigma_M^2)^{3/2}} \exp\left(-\frac{\mathcal{E}}{\sigma_M^2}\right) \tag{5}$$

where C_M is a normalisation constant.[1] If different stellar species are in equipartition (as was assumed by Bahcall & Wolf (1976, 1977)) then we expect

$$M\sigma_M^2 = M_\star \sigma_\star^2. \tag{6}$$

However, if the unbound stellar population has reached equilibrium by violent relaxation (Lynden-Bell, D. 1967), then all mass groups are expected to have similar velocity dispersions: this has been used by Alexander & Hopman (2009); O'Leary et al. (2009) and will be assumed here. The steady-state density function is largely insensitive to this choice of boundary condition (Bahcall & Wolf 1977; Alexander & Hopman 2009).

For bound orbits ($\mathcal{E} < 0$) the density function can be approximated as a power law

$$f_M(\mathcal{E}) = \frac{k_M n_\star}{(2\pi\sigma_M^2)^{3/2}} \left(-\frac{\mathcal{E}}{\sigma_M^2}\right)^{p_M}. \tag{7}$$

The exponent p_M varies depending upon the mass of the object, yielding mass segregation. For a system with a single mass component, Bahcall & Wolf (1976) find that $p = 1/4$. The normalisation constant k_M reflects the relative abundances of the different species.[2]

1.2 Model Parameters

We will be using the Fokker-Planck model of Hopman & Alexander (2006a,b); Alexander & Hopman (2009). This includes four stellar species: main sequence stars (MS), white

[1] C_M determines the population ratio of species M far from the black hole Alexander & Hopman (2009).
[2] For a single mass population ($p = 1/4$) $k = 2C$ gives a fit correct to within a factor a two (Bahcall & Wolf 1976; Keshet et al. 2009), we will assume this holds for the dominant species of stars as although it will vary slightly with p this change is small compared to errors introduced by fitting a simple power law (Hopman & Alexander 2006a; Alexander & Hopman 2009).

⋆ E-mail: cplb2@cam.ac.uk

Table 1. Stellar model parameters for the galactic centre using the results of Alexander & Hopman (2009) We use the main sequence star as our reference. The number fractions for unbound stars are estimates corresponding to a model of continuous star formation (Alexander 2005).

Star	$M/M_\odot$	$C_M/C_\star$	p_M	$k_M/k_\star$ [a]
MS	1.0	1	−0.1	1
WD	0.6	0.1	−0.1	0.09
NS	1.4	0.01	0	0.01
BH	10	0.001	0.5	0.008

[a] Toonen et al. (2009)

dwarfs (WD), neutron stars (NS), and black holes (BH). Their properties are summarised in table 1.

We assume a black hole mass of $M_\bullet = 4.31 \times 10^6 M_\odot$ (Gillessen et al. 2009) and a velocity dispersion of $\sigma = 103$ km s^{-1} (Tremaine et al. 2002). This gives a cusp radius of $r_c = 1.75$ pc. Using the results of Ghez et al. (2008) we would expect the total mass of stars in the core to be $m_\star(r_c) = 6.4 \times 10^6 M_\odot$ (which is within 10% of the value obtianed similarly from Genzel et al. (2003)). This gives a stellar density of $n_\star = 2.8 \times 10^5$ pc^{-3}.

1.3 Eccentricity & Periapsis

We wish to make a change of variables to characterise orbits by their eccentricity e and periapse radius r_p. The latter, unlike the semimajor axis, is always well defined regardless of eccentricity. For Keplerian orbits, the energy $\mathcal{E}$ and angular momentum J per unit mass are entirely characterised by these parameters

$$\mathcal{E} = -\frac{GM_\bullet(1-e)}{2r_p}, \tag{8}$$

$$J^2 = GM_\bullet(1+e)r_p. \tag{9}$$

We start by decomposing the velocity into three orthogonal components: radial v_r, azimuthal v_ϕ and polar v_θ. We will assume that the galactic core is spherically symmetric (Genzel et al. 2003; Schödel et al. 2007), therefore we are only interested in the combination

$$v_\perp^2 = v_\phi^2 + v_\theta^2 \tag{10}$$

$$= v^2 - v_r^2. \tag{11}$$

Under this change of variables

$$\mathrm{d}^2 v = \mathrm{d}v_r \mathrm{d}v_\phi \mathrm{d}v_\theta \to 2\pi v_\perp \mathrm{d}v_r \mathrm{d}v_\perp. \tag{12}$$

The specific energy and angular momentum are given by

$$\mathcal{E} = \frac{v_r^2 + v_\perp^2}{2} - \frac{GM_\bullet}{r}, \tag{13}$$

$$J^2 = r^2 v_\perp^2. \tag{14}$$

If we combine these with our earlier expressions in terms of e and r_p we find

$$v_\perp^2 = \frac{GM_\bullet(1+e)r_p}{r^2}, \tag{15}$$

$$v_r^2 = GM\left[\frac{2}{r} - \frac{(1-e)}{r_p} - \frac{(1+e)r_p}{r^2}\right]. \tag{16}$$

From the later we can verify that the turning points of an orbit occur at

$$r = r_p, \quad \frac{1+e}{1-e}r_p; \tag{17}$$

the periapse is the only turning point for orbits with $e > 1$. Since we now have expressions for $\{v_r, v_\perp\}$ in terms of $\{e, r_p\}$ we can calculate the Jacobian

$$\left|\frac{\partial(v_r, v_\perp)}{\partial(e, r_p)}\right| = \frac{\partial v_r}{\partial e}\frac{\partial v_\perp}{\partial r_p} - \frac{\partial v_r}{\partial r_p}\frac{\partial v_\perp}{\partial e} \tag{18}$$

$$= \frac{1}{2v_r v_\perp}\frac{e}{r_p}\left(\frac{GM}{r}\right)^2. \tag{19}$$

Using this, we may rewrite our velocity element as

$$\mathrm{d}^3 v = \mathrm{d}v_r \mathrm{d}v_\phi \mathrm{d}v_\theta \tag{20}$$

$$= 2\pi v_\perp \mathrm{d}v_r \mathrm{d}v_\perp \tag{21}$$

$$= \frac{\pi e}{v_r r_p}\left(\frac{GM}{r}\right)^2 \mathrm{d}e\mathrm{d}r_p. \tag{22}$$

As a consequence of our assumed spherical symmetry, the volume element can be expressed as

$$\mathrm{d}^3 r = 4\pi r^2 \mathrm{d}r. \tag{23}$$

Thus the phase space volume element can be expressed as

$$\mathrm{d}^3 r \mathrm{d}^3 v = \frac{4\pi^2 (GM)^2 e}{v_r r_p}\mathrm{d}r\mathrm{d}e\mathrm{d}r_p. \tag{24}$$

The number of stars in an element $\mathrm{d}r\mathrm{d}e\mathrm{d}r_p$ is

$$n(r, e, r_p) = \frac{4\pi^2 (GM)^2 e}{v_r r_p} f(\mathcal{E}). \tag{25}$$

From this, we can construct the expected number of stars to be on orbits with periapsis r_p and eccentricity e. We will define this locally, allowing it to vary with position. The number of stars found in a small radius range δr with given orbital properties should be given by the the total number of stars with these properties multiplied by the relative amount of time they spend in that range

$$n(r, e, r_p)\delta r = N(e, r_p; r)\frac{\delta t}{P(e, r_p)} \tag{26}$$

where $N(e, r_p; r)$ is the total number of stars with orbits given by $\{e, r_p\}$ defined at r, δt is the time spent in δr and $P(e, r_p)$ is the period of the orbit. We will defer the definition of this time for unbound orbits for now. The time spent in the radius range is

$$\delta t = 2\frac{\delta r}{v_r}, \tag{27}$$

where the factor of 2 is included to account for both inwards and outwards motion. Hence

$$N(e, r_p; r) = \frac{1}{2}v_r P n(r, e, r_p) \tag{28}$$

$$= \frac{2\pi^2 (GM)^2 P e}{r_p} f(\mathcal{E}). \tag{29}$$

The right hand side of this equation is independent of position, subject to the constraint that the radius is in the allowed range for the orbit $r_p \leqslant r \leqslant (1+e)r_p/(1-e)$, and so we may define $N(e, r_p) \equiv N(e, r_p; r)$. This is a consequence of the density function being dependent only upon an orbital constant of the motion.

If a burst of radiation is emitted each time a star passes through periapse, then the event rate for burst emission from orbits with parameters $\{e, r_p\}$, is given by

$$\Gamma(e, r_p) = \frac{N(e, r_p)}{P(e, r_p)} \quad (30)$$
$$= \frac{2\pi^2 (GM)^2 e}{r_p} f(\mathcal{E}). \quad (31)$$

The orbital period drops out from the calculation, so we do not have to worry about an appropriate definition for unbound orbits.

From the event rate we may define a probability of seeing a given number of events subject to the assumption that these events are uncorrelated. The probability is simply given by the Poisson distribution; the probability of there being r events is

$$\Pr(r|\Gamma(e, r_p)) = \frac{\Gamma^r \exp(-\Gamma)}{r!}. \quad (32)$$

The probability of there being a burst from an orbit with periapse r_p and eccentricity e is hence

$$\Pr(r \neq 0|\Gamma(e, r_p)) = 1 - \Pr(r = 0|\Gamma(e, r_p)). \quad (33)$$

To estimate the expectation of a quantity across all orbits we use

$$\langle X \rangle = \sum_r \int_0^\infty \mathrm{d}e \int_0^\infty \mathrm{d}r_p X(r; r_p, e) \Pr(r|\Gamma(e, r_p)). \quad (34)$$

Since the probability decays rapidly for large r, we may truncate the sum to give the required level of accuracy,

To generate a representative sample for the orbital parameters e and r_p, we use $\Gamma(e, r_p)$ as an unnormalised probability distribution and draw from it appropriately.

1.4 The Inner Cut-Off

From (31) we see that the event rate is highly sensitive to the smallest value of the periapsis. The inner cut-off for r_p could result from a number of different physical causes. Ultimately the orbits cannot encroach closer to the black hole than its last stable orbit. This depends upon the spin of the black hole, but is of the order of its Schwarzschild radius. Before we reach this point, however, there are other processes that may intervene to deplete the orbitting stars. Our treatment of these is approximate, however should hopefully produce reasonable estimates. We will consider three processes: tidal disruption, gravitational wave inspiral and collisional disruption. Tidal disruption imposes a definite definite cut-off, while the others use statistical arguments, they are therefore true for a typical star and it is unlikely that a star would be found beyond the imposed limits.

1.4.1 Tidal Disruption

Tidal forces from the black hole can disrupt stars. This occurs at the tidal radius

$$r_t \simeq \left(\frac{M_\bullet}{M}\right)^{1/3} R_M \quad (35)$$

where R_M is the radius of the star (Kobayashi et al. 2004). Tidal disruption is most significant for MS stars since they are least dense.

1.5 Gravitational Wave Inspiral

Stars orbitting about the black hole will continually radiate energy and angular momentum causing them to inspiral. Using the analysis of Peters (1964) for Keplerian binaries, for bound orbits the orbit-averaged rate of change of the periapsis and eccentricity are

$$\left\langle \frac{\mathrm{d}r_p}{\mathrm{d}t} \right\rangle = -\frac{64}{5} \frac{\Theta}{r_p^3} \frac{(1-e)^{3/2}}{(1+e)^{7/2}} \left(1 - \frac{7}{12}e + \frac{7}{8}e^2 + \frac{47}{192}e^3\right) \quad (36)$$
$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle = -\frac{304}{5} \frac{\Theta}{r_p^4} \frac{(1-e)^{3/2}}{(1+e)^{5/2}} \left(1 + \frac{121}{304}e^2 + \frac{47}{192}e^3\right), \quad (37)$$

where we have introduced

$$\Theta = \frac{G^3 M_\bullet M (M_\bullet + M)}{c^5} \quad (38)$$

For a circular orbit the inspiral time form initial periapsis r_{p0} is

$$\tau_c(r_{p0}) = \frac{5}{256} \frac{r_{p0}^4}{\Theta}. \quad (39)$$

For an orbit of finite eccentricity $(0 < e < 1)$, we can solve for the periapsis as a function of eccentricity

$$r_p(e) = A(1+e)^{-1} \left(1 + \frac{121}{304}e^2\right)^{870/2299} e^{12/19}, \quad (40)$$

where A is a constant fixed by the initial conditions: for an orbit with initial eccentricity e_0

$$A = (1+e_0) \left(1 + \frac{121}{304}e_0^2\right)^{-870/2299} e_0^{-12/19} r_{p0}. \quad (41)$$

The inspiral is complete when the eccentricity has decayed to zero. Consequently the inspiral time is

$$\tau(r_{p0}, e_0) = \int_0^{e_0} \frac{15}{304} \frac{A^4}{\Theta} \frac{e^{29/19}}{(1-e^2)^{3/2}} \left(1 + \frac{121}{304}e^2\right)^{1181/2299} \mathrm{d}e. \quad (42)$$

This is best evaluated numerically, however it may be written in closed from as

$$\tau(r_{p0}, e_0) = \tau_c(r_{p0})(1+e_0)^4 \left(1 + \frac{121}{304}e_0^2\right)^{-3480/2299} \quad (43)$$
$$\times F_1\left(\frac{24}{19}; \frac{3}{2}, \frac{-1181}{2299}; \frac{43}{19}; e_0^2, -\frac{121}{304}e_0^2\right), \quad (44)$$

using the Appell hypergeometric function of the first kind $F_1(\alpha; \beta, \beta'; \gamma; x, y)$.[3]

We will assume that an orbit is depleted of stars if the inspiral timescale is shorter than the relaxation timescale. This indicates the characteristic time for two-body collisions to change the velocity the star by order of itself (Binney & Tremaine 1987), and so indicates the time over which scattering may repopulate the orbit. Binney & Tremaine (1987) derive the relaxation timescale from the diffusion coefficient of the Fokker-Planck equation (section 8.4), for a system with a purely Maxwellian distribution

$$\tau_R \simeq 0.34 \frac{\sigma_\star^3}{G M_\star \rho_\star \ln\Lambda}, \quad (45)$$

where $\rho_\star$ is the mass density and the Coulomb logarithm is $\ln\Lambda = \ln(M_\bullet/M_\star)$. Bahcall & Wolf (1977) find a similar characteristic timescale, but with numerical prefactor

[3] For small eccentricities $\tau(r_{p0}, e_0) \simeq \tau_c(r_{p0})(1 + 4e_0)$.

$3/4\sqrt{8\pi} \simeq 0.15$. We shall adopt the former to be conservative. From this we estimate the relaxation time for an orbit as

$$T_R(r_p, e) = \left(\frac{\langle v(r)\rangle}{\sigma_\star}\right)^3 \frac{\rho_\star}{\langle\rho(r)\rangle}\tau_R \quad (46)$$

$$= \left(\frac{2E(e)}{\pi}\right)^3 \left(\frac{r_c(1-e)}{r_p}\right)^{3/2} \frac{\rho_\star}{\langle\rho(r)\rangle}\tau_R \quad (47)$$

using the velocity $\langle v(r)\rangle$ and density $\langle\rho(r)\rangle$ averaged over the duration of the orbit as characteristic quantities.[4] While the former may be written in closed form using the complete elliptic integral of the second kind, the latter is best evaluated numerically.

Unbound stars only undergo a single periapse passage and only radiate one burst of radiation, we shall therefore neglect any evolution in their orbital parameters.[5]

1.5.1 Collisions

As a consequence of the higher densities in the galactic core, stars may undergo a large number of close encounters with other stars. It takes 20–30 grazing collisions to disrupt a MS star (Freitag et al. 2006). We shall ignore the possibility of disruption for the other species since they have much smaller cross-sectional areas. The number of collisions a star will undergo in a time interval δt is

$$\delta K = n(r)\pi R_\star^2 v(r, e, r_p)\delta t. \quad (48)$$

For circular orbits we can find the radius at which collisions will lead to disruptions by setting $\delta K = 30$ and $\delta t = \tau_R$. We use the relaxation timescale for the system as this is the time over which stars are replenished from the reservoir. For non-circular orbits we must consider variation with position. Using $\delta r = v_r \delta t$, and then converting to an integral, we have for bound orbits

$$K = 2\pi R_\star^2 \frac{\tau_R}{P(r_p, e)} \int_{r_p}^{(1+e)r_p/(1-e)} n(r)\frac{v_r(r, e, r_p)}{v_r(r, e, r_p)}\,\mathrm{d}r, \quad (49)$$

where P is the period of the orbit. Again we may set $K = 30$ to find the orbits for which stars will be disrupted within a relaxation timescale. For unbound stars we are only interested in stars that would become disrupted before their periapse passage, so

$$K = \pi R_\star^2 \int_{r_p}^{r_c} n(r)\frac{v_r(r, e, r_p)}{v_r(r, e, r_p)}\,\mathrm{d}r, \quad (50)$$

assuming that the stars in the reservoir external to the core are unlikely to undergo close collisions.

[4] In calculating $\langle\rho(r)\rangle$ we neglect the possibility that the density may be decreased due to the depopulation of orbits. We therefore underestimate the relaxation timescale with this simple approach.
[5] Using the analysis of Turner (1977) it is possible to show that the change in eccentricity Δe, and fractional change in periapsis $\Delta r_p/r_p$ for an extreme mass-ratio binary are less than $\mathcal{O}(10\eta)$, where $\eta = M/M_\bullet$.

2 INTRODUCTION

It has been well established that RV Tauri variables possess infrared emission far in excess of their expected blackbody continuum, arising from their extended cool dust envelopes (Gerhz & Woolf 1970; Gerhz 1972; Gerhz & Ney 1972). Recently, Lloyd Evans (1985) and Goldsmith et al. (1987) have given detailed descriptions of the near-infrared properties of RV Tauri stars. In this paper we present an analysis of the *IRAS* data of RV Tauri stars with the help of the far-infrared two-colour diagram and a grid computed using a simple model of the dust envelope. Such two-colour plots have already been employed extensively by several investigators to study the circumstellar envelopes around oxygen-rich and carbon-rich objects which are in the late stages of stellar evolution (Hacking et al. 1985; Zuckerman & Dyck 1986; van der Veen & Habing 1988; Willems & de Jong 1988).

Table 1 summarizes the basic data on the 17 objects detected at 60 µm. Apart from the *IRAS* identification and the flux densities at 12-, 25-, 60- and 100-µm wavebands, it gives the spectroscopic groups of Preston et al. (1963), the light-curve classes of Kukarkin et al. (1969) and the periods of light variation. The list, which contains about 20 per cent of all the known RV Tauri stars, is essentially the same as that given by Jura (1986). The spectroscopic subgroups are from either Preston et al. (1963) or Lloyd Evans (1985).

3 DESCRIPTION OF THE ENVELOPE MODEL

If we assume that the dust grains in the envelope are predominantly of the same kind and are in thermal equilibrium, the luminosity at frequency ν in the infrared is given by

$$L(\nu) = \int_{\text{envelope}} \rho(r)Q_{\text{abs}}(\nu)B[\nu, T_{\text{g}}(r)]\exp[-\tau(\nu, r)]\,\mathrm{d}V, \quad (51)$$

where $Q_{\text{abs}}(\nu)$ is the absorption efficiency at frequency ν, $\rho(r)$ is the dust grain density, $T_{\text{g}}(\nu)$ is the grain temperature, $B[\nu, T_{\text{g}}(r)]$ is the Planck function, and $\tau(\nu, r)$ is the optical depth at distance r from the centre of the star.

The temperature $T_{\text{g}}(r)$ is determined by the condition of energy balance: amount of energy radiated = amount of energy absorbed. The amount of energy absorbed at any point is proportional to the total available energy at that point, which consists of:

(i) the attenuated and diluted stellar radiation;
(ii) scattered radiation, and
(iii) reradiation from other grains.

Detailed solutions of radiative transfer in circumstellar dust shells by Rowan-Robinson & Harris (1983a,b) indicate that the effect of heating by other grains becomes significant only at large optical depths at the absorbing frequencies [$\tau(\text{UV}) \gg 10$], and at optical depths $\tau(\text{UV}) < 1$ the grains have approximately the same temperature that they would have if they were seeing the starlight unattenuated and no other radiation.

The Planck mean optical depths of circumstellar envelopes around several RV Tauri stars, derived from the ratios of the luminosities of the dust shell (at infrared wavelengths) and the star, range from 0.07 to 0.63 (Goldsmith

Table 2. Data on the RV Tauri stars detected by *IRAS*.

Name Variable	*IRAS*	Flux density (Jy)[a] 12 µm	25 µm	60 µm	100 µm	Sp. group	Period (d)	Light-curve type	T_0 (K)
TW Cam	04166+5719	8.27	5.62	1.82	<1.73	A	85.6	a	555
RV Tau	04440+2605	22.53	18.08	6.40	2.52	A	78.9	b	460
DY Ori	06034+1354	12.44	14.93	4.12	<11.22	B	60.3		295
CT Ori	06072+0953	6.16	5.57	1.22	<1.54	B	135.6		330
SU Gem	06108+2734	7.90	5.69	2.16	<11.66	A	50.1	b	575
UY CMa	06160−1701	3.51	2.48	0.57	<1.00	B	113.9	a	420
U Mon	07284−0940	124.30	88.43	26.28	9.24	A	92.3	b	480
AR Pup	08011−3627	131.33	94.32	25.81	11.65	B	75.0	b	450
IW Car	09256−6324	101/06	96.24	34.19	13.07	B	67.5	b	395
GK Car	11118−5726	2.87	2.48	0.78	<12.13	B	55.6		405
RU Cen	12067−4508	5.36	11.02	5.57	2.01	B	64.7		255
SX Cen	12185−4856	5.95	3.62	1.09	<1.50	B	32.9	b	590
AI Sco	17530−3348	17.68	11.46	2.88	<45.62	A	71.0	b	480
AC Her	18281+2149	41.47	65.33	21.12	7.79	B	75.5	a	260
R Sct	18448−0545	20.88	9.30	8.10	<138.78	A	140.2	a	
R Sge	20117+1634	10.63	7.57	2.10	<1.66	A	70.6	b	455
V Vul	20343+2625	12.39	5.72	1.29	<6.96	A	75.7	a	690

[a] Observed by *IRAS*.

et al. 1987). There is much uncertainty in the nature of the optical properties of dust grains in the envelope. The carbon-rich RV Tauri stars are also reported to show the 10-µm silicate emission feature typical of oxygen-rich objects (Gerhz & Ney 1972; Olnon & Raimond 1986). The pure terrestrial silicates or lunar silicates are found to be completely unsuitable to account for the infrared emission from circumstellar dust shells around M-type stars (Rowan-Robinson & Harris 1983a). We assume that the absorption efficiency $Q_{\rm abs}(\nu)$ in the infrared varies as ν^{γ}. $\gamma=1$ appears to provide a reasonable fit in a variety of sources (Harvey, Thronson & Gatley 1979; Jura 1986). Under these circumstances the condition of energy balance implies that the dust temperature $T_{\rm g}$ will vary as r^{β}.

In view of the low value of the observed Planck mean optical depth for the stellar radiation and the nature of the assumed frequency dependence of the absorption efficiency, the extinction of the infrared radiation by the dust envelope can be neglected. If we consider the envelope to be spherically symmetric, equation (1) reduces to

$$L(\nu)=\int_{r_1}^{r_2} 4\pi r^2\rho(r)\, Q_{\rm abs}(\nu)B[\nu,T_{\rm g}(r)]\,{\rm d}r, \qquad (52)$$

where r_1 and r_2 are the inner and outer radii of the shell. For a dusty density distribution $\rho(r)\propto r^{\alpha}$ and $r_2\gg r_1$, equation (2) reduces to

$$L(\nu)\propto \nu^{2+\gamma-Q}\int_{X_0}^{\infty}\frac{x^Q}{{\rm e}^x-1}{\rm d}x, \qquad (53)$$

where $Q=-(\alpha+\beta+3)/\beta$ and $X_0=(h\nu/kT_0)$. T_0 represents the temperature at the inner boundary of the dust shell where grains start condensing. In a steady radiation pressure driven mass outflow in the optically thin case, values of α lie near -2 (Gilman 1972). γ and β are related by $\beta=-2/(\gamma+4)$.

In the *IRAS* Point Source Catalog (PSC, Beichman et al. 1985a), the flux densities have been quoted at the effective wavelengths 12, 25, 60 and 100 µm, assuming a flat energy spectrum $[\nu F(\nu)=1]$ for the observed sources. For each model given by equation (3), using the relative system response, the colour-correction factors (Beichman et al. 1985b) in each of the *IRAS* passbands were calculated and the fluxes were converted into flux densities expected for a flat energy distribution, as assumed in the *IRAS* PSC, so that the computed colours can be directly compared with the colours determined from the catalogue quantities. Such a procedure is more appropriate than correcting the *IRAS* colours for the energy distribution given by a particular model and then comparing them with those computed by the model.[6]

3.1 Colour–colour diagram

The IR colour is defined as

$$[\nu_1]-[\nu_2]=-2.5\log[f(\nu_1)/f(\nu_2)],$$

where ν_1 and ν_2 are any two wavebands and $f(\nu_1)$ and $f(\nu_2)$ are the corresponding flux densities assuming a flat energy spectrum for the source. In Fig. 1, we have plotted the [25]–[60] colours of RV Tauri stars against their corresponding [12]–[25] colours derived from the *IRAS* data. Filled circles represent stars of group A and open circles stars of group B. The two sets of near-parallel lines represent the loci of constant inner shell temperature T_0 and the quantity Q defined above. The models correspond to the case of absorption efficiency $Q_{\rm abs}(\nu)$ varying as ν (with $\gamma=1$ and hence $\beta=-0.4$). We have omitted R Sct in Fig. 1 because it shows a large deviation from the average relation shown by

[6] An example of a footnote.

Figure 1. Plot of [25]–[60] colours of RV Tauri stars against their [12]–[25] colours after normalizing as indicated in Beichman et al. (1985b). Some of the objects are identified by their variable-star names. Typical error bars are shown in the bottom right-hand corner. The lines represent the loci for constant inner shell temperature and the quantity Q. Note the separation of group A and B stars at $T_0 \sim 460$ K. Positions occupied by a sample of carbon and oxygen Miras are also shown. The $Q = 1.0$ line differs from the blackbody line by a maximum of ~ 0.05.

all the other objects. R Sct has a comparatively large excess at 60 µm, but the extent of a possible contamination by the infrared cirrus (Low et al. 1984) is unknown. Goldsmith et al. (1987) found no evidence of the presence of a dust envelope at near-IR wavelengths and the spectrum was consistent with a stellar continuum. This explains why R Sct lies well below the mean relation shown by stars of groups A and C between the [3.6]–[11.3] colour excess and the photometrically determined (Fe/H) (Dawson 1979). R Sct has the longest period of 140 d among the RV Tauri stars detected at far-infrared wavelengths and does not have the 10-µm emission feature seen in other objects (Gerhz 1972; Olnon & Raimond 1986). R Sct is probably the most irregular RV Tauri star known (McLaughlin 1932).

The inner shell temperatures (T_0) derived for the various objects are also given in Table 1 and we find the majority of them to have temperatures in the narrow range 400–600 K. If the dependences of $Q_{\rm abs}(\nu)$ on ν and $\rho(r)$ on r are similar in all the objects considered, then in the colour–colour diagram they all should lie along a line corresponding to different values of T_0 and in Fig. 1 we find that this is essentially the case. In view of the quoted uncertainties in the flux measurements, we cannot attach much significance to the scatter in Fig. 1.

At 100 µm the infrared sky is characterized by emission, called infrared cirrus, from interstellar dust on all spatial scales (Low et al. 1984), thereby impairing the measurements at far-infrared wavelengths. In Fig. 2, we have plotted the [60]–[100] colours of the six RV Tauri stars detected at 100 µm against their [25]–[60] colours, along with the grid showing the regions of different values for inner shell temperature T_0 and the quantity Q, as in Fig. 1. The results indicated by Fig. 2 are consistent with those derived from Fig. 1. AR Pup shows a large excess at 100 µm but, in view of the large values for the cirrus flags given in the catalogue, the intrinsic flux at 100 µm is uncertain.

3.2 Radial distribution of dust

From Fig. 1, it is evident that all RV Tauri stars lie between the lines corresponding to $Q = 1.5$ and 0.5. With

$$\alpha = -(1+Q)\beta - 3,$$

these values suggest limits of $r^{-2.0}$ and $r^{-2.4}$ for the dust density variation, indicating a near-constant mass-loss rate. Jura (1986) has suggested that the density in the circumstellar envelope around RV Tauri stars varies as r^{-1}, implying a mass-loss rate that was greater in the past than it is currently. By fitting a power law to the observed fluxes, such that $f(\nu)$ varies as ν^q, values of q determined by him for the various objects given in Table 1 lie in the range 0.6–1.2, with a mean $\bar{q} = 0.98$. The assumption of a power law corresponds to the case of $X_0 = 0$ in equation (3) and hence we get

$$q = 2 + \gamma - Q.$$

Since we assume that $Q_{\rm abs}(\nu)$ varies as ν, the resulting value for Q=2.0. None of the objects is found to lie in the corresponding region in the colour–colour diagram. Even this extreme value for Q implies a density which varies as $r^{-1.8}$.

Goldsmith et al. (1987) have reported that the simultaneous optical and near-IR data of AC Her can be fitted by a combination of two blackbodies at 5680 and 1800 K, representing, respectively, the stellar and dust shell temperatures, and suggested that in RV Tauri stars the grain formation is a sporadic phenomenon and not a continuous process. Apparently, they have been influenced by the remark by Gerhz & Woolf (1970) that their data in the 3.5–11 µm region of AC Her indicated a dust temperature of ∼300 K. We find that the K–L colours given by Gerhz (1972), Lloyd Evans (1985) and Goldsmith et al. (1987) are all consistent with each other. Surely, hot dust (∼ 1800 K), if present at the time of observations by Goldsmith et al. (1987), would have affected the K–L colour significantly. AC Her, like other members of its class, is found to execute elongated loops in the (U–B), (B–V) plane (Preston et al. 1963), indicating that significant departure of the stellar continuum from the blackbody is to be expected. Further, their data show only a marginal excess at the near-IR wavelengths. We feel that the case for the existence of hot dust around AC Her and hence for the sporadic grain formation around RV Tauri stars is not strong. In Fig. 3 we find that AC Her and RU Cen lie very close to R Sct which, according to Goldsmith et al. (1987), shows no evidence for the presence of a hot dust envelope.

Figure 2. Plot of the [60]–[100] colours of RV Tauri stars against their [25]–[60] colours after normalizing as indicated in Beichman et al. (1985b). The solid lines represent the loci for constant inner shell temperature and the quantity Q. The dashed line shows the locus for a blackbody distribution.

3.2.1 Comparison with oxygen and carbon Miras

In Fig. 1 we have also shown the positions of a sample of oxygen-rich and carbon-rich Miras. At the low temperatures characteristic of the Miras, a part of the emission at 12 μm comes from the photosphere. For a blackbody at 2000 K, the ratio of fluxes at wavelengths of 12 and 2 μm $(f_{12}/f_2) \sim 0.18$. The Miras shown in Fig. 1 have (f_{12}/f_2) ratios larger than twice the above value. It is clear that the three groups of objects populate three different regions of the diagram. Hacking et al. (1985) have already noticed that there are distinct differences between the *IRAS* colours of oxygen-rich and carbon-rich objects. On the basis of an analysis, using a bigger sample of bright giant stars in the *IRAS* catalogue, this has been interpreted by Zuckerman & Dyck (1986) as being due to a systematic difference in the dust grain emissivity index. U Mon shows the 10-μm silicate emission convincingly and, in most of the other objects for which low-resolution spectra in the near-infrared have been reported (Gerhz 1972; Olnon & Raimond 1986), the 10-μm emission may be partly attributed to silicates. Hence it is reasonable to expect that, in the envelopes around at least some of the RV Tauri stars, the dust grains are predominantly of silicates, as in the case of oxygen Miras (Rowan-Robinson & Harris 1983a). The fact that none of the RV Tauri stars is found in the region of the two-colour diagram occupied by the oxygen Miras indicates that the emissivity indices of the silicate grains in the two cases are different. Because of the higher temperatures and luminosities, the environment of grain formation will be different in RV Tauri stars.

3.2.2 Correlation with subgroups

Preston et al. (1963) have identified three spectroscopic subgroups, which are designated as groups A, B and C. Objects of group A are metal-rich; group C are metal-poor; group B objects are also metal-poor, but show carbon enhancements (Preston et al. 1963; Lloyd Evans 1974; Dawson 1979; Baird 1981). It is interesting to see that Table 1 contains no group C objects and that in Fig. 1 there is a clear separation of the two spectroscopic subgroups A and B, with the demarcation occurring at an inner shell temperature of about 450 K, group B stars having lower temperatures than group A. SX Cen is the only exception. Lloyd Evans (1974) has reported that metal lines are stronger in SX Cen than in other group B objects. It may be worth noting that SX Cen has the shortest period among the 100 or so objects with the RV Tauri classification. RU Cen has the coolest inner shell temperature, as already suggested by the near-infrared spectrum (Gerhz & Ney 1972).

Figure 3. Plot of (K–L) colours of RV Tauri stars detected by *IRAS* against their corresponding (J–K) colours. The position of AR Pup is indicated. The three objects lying close to the blackbody line are AC Her, RU Cen and R Sct.

Group B objects follow a different mean relationship from those of group A, having systematically larger 11-μm excess for a given excess at 3 μm (Lloyd Evans 1985). For a general sample of RV Tauri stars, the distinction between the oxygen-rich and carbon-rich objects is not that apparent in the *JHKL* bands. In Fig. 3 we have plotted the near-IR magnitudes of the objects given in Table 1 (except V Vul which has no available measurements) in the J–K, K–L plane. The colours, taken from Lloyd Evans (1985) and Goldsmith et al. (1987), are averaged if more than one observation exists, Landscape figure to go here. This figure was not part of the original paper and is inserted here for illustrative purposes. See the author guide for details (section 2.2 of `mn2eguide.tex`) on how to handle landscape figures or tables.

Figure 4.

because the internal agreements are found to be often of the order of observational uncertainties, in accordance with the earlier finding by Gerhz (1972) that variability has relatively little effect on colours. Barring RU Cen and AC Her, it is evident that stars belonging to group B show systematically larger excesses at L band for a given excess at K. The low excesses at near-IR wavelengths for AC Her and RU Cen are consistent with the very low dust temperatures indicated by the far-infrared colours.

It is already well established that from UBV photometry one can distinguish between groups A and B, members of group A being significantly redder than those of group B (Preston et al. 1963). Similarly, Dawson (1979) has found that the two spectroscopic groups are well separated in the DDO colour–colour diagrams when mean colours are used for the individual objects.

The clear separation of the spectroscopic subgroups A and B in the IR two-colour diagram suggests that the natures of dust grains in the envelopes in the two cases are not identical. This is to be expected because of the differences in the physical properties of the stars themselves. The average colours of group B stars are bluer than group A, but the envelope dust temperatures of B are cooler than those of A. The near-IR spectra of AC Her and RU Cen are extremely similar (Gerhz & Ney 1972). The striking similarities in the optical spectra of AC Her and RU Cen have been pointed out by Bidelman (O'Connell 1961). We feel that the physical properties, including the chemical composition, of the grains formed in the circumstellar envelope strongly depend on those of the embedded star. This, probably, explains the diversity of the energy distributions of RV Tauri stars in the near-infrared found by Gerhz & Ney (1972). On the basis of the observed differences in chemical abundances and space distribution of RV Tauri stars, Lloyd Evans (1985) has already pointed out that there is no direct evolutionary connection between group A and group B objects, thus ruling out the possibility that group B objects are the evolutionary successors of group A, in which grain formation has stopped and the cooler temperatures for the former are caused by an envelope expansion.

Kukarkin et al. (1969) have subdivided RV Tauri stars into two classes, RVa and RVb, on the basis of their light curves; the former shows a constant mean brightness, whereas the latter shows a cyclically varying mean brightness. Extensive observations in the near-infrared show that, on average, RVb stars are redder than RVa stars, and Lloyd Evans (1985) has suggested that in RVb stars dust shells are denser in the inner regions and hence radiate strongly in the 1–3 µm region. Fig. 3 confirms this; RVb objects show systematically larger (J–K) and (K–L) colours than RVa objects. Apparently, there is no distinction between objects of the two light-curve types at far-infrared wavelengths (Fig. 1).

4 CONCLUSIONS

In the [12]–[25], [25]–[60] colour diagram, RV Tauri stars populate cooler temperature regions ($T < 600\,\mathrm{K}$), distinctly different from those occupied by the oxygen and carbon Miras. Using a simple model in which

(i) the envelope is spherically symmetric,

(ii) the IR-emitting grains are predominantly of the same kind, and

(iii) in the infrared the absorption efficiency $Q_{\rm abs}(\nu) \propto \nu$,

we find that the *IRAS* fluxes are consistent with the density in the envelope $\rho(r) \propto r^{-2}$, where r is the radial distance. Such a dependence for the dust density implies that the mass-loss rates in RV Tauri stars have not reduced considerably during the recent past, contrary to the suggestion by Jura (1986). In the two-colour diagram, the blackbody line and the line corresponding to $\rho(r) \propto r^{-2.2}$ nearly overlap and the present data are insufficient to resolve between the two cases. The latter case is more physically reasonable, however.

The spectroscopic subgroups A and B are well separated in the *IRAS* two-colour diagram, with group B objects having systematically cooler dust envelopes. If we consider only the objects detected by *IRAS*, we find that stars belonging to group B show systematically larger excess at L band for a given excess at K. Apparently, there is no correlation between the light-curve types (RVa and RVb) and the far-infrared behaviour of these objects. It is fairly certain that the physical properties, including the chemical composition, of the embedded stars are directly reflected by those of the dust grains. Most probably, the grain formation process in RV Tauri stars is continuous and not sporadic as suggested by Goldsmith et al. (1987).

ACKNOWLEDGMENTS

I thank Professor N. Kameswara Rao for some helpful suggestions, Dr H. C. Bhatt for a critical reading of the original version of the paper and an anonymous referee for very useful comments that improved the presentation of the paper.

REFERENCES

Baird S.R., 1981, ApJ, 245, 208
Beichman C.A., Neugebauer G., Habing H.J., Clegg P.E., Chester T.J., 1985a, *IRAS* Point Source Catalog. Jet Propulsion Laboratory, Pasadena
Beichman C.A., Neugebauer G., Habing H.J., Clegg P.E., Chester T.J., 1985b, *IRAS* Explanatory Supplement. Jet Propulsion Laboratory, Pasadena
Dawson D.W., 1979, ApJS, 41, 97
Gerhz R.D., 1972, ApJ, 178, 715
Gerhz R.D., Ney E.P., 1972, PASP, 84, 768
Gerhz R.D., Woolf N.J., 1970, ApJ, 161, L213
Gilman R.C., 1972, ApJ, 178, 423
Goldsmith M.J., Evans A., Albinson J.S., Bode M.F., 1987, MNRAS, 227, 143
Hacking P. et al., 1985, PASP, 97, 616
Harvey P.M., Thronson H.A., Gatley I., 1979, ApJ, 231, 115
Jura M., 1986, ApJ, 309, 732
Kukarkin B.V. et al., 1969, General Catalogue of Variable Stars. Moscow
Lloyd Evans T., 1974, MNRAS, 167, 17P
Lloyd Evans T., 1985, MNRAS, 217, 493
Low F.J. et al., 1984, ApJ, 278, L19
McLaughlin D.B., 1932, Publ. Univ. Obs. Mich., 4, 135

O'Connell J.K., 1961, Specola Vaticana Ric. Astron., 6, 341
Olnon F.M., Raimond E., 1986, A&AS, 65, 607
Preston G.W., Krzeminski W., Smak J., Williams J.A., 1963, ApJ, 137, 401
Rowan-Robinson M., Harris S., 1983a, MNRAS, 202, 767
Rowan-Robinson M., Harris S., 1983b, MNRAS, 202, 797
van der Veen W.E.C.J., Habing H.J., 1988, A&A, 194, 125
Willems F.J., de Jong T., 1988, A&A, 196, 173
Zuckerman B., Dyck H.M., 1986, ApJ, 311, 345

REFERENCES

Alexander T., 2005, Physics Reports, 419, 65
Alexander T., Hopman C., 2009, The Astrophysical Journal, 697, 1861
Bahcall J. N., Wolf R. A., 1976, The Astrophysical Journal, 209, 214
Bahcall J. N., Wolf R. A., 1977, The Astrophysical Journal, 216, 883
Binney J., Tremaine S., 1987, Galactic Dynamics, first edn. Princeton University Press, Princeton
Freitag M., AmaroSeoane P., Kalogera V., 2006, The Astrophysical Journal, 649, 91
Genzel R., Schoedel R., Ott T., Eisenhauer F., Hofmann R., Lehnert M., Eckart A., Alexander T., Sternberg A., Lenzen R., Clenet Y., Lacombe F., Rouan D., Renzini A., Tacconi-Garman L. E., 2003, The Astrophysical Journal, 594, 812
Ghez A. M., Salim S., Weinberg N. N., Lu J. R., Do T., Dunn J. K., Matthews K., Morris M. R., Yelda S., Becklin E. E., Kremenek T., Milosavljevic M., Naiman J., 2008, The Astrophysical Journal, 689, 1044
Gillessen S., Eisenhauer F., Trippe S., Alexander T., Genzel R., Martins F., Ott T., 2009, The Astrophysical Journal, 692, 1075
Hopman C., Alexander T., 2006a, The Astrophysical Journal, 645, 1152
Hopman C., Alexander T., 2006b, The Astrophysical Journal, 645, L133
Keshet U., Hopman C., Alexander T., 2009, The Astrophysical Journal, 698, L64
Kobayashi S., Laguna P., Phinney E. S., Meszaros P., 2004, The Astrophysical Journal, 615, 855
Lynden-Bell, D. 1967, Monthly Notices of the Royal Astronomical Society, 136, 101
O'Leary R. M., Kocsis B., Loeb A., 2009, Monthly Notices of the Royal Astronomical Society, 395, 2127
Peters P. C., 1964, Physical Review, 136, B1224
Schödel R., Eckart A., Alexander T., Merritt D., Genzel R., Sternberg A., Meyer L., Kul F., Moultaka J., Ott T., Straubmeier C., 2007, Astronomy and Astrophysics, 469, 125
Toonen S., Hopman C., Freitag M., 2009, Monthly Notices of the Royal Astronomical Society, 398, 1228
Tremaine S., Gebhardt K., Bender R., Bower G., Dressler A., Faber S. M., Filippenko A. V., Green R., Grillmair C., Ho L. C., Kormendy J., Lauer T. R., Magorrian J., Pinkney J., Richstone D., 2002, The Astrophysical Journal, 574, 740
Turner M., 1977, The Astrophysical Journal, 216, 610

APPENDIX A: LARGE GAPS IN Lyα FORESTS DUE TO FLUCTUATIONS IN LINE DISTRIBUTION

(This appendix was not part of the original paper by A.V. Raveendran and is included here just for illustrative purposes. The references are not relevant to the text of the appendix, they are references from the bibliography used to illustrate text before and after citations.)

Spectroscopic observations of bright quasars show that the mean number density of Lyα forest lines, which satisfy certain criteria, evolves like $\mathrm{d}N/\mathrm{d}z = A(1+z)^\gamma$, where A and γ are two constants. Given the above intrinsic line distribution we examine the probability of finding large gaps in the Lyα forests. We concentrate here only on the statistics and neglect all observational complications such as the line blending effect (see Harvey et al. 1979, for example).

Suppose we have observed a Lyα forest between redshifts z_1 and z_2 and found $N-1$ lines. For high-redshift quasars z_2 is usually the emission redshift $z_{\rm em}$ and z_1 is set to $(\lambda_{{\rm Ly}\beta}/\lambda_{{\rm Ly}\alpha})(1+z_{\rm em}) = 0.844(1+z_{\rm em})$ to avoid contamination by Lyβ lines. We want to know whether the largest gaps observed in the forest are significantly inconsistent with the above line distribution. To do this we introduce a new variable x:

$$x = \frac{(1+z)^{\gamma+1} - (1+z_1)^{\gamma+1}}{(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}}. \tag{A1}$$

x varies from 0 to 1. We then have $\mathrm{d}N/\mathrm{d}x = \lambda$, where λ is the mean number of lines between z_1 and z_2 and is given by

$$\lambda \equiv \frac{A[(1+z_2)^{\gamma+1} - (1+z_1)^{\gamma+1}]}{\gamma+1}. \tag{A2}$$

This means that the Lyα forest lines are uniformly distributed in x. The probability of finding $N-1$ lines between z_1 and z_2, P_{N-1}, is assumed to be the Poisson distribution.

Figure A1. $P(> x_{\rm gap})$ as a function of $x_{\rm gap}$ for, from left to right, N = 160, 150, 140, 110, 100, 90, 50, 45 and 40. Compare this with Lloyd Evans (1985).

A1 Subsection title

We plot in Fig. A1 $P(> x_{\rm gap})$ for several N values. We see that, for $N = 100$ and $x_{\rm gap} = 0.06$, $P(> 0.06) \approx 20$ per cent. This means that the probability of finding a gap with a size larger than six times the mean separation is not significantly small. When the mean number of lines is large, $\lambda \sim N >> 1$, our $P(> x_{\rm gap})$ approaches the result obtained by Rowan-Robinson & Harris (1983b, fig. 4) for small (but still very large if measured in units of the mean separation) $x_{\rm gap}$, i.e., $P(> x_{\rm gap}) \sim N(1 - x_{\rm gap})^{N-1} \sim N\exp(-\lambda x_{\rm gap})$.

This paper has been typeset from a TEX/ LATEX file prepared by the author.