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Extreme-mass-ratio-bursts from extragalactic sources

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ABSTRACT

Extreme-mass-ratios bursts (EMRBs) are a potentially interesting gravitational wave signal. They are produced when a compact object passes through periapsis on a highly eccentric orbit about a much more massive object; we consider stellar mass objects orbiting the massive black holes (MBHs) found in the centre of galaxies. Such an object may emit many EMRBs before eventually inspiralling into the MBH. EMRBs from the Galaxy's MBH would be detectable with a space-borne gravitational wave detector. We investigate the possibility of detecting EMRBs from extragalactic sources.

Key words: black hole physics – Galaxy: centre – gravitational waves – methods: data analysis.

1 INTRODUCTION

It is well established that space is big (Adams 1979, chapter 8). The Milky Way, our own island universe, is but one of a multitude of galaxies. Each one of these may have a massive black hole (MBH) nestled at its core (Lynden-Bell & Rees 1971; Sołtan 1982).

In previous work (Berry & Gair 2012), we considered measuring the properties of the Galaxy's MBH using extreme-mass-ratio bursts (EMRBs). An EMRB is a short gravitational wave (GW) signal produced when a small object passes through periapsis on an orbit about a much more massive body; in our case this is a stellar mass compact object (CO) orbiting the MBH. If the periapse radius of the orbit is sufficiently small ($r_\mathrm{p} \lesssim 10 r_\mathrm{g}$ for a $10 M_\odot$ CO, where $r_\mathrm{g} = GM_\bullet/c^2$ is a gravitational radius), a single burst can be highly informative about the MBH, improving our knowledge of its mass and spin.

EMRBs could be considered as the precursors to the better studied extreme-mass-ratio inspirals (EMRIs; Amaro-Seoane et al. 2007). Close collisions in the dense nuclear cluster surrounding the MBH scatter COs onto highly eccentric orbits. They proceed to emit an EMRB each orbit (Rubbo, Holley-Bockelmann, & Finn 2006). If they survive for long enough without being scattered again, the loss of energy-momentum carried away by gravitational radiation shall lead the orbit to circularise; eventually the GW signal changes, so there is continuous significant emission and we have an EMRI which continues until the inevitable plunge into the MBH. EMRBs are much shorter than EMRIs; they do not have as much time to accumulate high signal-to-noise ratios (SNRs), and consequently they are neither detectable to the same range, or as informative as EMRIs. However, a CO could emit many EMRBs before transitioning to the EMRI regime, making EMRBs an interesting signal for GW detection.

In this work, we consider if EMRBs are detectable from other nearby galaxies. If so, they may be useful for constraining the properties of these galaxies' MBHs. Observations have shown MBH masses are correlated with properties of the host galaxies, such as bulge luminosity, mass, velocity dispersion and light concentration (Kormendy & Richstone 1995; Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000; Graham et al. 2001; Tremaine et al. 2002; Marconi & Hunt 2003; Häring & Rix 2004; Graham 2007; Graham et al. 2011). The two are linked via their shared history, such that one can inform us about the other.

Astrophysical black holes (BHs) are described by two quantities: mass M and (dimsensionless) spin a_* (Chandrasekhar 1998). The spin is related to the angular momentum J by

$$a_* = \frac{cJ}{GM^2}, \tag{1}$$

For many MBHs in the local neighbourhood, we have existing mass estimates. Measuring the spin would give us a complete picture, and would crucially give an insight into the formation history of the galaxy(Dotti et al. 2012; Volonteri et al. 2012).

MBHs accumulate mass and angular momentum through accretion and mergers (Volonteri 2010; Yu & Tremaine 2002); the spin encodes information about the mechanism that has most recently dominated the evolution. A gaseous disc spins up the MBH, resulting in high spin values (Volonteri et al. 2005); randomly orientated accretion events lead to low spin values (King & Pringle 2006; King, Pringle, & Hofmann 2008); minor mergers with smaller BHs can decrease the spin (Hughes & Blandford 2003; Gammie, Shapiro, & McKinney 2004), and major

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mergers between MBHs gives a likely spin $|a_*| \sim 0.7$ (Berti & Volonteri 2008; González et al. 2007). Determining how the spin evolved shall tell us about how the galaxy evolved(Barausse 2012).

We have some MBH spin measurements from X-ray observations of active galactic nuclei (Nardini et al. 2011; Patrick et al. 2011; Gallo et al. 2011; Lohfink et al. 2012, e.g.). Estimates span the entire range of allowed values, but are typically in the intermediate range of $a_* \sim 0.7$, with uncertainties of $\sim 10\%$. This population would be an interesting comparison for potential measurements from nearby galaxies.

EMRBs could be an interesting signal for a space-borne gravitational wave detector, such as the *Laser Interferometer Space Antenna* (*LISA*; Bender et al. 1998; Danzmann & Rüdiger 2003) or the *evolved Laser Interferometer Space Antenna* (*eLISA*; Jennrich et al. 2011; Amaro-Seoane et al. 2012).[1] At the time of writing, there is no currently funded mission. However, *LISA Pathfinder*, a technology demonstration mission, is due for launch at the end of 2014 (Anza et al. 2005; Antonucci et al. 2012). Hopefully, a full mission shall follow in the subsequent decade. Since there does not exist a definite mission design, we stick to the classic *LISA* design for the majority of this work.

EMRB waveforms are calculated and analysed as in Berry & Gair (2012), and we give only an outline of the techniques used. Waveform construction and the numerical kludge approximation are explained in Sec. 2. The basics of signal analysis are introduced in Sec. 3. In Sec. 4, the detectability of EMRBs from extragalactic MBHs is discussed. We show that bursts from other galaxies could be detected with *LISA* or *eLISA*. Following this, in Sec. 5 we discuss the information that could be extracted from these signals, and the constraints these could place.

Throughout this work we adopt a metric with signature $(+, -, -, -)$. Greek indices are used to represent spacetime indices $\mu = \{0, 1, 2, 3\}$ and lowercase Latin indices from the middle of the alphabet are used for spatial indices $i = \{1, 2, 3\}$. Lowercase Latin indices from the beginning of the alphabet are used for parameter space. Summation over repeated indices is assumed unless explicitly noted otherwise. Geometric units with $G = c = 1$ are used where noted, but in general factors of G and c are retained.

2 WAVEFORM GENERATION

We employ a semirelativistic approximation (Ruffini & Sasaki 1981): the CO travels along a geodesic in Kerr spacetime, but radiates as if it were in flat spacetime. This approach is known as a numerical kludge (NK). Comparison with more accurate, and computationally intensive, methods has shown that NK waveforms are reasonably accurate for extreme-mass-ratio systems (Gair et al. 2005; Babak et al. 2007): typical errors can few percent (Tanaka et al. 1993; Gair et al. 2005; Berry & Gair 2012). Binding the motion to a true geodesic ensures the signal has the correct frequency components, although neglecting the effects of background curvature ensures that these do not have the correct amplitudes. The geodesic parameters are kept fixed throughout the orbit, as there should be negligible evolution due to the emission of gravitational radiation.

All bursts are assumed to come from marginally bound, or parabolic, orbits. In this case, the CO starts at rest at infinity and has a single passage through periapsis. If the periapse radius is small enough, the orbit may still complete a number of rotations about the MBH; these are zoom-whirl orbits (Glampedakis & Kennefick 2002).

When integrating the Kerr geodesic equations, we use angular variables instead of the radial and polar Boyer-Lindquist coordinates (Drasco & Hughes 2004)

$$r = \frac{2r_{\mathrm{p}}}{1 + \cos\psi}; \tag{2}$$

$$\cos^2\theta = \frac{Q}{Q + L_z^2}\cos^2\chi = \sin^2\iota\cos^2\chi, \tag{3}$$

where Q is the Carter constant, L_z is the angular momentum about the z-axis and ι is the orbital inclination (Glampedakis, Hughes, & Kennefick 2002). This parametrization avoids complications associated with turning points of the motion.

Once the geodesic is constructed, we identify the Boyer-Lindquist co-ordinates with flat-space spherical polars (Gair et al. 2005; Babak et al. 2007). This choice is not unique, as a consequence of the arbitrary nature of the NK approximation. Using flat-space oblate spheroidal coordinates gives quantitatively similar results (Berry & Gair 2012). The quadrupole-octupole formula is used to derive the gravitational strain (Bekenstein 1973; Press 1977; Yunes et al. 2008). The inclusion of higher order terms modify the amplitudes of some frequency components for the more relativistic orbits by a few tens of percent.

3 SIGNAL ANALYSIS

IN this section we briefly cover the basics of GW analysis. A more complete discussion can be found in Finn (1992) and Cutler & Flanagan (1994). Those familiar with the subject may skip this section with impunity. In the following, uppercase Latin indices from the beginning of the alphabet are used for labelling detectors: $A = \{\mathrm{I}, \mathrm{II}\}$ for *LISA*, but $A = \{\mathrm{I}\}$ for *eLISA* which only has two arms, and so acts as a single detector.

The measured strain $\boldsymbol{s}(t)$ is the combination of the signal and the detector noise

$$\boldsymbol{s}(t) = \boldsymbol{h}(t) + \boldsymbol{n}(t); \tag{4}$$

we assume the noise is stationary and Gaussian, and noise in multiple data channels uncorrelated, but shares the same characterisation (Cutler 1998). We can then define a signal inner product(Cutler & Flanagan 1994)

$$(\boldsymbol{g}|\boldsymbol{k}) = 2\int_0^\infty \frac{\tilde{g}_A^*(f)\tilde{k}_A(f) + \tilde{g}_A(f)\tilde{k}_A^*(f)}{S_{\mathrm{n}}(f)}\,\mathrm{d}f, \tag{5}$$

introducing Fourier transforms

$$\tilde{g}(f) = \mathscr{F}\{g(t)\} = \int_{-\infty}^{\infty} g(t)\exp(2\pi i f t)\,\mathrm{d}t, \tag{6}$$

[1] The revised *eLISA* concept is the same revised design as the *New Gravitational-wave Observatory* (*NGO*) submitted to the European Space Agency for their L1 mission selection.

and $S_{\mathrm{n}}(f)$ is the noise spectral density. We use the noise model of Barack & Cutler (2004) for *LISA*, and the simplified sensitivity model from Jennrich et al. (2011) for *eLISA*.

The signal-to-noise ratio (SNR) is

$$\rho[\boldsymbol{h}] = (\boldsymbol{h}|\boldsymbol{h})^{1/2}. \tag{7}$$

The probability of a realization of noise $\boldsymbol{n}(t) = \boldsymbol{n}_0(t)$ is

$$p(\boldsymbol{n}(t) = \boldsymbol{n}_0(t)) \propto \exp\left[-\frac{1}{2}\left(\boldsymbol{n}_0|\boldsymbol{n}_0\right)\right]. \tag{8}$$

Therefore, if the incident waveform is $\boldsymbol{h}(t)$, the probability of measuring signal $\boldsymbol{s}(t)$ is

$$p(\boldsymbol{s}(t)|\boldsymbol{h}(t)) \propto \exp\left[-\frac{1}{2}\left(\boldsymbol{s}-\boldsymbol{h}|\boldsymbol{s}-\boldsymbol{h}\right)\right]. \tag{9}$$

4 DETECTABILITY

The detectability of a burst is determined upon its SNR. We assume a detection threshold of $\rho = 10$. The SNR of an EMRB depends upon many parameters. For a given MBH, the most important is the periapse radius r_{p}. There is a good correlation between ρ and r_{p}; other parameters specifying the inclination of the orbit or the orientation of the system with respect to the detector only produce scatter around this relation. The form of the ρ–r_{p} relation depends upon the noise curve.

We parametrize the detectability in terms of a characteristic frequency

$$f_* = \sqrt{\frac{GM}{r_{\mathrm{p}}^3}}. \tag{10}$$

This allows comparison between different systems where the same periapse does not correspond to the same frequency, and thus the same point of the noise curve.

We also expect the SNR to scale with other quantities. Let us define a characteristic strain amplitude for a burst h_0; we expect $\rho \propto h_0$, where the proportionality will be set by a frequency-dependent function than includes the effect of the noise curve. Assuming that the strain is dominated by the quadrupole contribution (Misner, Thorne, & Wheeler 1973, section 36.10; Hobson et al. 2006, section 17.9)

$$h_0 \sim \frac{G}{c^6}\frac{\mu}{R}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(r^2\right), \tag{11}$$

where μ is the CO mass, R is the distance to the MBH, t is time and r is a proxy for the position of the orbitting object. The characteristic rate of change is set by f_* and the characteristic length scale is set by r_{p}. Hence

$$h_0 \sim \frac{G}{c^6}\frac{\mu}{R}f_*^2 r_{\mathrm{p}}^2 \tag{12}$$

$$\sim \frac{G^{5/2}}{c^6}\frac{\mu}{R}f_*^{-2/3}M^{2/3}. \tag{13}$$

Using this, we can factor out the most important dependencies to give a scaled SNR

$$\rho_* = \left(\frac{\mu}{M_\odot}\right)^{-1}\left(\frac{R}{\mathrm{Mpc}}\right)\left(\frac{M}{10^6 M_\odot}\right)^{-2/3}\rho. \tag{14}$$

Space-based detectors are most sensitive from extreme-mass-ratio signals originating from MBHs with masses 10^5–10^6. Higher mass objects produce signals at too low frequencies. We considered several nearby MBHs that were likely candidates for detectable burst signals. Details are given in Table 1. For each, we calculated SNRs at $\sim 10^4$ different periapse distances, uniformly distributed in log-space between the innermost orbit and $100r_{\mathrm{g}}$. Each had a spin and orbital inclination randomly chosen from distributions uniform in a_* and $\cos\iota$.[2] For every periapse, five SNRs were calculated, each having a different set of intrinsic parameters specifying the relative orientation of the MBH, the orbital phase and the position of the detector, drawn from appropriate uniform distributions. The scaled SNRs are plotted in Fig. 1. The plotted points are the average values of $\ln\rho_*$ calculated for each periapse distance. The curve shows that EMRB SNR does scale as expected, and ρ_* can be describe as a one parameter curve. There remains some scatter about this (removing the averaging over intrinsic parameters increases this to about an order of magnitude); however, it is good enough for rough calculations.

Figure 1. Scaled signal-to-noise ratio for EMRBs as a function of characteristic frequency.

$$\rho_* = \alpha_1 f_*^{\beta_1}\left[1 + (\alpha_2 f_*)^{\beta_2}\right]\left[1 + (\alpha_3 f_*)^{\beta_4}\right]^{-\beta_5} \tag{15}$$

5 PARAMETER INFERENCE

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[2] The innermost orbit depends upon $a_a st$ and ι, hence these are drawn first.

Table 1. Sample of nearby MBHs that are candidates for producing detectable EMRBs.

Galaxy	$M/10^6 M_\odot$	R/Mpc	References
Milky Way (MW)	4.31	0.00833	Gillessen et al. (2009)
M32	2.5	0.770	
Andromeda (M31)	140	0.770	
Circinus	1.1	2.82	Graham (2008); Greenhill et al. (2003); Karachentsev et al. (2007)
NGC 4945	1.4	3.82	Greenhill et al. (1997); Karachentsev et al. (2007)
Sculptor (NGC 253)	10	3.5	Graham et al. (2011); Rodríguez-Rico et al. (2006); Rekola et al. (2005)
NGC 3368	7.3	10.1	Graham et al. (2011); Nowak et al. (2010); Tonry et al. (2001)
NGC 3489	5.8	11.7	Graham et al. (2011); Nowak et al. (2010); Tonry et al. (2001)
NGC 4395	0.36	4.0	Peterson et al. (2005); Thim et al. (2004)

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