GL.cpl: stabilization of Ginzburg Landau Equation with BoostConv algorithm

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Abstract

Program GL evaluate the unstable steady state solution of the Ginzburg Landau equation via time integration and application of the BoostConv algorithm.

1 Formulation

The knowledge of fixed points or periodic solutions of a dynamical system is important both for stability analysis and the development of flow control strategies. The aim of the present example is to show how the BoostConv algorithm can be used to efficiently compute the unstable steady fixed points of the GL equation. More generally, the algorithm can be efficiently used to calculate both unstable fixed point or periodic orbit of low or high dimensional systems: for more details on the algorithm and its use see [1]. BoostConv is based on the minimization of the residual norm at each integration step and can be applied as a black-box procedure in any iterative or time marching algorithm without negatively impacting the computational time of the original code. Here we use it to stabilise the solution of the Ginzburg–Landau equation which is has been widely used int past to model vortex shedding phenomena in the wake of bluff bodies ([1, 2]) Following Chomaz et al. [2], Chomaz [3], Bagheri et al. [4], we write the Ginzburg–Landau equation as

$$\frac{\partial A}{\partial t} + v\frac{\partial A}{\partial x} - \gamma\frac{\partial^2 A}{\partial x^2} - \mu(x)A + |A|^2A = 0 \tag{1}$$

This is a convection–diffusion equation characterised by complex convection and diffusion coefficients $v = U + 2ic_u$ and a$\gamma = 1 + ic_d$. In order to model non-parallel flows, following Hunt & Crighton [6], we assume μ to be a quadratic function of the form $\mu(x) = (\mu_0 - c_u)^2 + \mu_2 x^2/2$. Further details can be found in Bagheri et al. [5], where the meaning of each term is carefully explained. In this example, we focus our attention on the system dynamics for $\gamma = 1 - i$, $\mu_0 = 0.52$, $\mu_2 = -0.01$ and $v = 2 + 0.2i$; With these parameters, depending on the values of μ_2, the equation may admit, none, one or multiple unstable eigenvalues: for the user's convenience the program, before computing the evolution, displays for the selected parameters the first 3 eigenvalues (which for an infinite domain are known in closed form [7]). The parameter μ_2 plays also another role: for large values of μ_2 in fact the system is strongly nonparallel but weakly non-normal, while for very small values of μ_2 the system represents weakly nonparallel but strongly non-normal flow. Here, a fourth order Runge–Kutta scheme is used here to march the equation in time. The equation is discretised using second order central differences with periodic boundary conditions. The grid is uniform in the central region [-Lx..Lx] where only nx points are used for discretization; outside this interval the grid is stretched with a constant stretching factor α. In the example the values $nx = 800$, $\alpha = 1.0035$ and $Lx = 20$ have been used. For such parameters the computational domain extends from $x = -80$ to $x = +80$. The initial condition is chosen to be a δ function centred at $x = 0$.

The program stores the results in two files, "field.out" and "residual.out": in the first the solution for the real and imaginary part of A are saved every $plotfreq$ time steps at each point of the spatial domain, while the second contains the infinity norm of the residual saved every *plotres* time steps. Application of *BoostConv* is controlled by the parameters *boost* and *tboost* that specify if the stabilisation algorithm has to be used and, in case, the time after which it has to be switched on.

Finally the program uses the generated files to produce two plots by using the *gnuplot* pipeline: the first graph shows a contour map of the space-time evolution of A_r, while the second represents the evolution of the residual norm in time. Example of typical results are given in the figures below.

In particular figure 1 shows the application of BoostConv algorithm to recover the fixed point for $\mu_2 = -0.01$. In the first part of the simulation ($t < 100$) the system naturally evolves towards a limit cycle. Once a saturated periodic solution is reached, we apply BoostConv ($t > 100$) to our time-integration scheme: results show that the stabilisation procedure is able to rapidly recover the fixed point of the Ginzburg–Landau equation. Figure 2 documents the evolution of the norm of the residual for two different cases: the parameter μ_2 is changed to investigate the effect of the system non-normality and the numbers of unstable modes on the stabilisation procedure. In particular values of -10^{-2} and -10^{-3} have been used, passing from a moderately to a highly non-normal system. In the second case, moreover, there are two unstable modes acting on the system dynamics. Results show that BoostConv is always able to stabilise the system but, as expected, the non-normality and the numbers of unstable modes influences the convergence rate of the algorithm.

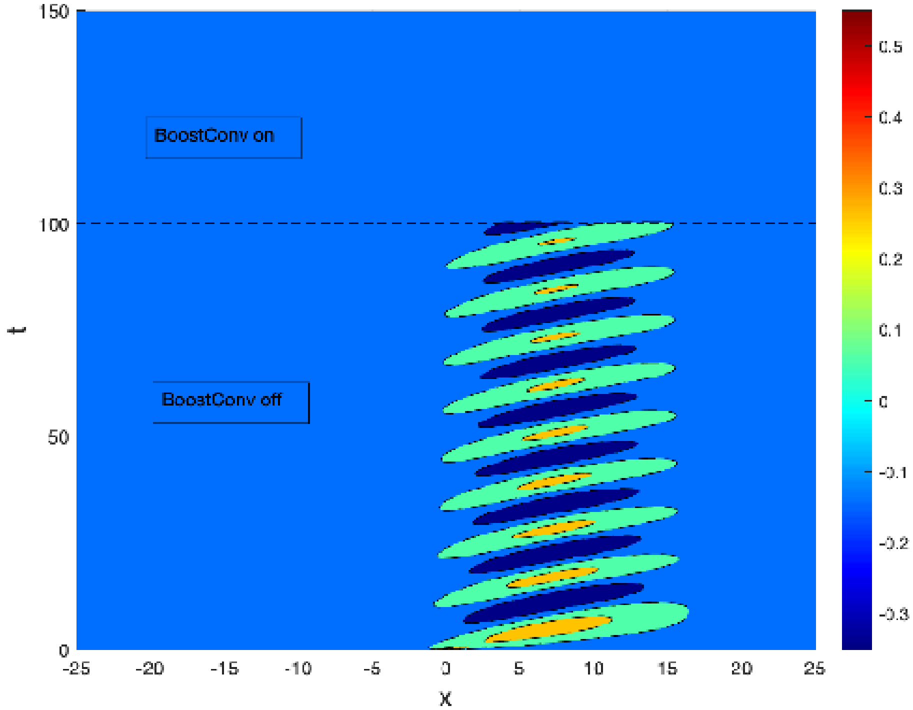

Figure 1: Stabilization of the Ginzburg–Landau equation using BoostConv. In the lower part of the figure (up to time t = 100), the system evolves toward a limit cycle. For $t > 100$, BoostConv algorithm is used to rapidly stabilize the system recovering its fixed point.

References

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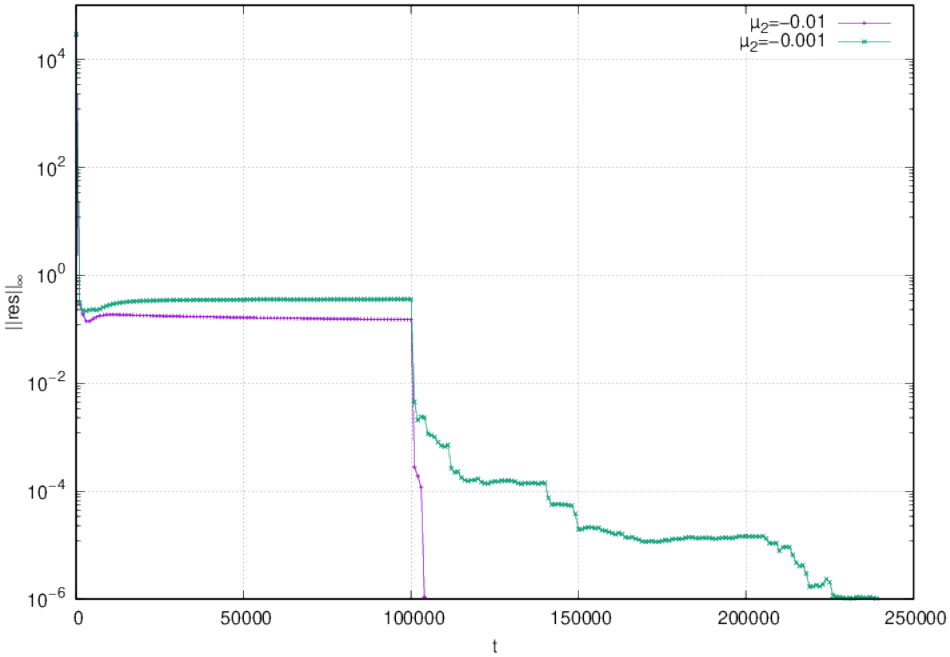

Figure 2: Evolution of the norm of the residual as a function of time for different values of μ_2.

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