Turbulent mean flow

Paolo Luchini
CPLcode.net

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Toolbox offering a mean turbulent velocity profile as a CPL FUNCTION that can be used for plotting or as a component of your own program, composed out of interpolations of the universal law of the wall and particular laws of the wake in various parallel geometries.

TurbMeanFlow.cpl provides the four functions defined in [2] for the interpolation of the universal law of the wall and of three different laws of the wake in turbulent parallel flow:

Universal wall function:

$$f(z^+) = \frac{\log(z^+ + 3.109)}{0.392} + 4.48 + \frac{7.3736 + (0.4930 - 0.02450z^+)z^+}{1 + (0.05736 + 0.01101z^+)z^+}\,\mathrm{e}^{-0.03385z^+}$$

Wake function for turbulent plane Couette flow:

$$G(Z) = (Z - 0.5)/(\exp(-25(Z - 0.5)) - 1)$$

Wake function for turbulent plane Poiseuille flow:

$$G(Z) = Z - 0.57Z^7$$

Wake function for turbulent circular pipe flow:

$$G(Z) = 2Z - 0.67Z^7$$

In the literature the universality of the mean component of turbulent parallel flow has been the subject of much debate. We have shown in [1, 2, 3] that, with an appropriate account of the matching between wall layer and defect layer, the classical theory of Prandtl, Von Kármán and Millikan can be restored to its full universality. In this approach a single, universal law of the wall is combined with a particular wake function for each different parallel flow, as shown in Figure 1. The accuracy of the wall function thus obtained is displayed in Figure 2.

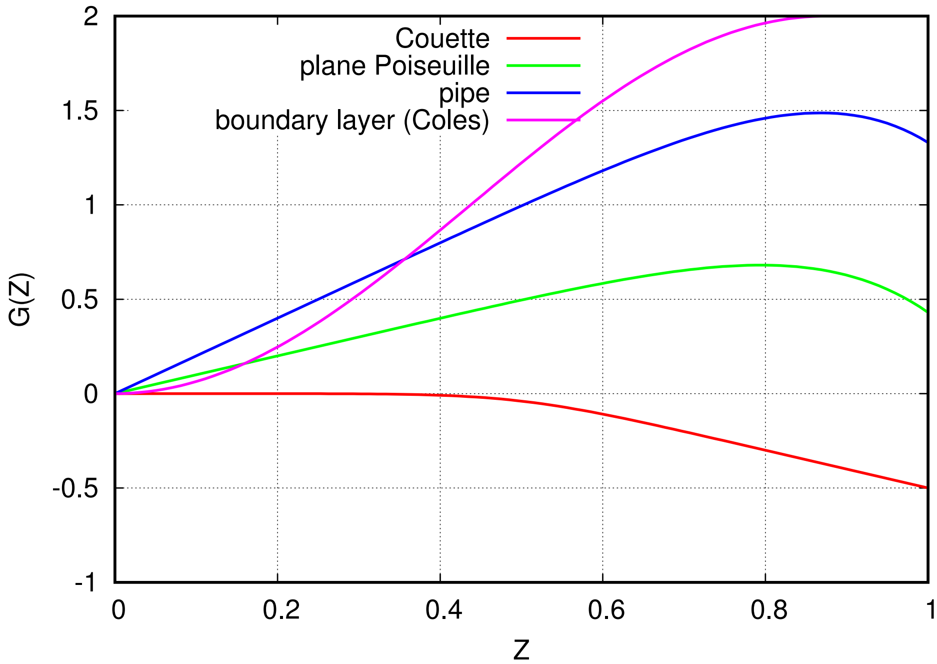

Figure 1: Wake functions of different geometries as extracted in [2] and displayed in [3].

These wake functions exhibit a linear leading term with respective coefficient 0, 1, 2 which equals the dimensionless pressure gradient, as predicted in [1]. From the functions provided by TurbMeanFlow.cpl the complete velocity profile in wall units can be reconstructed as

$$u^+ = \mathrm{wallaw}(z^+) + \mathrm{wake}(Z)$$

where $Z = z^+/\mathrm{Re}_\tau$ and wake can be cwake or dwake or pwake. The ability of this representation to closely match a wide range of existing numerical simulations and experiments was demonstrated in [2, 3].

TurbMeanFlow.cpl can be used in either compiled or interpreted CPL. For instance, in `icpl` you can plot the turbulent velocity profile in a pipe, say at $\mathrm{Re}_\tau = 1000$, by the commands

```
USE TurbMeanFlow
Retau=1000
plot [0:Retau] wallaw(x)+pwake(x/Retau) with lines
```

Alternately, you can `USE` these functions in your own CPL program.

References

[1] P. Luchini. Universality of the turbulent velocity profile. *Phys. Rev. Lett.* **118**, 224501 (2017). doi:10.1103/PhysRevLett.118.224501

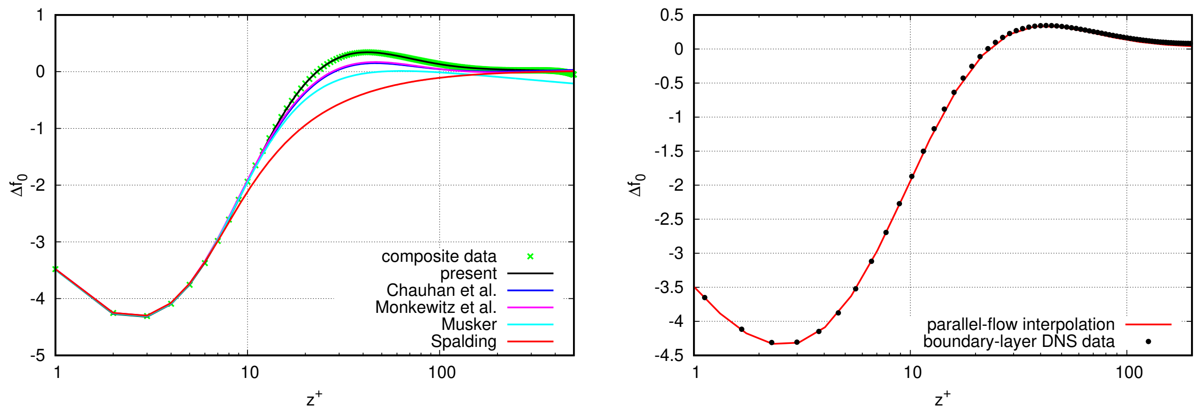

Figure 2: Various approximations of the law of the wall compared to parallel-flow data (left, from Fig. 31 of [2]), and to the boundary-layer DNS of [4] (right, from Fig. 33 of [2]). In ordinate is $\Delta f_0 = f(z^+) - \log(z^+)/0.392 - 4.48$.

[2] P. Luchini. Structure and interpolation of the turbulent velocity profile in parallel flow. *Eur. J. Mech. B/Fluids* **71**, 15–34 (2018). doi:10.1016/j.euromechflu.2018.03.006

[3] P. Luchini. Law of the Wall and Law of the Wake in Turbulent Parallel Flow. In: Örlü, R., Talamelli A., Peinke J., Oberlack M. (eds) Progress in Turbulence VIII. iTi 2018. Springer Proceedings in Physics, vol 226. Springer, Cham. doi:10.1007/978-3-030-22196-6_10

[4] J.A. Sillero, J. Jimenez, R.D. Moser (2013) One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$. *Phys. Fluids* **25**, 105102. doi:10.1063/1.4823831