Turbulent mean flow

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Toolkit offering a mean turbulent velocity profile as a CPL FUNCTION that can be used for plotting or as a component of your own program, comprising interpolations of the universal law of the wall and of particular laws of the wake in various parallel geometries.

TurbMeanFlow.cpl provides the five functions defined in [2, 4] for the interpolation of the universal law of the wall and of four different laws of the wake in turbulent parallel flow:

Universal wall function:

$$f(z^+) = \frac{\log(z^+ + 3.109)}{0.392} + 4.48 + \frac{7.3736 + (0.4930 - 0.02450z^+)z^+}{1 + (0.05736 + 0.01101z^+)z^+}\,\mathrm{e}^{-0.03385z^+}$$

Wake function for turbulent plane Couette flow:

$$G(Z) = (Z - 0.5)/(\exp(-25(Z - 0.5)) - 1)$$

Wake function for turbulent plane Poiseuille (closed channel) flow:

$$G(Z) = Z - 0.57Z^7$$

Wake function for turbulent circular pipe flow:

$$G(Z) = 2Z - 0.67Z^7$$

Wake function for turbulent open channel flow:

$$G(Z) = Z - 0.71Z^3$$

In the literature the universality of the mean component of turbulent parallel flow has been the subject of much debate. We have shown in [1, 2, 3, 4] that, with an appropriate account of the matching between wall layer and defect layer, the classical theory of Prandtl, Von Kármán and Millikan can be restored in its full universality. In this approach a single, universal law of the wall is combined with a particular wake function for each different parallel flow, as shown in Figure 1. The accuracy of the wall function thus obtained is displayed in Figure 2.

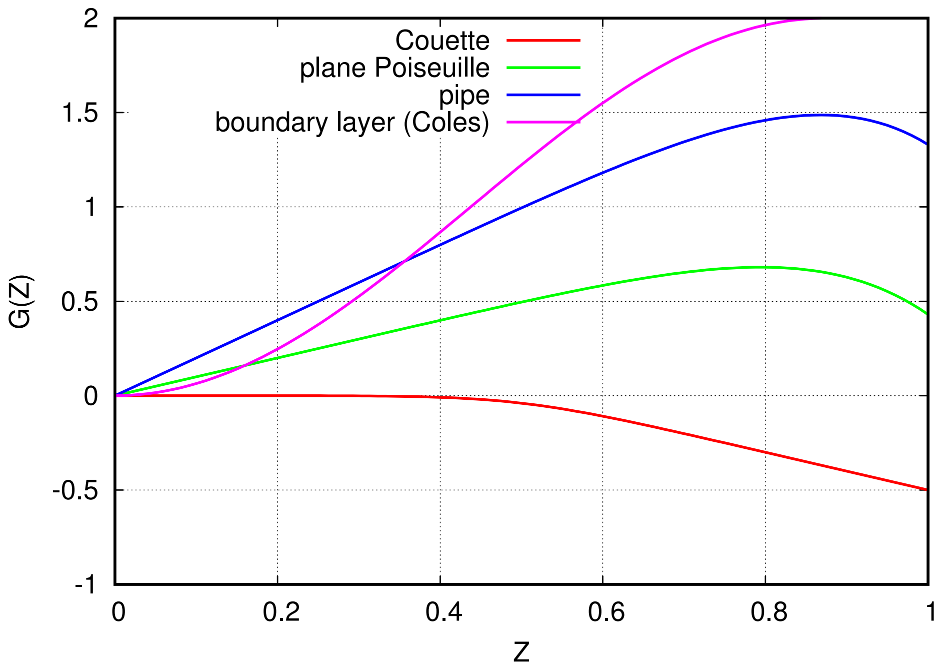

Figure 1: Wake functions of different geometries as extracted in [2] and displayed in [3].

These wake functions exhibit a linear leading term with respective coefficient 0, 1, 2 which equals the dimensionless pressure gradient, as predicted in [1]. From the functions provided by TurbMeanFlow.cpl the complete velocity profile in wall units can be reconstructed as

$$u^+ = \mathrm{wall}(z^+) + \mathrm{wake}(Z)$$

where $Z = z^+/\mathrm{Re}_\tau$ and wake can be cwake or dwake or pwake or ocwake. The ability of this representation to closely match a wide range of existing numerical simulations and experiments was demonstrated in [2, 3].

Additionally, TurbMeanFlow.cpl provides the friction laws described in §6 of [2], representing the head-loss coefficient f as a function of Re_τ through the functions pipefriction, Prandtlpipefriction and planeductfriction, and as a function of the bulk Reynolds number through function Moody.

TurbMeanFlow.cpl can be used in either compiled or interpreted CPL. For instance, in `icpl` you can plot the turbulent velocity profile in a pipe, say at $\mathrm{Re}_\tau = 1000$, by the commands

```
USE TurbMeanFlow
Retau=1000
plot [0:Retau] wall(x)+pwake(x/Retau) with lines
```

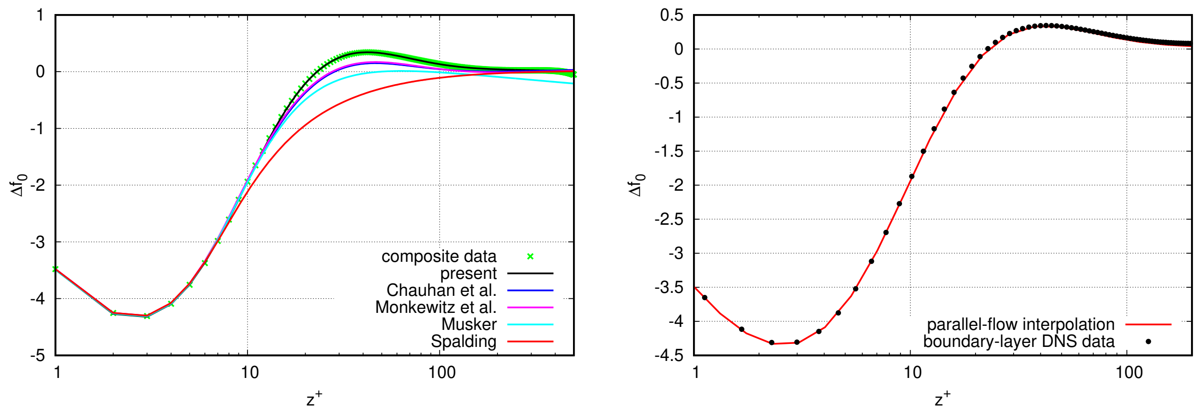

Figure 2: Various approximations of the law of the wall compared to parallel-flow data (left, from Fig. 31 of [2]), and to the boundary-layer DNS of [5] (right, from Fig. 33 of [2]). In ordinate is $\Delta f_0 = f(z^+) - \log(z^+)/0.392 - 4.48$.

Alternately, you can `USE` these functions in your own CPL program.

A gnuplot script TurbMeanFlow.gnu containing the same functions is also provided for convenience.

References

[1] P. Luchini (2017) Universality of the turbulent velocity profile. *Phys. Rev. Lett.* **118**, 224501. doi:10.1103/PhysRevLett.118.224501

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[3] P. Luchini (2019) Law of the Wall and Law of the Wake in Turbulent Parallel Flow. In: Örlü, R., Talamelli A., Peinke J., Oberlack M. (eds) Progress in Turbulence VIII. iTi 2018. Springer Proceedings in Physics, vol 226. Springer, Cham. doi:10.1007/978-3-030-22196-6_10

[4] P. Luchini (2023) Uniform representation of the turbulent velocity profile in an open channel. *arXiv preprint* arXiv:2310.11542 submitted to the Journal of Fluid Mechanics.

[5] J.A. Sillero, J. Jimenez, R.D. Moser (2013) One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$. *Phys. Fluids* **25**, 105102. doi:10.1063/1.4823831