Exploiting Channel Memory in Multi-User Wireless Scheduling without Channel Measurement: Capacity Regions and Algorithms

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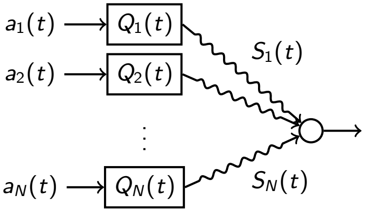
Q: How to exploit channel memory?

A Downlink Problem



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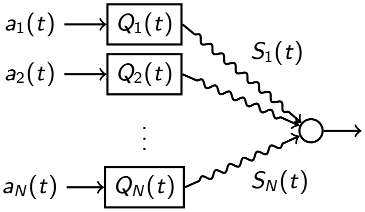

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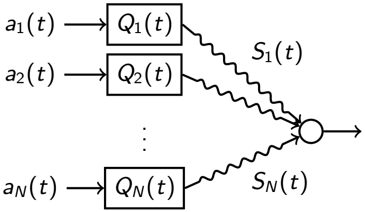

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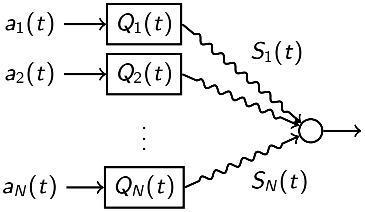

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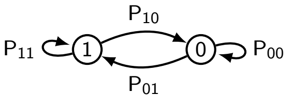

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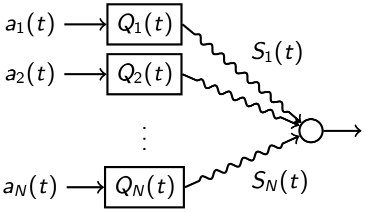

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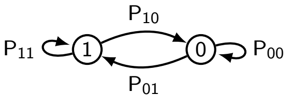

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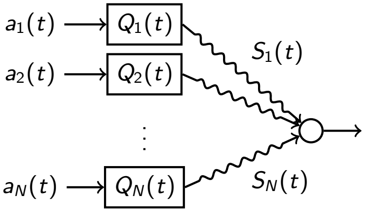

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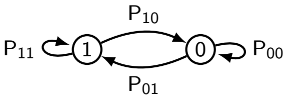

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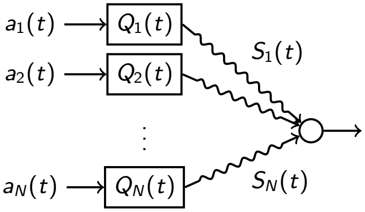

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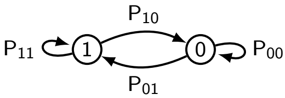

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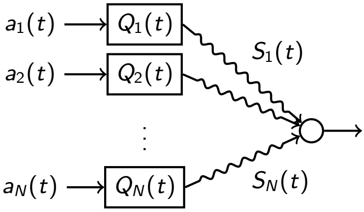

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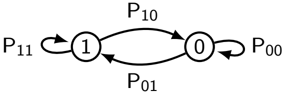
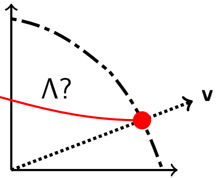


Goal: Network capacity region? Throughput-optimal policy?

Difficulty

- capacity region $\Lambda = N$-dimensional POMDP
- information state:

$$\omega_n(t) \triangleq \Pr\{S_n(t) = 1 \mid \text{observation history}\}$$



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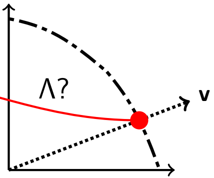
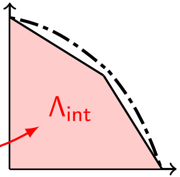
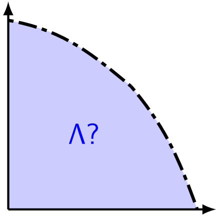


Idea

- forget POMDP
- Construct a good inner bound Λ_{int}



Road Map



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asymptotically tight when
(1) # of users is large
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For any known $\boldsymbol{\lambda}$ interior to Λ_{int} $\sim$ a polynomial-time algorithm

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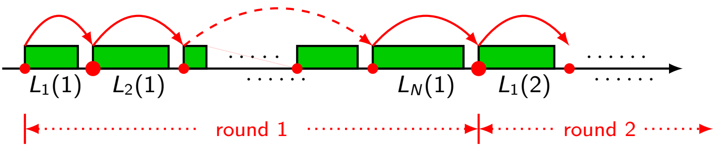
Round Robin (a building block)

Round robin v1:

- Serve users in circular order ($1 \to 2 \to \cdots \to N$); stay with each one until receiving a NACK.



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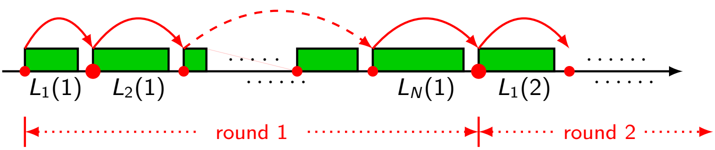

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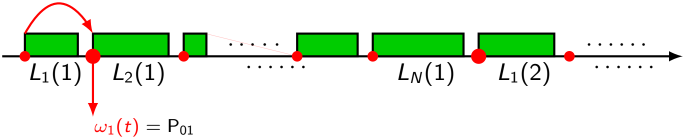

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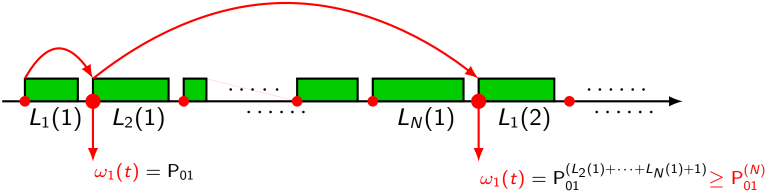

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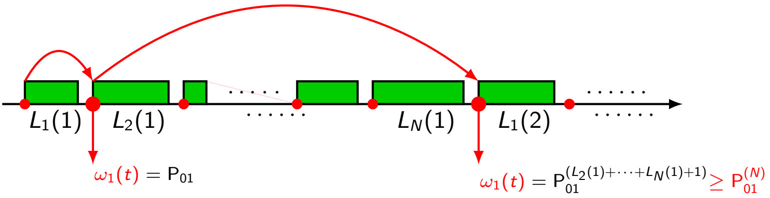

Round robin v2:

- When switch to a new channel, set $\omega_n(t) = \mathrm{P}_{01}^{(N)}$.

Throughput of RRv2

Theorem

Sum throughput of RRv2 is

$$c_N \triangleq \frac{\mathrm{P}_{01}(1-(1-x)^N)}{x\mathrm{P}_{10}+\mathrm{P}_{01}(1-(1-x)^N)}, \quad x \triangleq \mathrm{P}_{01}+\mathrm{P}_{10}<1,$$

and each user shares c_N/N.

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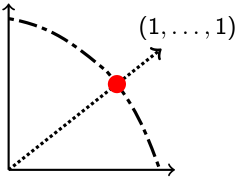
"asymptotically optimal for symmetry traffic"



Inner Capacity Bound

Randomized RRv2:

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- $\phi \sim N$-dim binary vector; user n is active if nth entry is 1.
- $M(\phi) \sim$ number of active users in ϕ.

$$\underbrace{\frac{c_{M(\phi)}}{M(\phi)}}_{\text{per-user throughput}} \times \phi$$

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- Random mixing $\sim$ time sharing

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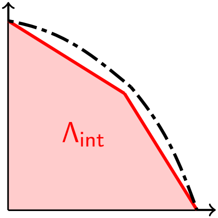
Theorem

The capacity region under randomized RRv2 is

$$\Lambda_{int} = \left\{ \boldsymbol{\lambda} \mid \boldsymbol{\lambda} \leq \boldsymbol{\mu},\ \boldsymbol{\mu} \in \mathrm{conv}\left(\left\{ \frac{c_{M(\phi)}}{M(\phi)} \phi \right\} \right) \right\}.$$



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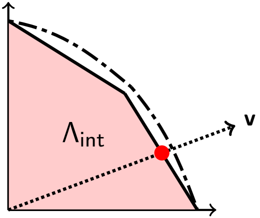
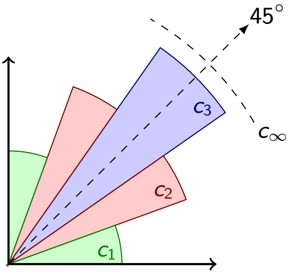


Lemma

If a directional vector $\mathbf{v} \sim$ *positive combination of binary vectors having* n *ones, then in that direction:*

max sum throughput $\geq c_n$

sum throughput loss $\leq c_\infty (1-x)^n$



Two-User Example

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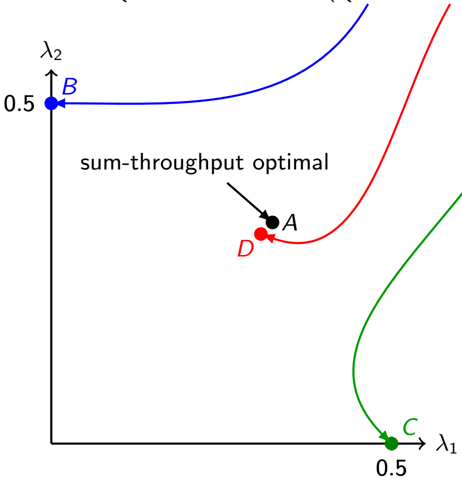

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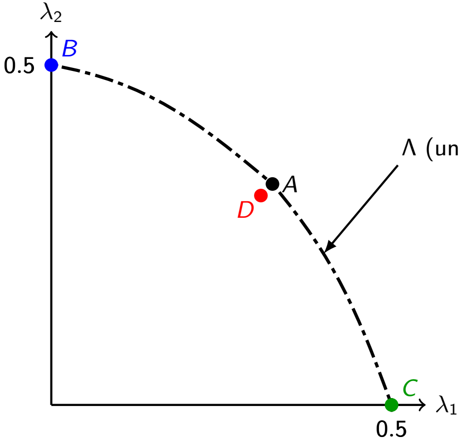

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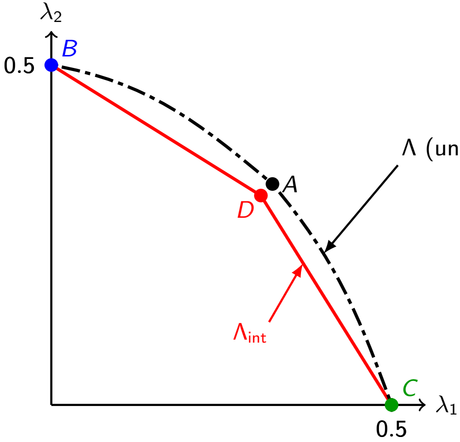

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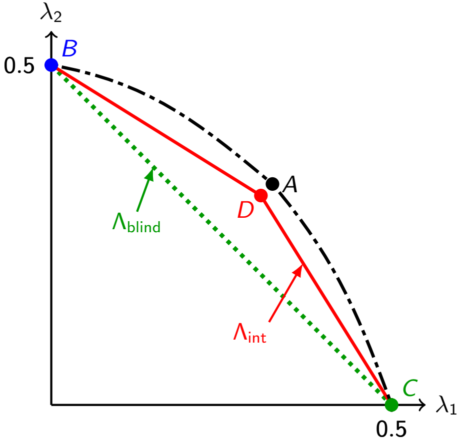

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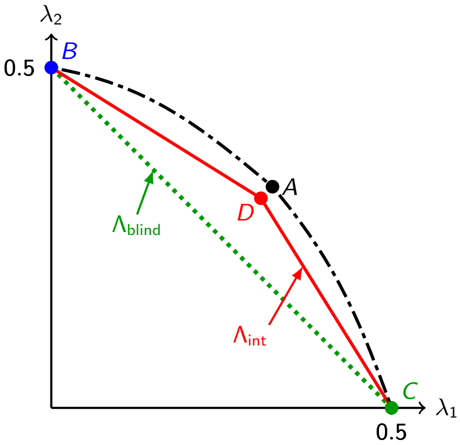

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  $$\text{sum throughput} \le \pi_{\text{ON}}$$

- Max throughput gain from channel memory:

  $$c_N - \pi_{\text{ON}} \xrightarrow{N \to \infty} \frac{P_{01}}{x\, P_{10} + P_{01}} - \frac{P_{01}}{P_{10} + P_{01}}$$

  $$x \triangleq P_{01} + P_{10}$$

Simple MaxWeight policy

Stabilize any $\boldsymbol{\lambda} \in \Lambda_{\text{int}}$:

- find a proper random mixture of $(2^N - 1)$ RRv2 policies so that $\boldsymbol{\mu} > \boldsymbol{\lambda}$
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Q-dependent dynamic round robin (QRR)

1. Observe $Q_n(t)$, and find ϕ^* that maximizes

$$f(\overrightarrow{Q}(t)) \triangleq \sum_{n:\text{active}} \left[\frac{Q_n(t) \mathsf{P}_{01}^{(M(\phi))}}{\mathsf{P}_{10}} - \frac{\mathsf{P}_{10} + \mathsf{P}_{01}^{(M(\phi))}}{\mathsf{P}_{10}} \sum_{n=1}^{N} Q_n(t) \lambda_n \right]$$

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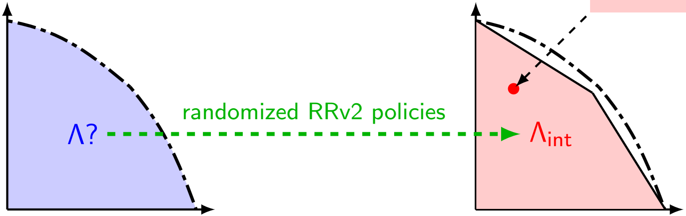
Theorem

For any $\boldsymbol{\lambda}$ interior to Λ_{int}, policy QRR *stabilizes the network.*

- proved by a frame-based variable-length Lyapunov drift argument
- QRR $\sim$ polynomial time algorithm

Wrap Up

QRR policy stabilizes the network for all $\boldsymbol{\lambda}$ interior to Λ_{int}.



network capacity region $\sim$ POMDP

asymptotically tight when
(1) # of users is large
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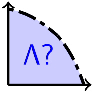
$$\text{max channel memory gain} = \frac{P_{01}}{x\,P_{10} + P_{01}} - \frac{P_{01}}{P_{10} + P_{01}} \quad (x = P_{01} + P_{10} < 1)$$

Applications / Future Work

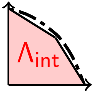
- Channel measurement and delayed information in multi-user wireless scheduling
  - Limited channel probing
  - Other QoS metrics
- Opportunistic spectrum access in cognitive radio networks
  - State of the art is POMDP for single-user case. Lots of potentials here!
- How channel memory can help in modern network protocols
  - Random access (e.g. CSMA) in wireless networks?
- Transform POMDP and restless bandit into stochastic network control problems

A New Methodology for Restless Bandit

In this paper:



all feasible solutions
to a restless bandit



feasible solutions
of practical interest

Let $\boldsymbol{\lambda}^*$ be the optimal solution to a restless bandit problem over Λ_{int}.
If $\boldsymbol{\lambda}^*$ is known, QRR achieves it in the network.

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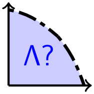
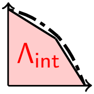


all feasible solutions to a restless bandit



feasible solutions of practical interest

Let $\boldsymbol{\lambda}^*$ be the optimal solution to a restless bandit problem over Λ_{int}.
If $\boldsymbol{\lambda}^*$ is known, QRR achieves it in the network.

What if....

.... a QRR-like policy achieves $\boldsymbol{\lambda}^*$ without knowing it in advance, then we are actually solving (approximately) a high-dimensional restless bandit!

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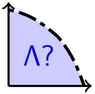

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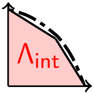

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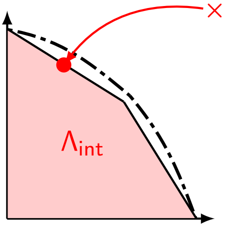

- pour unlimited traffic into the network
- perform admission control and utility maximization [Neely, Modiano, Li, ToN'08] to let the network learn the optimal solution