Searching 2D-Strings for Matching Frames

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The Maximum Matching Frame Problem

- Input: an $n \times n$ matrix M over an alphabet Σ
- Output: a maximum perimeter matching frame (u, d, ℓ, r)

0	n	V	W	l	a	m	Į	İ	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
Ο	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	i	t	е	r	a	l	S	a	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	а	t	O	t	У	O

О	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
Ο	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	i	t	е	r	a	l	S	a	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

Perimeter = $2(r - \ell + 1) + 2(d - u + 1) - 4$

0	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	i	t	е	r	a	l	S	a	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

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Perimeter = $2(r - \ell + 1) + 2(d - u + 1) - 4$ $\ell \qquad \qquad r$

O	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	i	t	е	r	a	l	S	a	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

Maximum Matching Frame – Our Results

- Exact $\tilde{O}(n^{2.5})$ time
- $(1-\epsilon)$ -approximation $\tilde{O}\left(\frac{n^2}{\epsilon^4}\right)$ time

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$$LCP_j(a,b) = 5$$

o n v w l a m l i s a c
r a l i t e r a l s s e
m p a e r s y a a u c t
o r b n e o h q b u e v
l l e e n a g n e o n q
b u e l i t e r a l s a c
d v r a l n t n e o n m
s e m e t k a t o t y o

reverse–
$$LCP_j(a,b) = 4$$

 a
 b

 o n v w l a m l i s a c

 r a l i t e r a l s s e

 m p a e r s y a a u c t

 o r b n e o h q b u e v

 l l e e n a g n e o n q

 u e l i t e r a l s a c

 d v r a l n t n e o n m

 s e m e t k a t o t y o

		a						b			
О	n	V	W	l	а	m	l	İ	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	а	а	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	0	n	q
u	е	ι	İ	t	е	r	а	l	S	a	С
d	V		а	l	n	t	n	е	O	n	m
S	е	m	е	t	k	а	t	O	t	У	O

$$LCP_i(a,b) = 2$$

		a						b			
0	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	а	l	S	S	е
m	p	а	е	r	S	У	а	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	i	t	е	r	а	l	S	a	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	0

reverse-
$$LCP_i(a, b) = 4$$

- Construction: $\tilde{O}(n^2)$ time
- Query: O(1) time

Tools – Fingerprints

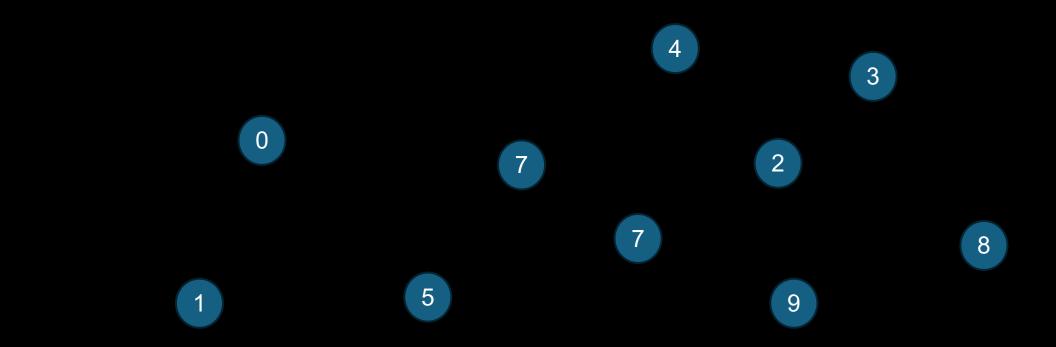
- fingerprint(S_1) = fingerprint(S_2) $\leftrightarrow S_1 = S_2$
- |fingerprint(x)| = O(1)
- O(1) to compute

		ш						U			
O	n	V	W	l	а	m	l	i	S	а	С
r	а	l	İ	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
0	r	b	n	е	0	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	l	İ	t	е	r	а	l	S	a	С
d	V	r	а	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	0	t	У	0

j

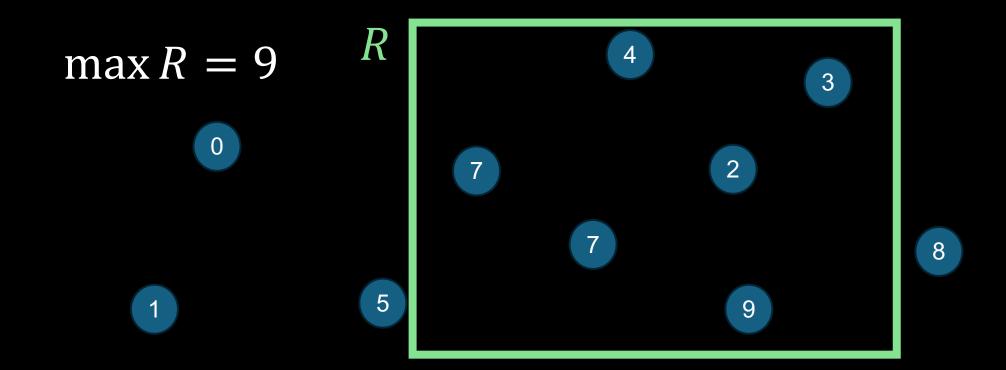
Tools – Orthogonal Range Queries

- $P \subseteq \mathbb{R}^d$ of d-dimensional points, each point p has value v(p)
- ORQDS supports max(R)



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Tools – Orthogonal Range Queries

- $P \subseteq \mathbb{R}^d$ of d-dimensional points, each point p has value v(p)
- ORQDS supports max(R)
- Construction time $\tilde{O}(|P|)$
- Query time $\tilde{O}(1)$

Maximum Matching Frame – Our Results

- Exact $\tilde{O}(n^{2.5})$ time
- $(1-\epsilon)$ -approximation $\tilde{O}\left(\frac{n^2}{\epsilon^4}\right)$ time

Maximum Matching Frame – Our Results

- Exact $\tilde{O}(n^{2.5})$ time
- $(1-\epsilon)$ -approximation $\tilde{O}\left(\frac{n^2}{\epsilon^4}\right)$ time

- Heavy light approach
- $\tilde{O}(n^{2.5})$ time for maximum short matching frame
 - Short: height **or** width $\leq \sqrt{n}$
- $\tilde{O}(n^{2.5})$ time for maximum tall matching frame
 - Tall: otherwise
- $\Rightarrow \tilde{O}(n^{2.5})$ time for maximum matching frame

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- $\widetilde{o}(n^{2.5})$ time for maximum short matching frame
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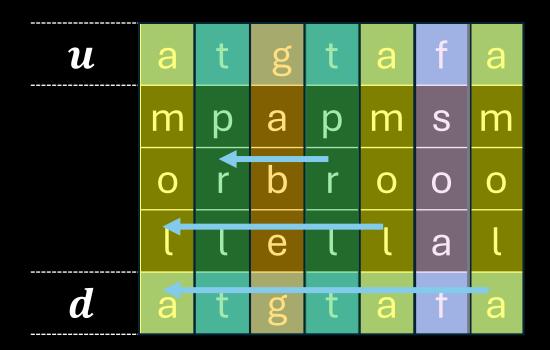
• Let $u, d \in [n]$ be two rows with distance $\leq \sqrt{n}$

1	u	a	t	g	t	a	f	a	Z	f	O	r	O
		m	p	а	p	m	S	m	a	p	u	С	u
$\leq \sqrt{n}$		O	r	b	r	O	O	O	q	r	u	е	u
		l	l	е	l	l	a	l	n	l	O	n	O
	d	a	t	g	t	a	f	a	a	W	O	r	O

• Decompose rows u, d into maximal equal segments

u	a	t	g	t	a	f	a	Z	f	О	r	O
	m	p	a	p	m	S	m	a	p	u	С	u
	Ο	r	b	r	O	O	O	q	r	u	е	u
	l	l	е	l	Į	а	l	n	l	O	n	O
d	a	t	g	t	a	f	a	а	W	O	r	O

- For every segment, find maximum matching frame
- For every column i, find a leftmost column i' < i such that the strings of i and i' match (using fingerprints)



- For every segment, find maximum matching frame
- For every column i, find a leftmost column i' < i such that the strings of i and i' match

u	a	t	g	t	а	f	a
	m	p	a	р	m	S	m
	0	r	b	r	O	O	O
	l	l	е	l	l	а	l
d	а	t	g	t	а	f	а

- $\tilde{O}(n)$ per rows u, d
- $O(n^{1.5})$ pairs of u, d with distance $\leq \sqrt{n}$
- (same for columns ℓ , r with distance $\leq \sqrt{n}$)
- $\Rightarrow \tilde{O}(n^{2.5})$ time for maximum short matching frame

- Heavy light approach
- $\tilde{O}(n^{2.5})$ time for maximum short matching frame
 - Short: height **or** width $\leq \sqrt{n}$
- $\widetilde{o}(n^{2.5})$ time for maximum tall matching frame
 - Tall: otherwise
- $\Rightarrow \tilde{O}(n^{2.5})$ time for maximum matching frame

- Input: $H \times W$ sub-matrix M' and a position $(i, j) \in M'$
- Output: maximum matching frame that contains (i, j)

o n a w l a m l a s a c r d l i l e r a l s s e m p g e g s y a a u c t o r l i l o h l b u e v l l e e g w j g e o n q u d l i l o h l l s a c

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- Input: $H \times W$ sub-matrix M' and a position $(i, j) \in M'$
- Output: maximum matching frame that contains (i, j)

					j						
О	n	a	W	l	a	m	l	a	S	a	С
r	d	l	i	l	е	r	a	l	S	S	е
m	р	g	е	g	S	У	a	a	u	С	t
0	r	l	i	l	0	h	l	b	u	е	V
l	l	е	е	g	W	j	g	е	0	n	q
u	d	l	i	l	0	h	l	l	S	a	С

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- Input: $H \times W$ sub-matrix M' and a position $(i, j) \in M'$
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О	n	a	W	l	a	m	l	a	S	a	С
r	d	l	i	l	е	r	a	l	S	S	е
m	p	g	е	g	S	У	a	a	u	С	t
O	r	l	i	l	O	h	l	b	u	е	V
l	l	е	е	g	W	j	g	е	0	n	q
u	d	l	i	l	O	h	l	l	S	a	С

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a w l a m l S a n d a S m p a al U e a C b n e o h q b U e n a g n $o \mid n$ d a sla t | n | a S $o \mid n \mid m$ a

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a |w| a m l S n a d a S m p a u e S a a C b n e o h q b U e n a g n $o \mid n$ a sla tn a S $o \mid n \mid m$ a

a m l a |w| S n a d a S m p a u e a a C o h q b b n e U e n a g n $o \mid n$ a S a |t|n| a nn m 0 a

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W u'a m l S n a ra S m p e a u C o h q n e l u a g n $o \mid n$ a S a d<u>t</u> n a n m 0 a

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Add to **ORQDS**:

 (ℓ, r, u', d')

Value:

$$(r-\ell+1)$$

O	n	a	W	l	a	m	l	a	S	a	С
r	d	ι	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
O	r	b	n	е	0	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	d		i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

a w l S a m l a n a S m p a e a u S a C b n e o h q b U e n a g|n|e| $o \mid n$ da sla t | n | a S $o \mid n \mid m$ a

S a w l a m l a n a m p a a u e S a C b n e o h q b U e n a g n $o \mid n$ da sla t | n | a S $o \mid n \mid m$ a

a w l s a n S m p a a u C e S a b n e o h q b U e n a g n e o n da a tn a S $o \mid n \mid m$ a

a w l a m s a n m p a a u e S a C b n e o h q U $o \mid n$ da a tn a S n m 0 a

max query ORQDS

such that:

$$\ell' \leq \ell$$

 $r' \geq r$

$$u \ge u'$$

 $d \leq d'$

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0	n	a	W	l	a	m	l	a	S	a	С
r	d	l	i	t	е	r	a	ι	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	0	n	q
u	d	l	i	t	е	r	a	ι	S	a	С
d	V	S	a	l	n	t	n	S	0	n	m
S	е	m	е	t	k	a	t	O	t	У	0

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$$u \geq u'$$

$$d \leq d'$$

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О	n	а	W	l	a	m	l	a	S	a	С
r	d	l	i	t	е	r	a	l	S	S	e
m	p	а	е	r	S	У	a	a	u	С	t
O	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	0	n	q
u	d	l	i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

max query ORQDS such that: $\ell' \leq \ell$ $r' \geq r$ $u \ge u'$ $d \leq d'$ d'

a W l a m l $o \mid n$ sa d S m p a U a e SV a C n e o h q b U e e n a g n $o \mid n$ e d u a a d t | n a n n m 0 a

max query ORQDS u' such that: u

 $\begin{aligned}
 \ell' &\leq \ell \\
 r' &\geq r \\
 u &\geq u' \\
 d &\leq d'
 \end{aligned}$

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Ο	n	а	W	l	a	m	l	а	S	a	С
r	d	l	i	t	е	r	a	l	S	S	е
m	p	а	е	r	S	У	a	a	u	С	t
O	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	d	l	i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	0

max query ORQDS u' such that: u

$$\ell' \leq \ell$$

$$r' \geq r$$

$$u \geq u'$$

$$d \leq d'$$

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0	n	а	W	l	a	m	l	а	S	a	С
r	d	l	i	t	е	r	a	l	S	S	e
m	p	а	е	r	S	У	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
	l	е	е	n	a	g	n	е	0	n	q
u	d	l	i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	0

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d'

max query ORQDS u' such that: u $\ell' \leq \ell$ $r' \geq r$ $u \geq u'$ $d \leq d'$ i d

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max query ORQDS u' such that: u

 $\ell' \leq \ell$ $r' \geq r$ $u \geq u'$ $d \leq d'$

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O	n	а	W	l	a	m	l	а	S	a	С
r	d	l	i	t	е	r	a	l	S	S	е
m	p	а	е	r	S	У	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	0	n	q
u	d	l	i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	О

max query ORQDS u' such that:

$$\ell' \leq \ell$$

$$r' \geq r$$

$$u \geq u'$$

$$d \leq d'$$

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O	n	a	W	l	a	m	l	a	S	a	С
r	d	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	У	a	a	u	С	t
O	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	0	n	q
u	d	l	i	t	е	r	a	l	S	a	С
d	V	S	a	l	n	t	n	S	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

$$\tilde{O}(H^2)$$

$$r-\ell+1$$

• $\tilde{O}(H^2 + W^2)$ for M' and (i, j)

- exponential guesses of H, W from \sqrt{n} to n
 - $O(\log^2 n) = \tilde{O}(1)$ guesses
- $O\left(\frac{n^2}{HW}\right)$ (i, j) positions for each H, W guess
 - such that every matching frame of size $[H/2, H] \times [W/2, W]$ contains a position from the set.

H	6
W	: 8

0	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	р	a	е	r	S	b	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	d	i	t	е	е	a	l	S	f	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

H=6

W = 8

H/2 = 3W/2 = 4

H/2	=	3
W/2		4

O	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	p	a	е	r	S	b	a	a	u	С	t
O	r	b	n	е	O	h	q	b	u	е	V
l	l	е	е	n	a	g	n	е	O	n	q
u	е	d	i	t	е	е	a	l	S	f	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	0

H/2	=	3
W/2	=	4

0	n	V	W	l	a	m	l	i	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	р	a	е	r	S	b	a	a	u	С	t
0	r	b	n	е	O	h	q	b	u	е	V
	l	е	е	n	a	g	n	е	0	n	q
u	е	d	i	t	е	е	a	l	S	f	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

H/2	=	3
W/2	=	4

С	е	t	V	q	С	m	О
a	S	С	е	n	f	n	У
S	S	u	u	0	S	O	t
i	l	а	b	е	l	е	O
l	a	а	q	n	a	n	t
m	r	b	h	g	е	t	a
a	е	S	O	a	е	n	k
l	t	r	е	n	t	Į	t
W	i	е	n	е	İ	a	е
V	l	a	b	е	d	r	m
n	a	p	r	l	е	V	е
O	r	m	O	l	u	d	S

H/2		3
W/2	=	4

0	n	V	W	l	a	m	l	•	S	a	С
r	a	l	i	t	е	r	a	l	S	S	е
m	р	a	е	r	S	b	a	а	u	С	t
O	r	b	n	е	O	h	q	b	u	е	V
l	l	е	e	n	a	g	n	е	O	n	q
u	е	d	i	t	е	е	a	l	S	f	С
d	V	r	a	l	n	t	n	е	O	n	m
S	е	m	е	t	k	a	t	O	t	У	O

- $\tilde{O}(H^2 + W^2)$ for M' and (i, j)
- exponential guesses of H,W from \sqrt{n} to n
 - $O(\log^2 n) = \tilde{O}(1)$ guesses
- $O\left(\frac{n^2}{HW}\right)$ (i, j) positions for each H, W guess
 - such that every matching frame of size $[H/2, H] \times [W/2, W]$ contains a position from the set.

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 - such that every matching frame of size $[H/2, H] \times [W/2, W]$ contains a position from the set.

Total runtime:

$$\tilde{O}\left(\frac{n^2}{HW}\cdot (H^2+W^2)\right)$$

• For (worst case)
$$H = \sqrt{n}$$
 and $W = n$:
$$\tilde{O}\left(\frac{n^2}{n\sqrt{n}}\cdot\left(n^2+\sqrt{n}^2\right)\right) = \tilde{O}(n^{2.5})$$

- Heavy light approach
- $\tilde{O}(n^{2.5})$ time for maximum short matching frame
 - Short: height **or** width $\leq \sqrt{n}$
- $\tilde{O}(n^{2.5})$ time for maximum tall matching frame
 - Tall: otherwise
- \Rightarrow $\widetilde{O}(n^{2.5})$ time for maximum matching frame

Maximum Matching Frame – Our Results

- Exact $\tilde{O}(n^{2.5})$ time
- (1ϵ) -approximation $\tilde{O}\left(\frac{n^2}{\epsilon^4}\right)$ time
 - This talk: $\varepsilon = O(1)$

$\tilde{O}(n^2)$ Time For $(1 - \varepsilon)$ -approximation

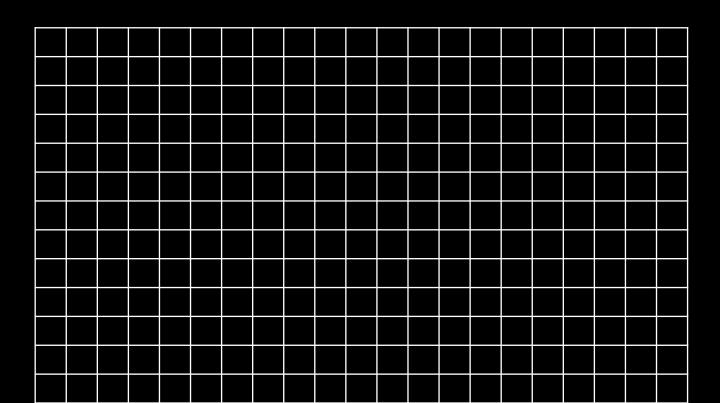
- $\tilde{O}(1)$ exponential guesses of H, W from 1 to n
- For each guess H, W, there are $O\left(\frac{n^2}{HW}\right)$ submatrices
- For each submatrix, find a big matching frame in time $\tilde{O}(HW)$
- \Rightarrow $\tilde{O}(n^2)$ algorithm for $(1-\varepsilon)$ -approximation to a maximum matching frame

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- For each guess H, W, there are $O\left(\frac{n^2}{HW}\right)$ submatrices
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- \Rightarrow $\tilde{O}(n^2)$ algorithm for $(1-\varepsilon)$ -approximation to a maximum matching frame

Find a Big Matching Frame

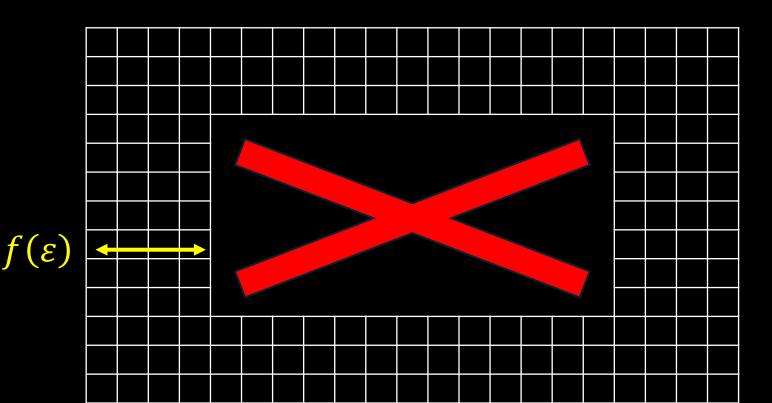
- Input: an $H \times W$ submatrix M'
- Output: a matching frame with sides close to the boundary of M'



Find a Big Matching Frame

- Input: an $H \times W$ submatrix M'
- Output: a matching frame with sides close to the boundary of M'

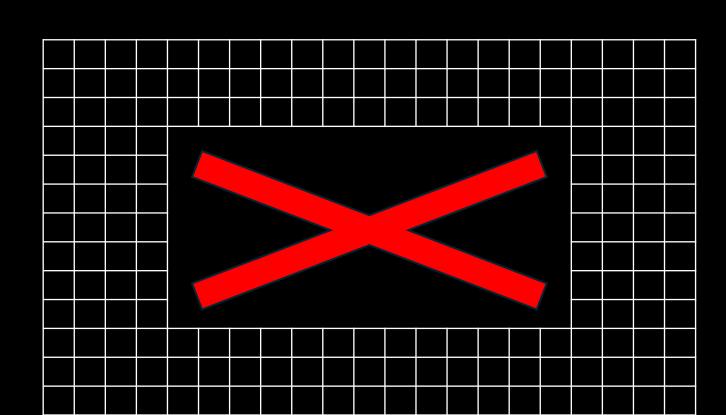
$$M'[i,j] \leftarrow \$_{i,j}$$



Find a Surrounding Matching Frame

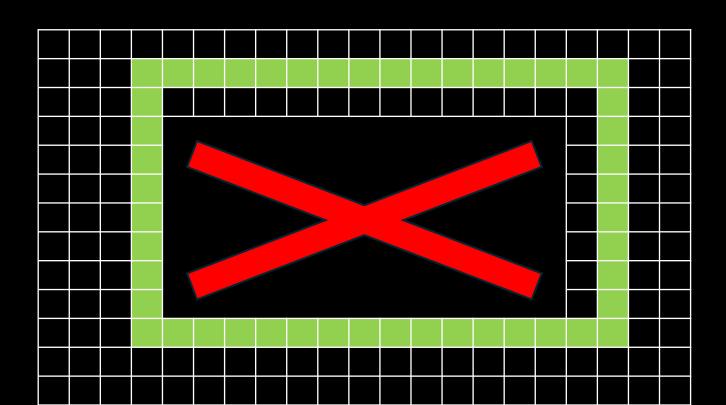
- Input: an $H \times W$ submatrix M'
- Output: a surrounding matching frame

$$M'[i,j] \leftarrow \$_{i,j}$$



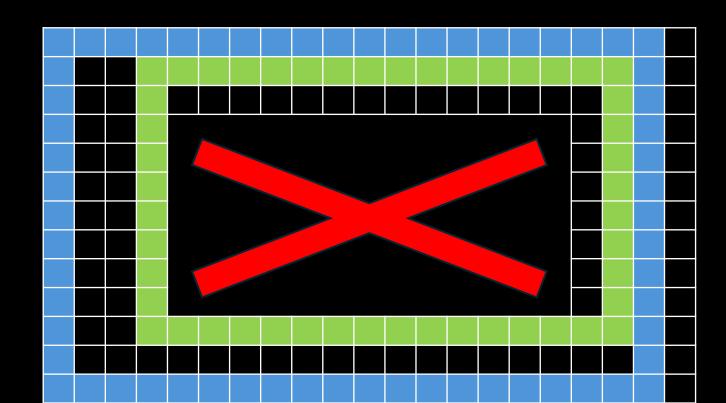
Find a Surrounding Matching Frame

- Input: an $H \times W$ submatrix M'
- Output: a surrounding matching frame



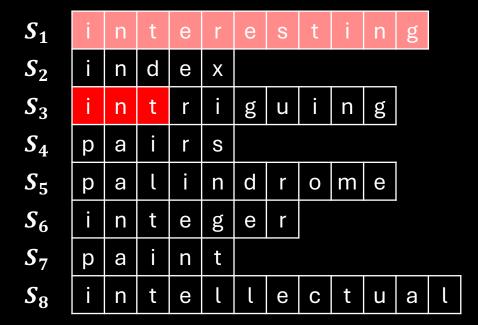
Find a Surrounding Matching Frame

- Input: an $H \times W$ submatrix M'
- Output: a surrounding matching frame



S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
S_3	i	n	t	r	i	g	u	i	n	g		
S_4	р	а	i	r	S							
S_5	р	а	l	i	n	d	r	O	m	е		
S_6	i	n	t	е	g	е	r					
<i>S</i> ₇	р	а	i	n	t							
S_8	i	n	t	е	ι	l	е	С	t	u	а	

S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
S_3	i	n	t	r	i	g	u	i	n	g		
S_4	р	а	i	r	S							
S_5	р	а	l	i	n	d	r	O	m	е		
S_6	i	n	t	е	g	е	r					
<i>S</i> ₇	р	а	i	n	t							
S_8	i	n	t	е	ι	ι	е	С	t	u	а	



S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
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S_5	р	а	l	i	n	d	r	Ο	m	е		
S_6	i	n	t	е	g	е	r					
<i>S</i> ₇	р	а	i	n	t							
S_8	i	n	t	е	ι	ι	е	С	t	u	а	l

• <u>Definition</u>: Given a tuple $(S_1, ..., S_m)$ of strings, we call a pair (i, j) interesting if for any ℓ such that $\ell \in [i + 1, j - 1]$ one has $LCP(S_i, S_\ell) < LCP(S_i, S_j)$.

S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
S_3	i	n	t	r	i	g	u	i	n	g		
S_4	р	а	i	r	S							
S_5	р	а	l	i	n	d	r	O	m	е		
S_6	i	n	t	е	g	е	r					
S_7	р	а	i	n	t							
S_8	i	n	t	е	ι	l	е	С	t	u	а	l

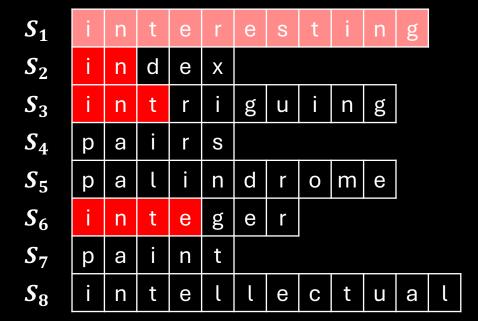
Not interesting!

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S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
S_3	i	n	t	r	i	g	u	i	n	g		
S_4	р	а	i	r	S							
S_5	р	а	l	i	n	d	r	0	m	е		
S_6	i	n	t	е	g	е	r					
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S_1	i	n	t	е	r	е	S	t	i	n	g	
S_2	i	n	d	е	Х							
S_3	i	n	t	r	i	g	u	i	n	g		
S_4	р	а	i	r	S							
S_5	р	а	l	i	n	d	r	O	m	е		
S_6	i	n	t	е	g	е	r					
<i>S</i> ₇	р	а	i	n	t							
S_8	i	n	t	е	ι	ι	е	С	t	u	а	l

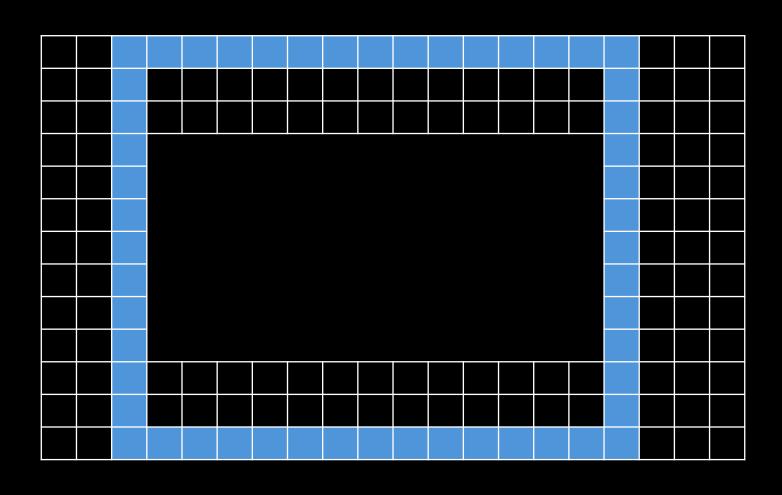
Interesting Triplets

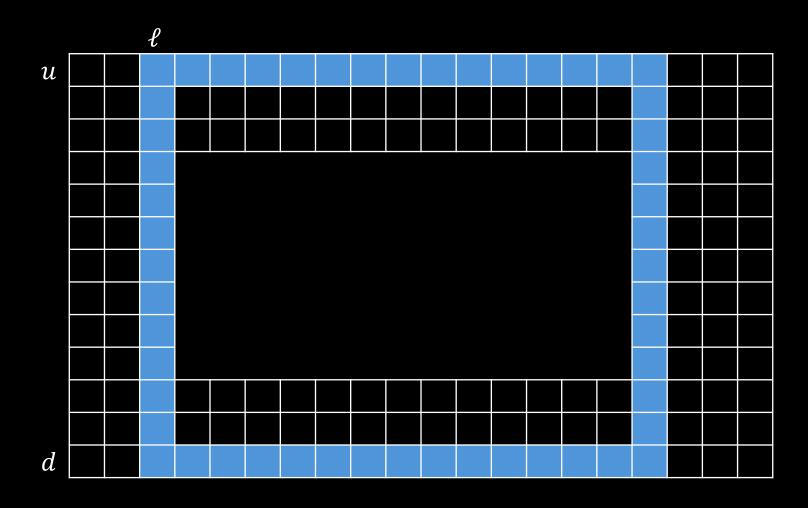
• <u>Definition</u>: Let M be an $H \times W$ matrix and $\ell \in [W]$. A triplet (u,d,ℓ) is called interesting if the pair (u,d) is interesting for the tuple $(M[1][\ell..W],...,M[n][\ell..W])$.

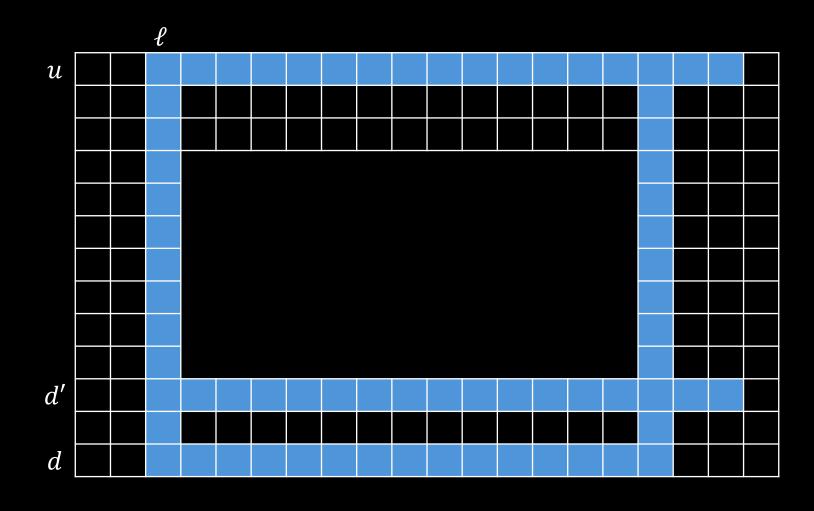
		ℓ											
	f	i	n	t	е	r	е	s	t	i	n	g	е
	j	i	n	d	е	Х	е	h	n	j	u	f	d
	t	i	n	t	r	i	g	u	i	n	g	i	r
u	е	р	а	i	r	s	t	t	У	u	j	g	b
	d	р	а	l	i	n	d	r	O	m	е	а	d
	s	i	n	t	е	g	е	r	У	m	r	d	g
d	b	р	а	i	n	t	s	f	b	r	t	d	d
	у	i	n	t	е	l	l	е	С	t	u	а	l

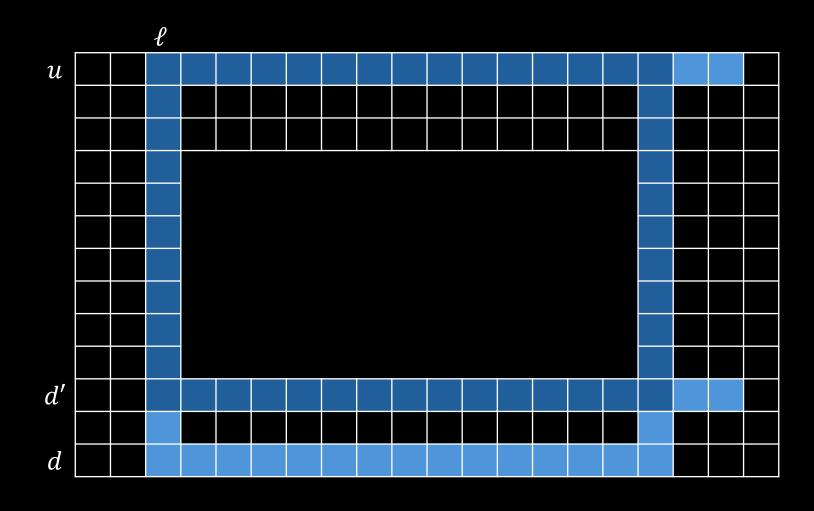
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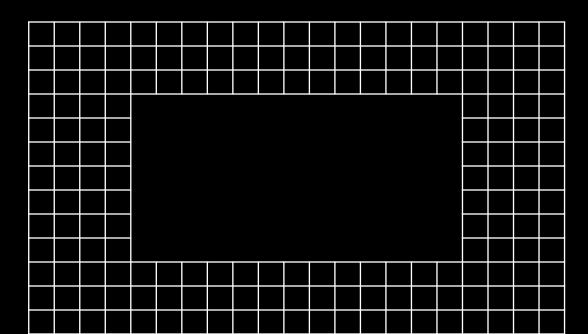






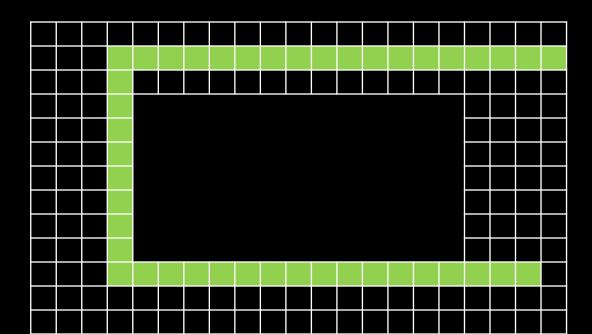
Find a Surrounding Matching Frame

- $\tilde{O}(|\text{interesting triplets}| + HW)$ time to find all interesting triplets
- $\tilde{O}(HW)$ interesting triplets



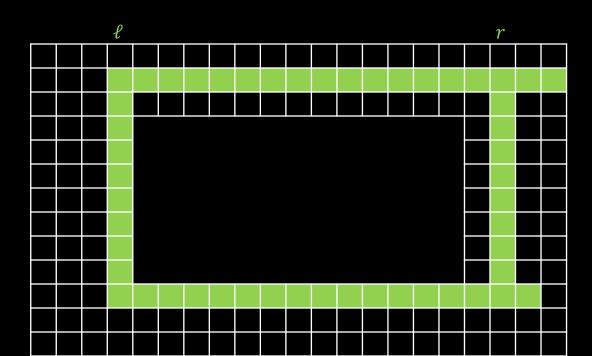
Find a Surrounding Matching Frame

- $\tilde{O}(|\text{interesting triplets}| + \overline{HW})$ time to find all interesting triplets
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- $\tilde{O}(1)$ time to find maximum r in an interesting triplet



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$\tilde{O}(n^2)$ Time For $(1-\varepsilon)$ -approximation

- ullet $\tilde{O}(1)$ exponential guesses of H,W from 1 to n
- For each guess H, W, there are $O\left(\frac{n^2}{HW}\right)$ submatrices
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Open Problems

- $\tilde{O}(n^2)$ for maximum matching frame
- Maximum palindromic frame
- Maximum weight frame
- Etc.

0	n	V	W	Į	a	m	l	i	S	а	С
t	Q	U	Ε	S	T		O	Ν	S	b	е
m	t	a	е	r	S	У	a	a	t	С	t
0	h	b	n	е	O	h	q	b	h	е	V
l	a	е	е	n	a	g	n	е	a	n	q
u	n	l	i	t	е	r	a	l	n	a	С
d	k	r	a	l	n	t	n	е	k	n	m
S	q	u	е	S	t	i	O	n	q	У	O

O	n	V	W	J	a	m	l	i	S	a	С
t	Q	U	Ε	S	Т	I	0	N	S	?	е
m	Т	a	е	r	S	У	a	a	t	С	t
O	Н	b	n	е	O	h	q	b	h	е	V
l	Α	е	е	n	a	g	n	е	a	n	q
u	N	l	i	t	е	r	a	l	n	a	С
d	K	r	a	l	n	t	n	е	k	n	m
S	Q	u	е	S	t	i	O	n	S	У	O