Statistical methods: Homework 3

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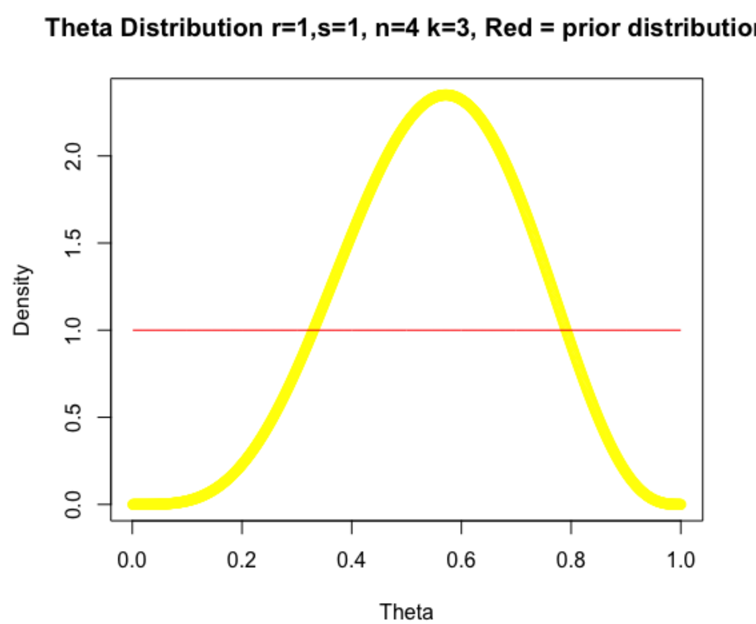
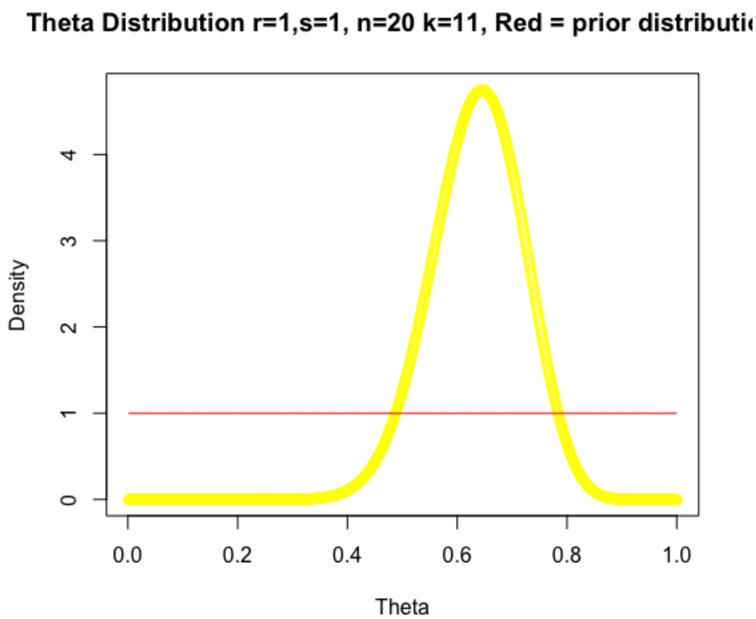
1 11.2.18

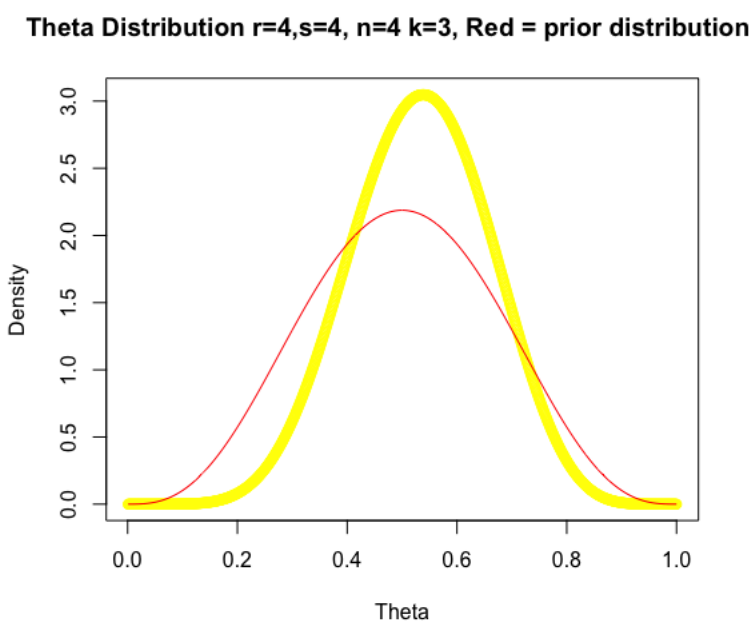
We are given that the distribution of θ is described by the PMF.

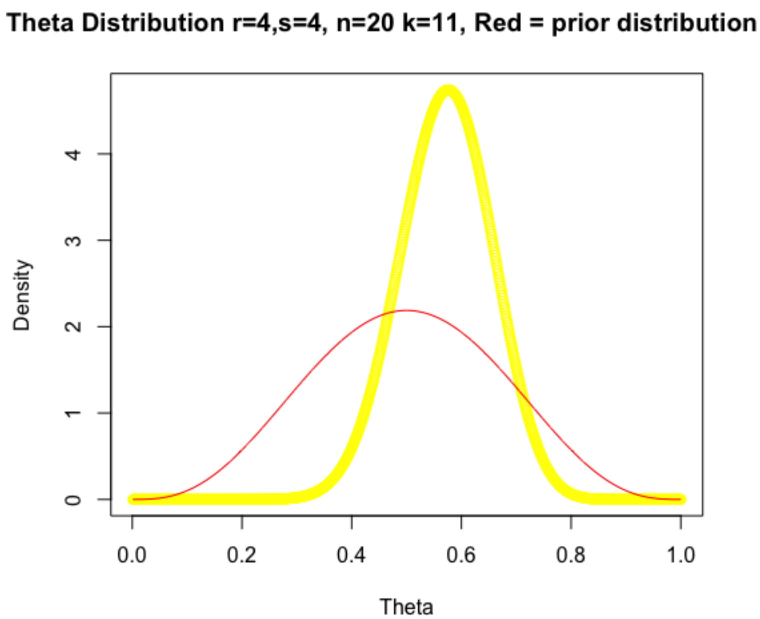
$$P(\theta|n,k) = \frac{\Gamma(n+r+s)}{\Gamma(r+k)\Gamma(s+n-k)}\theta^{r+k-1}(1-\theta)^{n+s-k-1}$$

a. Plot prior and posterior distributions for all 4 cases defined by the following values for parameters: $(r,s) = (1,1),(4,4)\&(n,k) = (4,3),(20,11)$. The distribution being used in this case is the conjugate prior of the gamma distribution.




Theta Distribution r=1,s=1, n=20 k=11, Red = prior distributi
Density
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0
Theta


Theta Distribution r=4,s=4, n=4 k=3, Red = prior distribution
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
0.2
0.4
0.6
0.8
1.0
Theta




```r
##r s (1,1) (4,4)
##n k (4,3) (20,11)


plot(c(1:1000)/1000,dbeta(c(1:1000)/1000,5, 4), col = 103, add = TRUE, xlab = 'Theta', ylab = 'Density', title('Theta Distribution r=1,s=1, n=4 k=3, Red = prior distribution'))
lines(c(1:1000)/1000,dbeta(c(1:1000)/1000,1, 1), col = 26,bg=405)

plot(c(1:1000)/1000,dbeta(c(1:1000)/1000,21, 12), col = 103, add = TRUE, xlab = 'Theta', ylab = 'Density', title('Theta Distribution r=1,s=1, n=20 k=11, Red = prior distribution') )
lines(c(1:1000)/1000,dbeta(c(1:1000)/1000,1, 1), col = 26,bg=405)

plot(c(1:1000)/1000,dbeta(c(1:1000)/1000,8, 7), col = 103, add = TRUE, xlab = 'Theta', ylab = 'Density', title('Theta Distribution r=4,s=4, n=4 k=3, Red = prior distribution') )
lines(c(1:1000)/1000,dbeta(c(1:1000)/1000,4, 4), col = 26,bg=405)


plot(c(1:1000)/1000,dbeta(c(1:1000)/1000,24, 15), col = 103, add = TRUE, xlab = 'Theta', ylab = 'Density', title('Theta Distribution r=4,s=4, n=20 k=11, Red = prior distribution') )
```

```
lines(c(1:1000)/1000,dbeta(c(1:1000)/1000,4, 4), col = 26,bg
    =405)
```

b. In each one of these 4 cases, determine the probability that the coin is biased towards heads (ie $P(\theta > 0.5)$) (You can use Beta distribution integration functionality of R)

```
$$P(\theta>.5 | r=1 ,s=1 ,n= 4, k=3)=0.6367187$$
> 1-pbeta(.5,5, 4)
[1] 0.6367187
```

$$P(\theta > .5|r = 1, s = 1, n = 20, k = 11) = 0.9449079$$

```
> 1-pbeta(.5,21, 12)
[1] 0.9449079
```

$$P(\theta > .5|r = 4, s = 4, n = 4, k = 3) = 0.6047363$$

```
> 1-pbeta(.5,8, 7)
[1] 0.6047363
```

$$P(\theta > .5|r = 4, s = 4, n = 20, k = 11) = 0.9283467$$

```
> 1-pbeta(.5,24,15)
[1] 0.9283467
```