32/50 total

1) a) ~~some~~ ~~Sometimes~~ always

$\|x\|_2 = \sigma_{max}(x) \leq \sqrt{\|x\|_F^2} = \sqrt{\sum \sigma_i^2}$

Correct.

8.5/12

if $\sigma_{max} = \sqrt{\sum \sigma_i^2}$ rank 1 so only one non-$\emptyset$ σ

b) Sometimes

$A: (q, \varepsilon)$-RIP if $\|x\|_0 \leq q \Rightarrow (1-\varepsilon)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\varepsilon)\|x\|_2^2$

X soltn ~~to~~ $\{\min \|z_0\| \mid Az = b\}$

Correct.

HW 4 had another example

c) $L = D - A$

~~Sometimes~~ Never

suppose $Lv = \lambda v$

$= (D - A)v$

~~if $D = I$ then~~

Only true sometimes in expectation.

Sometimes. True whenever D is a scaling of the identity. 1.5/3

d) Never

$\|Vx\|_2^2 = \|x\|_2^2 \neq \|V^T x\|_2^2$

Sometimes. True when n = k. 1/3

6.5/8

)

a) 3 - C , sharp drop off between 1st & 2nd

1 - a , smoother color gradient so less gap between σ_i

flipped a and b. 3/4

2 - b , by default , b/w then a bunch of grey-tones

b) ~~...~~ image C would yeild the cleanest picture although you could very easily approximate this image with a much lower rank approximation. $\|X\|_F^2 = \Sigma \sigma_i^2$

correct

error $\downarrow$ as $\sigma_i \downarrow$

c) power method converges fastest when the gap between σ_i is large.

Σ^3 has the largest gap, so it'll coverge the fastest.

Σ^1 has the smallest gap $\Rightarrow$ slow converg.

sigma^2 actually has the smallest gap, but correct reasoning. 1.5/2

3

7/8

a) $\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix} v = \lambda v$ no self-loops, $diag_{ii} = 0$

A is symmetric so by hint

$$tr(A) = \sum_{i=1}^{n} \lambda_i = 0$$

since all entries along the diagonal ~~a~~ are 0.

since $\sum_{i=1}^{n} \lambda_i = 0$, there must exist at least one $\lambda_j > 0$ and $\lambda_k < 0$.

what prevents all eigenvalues from being 0? 3/4

b) Laplacian $L = B^T B \Rightarrow$ is PSD

meaning all $\lambda_i \geq 0$

hence adjacency matrix A must have at least some eigenvalues which are different.

good!

4

3/10

a) by JL we have that

$$\|\Pi A v\|_2^2 \leq (1+\varepsilon)\|Av\|_2^2$$

and $\|\Pi v\|_2^2 \leq (1+\varepsilon)\|v\|_2^2$

$$\Pi A = U_d S_d V_d^T$$

need to use subspace embedding theorem. 2/6

$$\Pi A \tilde{v}_d = \lambda_d \tilde{v}_d$$

if λ is an eigen values for A then $f(\lambda)$ is an eigenvalue for $f(A)$

$$V = [v_1, \dots v_d]$$

Since $\|\Pi v\|_2^2 \leq (1+\varepsilon)\|v\|_2^2$

and $\|\Pi A v\|_2^2 \leq (1+\varepsilon)\|Av\|_2^2$

then $\|A\tilde{v}_2\|_2^2 \leq (1+\varepsilon) v_d$

b) $O(d^3)$ vs $O(nd^2)$ so yes, faster

using an iterative technique such as the Power Method or Krylov

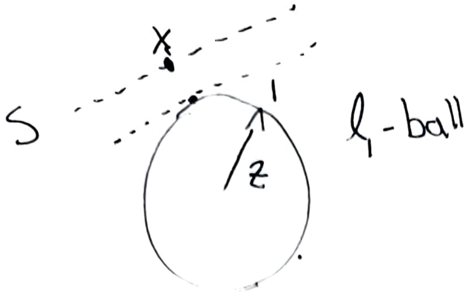
then we're in $O(d^2)(I)$ for I iterations

above method is O(mnd) time, so actually not faster. Can be speed up with subsampled randomized hadamard. 1/4

7/12

5

a)



correct answer, but incomplete reasoning. 4/6

$c = 1$

$$H = \{ y : \text{sign}(\cancel{x}) \cancel{x} = 1 \}$$

for a given point x, if $\|x\|_1 \leq 1$

then ~~sign(x)x~~ we're in our ~~se~~set

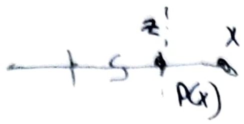
if $\|x\|_1 > 1$ then either $x > 1$ or $-x > 1$

$\text{sign}(x)\, x = 1$ will always be within our point x and the set S.

H is a tangent line to S so will always serve as a separating hyperplane.

5) b)

$a = x - p(x) = z \in S$ closest to x



$(x - p(x)) \cancel{z} y = C$

direction is $(x - p(x))$

want magnitude is $\|z\|_2$

$\|z - x\|_2^2 = \|z\|_2^2 - \|x\|_2^2 \quad \sqrt{P(x)^2 + \|x\|_2^2} = C$

gives you $H = \{ y : (x - p(x)) y = \sqrt{p(x)^2 + \|y\|_2^2}$

what is y here? (x-p(x))^Tp(x) works. 3/6