6/12 #1 **17.5/50 total**

(a) since all rows are a multiple of one, $\sqrt{tr(AA^T)} = \sqrt{\sum_{ij} a_{ij}^2}$

so ALWAYS

Correct answer, but not explanation. 2/3

(b) SOMETIMES

as X could be a non-unique solution we are not guaranteed to retrieve it

Correct answer, but not explanation. 2/3

(c) NEVER, since $L = D - A$ they have a different rank; ~~rank~~ so cannot have exactly same set of eigenvectors

Sometimes. True whenever D is a scaling of identity. 1/3

(d) VV^T is a projection matrix that has a property of norm preservation

therefore, ALWAYS $\|VV^T x\|_2 = \|x\|_2$

Sometimes. Not true when k < n. 1/3

#2 7.5/8

(a) sp. 3 image ~~b~~ c

sp. 2 image a

sp. 1 image b

Correct.

(b) spectrum 3 will be better approximated in rank 40 as the ~~remaining~~ 60 discarded singular values are smaller and do not contribute as much to an approximation error

Correct.

(c) Power Method convergence $T = O\left(\frac{\log d/\varepsilon}{\gamma}\right)$

in this case: d - same & ε is same => higher γ will contribute the most to the fastest convergence

$\gamma = \frac{\delta_1 - \delta_2}{\delta_1} = \frac{\delta_1}{\delta_1} - \frac{\delta_2}{\delta_1} = 1 - \frac{\delta_2}{\delta_1}$ => fastest will be the spectrum with $\min\left(\frac{\delta_2}{\delta_1}\right)$ => spectrum 3

forgot to say slowest, but correct reasoning. 1.5/2

#3 4/8

(a) A is symmetric by definition and b/c graph is undirected $\Rightarrow tr(A) = \sum_{i=1}^{n} \lambda_i$

since diagonal of A is $0 \Rightarrow tr(A) = 0$

what prevents all eigenvalues from being 0? 3/4

therefore, $tr(A) = 0 = \sum_{i=1}^{n} \lambda_i$

so, there will be $X = \{\lambda : \lambda > 0\}$ and $Y = \{\lambda : \lambda < 0\}$

$\sum_{j=1}^{|X|} \lambda_j + \sum_{i=1}^{|Y|} \lambda_i = 0$ therefore, both sets should be non-empty to get a sum 0

(b) $L = D - A$

if there is ~~was eig~~ the same set of eigenvalues

for each λ_i: $Av_2 = \lambda_i v_2$

$Lv_1 = \lambda_i v_1$ $\quad (D-A)v_1 = \lambda_i v_1$

$Dv - Av_1 = \lambda_i v_1 \Rightarrow Av_1 = Dv - \lambda_i v_1$

$Av_2 = \lambda_i v_2$

This argument doesn't rule out having same eigenvalues but different eigenvectors. 1/4.
We know that L is PSD so only has non-negative eigenvalues, whereas from part (a), A has a negative one.