NYU | TANDON SCHOOL OF ENGINEERING

New York University Tandon School of Engineering
Computer Science and Engineering

CS-UY 4563: Midterm Exam 1.
Monday, Mar. 9th, 2020, 9:00 - 10:15pm
50 Total Points

Directions

- Show all of your work to receive full (and partial) credit.
- If more space is required, you may use extra sheets of paper clearly marked with your name, netid, and the problem you are working on.

1. Always, Sometimes, Never. (12pts – 3pts each)

Indicate whether each of the following statements is ALWAYS true, SOMETIMES true, or NEVER true. **No justification is necessary to receive full credit for a correct answer**. To earn partial credit if you are wrong, you may provide a short justification or example to explain your choice.

(a) The empirical risk of a model is lower than the population risk.

ALWAYS SOMETIMES NEVER

(b) You train a multiple linear regression model with varying levels of ℓ_2 regularization. Let $\vec{\beta}^{(1)} = \arg\min_{\vec{\beta}} \|X\vec{\beta} - \vec{y}\|_2^2 + \lambda_1\|\vec{\beta}\|_2^2$ and let $\vec{\beta}^{(2)} = \arg\min_{\vec{\beta}} \|X\vec{\beta} - \vec{y}\|_2^2 + \lambda_2\|\vec{\beta}\|_2^2$.

If $\lambda_1 > \lambda_2$, is $\|X\vec{\beta}^{(1)} - \vec{y}\|_2^2 < \|X\vec{\beta}^{(2)} - \vec{y}\|_2^2$?

ALWAYS SOMETIMES NEVER

(c) The linear classifier found by logistic regression minimizes error rate (0-1 loss) on the training data.

ALWAYS SOMETIMES NEVER

(d) Consider a multiple linear regression problem where each data example has the form $(\vec{x}, y) = ([x_1, x_2], y)$. Transform the predictor variables by adding quadratic terms, so each new data example has the form $(\vec{x}_{trans}, y) = ([x_1, x_2, x_1^2, x_2^2, x_1x_2], y)$. Let L^* be the minimum training loss for the original problem and let L^*_{trans} be the minimum training loss for the transformed problem. Is $L^*_{trans} \le L^*$?

ALWAYS SOMETIMES NEVER

2. Model Diagnosis Short Answer (8pts)

You are trying to solve a prediction problem using a multiple linear regression model with ℓ_2 loss. You first split the data set into a train set (80%) and a test set (20%). You then train the model on the train set to obtain a parameter vector $\vec{\beta}$. Using $\vec{\beta}$, you evaluate the average squared loss of the regression model on the train and test set, separately.

For each of the following scenarios, circle all answers that apply. **No justification is necessary to receive full credit for a correct answer**. To earn partial credit if you are wrong, you may provide a short justification.

(a) (4pts) The average squared loss on the train set is 1.5 and the average squared loss on the test set is 12.6. **Which of the following techniques is likely to improve your average test loss?**

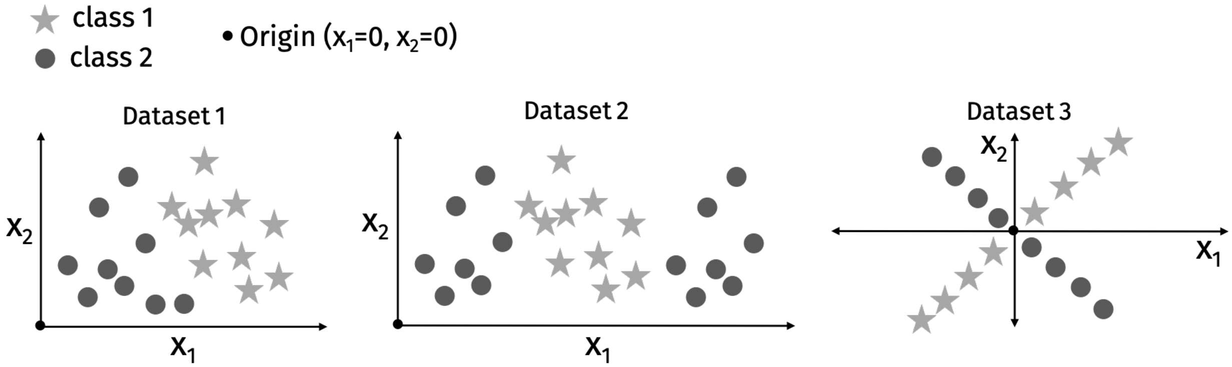
REGULARIZATION   FEATURE SELECTION   FEATURE TRANSFORM   DATA SCALING

(b) (4pts) The average squared loss on the train set is 10.2 and the average squared loss on the test set is 9.9. **Which of the following techniques is likely to improve your average test loss?**

REGULARIZATION   FEATURE SELECTION   FEATURE TRANSFORM   DATA SCALING

3. Model Diagnosis 2 (10pts)

Consider the following scatter plots of data for three binary classification problems. x_1 and x_2 are the independent variables and class labels are indicated by points with a different shape and shade.



(a) (4pts) Indicate which of the three clustering problems could be solved to high accuracy (small error rate) using a logistic regression model with no regularization and no feature transformations.

(b) (6pts) For any of the problems that you believe are not *directly solvable* with logistic regression, suggest a possible feature transformation which *would make it possible* to obtain a high accuracy solution with logistic regression. For each problem, your solution should be a set of new features $\phi_1(x_1, x_2), \phi_2(x_1, x_2), \ldots, \phi_q(x_1, x_2)$ that depend on the original features x_1 and x_2. You may use as large a q as you need.

4. Loss Minimization. (10pts)

For data with one predictor and one target: $(x_1, y_1), \ldots, (x_n, y_n)$, consider a linear regression model:

$$f_{\beta_0,\beta_1}(x) = \beta_0 + \beta_1 x$$

with *exponential loss*:

$$L(\beta_0, \beta_1) = \sum_{i=1}^{n} e^{(y_i - f_{\beta_0,\beta_1}(x_i))^2}$$

(a) (5pts) Write down an expression for the gradient of the loss L.

(b) (2pts) Name two algorithms/methods which could be used to minimize L.

(c) (3pts) In general, is this exponential loss more or less robust to outliers when compared to ℓ_2 loss? How about when compared to ℓ_∞ loss?

5. Bayesian Crab Classification (10pts)

A biologist is collecting specimens from two species of crabs, species S_0 and S_1. These species live in the same habitat and look similar to the human eye. To accelerate crab sorting by species, the biologist wants to develop a simple classification rule based on body measurements. She observes that the ratio of *forehead breadth* to overall *body length* differs between crabs in species S_0 and S_1. The biologist proposes to measure this ratio (denoted by R) and use it as a single predictor variable for classification.

The biologist assumes that the crab data comes from a "mixture of Gaussians" probabilistic model. In particular, she assumes that for each species, R follows a normal (Gaussian) probability distribution, with different parameters for each species. The biologist makes the following concrete observations:

- 35% of all crabs collected belong to S_0 and the remaining 65% belong to S_1.
- For crabs in S_0, the average value of R is .5. For crabs in S_1, the average value of R is .4.
- For both species, the standard deviation of R is .1.

(a) (6pts) Suppose we collect a new crab with forehead breadth to body length ratio R_{new}. The biologist would like to assign this crab to S_0 or S_1 using a maximum a posterior (MAP) classification rule. Denote this rule by $f : \mathbb{R} \rightarrow \{S_0, S_1\}$. The rule takes as input the ratio R_{new} and outputs S_0 or S_1.

Write down all mathematical expressions that would need to be evaluated to compute f for a given input R_{new}. Your expressions do not need to be simplified, but they should not involve unknown variables besides R_{new}. **Hint:** Use Bayes rule.

(b) (4pts) Show that, for this problem, the classification rule f has the following form:

$$f(R_{new}) = \begin{cases} S_0 \text{ if } R_{new} \geq \lambda \\ S_1 \text{ if } R_{new} < \lambda, \end{cases}$$

for some fixed threshold parameter λ (you do not need to explicitly compute λ).

(c) (3pts – extra credit) Given the biologist's data above, will the threshold λ for the MAP classification rule be EQUAL TO, LARGER, or SMALLER than .45? Justify your answer in a sentence or two. This problem can be solved without a calculator.