Probabilidad Visible Decay

Funciones previas, cálculo final y ejemplos numéricos.

Funciones previas

- dG/dE y $d\overline{G}/dE$ dependen de (m_i, m_f, E_i, E_f, c), definida como:

$$\frac{dG}{dE}(m_i, m_f, E_i, E_f, c) = \begin{cases} Si\ c = 1: & \begin{cases} Si\ e_i/x_{if} \leq e_f \cup\ e_f \leq e_i: & = \dfrac{1}{\sqrt{1-\dfrac{m_i}{\left(e_i \times 10^9\right)^2}}} \left(\dfrac{x_{if}}{x_{if}-1}\right)\left(\dfrac{1}{e_i\, e_f}\right)\dfrac{\left(e_i + e_f\sqrt{x_{if}}\right)^2}{\left(\sqrt{x_{if}}+1\right)^2} \\ Otro\ caso: & = 0 \end{cases} \\ Si\ c \neq 1: & \begin{cases} Si\ e_i/x_{if} \leq e_f \cup\ e_f \leq e_i: & = \dfrac{1}{\sqrt{1-\dfrac{m_i}{\left(e_i \times 10^9\right)^2}}} \left(\dfrac{x_{if}}{x_{if}-1}\right)\left(\dfrac{1}{e_i\, e_f}\right)\dfrac{\left(e_i - e_f\sqrt{x_{if}}\right)^2}{\left(\sqrt{x_{if}}-1\right)^2} \\ Otro\ caso: & = 0 \end{cases} \end{cases} \quad ...(1)$$

donde $x_{if} = m_i/m_f$, $e_i = E_i \times 10^{-9}, e_f = E_f \times 10^{-9}$.

Funciones previas

- dG/dE y $d\overline{G}/dE$ dependen de (m_i, m_f, E_i, E_f, c), definida como:

$$\frac{d\overline{G}}{dE}(m_i, m_f, E_i, E_f, c) = \begin{cases} Si\ c = 1: \begin{cases} Si\ e_i/x_{if} \le e_f \cup\ e_f \le e_i: & = \dfrac{1}{\sqrt{1 - \frac{m_i}{\left(e_i \times 10^9\right)^2}}} \left(\dfrac{x_{if}}{x_{if}-1}\right)\left(\dfrac{e_i - e_f}{e_i\, e_f}\right)\dfrac{\left(x_{if}\, e_f - e_i\right)}{\left(\sqrt{x_{if}}+1\right)^2} \\ Otro\ caso: & =\ 0 \end{cases} \\ Si\ c \ne 1: \begin{cases} Si\ e_i/x_{if} \le e_f \cup\ e_f \le e_i: & = \dfrac{1}{\sqrt{1 - \frac{m_i}{\left(e_i \times 10^9\right)^2}}} \left(\dfrac{x_{if}}{x_{if}-1}\right)\left(\dfrac{e_i - e_f}{e_i\, e_f}\right)\dfrac{\left(x_{if}\, e_f - e_i\right)}{\left(\sqrt{x_{if}}-1\right)^2} \\ Otro\ caso: & =\ 0 \end{cases} \end{cases} \quad ...(2)$$

donde $x_{if} = m_i/m_f$, $e_i = E_i \times 10^{-9}, e_f = E_f \times 10^{-9}$.

Partes del Integrando

- El cálculo del integrando requiere los siguientes elementos:

Matriz de Mezcla

$$U_{PMNS} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, s) \equiv U \quad , \qquad \dots (3)$$

donde $\theta_{12}, \theta_{13}, \theta_{23}, \delta \in IR$ y $s \in \{-1, 1\}$ y :

$$U_{11} = \cos\theta_{12}\cos\theta_{13} \;, U_{12} = \cos\theta_{13}\sin\theta_{12} \,, U_{13} = e^{-i\delta s}\sin\theta_{13} \,,$$
$$U_{21} = -\cos\theta_{23}\sin\theta_{12} - e^{-i\delta s}\cos\theta_{12}\sin\theta_{13}\sin\theta_{23} \,,$$
$$U_{22} = \cos\theta_{23}\cos\theta_{12} - e^{-i\delta s}\sin\theta_{12}\sin\theta_{13}\sin\theta_{23} \,,$$
$$U_{23} = \cos\theta_{13}\sin\theta_{23} \,,$$
$$U_{31} = -e^{-i\delta s}\cos\theta_{12}\cos\theta_{23}\sin\theta_{13} + \sin\theta_{12}\sin\theta_{23} \,,$$
$$U_{32} = -e^{-i\delta s}\cos\theta_{23}\sin\theta_{12}\sin\theta_{13} - \cos\theta_{12}\sin\theta_{23} \,,$$
$$U_{33} = \cos\theta_{13}\cos\theta_{23} \,.$$

Partes del Integrando

- El cálculo del integrando requiere los siguientes elementos:

(Hamiltoniano) $$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} - i\alpha_2 & 0 \\ 0 & 0 & \Delta m^2{}_{31} - i\alpha_3 \end{pmatrix} (U^*)^T + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, \alpha_2, \alpha_3\right) \quad , \qquad \ldots (4)$$

donde:

$A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,

$[\rho] = g/cm^3$,

$[E] = GeV = 10^9 eV$,

$[\Delta m^2{}_{21}] = [\Delta m^2{}_{31}] = eV^2$,

$[\alpha_2] = [\alpha_3] = eV$.

Partes del Integrando

- El cálculo del integrando requiere los siguientes elementos:

$$[v_1{}^T \; v_2{}^T \; v_3{}^T]\, H \,[v_1{}^T \; v_2{}^T \; v_3{}^T]^{-1} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

$$VHV^{-1} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad , \quad \ldots (5)$$

donde v_i y λ_i son los autovectores y autovalores de H respectivamente, además:

$$v_i = (v_{i1} \; v_{i2} \; v_{i3}) \text{ con i} \in \{1, 2, 3\} \quad \text{y} \quad V = \begin{pmatrix} v_{11} & v_{21} & v_{31} \\ v_{12} & v_{22} & v_{32} \\ v_{13} & v_{23} & v_{33} \end{pmatrix}.$$

Partes del Integrando

- El cálculo del integrando requiere los siguientes elementos:

$E_i = e_i \times 10^9$, $e_f = E_f \times 10^{-9}$, $L = 1300 \times 10^9 / C_{GeVtoKm}$, $C_{GeVtoKm} = 0.197327$, $\overline{U}_{PMNS} = (U_{PMNS}^T)^*$

$$H_i = H(E_i, \dots, s_i, \dots) \rightarrow V_i = V(E_i, \dots, s_i, \dots) \rightarrow V_i H_i V_i^{-1} = \begin{pmatrix} \lambda_1^i & 0 & 0 \\ 0 & \lambda_2^i & 0 \\ 0 & 0 & \lambda_3^i \end{pmatrix}$$

Definimos: $\tilde{m}_1^i = 2E_i Re\left(\lambda_1^i\right)$, $\tilde{m}_2^i = 2E_i Re\left(\lambda_2^i\right)$, $\tilde{m}_3^i = 2E_i Re\left(\lambda_3^i\right)$,

$\tilde{\alpha}_1^i = -2E_i Im\left(\lambda_1^i\right)$, $\tilde{\alpha}_2^i = -2E_i Im\left(\lambda_2^i\right)$, $\tilde{\alpha}_3^i = -2E_i Im\left(\lambda_3^i\right)$, $\tilde{C}_i = V_i^T . \overline{U}_{PMNS}$.

Además:

$$H_f = H\left(E_f, \dots, s_f, \dots\right) \rightarrow V_f = V\left(E_f, \dots, s_f, \dots\right) \rightarrow V_f H_f V_f^{-1} = \begin{pmatrix} \lambda_1^f & 0 & 0 \\ 0 & \lambda_2^f & 0 \\ 0 & 0 & \lambda_3^f \end{pmatrix}$$

Definimos: $\tilde{m}_1^f = 2E_f Re\left(\lambda_1^f\right)$, $\tilde{m}_2^f = 2E_f Re\left(\lambda_2^f\right)$, $\tilde{m}_3^f = 2E_f Re\left(\lambda_3^f\right)$,

$\tilde{\alpha}_1^f = -2E_f Im\left(\lambda_1^f\right)$, $\tilde{\alpha}_2^f = -2E_f Im\left(\lambda_2^f\right)$, $\tilde{\alpha}_3^f = -2E_f Im\left(\lambda_3^f\right)$, $\tilde{C}_f = V_f^T . \overline{U}_{PMNS}$.

Partes del Integrando

- El cálculo del integrando requiere los siguientes elementos:

Definimos : $\varsigma_{padre} \in \{2, 3\}$, $\varsigma_{hijo} \in \{1, 2\}$, $\varsigma_{hijo} < \varsigma_{padre}$, $q_{coup} \in \{1, -1\}$, $I^2 = -1$, f_i , $f_f \in \{1, 2, 3\}$,

$$mss = \begin{cases} Si \quad \Delta m^2{}_{31} > 0 : \ = \left\{ m^2_{light}, \Delta m^2{}_{21} - m^2_{light}, \Delta m^2{}_{31} - m^2_{light} \right\} \\ Si \quad \Delta m^2{}_{31} < 0 : \ = \left\{ m^2_{light}, -\Delta m^2{}_{31} + m^2_{light}, \Delta m^2{}_{21} - \Delta m^2{}_{31} + m^2_{light} \right\} \end{cases} \quad \ldots (6)$$

$$\Theta(s_i, s_f, q_{coup}) = \begin{cases} Si \quad s_i \times s_f > 0 : \ = \frac{dG}{dE}(mss_{\varsigma_{padre}}, mss_{\varsigma_{hijo}}, E_i, E_f, q_{coup}) \\ Si \quad s_i \times s_f < 0 : \ = \frac{d\bar{G}}{dE}(mss_{\varsigma_{padre}}, mss_{\varsigma_{hijo}}, E_i, E_f, q_{coup}) \end{cases} \quad \ldots (7)$$

$$\Psi(e_i) = 2\sum_{j=1}^{3}\sum_{p=1}^{3}\sum_{h=1}^{3}\sum_{n=1}^{3} Re\left([\tilde{C}_i^{-1}]_{jf_i}[\tilde{C}_i^{-1}]^*_{pf_i}[V_f]_{f_f h}[V_f]^*_{f_f n} \frac{\frac{e_f}{e_i}(\tilde{\alpha}^i_j + \tilde{\alpha}^i_p) - \left(\tilde{\alpha}^f_h + \tilde{\alpha}^f_n\right) - I\left(\frac{e_f}{e_i}\left(\tilde{m}^i_j - \tilde{m}^i_p\right) - \left(\tilde{m}^f_h - \tilde{m}^f_n\right)\right)}{\left(\frac{e_f}{e_i}(\tilde{\alpha}^i_j + \tilde{\alpha}^i_p) - \left(\tilde{\alpha}^f_h + \tilde{\alpha}^f_n\right)\right)^2 + \left(\frac{e_f}{e_i}\left(\tilde{m}^i_j - \tilde{m}^i_p\right) - \left(\tilde{m}^f_h - \tilde{m}^f_n\right)\right)^2} \times \left(e^{\frac{-i(\tilde{m}^f_h - \tilde{m}^f_n)}{2E_f L}} e^{\frac{-(\tilde{\alpha}^f_h + \tilde{\alpha}^f_n)}{2E_f L}} - e^{\frac{-i(\tilde{m}^i_j - \tilde{m}^i_p)}{2E_f L}} e^{\frac{-(\tilde{\alpha}^i_j + \tilde{\alpha}^i_p)}{2E_f L}} \right) [\tilde{C}_f]^{-1}_{\varsigma_{hijo} h}([\tilde{C}_f]^{-1}_{\varsigma_{hijo} n})^*[\tilde{C}_i]_{j\varsigma_{padre}}[\tilde{C}_i]^*_{p\varsigma_{padre}} \right) \frac{e_f}{e_i E_i} \alpha_{\varsigma_{padre}} \quad \ldots (8)$$

Probabilidad Visible Decay

- La integral de Ψ nos dará la probabilidad final:

$$P_{VisDcy} \equiv P\left(E, L, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta,\ \Delta m^2{}_{21}, \Delta m^2{}_{31}, m^2_{light}\ , \alpha_2\ ,\ \alpha_3\ , f_i\ , s_i, f_f\ , s_f\ , \varsigma_{hijo}\ , \varsigma_{padre}, q_{coup}\right)$$

$$P_{VisDcy}(E, \dots) = 10^9 \times \int_{E\times 1.00001}^{Min\left\{\ 20\ ,\ \frac{mss_{\varsigma_{padre}}}{mss_{\varsigma_{hijo}}}\times E\ \right\}} \Psi(e_i)de_i \quad \dots (9)$$

$$P_{FullDcy}(E, \dots) = P_{InvDcy}(E, \dots) + P_{VisDcy}(E, \dots) \quad \dots (10)$$

Nota: Tiempo promedio para calcular sólo los datos de ***Visible Decay*** de la tabla de referencia: 1 min 22,41 s .