Probabilidades

– Oscilación Estándar
– Decaimiento Invisible (Inv. Decay)
– Violación de Principio de Equivalencia (VEP)
– Interacción no Estandar (NSI)
– Ejemplos numéricos

Elementos del cálculo

OSCILACIÓN ESTÁNDAR

Formulación matemática (Osc. Standar)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

Matriz de Mezcla

$$U_{PMNS} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, s) \; \equiv \; U \quad , \qquad \text{...} \; (1)$$

donde $\theta_{12}, \theta_{13}, \theta_{23}, \delta \in IR$ y $s \in \{-1, 1\}$ y :

$$U_{11} = \cos\theta_{12}\cos\theta_{13} \; , U_{12} = \cos\theta_{13}\sin\theta_{12} \; , U_{13} = e^{-i\delta s}\sin\theta_{13} \; ,$$
$$U_{21} = -\cos\theta_{23}\sin\theta_{12} - e^{-i\delta s}\cos\theta_{12}\sin\theta_{13}\sin\theta_{23} \; ,$$
$$U_{22} = \cos\theta_{23}\cos\theta_{12} - e^{-i\delta s}\sin\theta_{12}\sin\theta_{13}\sin\theta_{23} \; ,$$
$$U_{23} = \cos\theta_{13}\sin\theta_{23} \; ,$$
$$U_{31} = -e^{-i\delta s}\cos\theta_{12}\cos\theta_{23}\sin\theta_{13} + \sin\theta_{12}\sin\theta_{23} \; ,$$
$$U_{32} = -e^{-i\delta s}\cos\theta_{23}\sin\theta_{12}\sin\theta_{13} - \cos\theta_{12}\sin\theta_{23} \; ,$$
$$U_{33} = \cos\theta_{13}\cos\theta_{23} \; .$$

Formulación matemática (Osc. Standar)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

(Hamiltoniano) $$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} & 0 \\ 0 & 0 & \Delta m^2{}_{31} \end{pmatrix} (U^*)^T + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}\right) \quad , \qquad \ldots (2)$$

donde:

$A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,
$[\rho] = g/cm^3$,
$[E] = GeV = 10^9 eV$,
$[\Delta m^2{}_{21}] = [\Delta m^2{}_{31}] = eV$.

Formulación matemática (Osc. Standar)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

$$[v_1{}^T\ v_2{}^T\ v_3{}^T]H[v_1{}^T\ v_2{}^T\ v_3{}^T]^{-1} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix},$$

$$VHV^{-1} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad \dots (3)$$

donde v_i y λ_i son los autovectores y autovalores de H respectivamente, además:

$$v_i = (v_{i1}\ v_{i2}\ v_{i3}) \text{ con i} \in \{1, 2, 3\} \quad \text{y} \quad V = \begin{pmatrix} v_{11} & v_{21} & v_{31} \\ v_{12} & v_{22} & v_{32} \\ v_{13} & v_{23} & v_{33} \end{pmatrix}.$$

Formulación matemática (Osc. Standar)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

$$S = V\begin{pmatrix} e^{-i\lambda_1 L^*} & 0 & 0 \\ 0 & e^{-i\lambda_2 L^*} & 0 \\ 0 & 0 & e^{-i\lambda_3 L^*} \end{pmatrix} V^{-1} \ , \quad \ldots (4)$$

$$P_{(\nu_\alpha \to \nu_\beta)} \equiv P_{\alpha\beta} = S_{\beta\alpha}(S_{\beta\alpha})^* = \left\| S_{\beta\alpha} \right\|^2 \ , \quad \ldots (5)$$

(Matriz de Probabilidad)

$$P = \begin{pmatrix} \|S_{11}\|^2 & \|S_{21}\|^2 & \|S_{31}\|^2 \\ \|S_{12}\|^2 & \|S_{22}\|^2 & \|S_{32}\|^2 \\ \|S_{13}\|^2 & \|S_{32}\|^2 & \|S_{33}\|^2 \end{pmatrix} = P\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, L\right) \ , \quad \ldots (6)$$

donde:

$$L^* = \frac{L \times 10^9}{C_{GeVtoKm}} \quad , \quad [L] = Km \quad , \quad C_{GeVtoKm} = h \times c \times 10^{15} \quad , \quad h = 6.58211928 \times 10^{-25} \quad , \quad c = 299792458 \, .$$

Elementos del cálculo

DECAIMIENTO INVISIBLE

Formulación matemática (Inv. Decay)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

(Hamiltoniano) $$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} - i\alpha_2 & 0 \\ 0 & 0 & \Delta m^2{}_{31} - i\alpha_3 \end{pmatrix} (U^*)^T + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, \alpha_2, \alpha_3\right) \quad , \qquad \ldots (7)$$

$$P_{InvDcy} \equiv P\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, L, \alpha_2, \alpha_3\right) \, , \qquad \ldots (8)$$

donde:

$A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,

$[\rho] = g/cm^3$,

$[E] = GeV = 10^9 eV$,

$[\Delta m^2{}_{21}] = [\Delta m^2{}_{31}] = eV$,

$[\alpha_2] = [\alpha_3] = eV$.

Elementos del cálculo

VIOLACIÓN DE PRINCIPIO DE EQUIVALENCIA (VEP)

Formulación matemática (VEP)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

(Hamiltoniano)

$$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} & 0 \\ 0 & 0 & \Delta m^2{}_{31} \end{pmatrix} (U^*)^T + 2E\times 10^9 U_g \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix} (U_g{}^*)^T + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, \gamma_2, \gamma_3\right) \quad , \qquad \ldots (9)$$

$$P_{VEP} \equiv P\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, L, \gamma_2, \gamma_3\right) \,, \qquad \ldots (10)$$

donde:

$A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,

$[\rho] = g/cm^3$,

$[E] = GeV = 10^9 eV$,

$[\Delta m^2{}_{21}] = [\Delta m^2{}_{31}] = eV$,

$[\gamma_2] = [\gamma_3] = 1$.

Elementos del cálculo

INTERACCIÓN NO ESTÁNDAR (NSI)

Formulación matemática (NSI)

- El cálculo de la probabilidad de oscilación estándar requiere los siguientes elementos:

(Hamiltoniano)

$$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} & 0 \\ 0 & 0 & \Delta m^2{}_{31} \end{pmatrix} (U^*)^T + A_m \begin{pmatrix} \epsilon_{ee} & |\epsilon_{e\mu}|e^{i\varphi_{e\mu}} & |\epsilon_{e\tau}|e^{i\varphi_{e\tau}} \\ |\epsilon_{e\mu}|e^{-i\varphi_{e\mu}} & \epsilon_{\mu\mu} & |\epsilon_{\mu\tau}|e^{i\varphi_{\mu\tau}} \\ |\epsilon_{e\tau}|e^{-i\varphi_{e\tau}} & |\epsilon_{\mu\tau}|e^{-i\varphi_{\mu\tau}} & \epsilon_{\tau\tau} \end{pmatrix} + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, \epsilon_{ee}, \epsilon_{\mu\mu}, \epsilon_{\tau\tau}, \epsilon_{e\mu}, \varphi_{e\mu}, \epsilon_{e\tau}, \varphi_{e\tau}, \epsilon_{\mu\tau}, \varphi_{\mu\tau}\right) , \quad \ldots (11)$$

$$P_{VEP} \equiv P\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}, L, \epsilon_{ee}, \epsilon_{\mu\mu}, \epsilon_{\tau\tau}, \epsilon_{e\mu}, \varphi_{e\mu}, \epsilon_{e\tau}, \varphi_{e\tau}, \epsilon_{\mu\tau}, \varphi_{\mu\tau}\right) , \quad \ldots (12)$$

donde:

$\epsilon_{ee}, \epsilon_{\mu\mu}, \epsilon_{\tau\tau} \in IR,\ [\epsilon_{ee}] = [\epsilon_{\mu\mu}] = [\epsilon_{\tau\tau}] = [\epsilon_{e\mu}] = [\epsilon_{e\tau}] = [\epsilon_{\mu\tau}] = 1$, $\quad A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,

$[\rho] = g/cm^3$,

$[E] = GeV = 10^9 eV$,

$[\Delta m^2{}_{21}] = [\Delta m^2{}_{31}] = eV$.

Probabilidades

EJEMPLOS NUMÉRICOS

Ejemplos Numéricos

- Parámetros de entrada: ($s = 1$, $\rho = 2.956740$, $L = 1300$, $\alpha = 2$, $\beta = 1$, $\nu_\mu \to \nu_e$)

Osc. Estándar: $\theta_{12} = 0.59$, $\theta_{13} = 0.15$, $\theta_{23} = 0.84$, $\delta = -1.57$, $\Delta m^2{}_{21} = 7.4\times 10^{-5}$, $\Delta m^2{}_{31} = 2.5\times 10^{-3}$.

Dec. Invisible: $\alpha_2 = 0, \alpha_3 = 5\times 10^{-5}$.

VEP: $U_g = U$, $\gamma_2 = 0$, $\gamma_3 = 2\times 10^{-24}$.

NSI: $\epsilon_{ee} = \epsilon_{\mu\mu} = \epsilon_{\tau\tau} = \epsilon_{e\tau} = \epsilon_{\mu\tau} = 0$, $|\epsilon_{e\mu}| = 0.05$, $\varphi_{e\mu} = -1.55$.

Cálculos:

Tiempo: 0.517s

E	P	P_{InvDcy}	P_{VEP}	P_{NSI}
0.1	0.333905	0.395235	0.333615	0.337769
0.5	0.137072	0.114237	0.13702	0.137292
1.0	0.0379297	0.0357768	0.039078	0.034009
2.5	0.0884374	0.083406	0.089026	0.0809008
5.0	0.0337275	0.0327747	0.0384718	0.0238453
7.5	0.0153735	0.0150987	0.0210784	0.00832915
10	0.00858383	0.0084758	0.0148167	0.00345215

Ejemplos Numéricos

- Parámetros de entrada: ($s = -1$, $\rho = 2.956740$, $L = 1300$, $\alpha = 2$, $\beta = 1$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$)

Osc. Estándar: $\theta_{12} = 0.59$, $\theta_{13} = 0.15$, $\theta_{23} = 0.84$, $\delta = -1.57$, $\Delta m^2{}_{21} = 7.4\times 10^{-5}$, $\Delta m^2{}_{31} = 2.5\times 10^{-3}$.

Dec. Invisible: $\alpha_2 = 0, \alpha_3 = 5\times 10^{-5}$.

VEP: $U_g = U$, $\gamma_2 = 0$, $\gamma_3 = 2\times 10^{-24}$.

NSI: $\epsilon_{ee} = \epsilon_{\mu\mu} = \epsilon_{\tau\tau} = \epsilon_{e\tau} = \epsilon_{\mu\tau} = 0$, $|\epsilon_{e\mu}| = 0.05$, $\varphi_{e\mu} = -1.55$.

Cálculos:

Tiempo: 0.462s

E	P	P_{InvDcy}	P_{VEP}	P_{NSI}
0.1	0.376591	0.280198	0.376685	0.375439
0.5	0.00148157	0.00013892	0.00150681	0.00269040
1.0	0.0212623	0.0169972	0.02123725	0.0165287
2.5	0.0183922	0.0169995	0.0178917	0.0166096
5.0	0.0171108	0.0164888	0.0187842	0.0107013
7.5	0.00987711	0.00962959	0.0128519	0.00454936
10	0.00617523	0.00605487	0.00996336	0.00201899