Probabilidad
Decoherencia

Elementos Previos

- Matrices de Gell - Mann:

$$\lambda_0 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ \lambda_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda_2 = \frac{1}{2}\begin{pmatrix} 0 & -I & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ \lambda_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -I \\ 0 & 0 & 0 \\ I & 0 & 0 \end{pmatrix},\ \lambda_6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ \lambda_6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -I \\ 0 & I & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

donde $I^2 = -1$.

Elementos Previos

- Matriz de Decoherencia:

$$MD = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_8 \end{pmatrix} \times 10^9 \ ,$$

donde:

$[\Gamma_j] = GeV \ , \ \forall\, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}.$

Elementos Previos

- Matriz de mezcla:

$$U_{PMNS} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} = U_{PMNS}\,(\theta_{12}, \theta_{13}, \theta_{23}, \delta, s) \;\equiv\; U \quad ,$$

donde $\theta_{12}, \theta_{13}, \theta_{23}, \delta \in IR$ y $s \in \{-1, 1\}$ y :

$U_{11} = \cos\theta_{12}\cos\theta_{13}$, $U_{12} = \cos\theta_{13}\sin\theta_{12}$, $U_{13} = e^{-i\delta s}\sin\theta_{13}$,
$U_{21} = -\cos\theta_{23}\sin\theta_{12} - e^{-i\delta s}\cos\theta_{12}\sin\theta_{13}\sin\theta_{23}$,
$U_{22} = \cos\theta_{23}\cos\theta_{12} - e^{-i\delta s}\sin\theta_{12}\sin\theta_{13}\sin\theta_{23}$,
$U_{23} = \cos\theta_{13}\sin\theta_{23}$,
$U_{31} = -e^{-i\delta s}\cos\theta_{12}\cos\theta_{23}\sin\theta_{13} + \sin\theta_{12}\sin\theta_{23}$,
$U_{32} = -e^{-i\delta s}\cos\theta_{23}\sin\theta_{12}\sin\theta_{13} - \cos\theta_{12}\sin\theta_{23}$,
$U_{33} = \cos\theta_{13}\cos\theta_{23}$.

Elementos Previos

- Hamiltoniano

$$H = \frac{1}{2E\times 10^9} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2{}_{21} & 0 \\ 0 & 0 & \Delta m^2{}_{31} \end{pmatrix} (U^*)^T + \begin{pmatrix} A_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

$$H \equiv H\left(E, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, s, \Delta m^2{}_{21}, \Delta m^2{}_{31}\right) \quad ,$$

donde:
$A_m = \rho \times 7.63247 \times 0.5 \times 10^{-14}$,
$[\rho] = g/cm^3$,
$[E] = GeV = 10^9 eV$,
$\left[\Delta m^2{}_{21}\right] = \left[\Delta m^2{}_{31}\right] = eV^2$.

Elementos Previos

- Matrices de estado inicial y final:

$$Mo = (U^*)^T \begin{pmatrix} \delta_\alpha^1 & 0 & 0 \\ 0 & \delta_\alpha^2 & 0 \\ 0 & 0 & \delta_\alpha^3 \end{pmatrix} U \;\; , \;\; Mf = (U^*)^T \begin{pmatrix} \delta_\beta^1 & 0 & 0 \\ 0 & \delta_\beta^2 & 0 \\ 0 & 0 & \delta_\beta^3 \end{pmatrix} U$$

donde:

$\alpha \in \{1, 2, 3\}$ $(sabor\ inicial)$,

$\beta \in \{1, 2, 3\}$ $(sabor\ final)$,

$$\delta_j^i = \begin{cases} 1 & , si \quad i = j \\ 0 & , si \quad i \neq j \end{cases} .$$

Elementos Previos

- Combinación lineal:

$$CL = a_0\lambda_0 + a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 + a_4\lambda_4 + a_5\lambda_5 + a_6\lambda_6 + a_7\lambda_7 + a_8\lambda_8 \ ,$$

$$CL = Mo \ , \quad ... \ (1)$$

donde $a_j \in Complejos \ \forall \ j \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\},$

resolvemos el sistema de (1) para hallar los valores de:

$a_j \ con \ j \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}.$

Elementos Previos

- Matriz de Densidad:

$$D(t) = \lambda_0\rho_0(t) + \lambda_1\rho_1(t) + \lambda_2\rho_2(t) + \lambda_3\rho_3(t) + \lambda_4\rho_4(t) + \lambda_5\rho_5(t) + \lambda_6\rho_6(t) + \lambda_7\rho_7(t) + \lambda_8\rho_8(t) \ ,$$

$$\frac{dD}{dt}(t) = \tilde{D}(t) = \lambda_0\frac{d\rho_0}{dt}(t) + \lambda_1\frac{d\rho_1}{dt}(t) + \lambda_2\frac{d\rho_2}{dt}(t) + \lambda_3\frac{d\rho_3}{dt}(t) + \lambda_4\frac{d\rho_4}{dt}(t) + \lambda_5\frac{d\rho_5}{dt}(t) + \lambda_6\frac{d\rho_6}{dt}(t) + \lambda_7\frac{d\rho_7}{dt}(t) + \lambda_8\frac{d\rho_8}{dt}(t) \ .$$

- Sistema de ecuaciones diferenciales :

$$\frac{dD}{dt}(t) = -I\{H.D(t) - D(t).H\} + \sum_{i=0}^{8}\sum_{j=0}^{8}\rho_i(t)(MD)_{ij}.\lambda_j \quad \ldots (2)$$

Elementos Previos

- Condiciones iniciales del sistema:

$$t = 0\ ,\ \rho_0(0) = a_0\ ,\ \rho_1(0) = a_1\ ,\ \rho_2(0) = a_2\ ,\ \rho_3(0) = a_3\ ,$$
$$\rho_4(0) = a_4\ ,\ \rho_5(0) = a_5\ ,\ \rho_6(0) = a_6\ ,\ \rho_7(0) = a_7\ ,\ \rho_8(0) = a_8.$$

- Matriz de densidad final:

$$t_f = \frac{L \times 10^9}{C_{GeVtoKm}} \quad , \quad D_f = D\left(t_f\right) = \sum_{j=0}^{8} \rho_j\left(t_f\right)\lambda_j\ ,\ \ldots (3)$$

donde:

$[L] = Km$,

$C_{GeVtoKm} = 0.197327.$

Probabilidad

- Cálculo final de la probabilidad:

$$P_{Dech} \equiv P\left(E, L, \rho, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \ \Delta m^2{}_{21}, \Delta m^2{}_{31}, MD, \alpha, \beta\right)$$

$$P_{Dech} = \left\| Traza\left[Mf.D_f \right] \right\| \quad \ldots (4)$$

Nota: Para los ejemplos $L = 1300$ y $\rho = 2.956740$.