Problem Sheet 1: Axiomatic Semantics

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Starred exercises $(*)$ are more challenging than the others.

1 Partial and Total Correctness

Recall the *Hoare triple* from lectures,

$$\{pre\}\ P\ \{post\}$$

where P denotes some program instructions, and $pre, post$ are respectively pre- and postconditions (*assertions* about the program state). The triple denotes a *specification* of the program, with respect to the given pre- and postconditions. Such a specification, in this course, can be interpreted in one of two different ways. If the specification holds in the sense of *partial correctness*, we will write:

$$\models \{pre\}\ P\ \{post\}$$

and if it holds in the sense of *total correctness*, we will write:

$$\models_{tot} \{pre\}\ P\ \{post\}$$

1.1 Exercises

Define, in English, the meaning of:

i. $\models \{pre\}\ P\ \{post\}$

ii. $\models_{tot} \{pre\}\ P\ \{post\}$

Which senses of correctness ($\models$, $\models_{tot}$, or neither) do the following triples hold under? Explain your judgements, giving counterexamples when they do not hold. (Formal proofs are *not* yet required. Assume a standard semantics for the program constructs. Assume that integer overflows cannot occur.)

iii. $\{x = 21 \wedge y = 5\}$ `skip` $\{y = 5\}$

iv. $\{y = 5\}$ `skip` $\{x = 21 \wedge y = 5\}$

v. $\{x > 10\}$ `x` $:=$ `2` $*$ `x` $\{x > 21\}$

vi. $\{x \geq 0 \wedge y > 1\}$ `while x < y do x :=` `x` $*$ `x` $\{x \geq y\}$

vii. $\{x = 5\}$ `while x > 0 do x :=` `x` $+$ `1` $\{x < 0\}$

2 A Hoare Logic for Partial Correctness

In the previous section we reasoned about the correctness of triples in an *ad hoc* way. In this section we will be more rigorous, instead *proving* such triples using a formal system of proof rules called a *Hoare logic*.

In particular, we will consider a Hoare logic for partial correctness. If a Hoare triple $\{pre\}\ P\ \{post\}$ can be *proven* using the rules of this Hoare logic, we will write:

$$\vdash \{pre\}\ P\ \{post\}.$$

Provability and Validity
Note carefully the distinction between $\vdash$, *provability*, and $\models$, *validity*. An important property, called *soundness*, is that every triple provable in a Hoare logic indeed holds in the desired sense of correctness. For our partial correctness Hoare logic, you can safely assume that it is *sound*, i.e.:

$$\vdash \{pre\}\ P\ \{post\} \quad \text{implies} \quad \models \{pre\}\ P\ \{post\}.$$

Figure 1 contains the proof rules of a Hoare logic for partial correctness that should be used in the exercises overleaf. Let x denote a program variable, and e denote a side-effect free expression. Let the lower-case symbols p, p', q, q', r denote assertions, b a Boolean expression, and the upper-case symbols P, Q arbitrary programs.

A triple can be proven in this Hoare logic if it can be instantiated from an axiom ([ass] or [skip]), or if it can be derived as the conclusion of an inference rule ([comp], [if], [while], or [cons]). (A reminder of how to apply axioms and inference rules is given in the appendix of this problem sheet.)

$$\text{[ass]} \quad \vdash \{p[e/x]\}\ x := e\ \{p\}$$

$$\text{[skip]} \quad \vdash \{p\}\ \texttt{skip}\ \{p\}$$

$$\text{[comp]}\ \frac{\vdash \{p\}\ P\ \{r\} \quad \vdash \{r\}\ Q\ \{q\}}{\vdash \{p\}\ P; Q\ \{q\}}$$

$$\text{[if]}\ \frac{\vdash \{b \wedge p\}\ P\ \{q\} \quad \vdash \{\neg b \wedge p\}\ Q\ \{q\}}{\vdash \{p\}\ \texttt{if}\ b\ \texttt{then}\ P\ \texttt{else}\ Q\ \{q\}}$$

$$\text{[while]}\ \frac{\vdash \{b \wedge p\}\ P\ \{p\}}{\vdash \{p\}\ \texttt{while}\ b\ \texttt{do}\ P\ \{\neg b \wedge p\}}$$

$$\text{[cons]}\ \frac{p \Rightarrow p' \quad \vdash \{p'\}\ P\ \{q'\} \quad q' \Rightarrow q}{\vdash \{p\}\ P\ \{q\}}$$

Figure 1: A Hoare logic for partial correctness

2.1 Exercises

In the exercises that require a proof (i.e. $\vdash$), you should use the proof rules of the Hoare logic in Figure 1.

Hint: the appendix contains a recap on the different types of proof rules—*axioms* and *inference rules*—and how to apply them in order to prove a Hoare triple.

i. What does the rule of consequence, [cons], allow us to do in proofs? Why must we show that $p \Rightarrow p'$ and $q' \Rightarrow q$ are valid implications?

ii. Explain the intuition behind the axiom of assignment, [ass].

iii. Show that $\vdash \{x > 0\}\ \texttt{x} := \texttt{x} + 1;\ \texttt{skip}\ \{x > 1\}$.

iv. Show that $\vdash \{x = a \wedge y = b\}\ \texttt{t} := \texttt{x};\ \texttt{x} := \texttt{x} + \texttt{y};\ \texttt{y} := \texttt{t}\ \{x = a + b \wedge y = a\}$.

v. Explain the intuition behind the proof rule [while].

vi. Let INV denote $in + m = 250$. Show that:

$$\vdash \{INV\}\ \texttt{while}\ (\texttt{i} > 0)\ \texttt{do}\ \texttt{m} := \texttt{m} + \texttt{n};\ \texttt{i} := \texttt{i} - 1\ \{INV\}.$$

vii. Define a proof rule for the Pascal construct:

$$\texttt{repeat}\ P\ \texttt{until}\ b$$

without reference to the [while] rule.

Hint: the loop body is always executed *at least* once.

viii. A new command has been added to the language: `surprise`. It has a very unusual semantics. Upon execution, `surprise` randomly chooses a variable in the program state, and randomly changes its assignment. Propose and justify an axiom for this command.

ix. (∗) Propose an alternative, "forward" axiom of assignment in which a substitution is not applied to the precondition, i.e. replace the question marks in:

$$\vdash \{p\}\ x := e\ \{??\}.$$

Hint: the postcondition contains an existential quantifier.

Appendix: Applying Proof Rules

We review in this appendix how to prove triples in Hoare logics through the application of axioms and inference rules.

The proof rules [skip] and [ass] of Figure 1 are examples of *axioms*[1]. Axioms describe "self-evident" facts about programs, in the sense that they can be applied without additional reasoning. A triple $\{pre\}\ P\ \{post\}$ is *proven* in a Hoare logic, denoted $\vdash \{pre\}\ P\ \{post\}$, if it can be instantiated from such an axiom. For example, from the [skip] axiom, we can immediately prove any triple for `skip` that has the same pre- and postcondition, for example:

$$\vdash \{x = 5\}\ \texttt{skip}\ \{x = 5\}$$

$$\vdash \{\text{true}\}\ \texttt{skip}\ \{\text{true}\}$$

Most triples, of course, cannot simply be instantiated from axioms. For these, we must apply *inference rules*, such as [comp], [if], [while], and [cons] from Figure 1. Inference rules consist of at least one premise (given above the horizontal line), and exactly one conclusion (given underneath). If we can prove the premises, then we can deduce the triple in the conclusion. For example, to prove the triple:

$$\{x > 0\}\ \texttt{x} := \texttt{x} + \texttt{1};\ \texttt{skip}\ \{x > 1\}$$

simply instantiate [comp] with this triple as the conclusion, and then proceed to prove the premises:

$$\vdash \{x > 0\}\ \texttt{x} := \texttt{x} + \texttt{1}\ \{r\}$$

$$\text{and} \quad \vdash \{r\}\ \texttt{skip}\ \{x > 1\}.$$

Here r is some assertion, to be determined, that holds at the midpoint of the two subprograms.

Visualising proofs is a matter of personal taste. For smaller proofs, I usually find it helpful to visualise them as *proof trees*, with axiom instantiations as the leaves, applications of inference rules as the body, and the triple we want to prove as the root. For example, the above could be visualised as follows (note that we have taken r to be $x < 1$):

$$\dfrac{\dfrac{??}{\vdash \{x > 0\}\ \texttt{x} := \texttt{x} + \texttt{1}\ \{x > 1\}}[??] \qquad \dfrac{}{\vdash \{x > 1\}\ \texttt{skip}\ \{x > 1\}}[\text{skip}]}{\vdash \{x > 0\}\ \texttt{x} := \texttt{x} + \texttt{1};\ \texttt{skip}\ \{x > 1\}}[\text{comp}]$$

(The left-hand side of this proof tree is left incomplete because it forms part of an exercise!)

An inference rule we should give some special attention to is [cons], the rule of consequence. Two of its premises require showing—outside of the Hoare logic—that some logical implications are valid. We will discuss this further in the exercises.

[1] More precisely, *axiom schemata*, but most authors drop the latter part in their terminology.

Problem Sheet 2: Auto-Active Verification

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"Beware of bugs in the above code; I have only proved it correct, not tried it."
– Donald E. Knuth

Starred exercises (∗) are more challenging than the others.

1 Boogie

In this first set of exercises, we will work directly with Boogie [1], the intermediate verification language and automatic verification framework developed by Microsoft Research. For an introduction to the language and verifier, consult the following slides and manual:

- The lecture slides:
  http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/05-AutoActiveVerification.pdf
- Boogie manual:
  http://research.microsoft.com/en-us/um/people/leino/papers/krml178.pdf

1.1 Setting Up

Boogie can be interacted with through your web browser:

http://rise4fun.com/boogie

Should you prefer, you can also install it locally:

http://research.microsoft.com/en-us/projects/boogie/

*With input from Nadia Polikarpova and Julian Tschannen.

1.2 Exercises

i. Prove the following specification in Boogie:

$$\{\text{true}\}\ t := x;\ x := x + y;\ y := t\ \{x = x' + y' \wedge y = x'\}$$

where x', y' respectively denote the values of x and y in the pre-state.

ii. Consider the Boogie program `ArraySum` which is supposed to recursively compute the sum of array elements:

`http://rise4fun.com/Boogie/2kR`

Fix the program and then verify it in Boogie.

Hint: don't forget about loop invariants! Without an invariant, any loop in Boogie is treated as equivalent to assigning arbitrary values to program variables.

iii. Implement and verify the algorithm `FindZero` (its signature is given in `http://rise4fun.com/Boogie/SciP`), that linearly searches an array for the element 0:

> *Input:* an integer array a, and its length N.
>
> *Output:* an index $k \in \{0, \ldots, N-1\}$ into the array a such that $a[k] = 0$; otherwise $k = -1$.

The specification should guarantee that if there exists an array element $a[i] = 0$ with $0 \leq i < N$, then `FindZero` will always return a k such that $k \geq 0$ and $a[k] = 0$.

iv. (∗) Copy the procedure `FindZero` you wrote in part (iv), rename it to `FindZeroPro`, and add the following two preconditions:

```
requires (forall i: int :: 0 <= i && i < N ==> 0 <= a[i]);
requires (forall i: int :: 0 <= i-1 && i < N ==> a[i-1]-1 <= a[i]);
```

These additional preconditions require that along the array, values never decrease by more than one. Adapt your linear search algorithm such that after an iteration of its loop, instead of incrementing the current index k by 1, it now increments it by $a[k]$. Verify that the procedure still establishes the same postconditions as in (iv).

Hint: you will need to prove that all array values between $a[k]$ and $a[k+a[k]]$ are non-zero (i.e. that 0-values are not skipped over by the search) and use this property in the loop. For this you will need to write more than simply a loop invariant, e.g. some "ghost" (or "proof") code.

v. (∗) Take a look at the Boogie program `BinarySearch` which is supposed to perform a binary search[1]:

`http://rise4fun.com/Boogie/Ilf`

Debug the implementation and add the missing loop invariants.

[1] See: `http://en.wikipedia.org/wiki/Binary_search_algorithm`

2 AutoProof

This second set of exercises is concerned with AutoProof [2], an auto-active verifier for programs written in (a subset of) the object-oriented language Eiffel. The tool takes an Eiffel program annotated with *contracts* (i.e. executable pre-/postconditions, class invariants, intermediate assertions), and translates it into a Boogie program for verification. Errors returned by the Boogie verifier are traced back to the relevant parts of the original Eiffel program (see Figure 1).

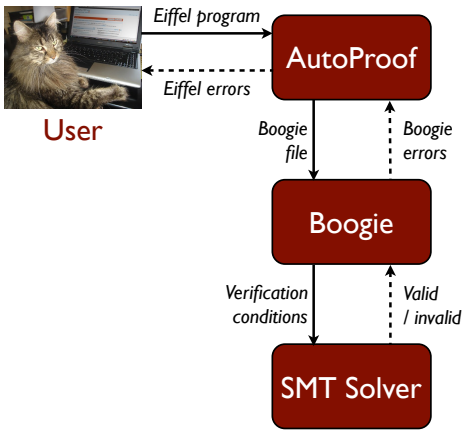

Figure 1: The AutoProof workflow

Extensive documentation (including a manual, tutorial, and software repository) is available online:

http://se.inf.ethz.ch/research/autoproof/

2.1 Exercises

The Eiffel programs for the following exercises are all available online in a web-based interface to AutoProof. Simply follow the given links for each exercise, edit the code and contracts in your browser, then hit the "Verify" button to run the tool. Note that the web interface to AutoProof does not permanently save your changes, so please make sure to save local copies of your solutions.

i. Consider the class `WRAPPING_COUNTER` in:

   http://cloudstudio.ethz.ch/e4pubs/#sv-task1

   The method `increment` increases its integer input by one, *except* if the input is 59, in which case it wraps it round to 0. Verify the class in AutoProof *without* changing the implementation, i.e. adding only the necessary preconditions. Strengthen the postcondition further as suggested in the comments, and check that the proof still goes through.

ii. In the axiomatic semantics problem sheet, we encountered several simple program specifications expressed as Hoare triples. Using the class `AXIOMATIC_SEMANTICS` in:

   http://cloudstudio.ethz.ch/e4pubs/#sv-task2

   write some simple contract-equipped methods and show the following in AutoProof:

(A) $\models$ $\{x = 21 \wedge y = 5\}$ `skip` $\{y = 5\}$
(B) $\models$ $\{x > 10\}$ `x` $:=$ `2` $*$ `x` $\{x > 21\}$
(C) $\models$ $\{x \geq 0 \wedge y > 1\}$ `while x < y do x := x * x` $\{x \geq y\}$
(D) $\models$ $\{x = 5\}$ `while x > 0 do x := x + 1` $\{x < 0\}$
(E) $\models$ $\{x = a \wedge y = b\}$ `t := x; x := x + y; y := t` $\{x = a + b \wedge y = a\}$
(F) $\models$ $\{in + m = 250\}$ `while (i > 0) do m := m + n; i := i - 1` $\{in + m = 250\}$

Hint: Eiffel does not offer a while construct. Try experimenting with from-until-loop instead, as well as if-then-else with recursion (note that recursive calls should be surrounded by `wrap` and `unwrap` so that the verifier checks the class invariant—see the code comments).

iii. Consider the class `MAX_IN_ARRAY` in:

http://cloudstudio.ethz.ch/e4pubs/#sv-task3

What does the `max_in_array` method do? Prove the class correct in AutoProof by determining a suitable precondition and loop invariant.

Hint: you might find Eiffel's across-as-all loop construct[2] helpful for expressing loop invariants.

iv. (∗) Consider the class `SUM_AND_MAX` in:

http://cloudstudio.ethz.ch/e4pubs/#sv-task4

What does the method `sum_and_max` do? What can you prove about it using AutoProof?

v. (∗∗) Consider the class `LCP` in:

http://cloudstudio.ethz.ch/e4pubs/#sv-task5

The method `lcp` implements a Longest Common Prefix (LCP) algorithm[3] with input and output as follows:

> *Input:* an integer array a, and two indices x and y into this array.
>
> *Output:* length of the longest common prefix of the subarrays of a starting at x and y respectively.

What can you prove about the class in AutoProof?

References

[1] K. Rustan M. Leino. This is Boogie 2. Technical report, 2008. http://research.microsoft.com/en-us/um/people/leino/papers/krml178.pdf.

[2] Julian Tschannen, Carlo A. Furia, Martin Nordio, and Nadia Polikarpova. AutoProof: Auto-active functional verification of object-oriented programs. In *Proc. TACAS 2015*, volume 9035 of *LNCS*, pages 566–580. Springer, 2015. http://se.inf.ethz.ch/people/tschannen/publications/tfnp-tacas15.pdf.

[2] See: http://bertrandmeyer.com/2010/01/26/more-expressive-loops-for-eiffel/
[3] From the FM 2012 verification challenge.

Problem Sheet 3: Data Flow Analysis

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Starred exercises (∗) are more challenging than the others.

1 Reaching Definitions Analysis

These exercises are based on the material from the "Reaching Definitions Analysis" section of this lecture:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/06-DataFlowAnalysis.pdf

Consider the following program fragment:

```
x := 10;
while x > 0 do
     y := 2 * y;
     if y > 10 do
          x := x - 1;
     else
          y := x + 2;
     end
     x := x - 1;
end
x := x - 1;
```

i. Draw the *control flow graph* of the program fragment.

ii. Annotate the control flow graph with the results of a *reaching definitions analysis*.

iii. Provide (or draw) the *use-definition* information for program variables `x` and `y`.

*Exercises adapted from an earlier version of the course, when Stephan van Staden was the teaching assistant.

2 Live Variables Analysis

These exercises are based on the material from the "Live Variables Analysis" and "Equation Solving" sections of this lecture:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/06-DataFlowAnalysis.pdf

Consider the following program fragment:

```
x := y;
x := x - 1;
x := 4;
while y < x do
     y := y + x;
end
y := 0;
```

i. Identify the elementary blocks of the program and label them.

ii. Write down the *equations* for a live variables analysis of the program.

iii. Solve the data flow equations using *chaotic iteration*.

iv. Using the result obtained in (iii), perform *dead code elimination* on the program fragment.

v. (∗) Is the program resulting in step (iv) free of dead variables? If not, explain why and modify the live variables analysis so that it can be used to produce a program free of dead variables.

Problem Sheet 4: Program Slicing and Abstract Interpretation

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Starred exercises (∗) are more challenging than the others.

1 Program Slicing

These exercises are based on the material from the "Program Slicing" section of this lecture:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/07-Slicing.pdf

Consider the following program fragment:

```
x := 0;
y := 0;
i := n;
j := n;
while i > 0 do
     x := x + 1;
     i := i - 1;
     j := i;
     while j > 0 do
          y := y + 1;
          j := j - 1;
     end
end
print(x);
print(y);
```

i. Draw the *program dependence graph* for this fragment.

ii. Compute the backward slice of the program fragment for the *slicing criteria* `print(x)` and `print(y)`.

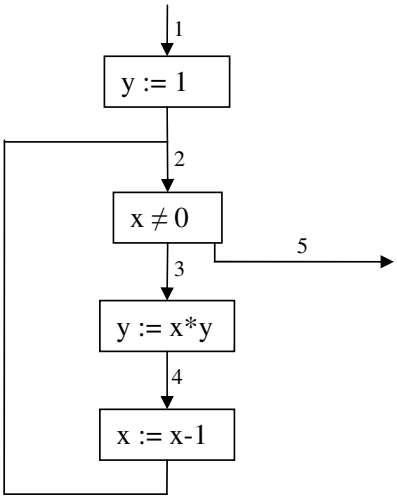
*Exercises adapted from an earlier version of the course, when Stephan van Staden was the teaching assistant.

2 Abstract Interpretation

These exercises are based on the material from the "Abstract Interpretation" lecture:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/08-AbstractInterpretation.pdf

Consider again the factorial algorithm from the lecture with sign analysis equations:



$$
\begin{aligned}
A_1 &= [x \mapsto +,\ y \mapsto \top] \\
A_2 &= A_1[y \mapsto +] \sqcup A_4[x \mapsto A_4(x) \ominus +] \\
A_3 &= A_2 \\
A_4 &= A_3[y \mapsto A_3(x) \otimes A_3(y)] \\
A_5 &= A_2 \sqcap [x \mapsto 0,\ y \mapsto \top]
\end{aligned}
$$

i. Compute the analysis result by *chaotic iteration*.

ii. Is the analysis precise? What is it unable to prove about the program?

iii. Improve the precision by:

   (a) Changing the program but not the analysis (i.e. compute the factorial in a way that is more "friendly" for the analysis).

   (b) (∗) Changing the analysis but not the program.

Problem Sheet 5: Model Checking

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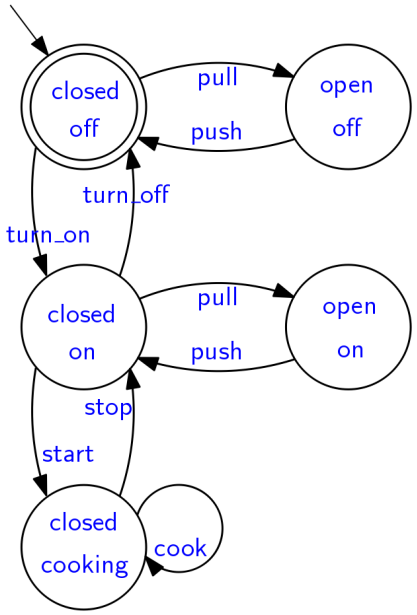
The exercises in this problem sheet are all based on the first set of model checking slides:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/09-ModelChecking.pdf

1 Evaluating LTL Formulae on Automata

Consider the the following microwave oven automaton:



Do the following properties hold? Justify your judgements.

i. $\Box$ (start $\rightarrow \Diamond$ stop)

ii. $\Box \Diamond$ turn_off

iii. $\Box \Diamond$ (turn_off $\vee$ push)

iv. $\Diamond$ (turn_off $\vee$ push)

v. ($\Box$ false) $\vee \Diamond$ (turn_off $\vee$ push)

vi. (turn_on U start) $\vee$ (pull U push)

2 Equivalence of LTL Formulae

These exercises are about proving the equivalence of LTL formulae. In the first two of these exercises, you will be proving that the operators $\Diamond$ and $\Box$ are *derived*, i.e. they can be defined entirely in terms of the core LTL operators.

i. Recall that $w, i \models \Diamond F$ if there exists some j such that $i \leq j \leq n$ and $w, j \models F$.

Prove that this is also the case for $w, i \models \text{true } \mathsf{U}\ F$ (i.e. that this is an equivalent way of expressing $\Diamond F$ as a derived operator).

ii. Recall that $w, i \models \Box F$ if for all j such that $i \leq j \leq n$, $w, j \models F$.

Prove that this is also the case for $w, i \models \neg\Diamond\neg F$ (i.e. that this is an equivalent way of expressing $\Box F$ in terms of other LTL operators).

iii. Prove that $\Diamond$ is *idempotent*, i.e. that $\Diamond p$ is equivalent to $\Diamond\Diamond p$.

3 Automata-Based Model Checking

Let us prove by *model checking* that $\Box\ \Diamond$ turn_off is not a property of the microwave oven automaton in Section 1.

i. Build an automaton with the same language as $\neg(\Box\ \Diamond$ turn_off).

Hint: start with the non-negated formula and then invert the accepting and non-accepting states of its automaton.

ii. Compute the intersection of the automaton you built in part (i) and the microwave oven automaton.

iii. Check the intersection automaton for accepting runs, using them to prove that $\Box\ \Diamond$ turn_off is not a property of the microwave oven automaton.

Problem Sheet 6: Software Model Checking

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The exercises in this problem sheet are based on the software model checking slides:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/11-SoftwareModelChecking.pdf

1 Predicate Abstraction

Recall that:

- $Pred(f)$ denotes the weakest under-approximation of the expression f expressible as a Boolean combination of the given predicates.
- The Boolean abstraction of an `assume` c `end` statement is `assume not` $Pred($`not` $c)$ `end` followed by a parallel conditional assignment updating the predicates with respect to the original `assume` statement.
- The Boolean abstraction of an `assert` c `end` statement is `assert` $Pred(c)$ `end`.

Exercises

i. Justify, using Venn diagrams, the use of double negation in the Boolean abstraction of `assume` statements.

ii. Consider the following code snippet C_1, where x, y, z are integer variables:

```
assume x > 0 end
z := (x*y) + 1
assert z >= 1 end
```

Build the Boolean abstraction A_1 of the code snippet C_1 with respect to the following set of predicates:

$$p \triangleq x > 0$$
$$q \triangleq y > 0$$
$$r \triangleq z > 0$$

*Some exercises were adapted from earlier ones written by Stephan van Staden and Carlo A. Furia.

iii. Consider the following code snippet C_2, where x, y are integer variables:

```
if x > 0 then
     y := x + x
else
     if x = 0 then
          y := 1
     else
          y := x * x
     end
end
assert y > 0 end
```

(a) Normalise the guards of conditionals using nondeterminism and `assume` statements.

(b) Build the Boolean abstraction A_2 of the normalised code snippet C_2 with respect to the following set of predicates:

$$p \triangleq x > 0$$
$$q \triangleq y > 0$$

2 Error Traces

i. Provide an annotated trace for the Boolean abstraction A_1, and a corresponding (feasible) annotated trace for the concrete program C_1 in which `assert z >= 1 end` evaluates to false when reached.

ii. Can you verify the Boolean abstraction A_2? If not, give a trace as a counterexample and prove whether or not it is *spurious*.

Problem Sheet 7: Verification of Real-Time Systems

Carlo A. Furia and Chris Poskitt

Starred exercises (∗) are more challenging than the others.

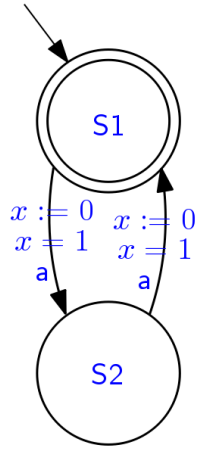
The following exercises are based on the lectures about verifying real-time systems:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/13-RealTimeSystems.pdf

Assume that the time domain consists of the non-negative real numbers.

1 MTL Property Checking

Consider first the following timed automaton:



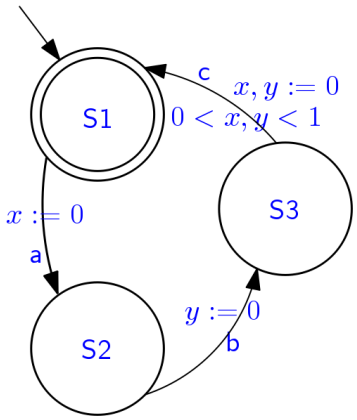
Do the following properties hold?

i. $\Box\ a$

ii. $\Box\ (\Diamond{=}1\ a)$

iii. $\Box\ (\Box{=}1\ a)$

Consider now the following timed automaton:



Do the following properties hold?

iv. $\square\ (a \rightarrow \Diamond(0,1)\ c)$

v. $\square\ (a \rightarrow \Diamond(0,1)\ b)$

vi. $\square\ (a \rightarrow (a \vee b)\ \mathsf{U}(0,1)\ c)$

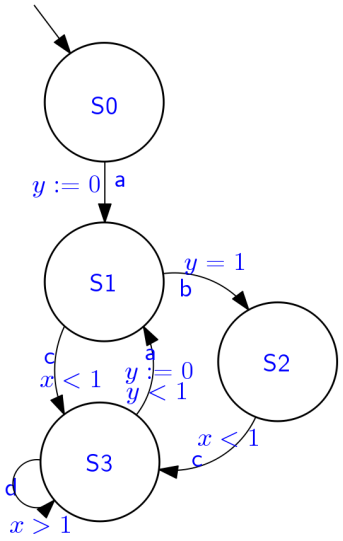
vii. $\square\ (a \rightarrow (a \vee b)\ \mathsf{U}(1,2)\ c)$

2 Region Automaton Construction

i. Construct the region automaton for the first timed automaton in Section 1.

ii. Construct the region automaton for the second timed automaton in Section 1.

iii. (∗) Construct the region automaton for the following timed automaton (exercise taken from *Alur & Dill, 1994*):



3 Semantics of MTL Formulae

i. Is the formula $\Box \Diamond_{>0}$ true satisfied by any timed word?

ii. Is the formula $\Box \Diamond_{\geq 0}$ true satisfied by any timed word?

iii. Is $\Diamond_{[a,b]} \Diamond_{[c,d]} q$ equivalent or non-equivalent to $\Diamond_{[a+c,b+d]} q$ for all $0 \leq a \leq b \leq c \leq d$?

Problem Sheet 8: Separation Logic

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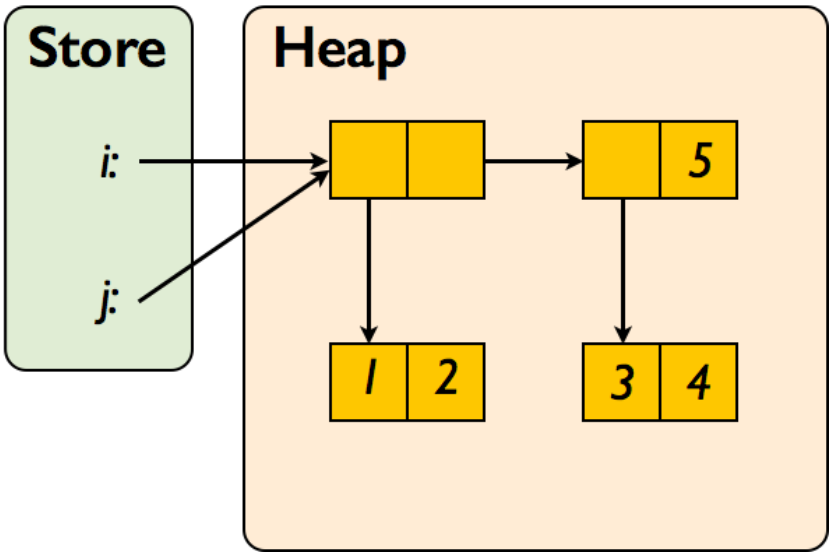
Starred exercises (∗) are more challenging than the others.

1 Separation Logic Assertions

These exercises are about assertions in separation logic, in particular, the *separating conjunction* $*$. For a reminder of the semantics of assertions (as well as numerous examples), please refer to the first set of separation logic lecture slides:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/15-SeparationLogic-Part1.pdf

i. Consider the following state:



Which of the following assertions hold in this state? For the ones that do not: why not?

(a) $\exists z.\ i \mapsto z$
(b) $\exists z.\ i \mapsto z * j \mapsto z$
(c) $\exists z.\ i \mapsto z * \text{true}$
(d) $\exists z.\ (i \mapsto z * \text{true}) \wedge i = j \wedge (z \mapsto 1, 2 * \text{true})$
(e) $\exists x, x', y, z.\ i \mapsto x * j \mapsto x' * x \mapsto 1, 2 * j + 1 \mapsto y * y \mapsto z, 5 * z \mapsto 3, 4$
(f) $\text{true} * \text{emp}$

ii. Do the following implications hold for all states and predicates p, q? If not, why not?

$$p \text{ implies } p * p$$
$$p * q \text{ implies } [(p \wedge q) * \text{true}]$$

2 Separation Logic Proofs

These exercises involve *proving* the correctness of heap-manipulating programs using separation logic. For some Hoare-style proof rules, please refer back to the first problem sheet. For the small axioms and the frame rule of separation logic, please refer to the first set of separation logic lecture slides:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/15-SeparationLogic-Part1.pdf

The second set of lecture slides demonstrates their use on two simple heap-manipulating programs:

http://se.inf.ethz.ch/courses/2015b_fall/sv/slides/16-SeparationLogic-Part2.pdf

i. Consider the following program:

```
l := cons(1);
r := cons(2,3);
temp1 := [r+1];
temp2 := [l];
[l] := temp1;
[r] := temp2;
```

Starting from precondition {emp}, apply the axioms and inference rules of separation logic to derive a postcondition expressing exactly the contents of the store and heap at termination. Then, depict this state using the store and heap diagrams presented in the lectures.

ii. We can assert that a heap contains a linked list by using the following inductively-defined predicate:

$$\begin{aligned} \text{list}([], i) &\iff \text{emp} \wedge i = nil \\ \text{list}(a :: as, i) &\iff \exists j.\ i \mapsto a, j * \text{list}(as, j) \end{aligned}$$

where nil is a constant used to terminate the list.

Using the list predicate, verify the following program, which deletes the first item of a non-empty linked list (assume that i points to the list).

```
dispose(i);
k := [i+1];
dispose(i+1);
i := k;
```

iii. **(∗)** Consider the following inductively-defined predicate:

$$\begin{aligned}\text{tree}(e,\tau) \iff & \texttt{if } (\text{isAtom}(e) \land e = \tau) \texttt{ then emp} \\ & \texttt{else } \exists x, y, \tau_1, \tau_2.\ \tau = \langle \tau_1, \tau_2 \rangle \\ & \land\ e \mapsto x, y * \text{tree}(x, \tau_1) * \text{tree}(y, \tau_2)\end{aligned}$$

where $\text{tree}(p, \tau)$ holds if p points to a data structure in memory representing the "mathematical" tree τ (i.e. the tree as an abstract mathematical object; not a representation in computer memory). Consider the following specification:

$$\{\text{tree}(p,\tau)\}\ \text{CopyTree}(p,q)\ \{\text{tree}(p,\tau) * \text{tree}(q,\tau)\}$$

which expresses that the procedure CopyTree stores, in a separate portion of memory pointed to by q, a mathematically equivalent tree to the one pointed to by p.

Define the procedure CopyTree and verify the specification.

Hint: the following derived axiom for heap lookups may simplify the proof:

$$\vdash \{e \mapsto e'\}\ x := [e]\ \{e \mapsto e' \land x = e'\}$$

provided that x does not appear free in e or e'.

Problem Sheet 9: Program Proofs

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1 Axiomatic Semantics

This section provides some additional questions on Hoare logic. Some proof rules are provided in Figure 1.

i. Devise an axiom for the command $\texttt{havoc}(\texttt{x}_0, \ldots, \texttt{x}_n)$, which assigns arbitrary values to the variables $\texttt{x}_0, \ldots, \texttt{x}_n$.

ii. Write a program that computes the factorial of a natural number stored in variable `x` and assigns the result to variable `y`. Prove that the program is correct using our Hoare logic.

iii. Define a proof rule for the `from-until-loop` construct.

iv. Consider the following annotated program, where A is an array indexed from 1 with elements of type G, n is an integer variable storing A's size, k is another integer variable, v is a variable of type G initialised to some fixed value, and *found* is a Boolean variable.

$\{n \geq 0\}$

```
from
    k := n
    found := False
until found or k < 1 loop
  if A[k] = v then
    found := True
  else
    k := k − 1
  end
end
```

$\{(found \implies 1 \leq k \leq n \wedge A[k] = v) \wedge (\neg found \implies k < 1)\}$

(a) What does the program do? In particular, what does the value of k represent on exit?

(b) Prove the triple using the axioms and inference rules of Hoare logic.

v. Sarah Proofgood has successfully shown that given an arbitrary program P and postcondition *post*, the triple:

$$\{\text{WP}[P, post]\}\ P\ \{post\}$$

can be proven in our Hoare logic, i.e. $\vdash \{\text{WP}[P, post]\}\ P\ \{post\}$. Here, $\text{WP}[P, post]$ is an assertion expressing the *weakest (liberal) precondition* relative to P and *post*; that is, the weakest condition that must be satisfied for P to establish *post* (without guaranteeing termination).

Using Sarah's result, show that *any* valid triple $\models \{p\}\ P\ \{q\}$ is provable in our Hoare logic, i.e. $\vdash \{p\}\ P\ \{q\}$.

2 Separation Logic

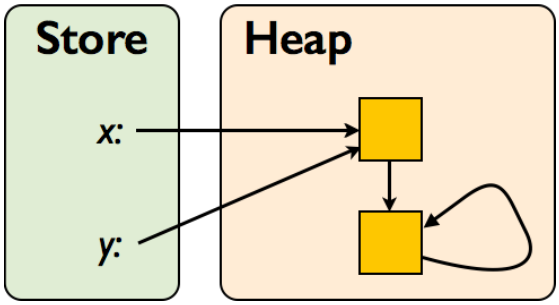
This section provides some additional practice on using separation logic. The small axioms and frame rule of separation logic are given in Figure 2.

i. Are the following assertions satisfiable? Justify your answers.

$$p * \neg p$$

$$x = y * \neg(x = y)$$

ii. Consider the following program state:



Which of the following assertions does this state satisfy? For the assertions it does not satisfy: why not?

(a) $\exists v.\ x \mapsto v * v \mapsto v$

(b) $\exists v.\ x \mapsto v * v \mapsto v * y \mapsto v$

(c) $y \mapsto \_$

(d) $(x = y) \wedge (y \mapsto \_ * \text{true})$

(e) $(x = y) * \text{true}$

iii. Starting from precondition {emp}, apply the axioms and inference rules of separation logic to derive a postcondition expressing exactly the contents of the store and heap at termination (assume that `x` and `y` are the only variables). Then, depict the post-state using the store and heap diagrams presented in the lectures.

```
x := cons(5,9);
y := cons(6,7);
x := [x];
[y+1] := 9;
dispose(y);
```

iv. A well-formed binary tree t is defined by the grammar:

$$t \triangleq n \mid (t_1, t_2)$$

i.e. t can be either a leaf, which is a single number n, or an internal node with a left subtree t_1 and a right subtree t_2. Consider the following definition of the inductive predicate $tree(t, i)$ which asserts that i is a pointer to a well-formed binary tree t:

$$\begin{aligned} tree(n, i) &\triangleq i \mapsto n \\ tree((t_1, t_2), i) &\triangleq \exists l, r.\ i \mapsto l, r * tree(t_1, l) * tree(t_2, r) \end{aligned}$$

Using these definitions, give a proof outline of the following triple. There must be at least one assertion between every two commands.

$\{tree((1, t), i)\}$

```
x := [i];
[i] := 2;
y := [i+1];
dispose(i);
dispose(x);
dispose(i+1);
```

$\{tree(t, y)\}$

Appendix: Proof Rules

$$[\text{ass}] \quad \vdash \{p[e/x]\}\ x := e\ \{p\}$$

$$[\text{skip}] \quad \vdash \{p\}\ \texttt{skip}\ \{p\}$$

$$[\text{comp}]\ \frac{\vdash \{p\}\ P\ \{r\} \quad \vdash \{r\}\ Q\ \{q\}}{\vdash \{p\}\ P; Q\ \{q\}}$$

$$[\text{if}]\ \frac{\vdash \{b \wedge p\}\ P\ \{q\} \quad \vdash \{\neg b \wedge p\}\ Q\ \{q\}}{\vdash \{p\}\ \texttt{if}\ b\ \texttt{then}\ P\ \texttt{else}\ Q\ \{q\}}$$

$$[\text{while}]\ \frac{\vdash \{b \wedge p\}\ P\ \{p\}}{\vdash \{p\}\ \texttt{while}\ b\ \texttt{do}\ P\ \{\neg b \wedge p\}}$$

$$[\text{cons}]\ \frac{p \Rightarrow p' \quad \vdash \{p'\}\ P\ \{q'\} \quad q' \Rightarrow q}{\vdash \{p\}\ P\ \{q\}}$$

Figure 1: A Hoare logic for partial correctness

$$\vdash \{e \mapsto \_\}\ [e] := f\ \{e \mapsto f\}$$

$$\vdash \{e \mapsto \_\}\ \texttt{dispose}(e)\ \{\text{emp}\}$$

$$\vdash \{X = x \wedge e \mapsto Y\}\ x := [e]\ \{e[X/x] \mapsto Y \wedge Y = x\}$$

$$\vdash \{\text{emp}\}\ x := \texttt{cons}(e_0, \ldots, e_n)\ \{x \mapsto e_0, \ldots, e_n\}$$

$$\frac{\vdash \{p\}\ P\ \{q\}}{\vdash \{p * r\}\ P\ \{q * r\}}$$

side condition: no variable modified by P appears free in r

Figure 2: The small axioms and frame rule of separation logic

Problem Sheet 10: Testing

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For both of these excerpts of code, answer the coverage-related questions that follow:

Algorithm 1:

```
String function(int x, int y)
{
        boolean z;

        if (x < y)
                z := true
        else
                z := false

        if (z && x+y == 10)
                result := "a"
        else
                result := "b"
}
```

Algorithm 2:

```
if x > 0 then
   y := x + x
   while y < 15 do
      y := y + 2
   end
else
   if x = 0 then
      y := 1
   else
      y := x * x
   end
end
```

1 Branch and Path Coverage

i. How many branches are present?

ii. Is it possible to test every branch? Provide a set of tests to exercise as many branches as possible.

iii. How many paths are present?

iv. Which path(s) remains untested by your tests in part (ii)?

v. Add tests, if needed, to achieve path coverage.

2 Logic Coverage

i. Write down the predicates that occur in the code.

ii. Is it possible to obtain full predicate coverage? Provide a set of tests that will obtain the highest possible predicate coverage.

iii. Write down the clauses appearing in the code.

iv. Can we exercise full clause coverage? Write tests for maximal clause coverage.

v. In general, does predicate and/or clause coverage imply branch coverage?

*Exercise sheet adapted from an earlier version by Stephan van Staden.