The tables below report the estimated mean and standard error of the in-control ARL based on $K = 50,000$ simulated samples from the indicated distributions.

Wrapped symmetric t

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	240 (0.63)	242 (0.63)	242 (0.64)	234 (1.02)	234 (1.04)	228 (1.03)
500	490 (1.27)	490 (1.27)	490 (1.28)	491 (2.18)	483 (2.16)	477 (2.15)
1000	1033 (2.69)	1039 (2.69)	1039 (2.69)	1019 (4.54)	999 (4.50)	976 (4.43)

Wrapped Student's t with $df = 2$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	241 (0.64)	241 (0.63)	235 (1.03)	233 (1.04)	229 (1.04)
500	490 (1.27)	490 (1.28)	487 (1.28)	494 (2.19)	482 (2.16)	475 (2.15)
1000	1038 (2.70)	1037 (2.72)	1035 (2.70)	1017 (4.54)	1001 (4.52)	988 (4.46)

Wrapped Student's t with $df = 3$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.64)	242 (0.63)	244 (0.64)	240 (1.03)	239 (1.04)	235 (1.04)
500	493 (1.28)	492 (1.27)	492 (1.27)	496 (2.19)	488 (2.15)	482 (2.14)
1000	1038 (2.67)	1040 (2.67)	1040 (2.70)	1024 (4.53)	1003 (4.51)	988 (4.44)

Wrapped Student's t with $df = 2$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	243 (0.63)	244 (0.64)	240 (1.03)	239 (1.04)	236 (1.05)
500	490 (1.26)	489 (1.27)	490 (1.28)	501 (2.19)	489 (2.16)	483 (2.17)
1000	1034 (2.68)	1037 (2.70)	1040 (2.70)	1022 (4.57)	1007 (4.50)	994 (4.43)

Wrapped Student's t with $df = 3$, warmup = 25

Wrapped symmetric stable

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	241 (0.63)	240 (0.63)	236 (1.03)	233 (1.04)	229 (1.03)
500	490 (1.27)	488 (1.28)	488 (1.28)	490 (2.17)	487 (2.19)	479 (2.17)
1000	1034 (2.68)	1034 (2.69)	1036 (2.70)	1020 (4.57)	1009 (4.54)	991 (4.51)

Wrapped Stable with $\alpha = 2$, $\beta = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	243 (0.64)	243 (0.64)	237 (1.04)	234 (1.04)	225 (1.03)
500	488 (1.26)	492 (1.28)	493 (1.28)	492 (2.17)	484 (2.16)	464 (2.12)
1000	1036 (2.69)	1039 (2.70)	1043 (2.70)	1014 (4.50)	996 (4.45)	959 (4.34)

Wrapped Stable with $\alpha = 1$, $\beta = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.64)	246 (0.65)	244 (0.66)	238 (1.05)	232 (1.05)	209 (1.03)
500	491 (1.27)	494 (1.29)	497 (1.30)	499 (2.20)	479 (2.18)	427 (2.05)
1000	1042 (2.70)	1048 (2.72)	1055 (2.73)	1021 (4.53)	979 (4.40)	874 (4.10)

Wrapped Stable with $\alpha = 0.5$, $\beta = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	244 (0.63)	241 (0.63)	243 (0.64)	241 (1.04)	238 (1.03)	235 (1.03)
500	491 (1.28)	492 (1.28)	489 (1.27)	496 (2.18)	495 (2.20)	488 (2.19)
1000	1034 (2.70)	1042 (2.70)	1044 (2.72)	1031 (4.60)	1017 (4.54)	996 (4.46)

Wrapped Stable with $\alpha = 2$, $\beta = 0$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	244 (0.63)	245 (0.64)	244 (0.64)	244 (1.05)	239 (1.04)	234 (1.04)
500	493 (1.29)	494 (1.29)	494 (1.29)	493 (2.16)	491 (2.17)	476 (2.13)
1000	1045 (2.70)	1038 (2.68)	1044 (2.73)	1018 (4.55)	1005 (4.48)	964 (4.29)

Wrapped Stable with $\alpha = 1$, $\beta = 0$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	245 (0.63)	247 (0.64)	250 (0.64)	244 (1.05)	241 (1.06)	228 (1.03)
500	494 (1.29)	496 (1.29)	499 (1.29)	502 (2.21)	490 (2.18)	460 (2.07)
1000	1044 (2.70)	1047 (2.68)	1057 (2.73)	1023 (4.53)	991 (4.40)	914 (4.07)

Wrapped Stable with $\alpha = 0.5$, $\beta = 0$, warmup = 25

Wrapped skew-normal and skew-t

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	241 (0.63)	238 (0.64)	237 (1.04)	227 (1.03)	208 (0.99)
500	490 (1.28)	487 (1.28)	484 (1.28)	492 (2.19)	472 (2.14)	430 (2.02)
1000	1044 (2.71)	1038 (2.70)	1030 (2.70)	1013 (4.51)	971 (4.41)	871 (4.05)

Wrapped Skew-t with $df = 2$, $\lambda = 2$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	239 (0.63)	235 (0.64)	234 (1.04)	222 (1.02)	202 (0.98)
500	490 (1.28)	486 (1.28)	480 (1.27)	487 (2.17)	464 (2.11)	408 (1.97)
1000	1040 (2.70)	1037 (2.70)	1027 (2.69)	1008 (4.51)	946 (4.33)	815 (3.88)

Wrapped Skew-t with $df = 2$, $\lambda = 7$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	239 (0.63)	234 (0.64)	233 (1.03)	222 (1.02)	202 (0.97)
500	489 (1.27)	485 (1.28)	479 (1.28)	486 (2.17)	459 (2.10)	405 (1.96)
1000	1042 (2.71)	1032 (2.68)	1024 (2.68)	1017 (4.54)	942 (4.33)	812 (3.90)

Wrapped Skew-t with $df = 2$, $\lambda = \infty$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	243 (0.63)	242 (0.64)	241 (1.04)	233 (1.03)	222 (1.00)
500	488 (1.27)	490 (1.28)	489 (1.27)	496 (2.18)	482 (2.15)	445 (2.04)
1000	1044 (2.68)	1043 (2.71)	1042 (2.73)	1017 (4.49)	979 (4.41)	889 (4.06)

Wrapped Skew-t with $df = 2$, $\lambda = 2$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.64)	241 (0.63)	240 (0.64)	238 (1.03)	233 (1.02)	215 (0.98)
500	491 (1.28)	490 (1.28)	487 (1.27)	494 (2.18)	471 (2.10)	427 (1.97)
1000	1038 (2.68)	1039 (2.70)	1038 (2.69)	1011 (4.50)	950 (4.28)	854 (3.92)

Wrapped Skew-t with $df = 2$, $\lambda = 7$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.64)	241 (0.63)	241 (0.64)	238 (1.03)	231 (1.02)	215 (0.98)
500	490 (1.27)	490 (1.29)	485 (1.28)	492 (2.17)	469 (2.10)	427 (1.98)
1000	1039 (2.69)	1040 (2.71)	1033 (2.70)	1021 (4.52)	948 (4.26)	843 (3.89)

Wrapped Skew-t with $df = 2$, $\lambda = \infty$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	242 (0.64)	238 (0.63)	236 (1.03)	233 (1.03)	224 (1.02)
500	490 (1.28)	489 (1.27)	484 (1.27)	494 (2.17)	488 (2.20)	468 (2.15)
1000	1033 (2.70)	1039 (2.70)	1037 (2.71)	1017 (4.52)	1003 (4.49)	964 (4.43)

Wrapped Skew-t with $df = 3$, $\lambda = 2$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	240 (0.63)	237 (0.63)	236 (1.03)	230 (1.03)	221 (1.02)
500	488 (1.28)	487 (1.27)	484 (1.28)	490 (2.17)	475 (2.15)	455 (2.09)
1000	1036 (2.69)	1034 (2.69)	1033 (2.71)	1016 (4.56)	990 (4.48)	939 (4.32)

Wrapped Skew-t with $df = 3$, $\lambda = 7$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	239 (0.63)	237 (0.64)	232 (1.03)	228 (1.02)	220 (1.02)
500	487 (1.26)	483 (1.27)	483 (1.28)	486 (2.14)	474 (2.14)	453 (2.09)
1000	1036 (2.69)	1035 (2.71)	1031 (2.69)	1014 (4.54)	989 (4.52)	928 (4.28)

Wrapped Skew-t with $df = 3$, $\lambda = \infty$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.62)	243 (0.63)	243 (0.63)	241 (1.04)	238 (1.03)	235 (1.04)
500	493 (1.27)	492 (1.27)	490 (1.28)	496 (2.17)	493 (2.17)	479 (2.14)
1000	1032 (2.69)	1038 (2.69)	1037 (2.70)	1020 (4.48)	1013 (4.52)	978 (4.37)

Wrapped Skew-t with $df = 3$, $\lambda = 2$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	241 (0.63)	242 (0.64)	239 (1.04)	237 (1.04)	231 (1.03)
500	489 (1.26)	490 (1.28)	490 (1.28)	496 (2.17)	486 (2.16)	466 (2.09)
1000	1039 (2.68)	1039 (2.69)	1041 (2.71)	1020 (4.54)	1002 (4.50)	952 (4.33)

Wrapped Skew-t with $df = 3$, $\lambda = 7$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	241 (0.63)	242 (0.64)	238 (1.03)	237 (1.04)	229 (1.02)
500	489 (1.27)	490 (1.28)	489 (1.27)	493 (2.17)	487 (2.17)	463 (2.09)
1000	1045 (2.71)	1034 (2.69)	1038 (2.69)	1018 (4.56)	995 (4.46)	945 (4.31)

Wrapped Skew-t with $df = 3$, $\lambda = \infty$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	240 (0.63)	241 (0.63)	240 (0.63)	235 (1.04)	231 (1.03)	227 (1.03)
500	487 (1.27)	488 (1.28)	488 (1.28)	492 (2.20)	487 (2.19)	476 (2.18)
1000	1040 (2.68)	1039 (2.69)	1032 (2.68)	1018 (4.54)	1000 (4.49)	990 (4.49)

Wrapped Skew–Normal with $\lambda = 2$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.64)	239 (0.63)	239 (0.63)	235 (1.04)	229 (1.02)	225 (1.02)
500	487 (1.27)	488 (1.27)	485 (1.27)	488 (2.18)	474 (2.13)	470 (2.14)
1000	1039 (2.70)	1034 (2.69)	1033 (2.68)	1009 (4.52)	991 (4.47)	974 (4.44)

Wrapped Skew–Normal with $\lambda = 7$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	240 (0.63)	238 (0.63)	239 (0.64)	234 (1.03)	229 (1.02)	224 (1.02)
500	486 (1.28)	488 (1.28)	485 (1.27)	487 (2.18)	475 (2.14)	469 (2.13)
1000	1038 (2.68)	1040 (2.70)	1032 (2.66)	1005 (4.52)	991 (4.49)	965 (4.39)

Wrapped Skew–Normal with $\lambda = \infty$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	242 (0.63)	242 (0.63)	241 (1.04)	238 (1.04)	237 (1.04)
500	492 (1.28)	490 (1.28)	490 (1.27)	499 (2.21)	494 (2.18)	486 (2.18)
1000	1044 (2.71)	1037 (2.68)	1039 (2.69)	1024 (4.54)	1009 (4.49)	1004 (4.51)

Wrapped Skew–Normal with $\lambda = 2$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.64)	242 (0.64)	242 (0.64)	240 (1.03)	235 (1.03)	235 (1.03)
500	490 (1.27)	488 (1.26)	488 (1.27)	494 (2.17)	486 (2.14)	482 (2.15)
1000	1041 (2.70)	1036 (2.70)	1036 (2.67)	1016 (4.53)	1002 (4.48)	991 (4.47)

Wrapped Skew–Normal with $\lambda = 7$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.64)	242 (0.64)	241 (0.63)	239 (1.03)	236 (1.03)	234 (1.02)
500	492 (1.28)	486 (1.26)	486 (1.27)	492 (2.18)	486 (2.16)	482 (2.15)
1000	1038 (2.69)	1037 (2.68)	1036 (2.69)	1009 (4.49)	999 (4.49)	980 (4.43)

Wrapped Skew–Normal with $\lambda = \infty$, warmup = 25

Wrapped skew-stable

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	243 (0.64)	238 (0.65)	235 (1.03)	226 (1.03)	202 (1.00)
500	491 (1.28)	489 (1.29)	487 (1.30)	489 (2.19)	460 (2.10)	413 (2.00)
1000	1043 (2.72)	1043 (2.71)	1044 (2.73)	1007 (4.53)	937 (4.26)	831 (3.96)

Wrapped Stable with $\alpha = 0.5$, $\beta = 0.75$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	240 (0.64)	235 (0.64)	234 (1.03)	221 (1.01)	197 (0.97)
500	488 (1.27)	489 (1.28)	483 (1.30)	484 (2.16)	454 (2.09)	397 (1.94)
1000	1039 (2.69)	1036 (2.70)	1037 (2.73)	1007 (4.56)	924 (4.21)	787 (3.80)

Wrapped Stable with $\alpha = 0.5$, $\beta = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	244 (0.64)	244 (0.65)	239 (1.04)	234 (1.04)	221 (1.01)
500	491 (1.28)	494 (1.28)	495 (1.29)	495 (2.20)	472 (2.11)	439 (2.02)
1000	1040 (2.69)	1045 (2.70)	1046 (2.71)	1011 (4.54)	952 (4.28)	869 (3.96)

Wrapped Stable with $\alpha = 0.5$, $\beta = 0.75$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	244 (0.64)	243 (0.64)	239 (1.03)	229 (1.01)	216 (0.98)
500	489 (1.26)	492 (1.28)	492 (1.30)	491 (2.16)	469 (2.11)	423 (1.96)
1000	1039 (2.68)	1042 (2.69)	1045 (2.74)	1017 (4.58)	939 (4.19)	837 (3.84)

Wrapped Stable with $\alpha = 0.5$, $\beta = 1$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	240 (0.63)	241 (0.64)	241 (0.64)	234 (1.03)	228 (1.02)	217 (1.01)
500	487 (1.27)	486 (1.26)	487 (1.28)	490 (2.18)	476 (2.15)	450 (2.06)
1000	1038 (2.68)	1037 (2.69)	1034 (2.71)	1013 (4.54)	975 (4.42)	907 (4.16)

Wrapped Stable with $\alpha = 1$, $\beta = 0.75$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	240 (0.63)	239 (0.63)	238 (0.64)	232 (1.02)	228 (1.03)	216 (1.00)
500	488 (1.27)	489 (1.28)	485 (1.29)	491 (2.17)	469 (2.12)	440 (2.05)
1000	1041 (2.70)	1031 (2.68)	1030 (2.68)	1014 (4.55)	971 (4.39)	891 (4.15)

Wrapped Stable with $\alpha = 1$, $\beta = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	244 (0.64)	244 (0.64)	241 (1.05)	236 (1.03)	231 (1.02)
500	490 (1.27)	491 (1.26)	492 (1.28)	494 (2.18)	488 (2.18)	470 (2.10)
1000	1038 (2.68)	1036 (2.69)	1044 (2.73)	1026 (4.57)	990 (4.44)	939 (4.20)

Wrapped Stable with $\alpha = 1$, $\beta = 0.75$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	242 (0.63)	241 (0.63)	238 (1.03)	232 (1.02)	227 (1.01)
500	490 (1.27)	488 (1.27)	488 (1.28)	492 (2.16)	482 (2.17)	457 (2.08)
1000	1034 (2.67)	1033 (2.69)	1037 (2.68)	1025 (4.55)	980 (4.41)	909 (4.11)

Wrapped Stable with $\alpha = 1$, $\beta = 1$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	242 (0.63)	241 (0.64)	236 (1.04)	233 (1.03)	228 (1.04)
500	491 (1.29)	489 (1.29)	491 (1.29)	488 (2.16)	480 (2.16)	475 (2.14)
1000	1034 (2.67)	1034 (2.70)	1042 (2.72)	1017 (4.52)	1001 (4.53)	978 (4.43)

Wrapped Stable with $\alpha = 1.5$, $\beta = 0.75$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	241 (0.64)	239 (0.63)	236 (1.04)	231 (1.04)	226 (1.02)
500	490 (1.28)	487 (1.27)	489 (1.28)	487 (2.17)	483 (2.19)	471 (2.14)
1000	1035 (2.70)	1035 (2.70)	1039 (2.70)	1016 (4.54)	998 (4.48)	965 (4.41)

Wrapped Stable with $\alpha = 1.5$, $\beta = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	243 (0.63)	243 (0.63)	240 (1.04)	236 (1.03)	236 (1.04)
500	489 (1.27)	491 (1.28)	490 (1.28)	493 (2.16)	491 (2.18)	487 (2.15)
1000	1037 (2.69)	1036 (2.68)	1044 (2.72)	1030 (4.57)	1011 (4.54)	984 (4.40)

Wrapped Stable with $\alpha = 1.5$, $\beta = 0.75$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	242 (0.63)	243 (0.63)	241 (1.05)	237 (1.04)	235 (1.03)
500	488 (1.27)	490 (1.28)	490 (1.28)	493 (2.17)	489 (2.18)	484 (2.15)
1000	1035 (2.68)	1037 (2.67)	1040 (2.72)	1023 (4.53)	1008 (4.51)	975 (4.38)

Wrapped Stable with $\alpha = 1.5$, $\beta = 1$, warmup = 25

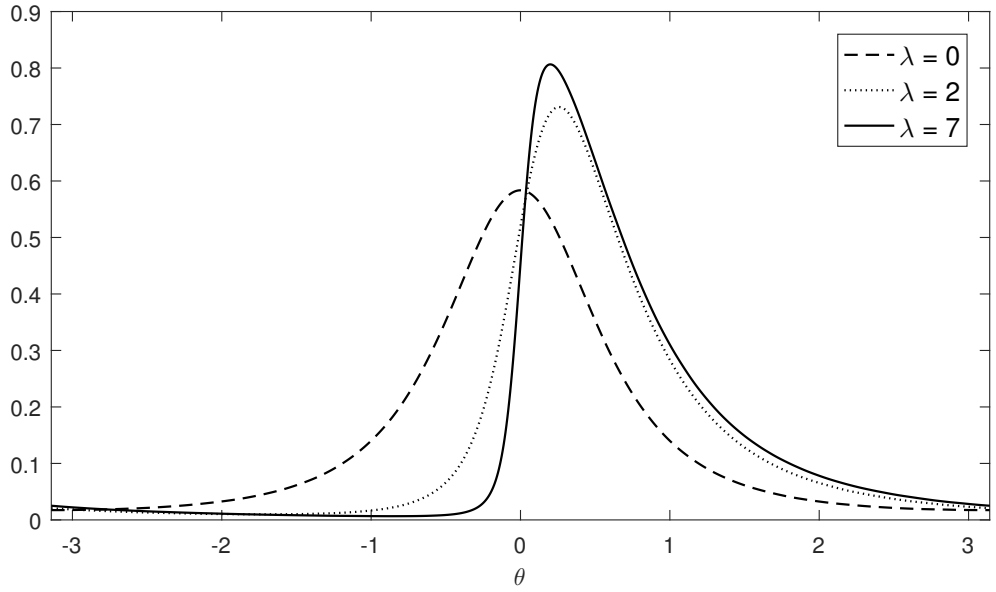

Figure 1: Densities of skew-t distribution with 2 degrees of freedom and skewness parameters λ

Wrapped skew-t

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	243 (0.64)	242 (0.63)	241 (1.04)	235 (1.02)	222 (0.99)
500	491 (1.28)	489 (1.28)	489 (1.28)	495 (2.16)	477 (2.11)	440 (1.99)
1000	1044 (2.70)	1037 (2.69)	1037 (2.70)	1021 (4.50)	965 (4.33)	850 (3.87)

Wrapped Skew-t with $df = 2$, $\lambda = \infty$, warmup = 50

Wrapped skew-stable

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	244 (0.63)	246 (0.64)	245 (0.64)	242 (1.04)	236 (1.04)	225 (0.99)
500	492 (1.27)	493 (1.28)	494 (1.29)	493 (2.16)	471 (2.08)	438 (1.96)
1000	1043 (2.71)	1038 (2.69)	1043 (2.73)	1022 (4.58)	943 (4.18)	850 (3.84)

Wrapped Stable with $\alpha = 0.5$, $\beta = 1$, warmup = 50

The pdf of the Jones-Pewsey sine-skewed family of distributions developed in Abe and Pewsey (2011) is given by

$$f(\theta) = \frac{\cosh^{1/\psi}(\kappa\psi)(1+\tanh(\kappa\psi)\cos\theta)^{1/\psi}(1+\lambda\sin\theta)}{2\pi P_{1/\psi}(\cosh(\kappa\psi))}, \ |\theta| \leq \pi.$$

This includes as special cases several known distributions, including the sine-skewed von Mises (limit as $\psi \to 0$), the sine-skewed cardioid ($\psi = 1$) and the sine-skewed wrapped Cauchy ($\psi = -1$). For this general family of distributions, let $f_0(\theta)$ denote the pdf of the associated symmetric family (recovered by setting $\lambda = 0$) and define

$$\alpha_{0,p} = E[\cos(p\theta_0)]$$

where θ_0 has pdf f_0. The circular variance of random variable θ with pdf f is then given by

$$V = 1 - \sqrt{\alpha_{0,1}^2 + \lambda^2(1-\alpha_{0,2})^2}/4.$$

Let

$$\rho = 1 - V = \sqrt{\alpha_{0,1}^2 + \lambda^2(1-\alpha_{0,2})^2}/4$$

and let κ_* be the value such that $A(\kappa_*) = \rho$ with $A(\kappa) = I_1(\kappa)/I_0(\kappa)$. Note that κ_* is the effective concentration corresponding to the von Mises distribution definition. Outlined below are the computations leading to expressions for $\rho = 1 - V$ for the three special cases mentioned.

For the sine-skewed von Mises distribution, the pdf is

$$f(\theta) = \frac{e^{\kappa\cos\theta}}{2\pi I_0(\kappa)}(1+\lambda\sin\theta)$$

with p^{th} cosine moment

$$\alpha_{0,p} = \frac{I_p(\kappa)}{I_0(\kappa)}$$

from which

$$\rho = \sqrt{\frac{I_1^2(\kappa)}{I_0^2(\kappa)} + \frac{\lambda^2}{4}\left(1 - \frac{I_2(\kappa)}{I_0(\kappa)}\right)^2}.$$

For the sine-skewed cardioid distribution, the pdf is

$$f(\theta) = \frac{1}{2\pi}(1+k\cos\theta)(1+\lambda\sin\theta)$$

with 1^{st} and 2^{nd} cosine moments $\alpha_{0,1} = k/2$ and $\alpha_{0,2} = 0$ from which

$$\rho = \frac{1}{2}\left(k^2+\lambda^2\right)^{1/2}.$$

For the sine-skewed wrapped Cauchy distribution (where γ denotes the Cauchy scale parameter), the pdf is

$$f(\theta) = \frac{1}{2\pi}\frac{1-\gamma^2}{1+\gamma^2-2\gamma\cos\theta}(1+\lambda\sin\theta)$$

with p^{th} cosine moment

$$\alpha_{0,p} = \exp(-p\gamma),\ p = 1, 2, \ldots$$

from which

$$\begin{aligned}\rho &= \sqrt{\exp(-2\gamma) + \frac{\lambda^2}{4}(1 - 2\exp(-2\gamma) + \exp(-4\gamma))} \\ &= \sqrt{\left(1 - \frac{\lambda^2}{2}\right)\exp(-2\gamma) + \frac{\lambda^2}{4}\{1 + \exp(-4\gamma)\}}.\end{aligned}$$

All tables below report the mean and standard error of the in-control ARL based on $K = 50,000$ simulated samples.

Sine-skew cardioid

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	241 (0.64)	242 (0.63)	237 (1.03)	235 (1.03)	233 (1.03)
500	488 (1.27)	488 (1.27)	490 (1.28)	490 (2.19)	488 (2.18)	487 (2.17)
1000	1033 (2.64)	1036 (2.69)	1038 (2.70)	1011 (4.50)	1012 (4.53)	1018 (4.58)

Sine-skew cardioid distribution, $\lambda = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	242 (0.63)	240 (0.63)	240 (0.63)	237 (1.05)	234 (1.03)	233 (1.04)
500	490 (1.28)	487 (1.28)	487 (1.27)	492 (2.19)	483 (2.16)	482 (2.16)
1000	1035 (2.71)	1038 (2.72)	1036 (2.70)	1017 (4.54)	1008 (4.52)	999 (4.51)

Sine-skew cardioid distribution, $\lambda = 0.8$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	241 (0.63)	239 (0.63)	-	233 (1.04)	234 (1.05)
500	-	488 (1.27)	486 (1.28)	-	480 (2.15)	480 (2.17)
1000	-	1036 (2.68)	1038 (2.70)	-	1012 (4.59)	998 (4.51)

Sine-skew cardioid distribution, $\lambda = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	243 (0.63)	243 (0.63)	240 (1.04)	239 (1.04)	240 (1.04)
500	492 (1.27)	489 (1.26)	489 (1.26)	496 (2.20)	496 (2.20)	495 (2.18)
1000	1034 (2.67)	1040 (2.70)	1044 (2.70)	1017 (4.50)	1017 (4.51)	1023 (4.58)

Sine-skew cardioid distribution, $\lambda = 0$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	242 (0.63)	242 (0.63)	241 (1.05)	238 (1.03)	239 (1.05)
500	492 (1.28)	490 (1.29)	489 (1.27)	495 (2.20)	492 (2.17)	492 (2.17)
1000	1037 (2.69)	1042 (2.71)	1039 (2.70)	1017 (4.53)	1014 (4.50)	1011 (4.52)

Sine-skew cardioid distribution, $\lambda = 0.8$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	242 (0.63)	242 (0.63)	-	240 (1.05)	240 (1.05)
500	-	490 (1.27)	490 (1.27)	-	489 (2.15)	489 (2.17)
1000	-	1041 (2.71)	1037 (2.68)	-	1021 (4.58)	1010 (4.49)

Sine-skew cardioid distribution, $\lambda = 1$, warmup = 25

Sine-skew Von Mises

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.64)	240 (0.63)	238 (0.64)	236 (1.04)	233 (1.04)	229 (1.04)
500	486 (1.26)	488 (1.28)	486 (1.28)	492 (2.18)	484 (2.17)	474 (2.16)
1000	1038 (2.68)	1034 (2.69)	1040 (2.71)	1017 (4.55)	1003 (4.56)	980 (4.43)

Sine-skew Von Mises distribution, $\lambda = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	241 (0.63)	241 (0.63)	239 (0.63)	238 (1.04)	234 (1.04)	228 (1.04)
500	490 (1.28)	487 (1.27)	490 (1.28)	496 (2.19)	482 (2.16)	480 (2.17)
1000	1033 (2.67)	1039 (2.70)	1041 (2.71)	1018 (4.54)	1005 (4.54)	986 (4.48)

Sine-skew Von Mises distribution, $\lambda = 0.8$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	240 (0.63)	240 (0.63)	-	229 (1.02)	229 (1.03)
500	-	487 (1.27)	487 (1.28)	-	484 (2.18)	476 (2.17)
1000	-	1036 (2.70)	1035 (2.71)	-	1001 (4.52)	983 (4.49)

Sine-skew Von Mises distribution, $\lambda = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	242 (0.63)	243 (0.63)	240 (1.04)	239 (1.05)	237 (1.05)
500	489 (1.27)	492 (1.29)	490 (1.27)	498 (2.18)	491 (2.17)	485 (2.15)
1000	1037 (2.68)	1038 (2.70)	1043 (2.71)	1024 (4.56)	1014 (4.56)	997 (4.46)

Sine-skew Von Mises distribution, $\lambda = 0$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	243 (0.63)	243 (0.63)	243 (0.63)	241 (1.04)	238 (1.04)	235 (1.04)
500	490 (1.27)	490 (1.28)	491 (1.27)	499 (2.19)	491 (2.17)	487 (2.16)
1000	1036 (2.70)	1037 (2.70)	1042 (2.70)	1023 (4.54)	1010 (4.51)	1001 (4.52)

Sine-skew Von Mises distribution, $\lambda = 0.8$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	243 (0.63)	242 (0.63)	-	236 (1.02)	235 (1.04)
500	-	493 (1.29)	488 (1.28)	-	492 (2.19)	486 (2.17)
1000	-	1042 (2.71)	1039 (2.71)	-	1003 (4.49)	996 (4.49)

Sine-skew Von Mises distribution, $\lambda = 1$, warmup = 25

Sine-skew wrapped Cauchy

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	233 (0.63)	233 (0.64)	234 (0.64)	226 (1.03)	221 (1.03)	211 (1.02)
500	476 (1.28)	478 (1.28)	478 (1.28)	477 (2.17)	468 (2.17)	448 (2.12)
1000	1016 (2.68)	1023 (2.71)	1029 (2.74)	994 (4.51)	972 (4.42)	922 (4.33)

Sine-skew wrapped Cauchy distribution, $\lambda = 0$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	233 (0.63)	232 (0.64)	230 (0.64)	226 (1.03)	218 (1.02)	207 (1.02)
500	475 (1.27)	472 (1.27)	475 (1.29)	475 (2.18)	459 (2.16)	433 (2.11)
1000	1023 (2.69)	1012 (2.67)	1020 (2.72)	988 (4.50)	961 (4.46)	895 (4.22)

Sine-skew wrapped Cauchy distribution, $\lambda = 0.8$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	231 (0.64)	228 (0.64)	-	218 (1.03)	204 (1.00)
500	-	472 (1.28)	472 (1.29)	-	457 (2.14)	425 (2.08)
1000	-	1017 (2.72)	1014 (2.70)	-	949 (4.41)	879 (4.18)

Sine-skew wrapped Cauchy distribution, $\lambda = 1$, warmup = 10

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	233 (0.64)	234 (0.64)	234 (0.64)	232 (1.04)	228 (1.03)	221 (1.03)
500	472 (1.28)	474 (1.29)	473 (1.29)	478 (2.17)	474 (2.16)	459 (2.14)
1000	1008 (2.68)	1012 (2.70)	1019 (2.74)	990 (4.50)	975 (4.39)	937 (4.34)

Sine-skew wrapped Cauchy distribution, $\lambda = 0$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	232 (0.63)	232 (0.63)	232 (0.64)	230 (1.02)	225 (1.03)	220 (1.03)
500	471 (1.27)	471 (1.28)	472 (1.28)	479 (2.18)	466 (2.14)	449 (2.12)
1000	1013 (2.72)	1006 (2.70)	1012 (2.73)	986 (4.47)	968 (4.47)	911 (4.21)

Sine-skew wrapped Cauchy distribution, $\lambda = 0.8$, warmup = 25

Nominal	$\zeta = 0$			$\zeta = 0.25$		
ARL	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
250	-	231 (0.63)	230 (0.64)	-	225 (1.04)	218 (1.01)
500	-	470 (1.27)	471 (1.29)	-	467 (2.15)	441 (2.08)
1000	-	1008 (2.71)	1005 (2.70)	-	962 (4.43)	897 (4.20)

Sine-skew wrapped Cauchy distribution, $\lambda = 1$, warmup = 25