Richards model for simulations of water infiltration in agricultural soil using DuMu^x

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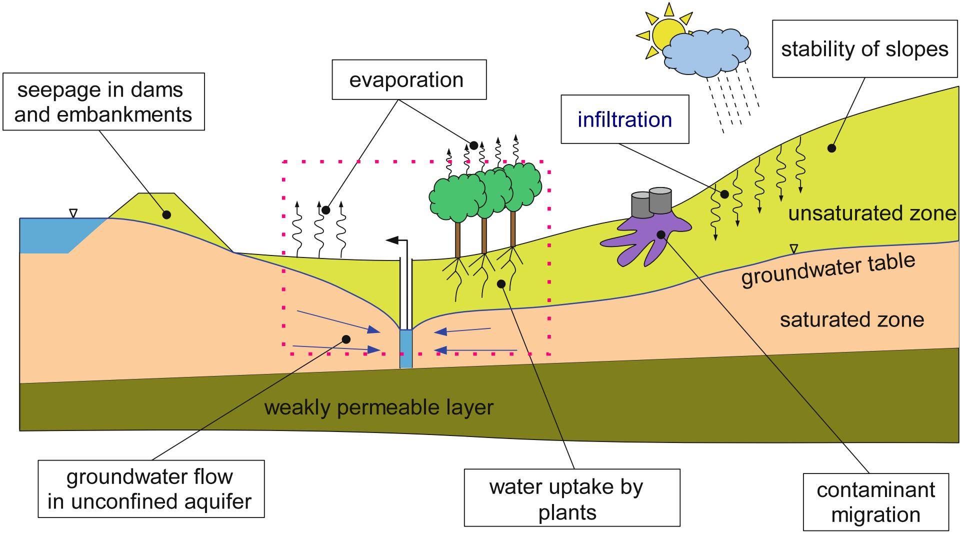
Ecological footprint of the water cycle on agriculture



Presentation of water infiltration problem

- Population and urban growth for years, has increased both the demand for food and the exploitation of resources such as water and soil for agricultural activities. However, in an effort to satisfy the demand, we have neglected good management and stability of the surronunding ecosystems [3].

- One of the main problems with the greatest environmental impact is the inefficient use of water management when implementing irrigation systems. As a consequence, large amounts of water are consumed, groundwater levels are polluted, affecting the water footprint, and soils deteriorate.
  Soil is a non-renewable resource!.

Presentation of water infiltration problem

- In Valle del Cauca, for example, one of the most common crops is sugar cane, which generates contamination problems for both the soil and groundwater (aquifers).
- Currently, these aquifers supply more than 80% of the useful water used throughout the region, and the impacts of fertilizers and inefficient irrigation systems that currently exist are unknown.

Advantage of a Simulation

- The objective is to optimize the use of water in agricultural activities, including mathematical modelling. The purpose is to study the Richard's equation and simulate the behavior in the water-soil interaction, trying to deepen the understanding of the infiltration phenomenon.

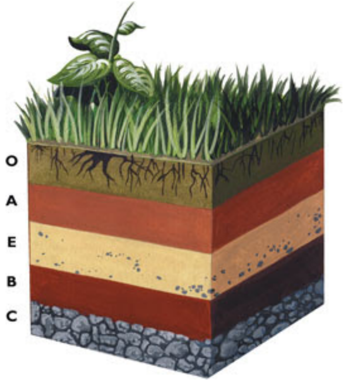
- It also saves time and money, since several variables and conditions of interest, such as temperature, humidity, saturation, etc., can be included prior to field testing.

- The simulation also allows studying various types of soil and different interactions, for example with organic matter, plants, nutrients, etc. These studies will allow the improvement of different plant species and agricultural methods.

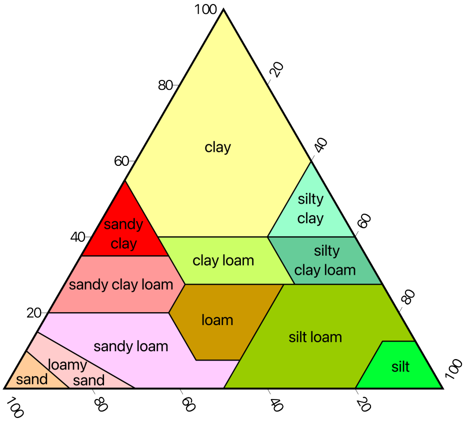
- It is also necessary to study how and for how long contaminants infiltrate the soil and reach the aquifers.

The soil and his features

- Soils have different properties like physical, mechanical, electrical, among others.
- They also have different pore sizes, and thus are classified, for which the textural triangle is used.
- Not all soils are the same, nor are they used for the same purposes, so soil studies must be carried out to determine their characteristics and properties.



The soil and his features



Listing: Spatial parameters `soil.params`.

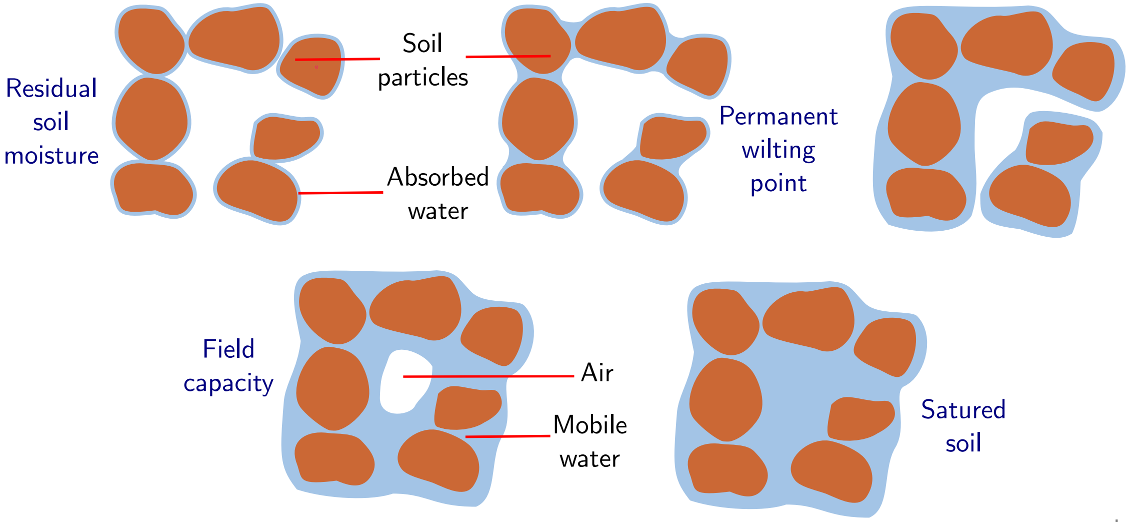
```
Densidad Aparente: 1,36 gr/cm3
Densidad real: 2,54 gr/cm3
Humedad a diferentes tensiones:
Saturacion: 65,89
0,1 bar: 58,20
cc: 40,25
1 bar: 39,43
3 bares: 36,95
5 bar: 34,48
10 bar: 28,30
P.M.P: 22,12
Textura:
Arenas: 30,76
Limos: 26
Arcillas: 43,04
Clase textural: Arcilloso
Porosidad: 46,46%
Conductividad Hidraulica: 2,83 mm/Hora
```

Soil water content

Depending on the porosity and characteristics, soils can retain water in different ways, the amount of water that cannot be easily obtained is called residual water, when there is a little more water, it is called permanent wilting point because the plants cannot yet dispose of that water, it is strongly trapped by the pores.

When the soil has water available to the plant, but still has empty air spaces, it is called field capacity. And finally, when it is completely filled with water, the soil is said to be saturated.

Soil water content



Capillarity

$\boxed{\text{What is the capillarity water soil?}}$

Capillarity

Capillarity is a property of liquids that depends on their surface tension, which, in turn, depends on the cohesion of the liquid, and which gives it the ability to move up or down a capillary tube.

When a liquid moves up a capillary tube, it is because the intermolecular force or intermolecular cohesion is less than the adhesion of the liquid to the tube material; that is, it is a wetting liquid. The liquid continues to rise until the surface tension is balanced by the weight of the liquid filling the tube.

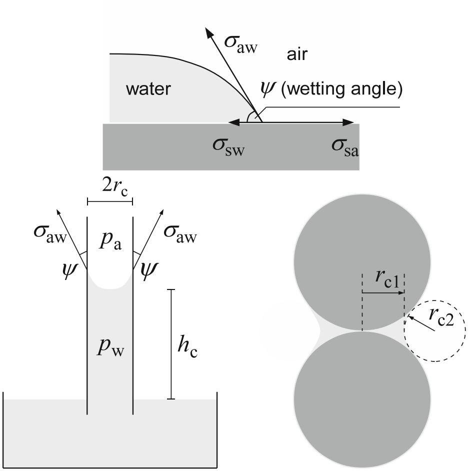
This is the case of water, and it is this property that partially regulates its ascent within plants, without expending energy to overcome gravity.

Capillarity

$$\Delta p = p_a - p_w = \sigma_{aw}\left(\frac{1}{r_{c1}} + \frac{1}{r_{c2}}\right) = \frac{2\sigma_{aw}\cos\psi}{r_c}$$

$$\sigma_{aw}\cos\psi = \sigma_{Sa} - \sigma_{Sw}, \quad h_c = \frac{1.5\times 10^{-5}}{r_c}$$

- σ_{aw} is the surface tension of the air-water interface.
- σ_{Sa} is the surface tension of the solid-air interface.
- σ_{Sw} is the surface tension of the solid-water interface.
- ψ is the wetting angle.



Mathematical Model of Infiltration

Richards equation 1D

Derived the conservation of mass, the one-dimensional continuity equation of infiltration is given as

$$\frac{\partial \theta}{\partial t} + \nabla \cdot \vec{q} = 0. \tag{1}$$

By Darcy's law, q_z represents

$$q_z = -K\frac{\partial H}{\partial z} = -K\frac{\partial (h-z)}{\partial z} = K\left(1 - \frac{\partial h}{\partial z}\right). \tag{2}$$

Joining the equations (1) and (2) we obtain the mixed Richards formulation

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial z}\left[K\left(1 - \frac{\partial h}{\partial z}\right)\right] = 0.$$

where

- θ is the soil water content $\left(m^3/m^{-3}\right)$.
- z is the soil depth
- $\vec{q}$ is the water infiltration rate (m/s).
- K is hydraulic conductivity.
- H is the total water potential in axis z.
- h is the preassure head.

Software simulator for Porous Media Problems

DUNE for Multi-{Phase, Component, Scale, Physics, ...} flow and transport in porous media (DuMu^x)

- DuMu^x is a multipurpose open-source simulator under the GNU Lesser General Public License 3 .
- DUNE is available on macOS, Debian , Ubuntu , openSUSE , Arch Linux and FreeBSD .
  🎉 DUNE Release 2.9.0 is planned for end of October 2022.
- Porous-Medium Flow, Non-Isothermal, Free Flow, Geomechanics, Pore-Network models. Multidomain, multi-component, multi-phase. Parallel, Grid Adaptivity.
  🎉 Release 3.6.0 is planned for October 7, 2022.

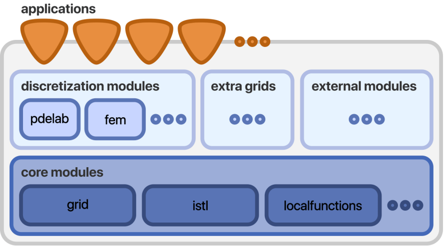


Figure: Taken from `https://dune-project.org`.

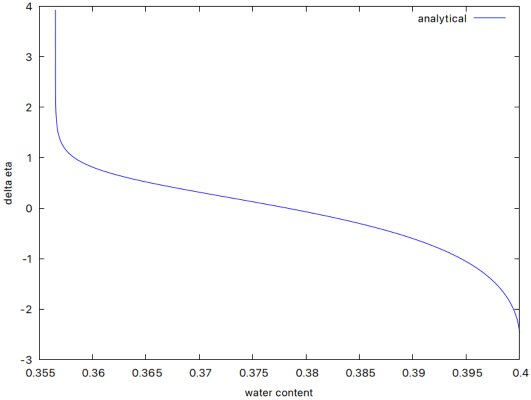
Figure: Binaries are available in arch4edu repository. (Jingbei Li, Carlos Aznarán, 2022.)

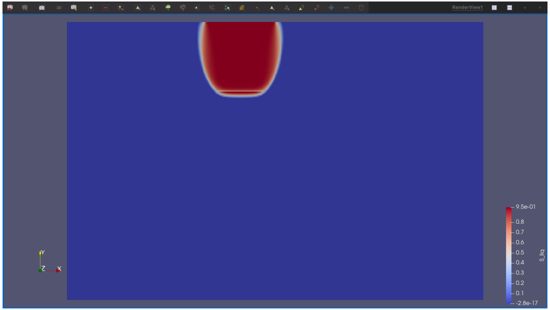
Spatial parameters file

```cpp
template<class GridGeometry, class Scalar>
class RichardsLensSpatialParams
: public FVPorousMediumFlowSpatialParamsMP<GridGeometry, Scalar, RichardsLensSpatialParams<GridGeometry, Scalar>>
{
    using ThisType = RichardsLensSpatialParams<GridGeometry, Scalar>;
    using ParentType = FVPorousMediumFlowSpatialParamsMP<GridGeometry, Scalar, ThisType>;
    using GridView = typename GridGeometry::GridView;
    using FVElementGeometry = typename GridGeometry::LocalView;
    using SubControlVolume = typename FVElementGeometry::SubControlVolume;
    using Element = typename GridView::template Codim<0>::Entity;
    using GlobalPosition = typename Element::Geometry::GlobalCoordinate;

    enum { dimWorld = GridView::dimensionworld };

    using PcKrSwCurve = FluidMatrix::VanGenuchtenDefault<Scalar>;

public:
    using PermeabilityType = Scalar;

    RichardsLensSpatialParams(std::shared_ptr<const GridGeometry> gridGeometry)
    : ParentType(gridGeometry)
    , pcKrSwCurveLens_("SpatialParams.Lens")
    , pcKrSwCurveOuterDomain_("SpatialParams.OuterDomain")
    {
        lensLowerLeft_ = {1.0, 2.0};
        lensUpperRight_ = {4.0, 3.0};
        lensK_ = 1e-12;
        outerK_ = 5e-12;
    }
```

```cpp
private:
    bool isInLens_(const GlobalPosition &globalPos) const
    {
        for (int i = 0; i < dimWorld; ++i)
            if (globalPos[i] < lensLowerLeft_[i] - eps_ ||
globalPos[i] > lensUpperRight_[i] + eps_)
                return false;

        return true;
    }

    static constexpr Scalar eps_ = 1e-6;

    GlobalPosition lensLowerLeft_;
    GlobalPosition lensUpperRight_;

    Scalar lensK_;
    Scalar outerK_;
    const PcKrSwCurve pcKrSwCurveLens_;
    const PcKrSwCurve pcKrSwCurveOuterDomain_;
};
```

Results


analytical
delta eta
water content
4
3
2
1
0
-1
-2
-3
0.355
0.36
0.365
0.37
0.375
0.38
0.385
0.39
0.395
0.4


RenderView1
9.5e-01
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-2.8e-17
S_liq
Y
Z
X

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Acknowledgment

Thank you so much!

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Sede Palmira, Colombia.

Slides available on:

```
https://cpp-review-dune.github.io/
    flow-test-dumux/slides.pdf
```

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