Human Capital Model

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1 Introduction

Considering the shortage of the talent, it's essential for companys to retain good people and make them well-trained. However, current situation is not satisfactory while many talents always tend to get a good job via job-hopping, causing orgnizational churn in employees who are closely connected to them. In order to simulate this process and improve it, we build a human capital model based on Social Network Analysis and Markov process.

$$Q = \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz$$

$$\begin{aligned}
\int_{t_1}^{t_2} \iint\limits_{\Gamma} k \frac{\partial u}{\partial n} dsdt &= \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \\
&= \int_{t_1}^{t_2} \iiint\limits_{\Omega} [\frac{\partial}{\partial x}(k\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial u}{\partial z})] dxdydx \\
&= \iiint\limits_{\Omega} \rho C_w (\int_{t_1}^{t_2} \frac{\partial u}{\partial t} dt) dxdydz
\end{aligned}$$

1.1 Restatement of the Problem

We are required to build a mathematical model to solve the problem of heating water in bathtub. Thus we have some subproblems:

- Build a basic model that demonstrate the change of the temperature in the bathtub in space and time without other intervention.
- Figure out the influence of parameters such as the shape of the bathtub or the motions made by the person or so.
- Propose a strategy to keep the temperature as even as possible.

In the first step, we build the simplest model. The shape of the bathtub is a cuboid, and the person will stay still. In the second step, the person move slowly and the shape of the bathtub changes. In the third step, we develop the conclusion of the strategy.

1.2 Literature Review

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2 Terminology

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3 Assumptions and Justifications

- **assumption 1** If an employee has probability to promote, he won't churn.

  The possibility of the unforeseen accidents, which could force an employee to leave his position,is neglected.Human nature, an employee will stay at his position to chase for higher level.

- **assumption 2** For a vacancy,if there exists an employee measures up to it already,ICM won't recruit for it.

  Since recruiting good people is difficult, time consuming and expensive according to issue 5,it is wasteful to recruit for a position if an employee can promote to it.

- **assumption 3** Demotion won't occur.

- **assumption 4** Administrative clerk won't promote or be transferred.

- **assumption 5** For the promotion probability and organization change, the other factors effects the churn probability is invariable.

  Though churn derives from varieties of reasons and they are actually lacking of known conditions and data to estimate them, we have to regard it as stable in our model.

- **assumption 6** Each division or office have at least one middle manager or senior manager.

4 Human Capital Model

4.1 Model Overview

Partial differential equations are widely used in natural science, engineering and economical management. It is natural to use partial differential equation to solve this physical problem.

Definitions of symbols employed in this paper are listed in **Table 1**.

4.2 Human Performance Model

In this part, we buid a people model to evaluate an employee in four aspects, in terms of **Quantitative Management Performance** — work achievement, work ability, work attitude and potential. These four aspects are supposed to be quantized according to annual evaluation based on performance as judged by the

Variable	Description
i	Index of an employee
$L_{i,t}$	The level of i
$s_{ij,t}$	The relation strength from i to j
$a_{ij,t}$	Represent the influence caused by relationship between superior and subordinate
$f_{ij,t}$	Represent the influence caused by relationship between person with many friends and p
$c_{i,t}$	Clustering Coefficient of i

supervisor and we take these independent variables as A_{ac_i}, A_{ab_1}, A_{at_i} and A_{po_i} for each employee i. For each of the four parameters, it goes from 0 to 1. The statues are used for calculating the probability that employee can promote. Meanwhile, they influence leaving probability and team cohesiveness as well.

It is obvious that some of the parameters are somehow more important than others. So in an effort to make our model more accurate and reliable,we introduce a weighted index of deviation AD_i, with

$$A_i = w_{ac} \cdot A_{ac_i} + w_{ab} \cdot A_{ab_i} + w_{at} \cdot A_{at_i} + w_{po} \cdot A_{po_i} \quad (1)$$

We determine weights via the Analytical Hierarchy Process(AHP) [Saaty 1982]. We build a 4×4 reciprocal matrix by pair comparison:

	A_{ac}	A_{ab}	A_{at}	A_{po}
A_{ac}	1	5	2	1
A_{ab}	$\frac{1}{5}$	1	$\frac{1}{3}$	$\frac{1}{4}$
A_{at}	$\frac{1}{2}$	3	1	1
A_{po}	1	4	1	1

The meaning of the number in each cell is explained in []. The numbers themselves are based on our own subjective decisions.

We then get the weight of each factor by calculating the bigest eigenvalue and it's corresponding eigenvector, as given in Table.

Factor	A_{ac}	A_{ab}	A_{at}	A_{po}
Weight	0.3805	0.0709	0.2371	0.3030

We test the consistency of the preferences for this instance of the AHP. For good consistency [Alonso and Lamata 2006, 446 - 447]:

- The principal eigenvalue λ_{max} of the matrix should be close to the number n of alternatives, here 4; we get $\lambda_{max} = 4.047$.
- The consistency index $CI = (\lambda_{max} - n)/(n-1)$ should be close to 0; we get $CI = 0.0157$.
- The consistency ratio $CR = CI/RI$ (where RI is the average value of CI for random matrices) should be less than 0.1; we get $CR = 0.0182$.

Hence, our decision method displays perfectly acceptable consistency and the weights are reasonable.

4.3 Social Network Model

The social network model contains a directed weighted graph $G(V, E)$ in which V denote the employees and E denote the connection between employees. Since there are personnel changes, $G(V, E)$ will change with time goes by. In order to simulate this situation, we use $G_t(V_t, E_t)$ instead of $G(V, E)$ where t is a discrete variable. So $G_t(V_t, E_t)$ denote the social network in the t-th month.

First, we explain the way we build edges of $G_t(V_t, E_t)$.

When $t = 0$, there are about $370 \times 85\%$ nodes (employees) in G_t. We build edges between employees in the same division or office, since employees in the same division or office certainly know each other. So each division or office form a complete graph and employees in the differient division or office don't know others, which is impossible. To solve this problem, we build 10 edges for each employee with employees in other divisions with equal probability. Then we build the other edges with probability $p = \frac{|N_{i,t} \cap N_{j,t}|}{|N_{i,t} \cup N_{j,t}|}$ which is called Jaccard similarity coefficient[Jaccard 1901].

When $t > 0$, there will be employees leaving or joining the company. If an employee leave, all his edges with other employees with be deleted. If an employee newly join the company, he will follow steps which employees at $t = 0$ take. This is the dynamic process of graph $G_t(V_t, E_t)$.

Let $s_{ij,t}$ denote the weight from i to j at time t. We have these properties of $G_t(V_t, E_t)$:

- $s_{ij,t} \neq s_{ij,t}$

  We made this graph directed and weighted because one person may consider another person his best friend while that person doesn't consider him a good friend. This situation may appear because of the relationship between superior and subordinate and the relationship between person with more friend and person with less friend. In general, $s_{ij,t} \neq s_{ij,t}$ for the reason above.

- $s_{ij,t} = \dfrac{a_{ij,t} + f_{ij,t}}{2}$

  $a_{ij,t}$ denote the influence caused by the relationship between superior and subordinate, $f_{ij,t}$ denote the influence caused by the amount of friends for i and j. We define

$$a_{ij,t} = \begin{cases} \frac{1}{2+|L_{i,t}-L_{j,t}|}, & L_{i,t} \geq L_{j,t} \\ 1 - \frac{1}{2+|L_{i,t}-L_{j,t}|}, & L_{i,t} < L_{j,t} \end{cases}$$

  , where $L_{i,t}$ denote the level of i at time t, and

$$f_{ij,t} = \frac{|N_{i,t} \cap N_{j,t}|}{|N_{i,t}|}$$

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Finally, we explain the concept of clustering coefficient [Duncan J. Watts and Steven Strogatz 1998] of i denoted as $c_{i,t}$, which is a measure of the degree to which nodes in a graph tend to cluster together. $c_{i,t}$ is defined as:

$$c_{i,t} = \frac{|\{s_{jk,t} : v_j, v_k \in M_i, s_{jk,t} \in E\}|}{k_{i,t}(k_{i,t} - 1)}$$

where $M_{i,t} = \{v_j : s_{ij,t} \in E s_{ji,t} \in E\}$ and $k_{i,t} = |N_{i,t}|$.

4.4 Promote and Churn Model

The main part of our model is the algorithm which controls the behavior of employees. According to assumption 1 and assumption 2, we have the methodology as follow:

step 1

The network of the company we've built changes in terms of orgnizational churn and promotion. Considering various of factors in reality, we build an orgnizational churn and promotion model to predict the dynamic process.

The first part of the model is churn model.We define the churn rate of an employee i as l_i to evaluate the probability to churn. l_i is usually controlled by sorts of factors to different degrees. We divides l_i into three parts:l_{i1}, l_{i2} andl_{i3}. l_{i1} represents the churn rate because of lacking of promotion opportunity. l_{i2} represents the churn rate because of the changes of other employees related to employee i. To simplify our model, we presume that l_i1, l_i2 is linear correlated with $(1 - p_i)$ and s_i, which means

$$l_{i1} = \lambda_1(1 - p_i), l_{i2} = \lambda_2 s_i \tag{2}$$

λ_1 and λ_2 could be ensured in later calculation.

l_{i3} represents the other factors we can't get any information from the known conditons so that we regard it as stable. Thus

$$l_i = \lambda_1(1 - p_i) + \lambda_2 s_i + l_i3 \tag{3}$$

After analyzing a great deal of churn rate reports, we get the composition of the three parts are 10.9%, 2.2% and 75.4% respectively. According to the percentage and the general churn rate 18%, we can calculate λ_1, λ_2 and l_{i3}. As an orinigal condition,it satisfies

$$\begin{cases} \lambda_1 \sum_{i=1}^{370}(1 - p_i) & = 10.9\% \times 370 \times 1.5\% \\ \lambda_2 \sum_{i=1}^{370} s_i & = 2.2\% \times 370 \times 1.5\% \\ l_i3 & = 1.5\% \times 75.4\% \end{cases} \tag{4}$$

Thus we can use equation (4) to calculate the churn rate l_i.

The second part of the model is promotion model aimed to predict the promotion condition.We define the promotion rate of an employee i as p_i to evaluate the probability to promote.As a matter of fact,if there is a vacancy, judging if an employee suits the site involves work experience and ability.It is essential that he is supposed to have several years of work experience according to issue 6.If an employee satisfies the experience condition,it turns out to think about his ability.Since in people model,each employee's ability is evaluated by a parameter A_{D_i}. For each level of position,it has an ability standard,as shows in Figure[].

The ability of an employee are supposed to reach the four standard parameters respectively,otherwise its p_i is 0.For those who reach the standard, the promotion probability can be calculated by the equation: $p_i = \frac{A_{D_i}}{\sum_{\alpha} A_{D_\alpha}}$ where α is employee who have probability to promote.

5 Performance and Analysis

5.1 Analysis for Task 2

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5.2 Analysis for Task 3

We assume that the company offers training programs for its employees monthly and newly hired employees start to get their salaries next month after they enter the company. With these two assumption, results can be drawn according to our model through simulation.

Budget can be divided into threee parts: salary budget, training budget and recruiting budget. The budget requirement predicted for next two years is listed in the table below in terms of σ.

Total Budget	Salary Budget	Training Budget	Recruiting Budget
1170.8σ	951.387σ	164.423σ	55.08σ

5.3 Analysis for Task 4

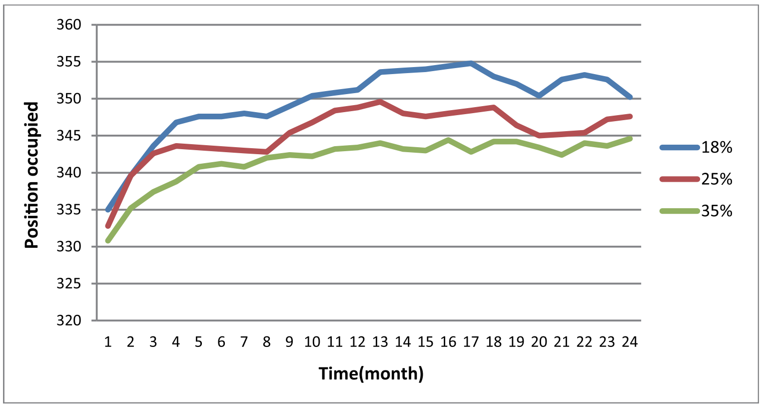


Figure 1: Status of positions

To analyze the status of positions under different churn rate, we use our model to simulate dynamic processes with these churn rate constraints. We execute our program 100 times for each churn rate and average the predicted values. Figure 1 shows the averaged results our model predicted. Under all of these three conditions, the number of employees in the company keeps rising. The higher the churn rate, the lower the final full rate the company reaches after two years. But ICM can sustain its 80% for positions even if the churn rate goes to 35% according to our model's precition.

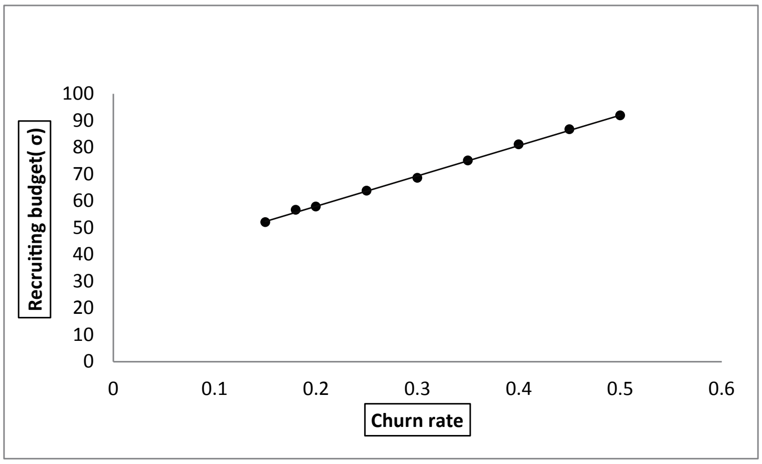
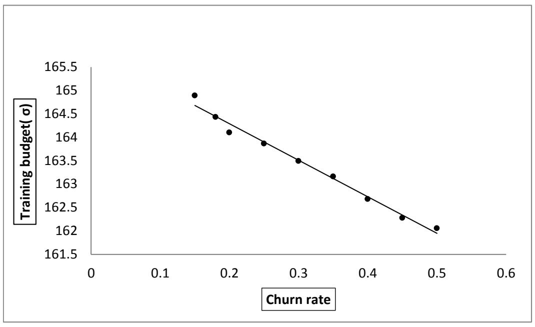


Figure 2: Recruiting budget



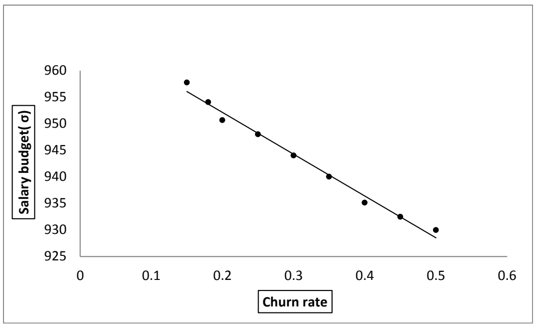


Figure 3: Training budget and salary budget

The churn rate effect the budget of the company as well. The three parts of the bugdet behave differently when churn rate increases. The calculated budget is shown in Figure 2 and Figure 3. Each data point in three charts is an averaged result of 10 predictions and a linear trendline is added to each chart. It is clear that recruiting budget showed in Figure 2 is likely to be proportional to the churn rate while salary budget and training budget showed in Figure 3 are likely to be inversely proportional to the churn rate.

To maintain enough employees, the company has to spend more on recruiting. So high turnover rate directly increase the recruiting budget. High turnover rate's effect on training budget and salary budget is more complex. On the one hand, when churn rate goes up, vacancies in the middle level keeps rising due to long recruiting time and low promote rate. On the other hand, the vacancies in lower level remains low because of the short recruiting time. So the full rate of the company decreases when churn rate rises. Since training budget and salary budget are closely related to full rate, both of them decrease when turnover rate goes up.

5.4 Analysis for Task 5

We apply following changes to our model to simulate the required process:

- Change the churn rate of junior managers and experienced supervisors to 30%

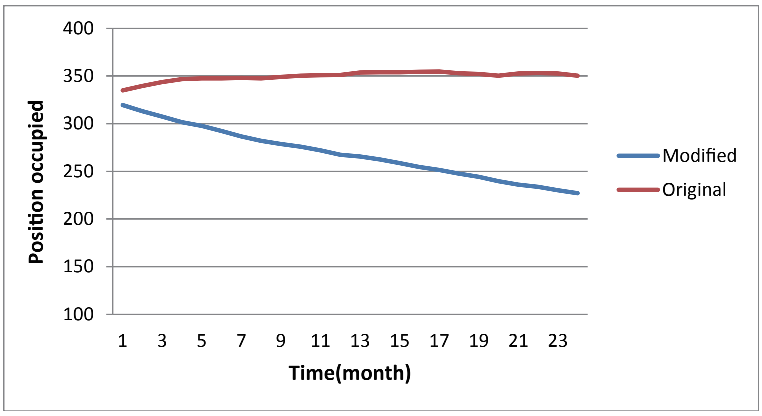


Figure 4: Status of position

Level of Position	Modified	Original
Senior manager/Executive	5.6	8.4
Junior manager/Executive	9.0	18.4
Experienced supervisor	7.4	23.0
Inexperienced supervisor	9.2	23.2
Experienced employee	72.6	107.4
Inexperienced employee	99.6	149.6
Administrative clerk	24.0	24.0

Table 1: Status of position

- Prohit external recruiting
- Promoting only qualified employees

The result of simulation is shown in Figure 4 and Table 1. All the data shown is an average of ten predictions. While the number of positions occupied remains stable with origial conditions, it drops remarkably with modified conditions.

We list specific data of each level in Table 1. In the modified case, the numbers of employees are lower than original case especially the those of middle levels. Since there is no external recruiting in modified case, it is obivious that the full rate will decrease due to employees' leave. Although some qualified employees can be promoted into higher level, the high churn rate of the middle level and difficulty of satisfying the promotion conditions make the numbers of middle level employees relatively low. The situation given in that task 5 will cause unrecoverable damage to ICM's HR health. With the full rate of middle level employees lower than 50%, the HR structure is broken into fragments and the company won't be able to function normally.

6 Advice for HR

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6.1 Incentive Mechanism

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6.2 Matching Employees to the Right Position

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7 Team Science

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8 Sensitivity Analysis

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9 Strengths and Weaknesses

Strengths

- Our model make fully use of the theory of multilayer networks so that it quantizes the relation accurately and reasonablly.
- Our model exellently proves the interaction among these factors:leave probability, promotion probability and productivity.
- The network we built include both microcosmic part and macrocosmic part, and they react to each other.
- Our model proves the effection of time.

Weaknesses

- We assume that the water is still. In fact, streams flow and fluid dynamic is better taken into consideration. The actual situation is much complicated.
- The model of the person should be more realistic. In fact, model of human body includes hands, legs or other parts, and the surface of human skin isn't even.
- The condensation of water should be taken into consideration. Water that evaporate become water vapour, and some will condense into water on the bathtub.

10 Conclusions

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References

[1]