Human Capital Model

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Contents

1 Introduction **2**
1.1 Restatement of the Problem 2

2 Assumptions and Justifications **3**

3 Water in the Bathtub Model **3**
3.1 Model Overview 4
3.2 Heat Equation 4
3.3 Initial-Boundary Value Condition 6
3.4 PDE Solution 6

4 Results and Analysis **6**

5 Sensitivity Analysis **7**
5.1 Parameters of Bathtub 7
5.2 Parameters of Person 7
5.3 Effects of Motion 7
5.4 Effects of Bubble Bath Additive 7

6 Strengths and Weaknesses **7**

7 Non-technical explanation **8**

1 Introduction

$$Q = \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz$$

$$\begin{aligned}\int_{t_1}^{t_2} \iint\limits_{\Gamma} k \frac{\partial u}{\partial n} dsdt &= \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \\ &= \int_{t_1}^{t_2} \iiint\limits_{\Omega} [\frac{\partial}{\partial x}(k\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial u}{\partial z})] dxdydx \\ &= \iiint\limits_{\Omega} \rho C_w (\int_{t_1}^{t_2} \frac{\partial u}{\partial t} dt) dxdydz\end{aligned}$$

$$\rho C_w \frac{\partial u}{\partial t} = k \triangle u + \frac{C_w V_h (u_h - u)}{V} - \frac{C_v g}{V}$$

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1.1 Restatement of the Problem

We are required to build a mathematical model to solve the problem of heating water in bathtub. Thus we have some subproblems:

- Build a basic model that demonstrate the change of the temperature in the bathtub in space and time without other intervention.
- Figure out the influence of parameters such as the shape of the bathtub or the motions made by the person or so.
- Propose a strategy to keep the temperature as even as possible.

In the first step, we build the simplest model. The shape of the bathtub is a cuboid, and the person will stay still. In the second step, the person move slowly and the shape of the bathtub changes. In the third step, we develop the conclusion of the strategy.

Constant	Description
k	Thermal conductivity of water
ρ	Density of water
γ_1	Heat exchange coefficient between water and air
γ_2	Heat exchange coefficient between water and person
γ_3	Heat exchange coefficient between water and bathtub
C_w	Specific heat capacity of water
V	Total volume of water
x_s	Humidity ratio in saturated air
x_0	Initial humidity ratio in the air
A	Square of water surface
V_a	Volume of surrounding air
C_v	Evaporation heat of water
u_2	Temperature of person's surface
Variable	Description
u	Temperature of water
u_1	Temperature of air
u_3	Temperature of bathtub

2 Assumptions and Justifications

- **assumption 1** If an employee has probability to promote, he won't churn.

  The possibility of the unforeseen accidents, which could force an employee to leave his position,is neglected.Human nature, an employee will stay at his position to chase for higher level.

- **assumption 2** For a vacancy,if there exists an employee measures up to it already,ICM won't recruit for it.

  Since recruiting good people is difficult, time consuming and expensive according to issue 5,it is wasteful to recruit for a position if an employee can promote to it.

- **assumption 3** Demotion won't occur.

- **assumption 4** Administrative clerk won't promote or be transferred.

- **assumption 5** For the promotion probability and organization change, the other factors effects the churn probability is invariable.

  Though churn derives from varieties of reasons and they are actually lacking of known conditions and data to estimate them, we have to regard it as stable in our model.

- **assumption 6** Each division or office have at least one middle manager or senior manager.

3 Water in the Bathtub Model

Definitions of symbols employed in this paper are listed in **Table 1**.

3.1 Model Overview

Partial differential equations are widely used in natural science, engineering and economical management. It is natural to use partial differential equation to solve this physical problem.

In our model, five factors account for heat transfer of water: heat exchange between water and air, heat exchange between water and the person, heat exchange between water and the bathtub, heat energy of hot water from the faucet and heat energy of water flow outside. According to thermology, we deduce the equation of water temperature in the bathtub. The initial-boundary value conditions are deduced according to Fourier theorem, vaporization equation[ ].

Since the equation is a relatively complex 3D partial differential equation, we use matlab to solve it. We build a geometry as the bathtub. By finite element method, the geometry is divided into several parts. The solution of each part is considered to be a relatively precise value solution.

3.2 Heat Equation

In this part, we use physical theorem to deduce the heat conduction equation in the bathtub.

Figure 1 shows thermal analysis of water in bathtub.

First, we assume that the heat change of all the water in a tiny time Δt from t_1 to t_2 is ΔQ. As it shown in Figure 1, heat of water in the bathtub exchanges with air, the person and the bathtub. In addition, heat changes when the hot water flow in and the excess water flow out. Thus ΔQ is divided into five parts, i.e.

$$\Delta Q = \Delta Q_1 + \Delta Q_2 - \Delta Q_3 - \Delta Q_4 \tag{1}$$

in which ΔQ_1 is the heat transferred from air, person and bathtub, ΔQ_2 is the heat of the hot water from the faucet in Δt, ΔQ_3 is the heat of the water flow away in Δt, ΔQ_4 is the heat loss because of vaporization.

Totally, internal energy of water changes when heat exchanges happen. Then the temperature changes according to the equation

$$Q = \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \tag{2}$$

, in which Ω is the region of water, ρ represents the density of water, u_f stands for the final temperature when t=t_2, and u_i is the initial temperature when t=t_1.

Considering the right side of equation 1. Fourier theorem indicates that,

$$dQ = -k\frac{\partial u}{\partial n} dSdt \tag{3}$$

holds for the heat flow through surface with square of dS, as it shown in Figure 2. In the equation, k is thermal conductivity, $\frac{\partial u}{\partial n}$ is the derivative of u along the normal line. Thus, after integration of dQ by time and space

$$\Delta Q_1 = \int_{t_1}^{t_2} [\int_{\Gamma} k\frac{\partial u}{\partial n} dS] dt \tag{4}$$

in which Γ is the boundary of Ω, the heat flow into the water by thermal conduction could be calculated.

ΔQ_2 is determined by the internal energy of hot water from the faucet in Δt. And ΔQ_2 can be calculated by $\Delta Q_2 = \Delta m C u_h$. Since the hot water is constant, $\Delta m = v\Delta t$.

It is obvious that the excess water is the same value of hot water in any time period. Thus ΔQ_3 can be calculated by $\Delta Q_3 = \Delta m C_w u$. Also, $\Delta m = v\Delta t$. Δv is the weight of hot water per second, Δm is the weight of hot water flows in during Δt.

According to thermal theorem, heat loss in vaporization is proportional to the weight of vapor water. $\Delta Q_4 = C_v \Delta m_v$. Δm_v is the weight of evaporation water in Δt, C_v is evaporation heat of water per kilogram.

Since water could be considered as manifold, according to Green formula, we deduce the formula

$$\begin{aligned}\int_{t_1}^{t_2} \iint\limits_{\Gamma} k\frac{\partial u}{\partial n} ds dt &= \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \\ &= \int_{t_1}^{t_2} \iiint\limits_{\Omega} [\frac{\partial}{\partial x}(k\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial u}{\partial z})] dxdydx \\ &= \iiint\limits_{\Omega} \rho C_w (\int_{t_1}^{t_2} \frac{\partial u}{\partial t} dt) dxdydz\end{aligned}$$

Combining the equations above, we conclude that

$$\rho C_w \frac{\partial u}{\partial t} = k \triangle u + \frac{C_w V_h (u_h - u)}{V} - \frac{C_v g}{V} \tag{5}$$

In this heat equation, u_h and v_h are two variables that control the strategy.

Now let's discuss g, evaporation rate per square. As Figure 3 shows, water evaporates into air and air humidity increases, until it is saturate. According to empirical formula in [], we know that

$$g = \frac{(25 + 19v)(x_s - x)}{3600} \tag{6}$$

in which v is the velocity of air, x_s is humidity ratio in saturated air, x is unsaturated humidity ratio in air. Considering the humidity ratio in the air, it indicates

$$x = x_0 + \int \frac{Ag}{\rho V_a} dt \tag{7}$$

in which x_0 is the initial humidity ratio in the air, ρ is the density of water, V_a is the total volume of surrounding air.

Combing and differentiating two equations, we get an ordinary partial equation

$$\frac{3600}{25 + 19v} \frac{dg}{dt} = \frac{A}{\rho V_a} \tag{8}$$

Solving the equation, we get the solution

$$g = \frac{(25 + 19v)(x_s - x_0)}{3600} e^{-\lambda t}, \lambda = \frac{A(25 + 19v)}{3600 \rho V_a} \tag{9}$$

3.3 Initial-Boundary Value Condition

Boundary value conditions are quite essential in partial differential equations. In our model, water is not a regular research object. Thus boundary value condition is complex.

The boundary of water consists of four parts: water-air boundary, water-person boundary, water-bathtub boundary, and the square for faucet. We denote them as Γ_1, Γ_2, Γ_3, Γ_4 respectively.

Since person is temperature-constant, the boundary value condition of Γ_1 and Γ_3 could be considered as the Dirichlet value condition. That is

$$u = u_i, (x, y, z) \in \Gamma_i, i = 2.4 \tag{10}$$

In the equation, u_2 is the surface temperature of the person. We assume it is constant. u_4 is the temperature of hot water.

The boundary value condition of Γ_1 and Γ_3 could be considered as the third boundary value condition approximately. That is

$$k\frac{\partial u}{\partial n} + \gamma_i u = \gamma_i u_i, (x, y, z) \in \Gamma_i \tag{11}$$

In the equation, i=1 or 3 and u_i is the temperature of contact material, γ_i is the heat exchange coefficient of Γ_i.

In the equation above, u_1, u_3 are temperatures of air and bathtub, and they are time-dependant functions. It takes several steps to calculate them.

Consider the temperature of air, u_1. Heat exchange of air consists of two ways, heat conduction and evaporation.

In our model, we assume the temperature of water is equilibrium throughout the bathtub initially.

3.4 PDE Solution

Since our equations are complex and the region is quite irregular, it's impossible to get a regular solution. Finite element method is normally adopted to deal with this kind of problem.

We use Partial Differential Equation Toolbox in matlab to solve our equations. Partial Differential Equation Toolbox makes use of finite element method to separate irregular geometry and calculate a relatively precise solution, get solution of temperature corresponding to different u_h and v_h. By comparing temperature distribution, we choose proper parameters as the best strategy.

First, we build a 3D geometry model to simulate a proper-size bathtub, as it shown in Figure 2. Matlab divided the geometry into several elements.

Then we input the heat equation and initial-boundary value conditions. Boundary value conditions are different in three parts.

4 Results and Analysis

Our strategy focus on two variables, v_h and u_h. Several pairs of data are tested to find the best strategy.

	v_h	u_h
1		5
2		1
3		3
4		4

5 Sensitivity Analysis

5.1 Parameters of Bathtub

- We change the shape of the bathtub to sphere and elliptical with the same strategy. The temperature graphic shows that sphere is the most proper shape for keeping water temperature. Square performs the worst on keeping temperature. However, temperature of square bathtub is more evenly than others.
- We change the volume of the bathtub without changing the proportion. The result as following Table [] shows that with the same v_h, u_h increases as volume increases; with the same u_h, v_h increased as volume increases.

5.2 Parameters of Person

-
-
-

5.3 Effects of Motion

When the person in the bathtub make motions, the heat exchange coefficient between person and water γ_2 increases. That means heat exchange between water and person increases.

In another aspect, the air above flow, so that evaporation heat of water increases.

5.4 Effects of Bubble Bath Additive

6 Strengths and Weaknesses

Strengths

- Our model make good use of thermology knowledge so it can reflect the situation in the real world realistically.
- Our model build a 3D geometry to simulate the research object so
- The network we built include both microcosmic part and macrocosmic part, and they react to each other.
- Our model proves the effect of time.

Weaknesses

- We assume that the water is still. In fact, streams flow and fluid dynamic is better taken into consideration. The actual situation is much complicated.
- The model of the person should be more realistic. In fact, model of human body includes hands, legs or other parts, and the surface of human skin isn't even.
- The condensation of water should be taken into consideration. Water that evaporate become water vapour, and some will condense into water on the bathtub.

References

[1]

7 Non-technical explanation

Users,
If you'd like to use the bathtub, it's better to acknowledge that:

- Obviously, the water exchange thermal energy directly with the air. That's why you can see water vapour float above the water.
- On the other hand, the bathtub is cooler than the water on most occasions. It's reasonable that the bathtub absorbs heat from the water. However, the bathtub conduct heat easily, while in this way the air's temperature rises.

From statement above, you may understand our bathtub can't stay in a even temperature evenly. Consequently we have some suggestions for you, if you'd like to enjoy more in your bath :

- To keep the water warm and relax yourself, you may turn the faucet on. Water-in in high temperature loses it's energy quickly, and water-in in low temperature is unable to warm your surrounding. It's best that you set the speed of the water-in at xxx, and temperature at yyy.
- Further more, your movement effects the water. It would be great if you stay still, but it doesn't count too much. For a better temperature-keeping situation, you may add some bubble bath additive to decrease the temperature loss.