Human Capital Model

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Contents

1 Introduction

$$Q = \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz$$

$$\begin{aligned}
\int_{t_1}^{t_2} \iint\limits_{\Gamma} k \frac{\partial u}{\partial n} dsdt &= \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \\
&= \int_{t_1}^{t_2} \iiint\limits_{\Omega} [\frac{\partial}{\partial x}(k \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial u}{\partial z})] dxdydx \\
&= \iiint\limits_{\Omega} \rho C_w (\int_{t_1}^{t_2} \frac{\partial u}{\partial t} dt) dxdydz
\end{aligned}$$

$$\rho C_w \frac{\partial u}{\partial t} = k \triangle u + \frac{C_w V_h (u_h - u)}{V} - \frac{C_v g}{V}$$

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1.1 Restatement of the Problem

We are required to build a mathematical model to solve the problem of heating water in bathtub. Thus we have some subproblems:

- Build a basic model that demonstrate the change of the temperature in the bathtub in space and time without other intervention.
- Figure out the influence of parameters such as the shape of the bathtub or the motions made by the person or so.
- Propose a strategy to keep the temperature as even as possible.

In the first step, we build the simplest model. The shape of the bathtub is a cuboid, and the person will stay still. In the second step, the person move slowly and the shape of the bathtub changes. In the third step, we develop the conclusion of the strategy.

Constant	Description
k	Thermal conductivity of water
ρ	Density of water
γ_1	Heat exchange coefficient between water and air
γ_2	Heat exchange coefficient between water and person
γ_3	Heat exchange coefficient between water and bathtub
C_w	Specific heat capacity of water
V	Total volume of water
x_s	Humidity ratio in saturated air
x_0	Initial humidity ratio in the air
A	Square of water surface
V_a	Volume of surrounding air
C_v	Evaporation heat of water
u_2	Temperature of person's surface
Variable	Description
u	Temperature of water
u_1	Temperature of air
u_3	Temperature of bathtub

2 Assumptions and Justifications

- **assumption 1** If an employee has probability to promote, he won't churn.

  The possibility of the unforeseen accidents, which could force an employee to leave his position,is neglected.Human nature, an employee will stay at his position to chase for higher level.

- **assumption 2** For a vacancy,if there exists an employee measures up to it already,ICM won't recruit for it.

  Since recruiting good people is difficult, time consuming and expensive according to issue 5,it is wasteful to recruit for a position if an employee can promote to it.

- **assumption 3** Demotion won't occur.

- **assumption 4** Administrative clerk won't promote or be transferred.

- **assumption 5** For the promotion probability and organization change, the other factors effects the churn probability is invariable.

  Though churn derives from varieties of reasons and they are actually lacking of known conditions and data to estimate them, we have to regard it as stable in our model.

- **assumption 6** Each division or office have at least one middle manager or senior manager.

3 Water in the Bathtub Model

Definitions of symbols employed in this paper are listed in **Table 1**.

3.1 Model Overview

Partial differential equations are widely used in natural science, engineering and economical management. It is natural to use partial differential equation to solve this physical problem.

In our model, five parts accounts for heat transfer of water: heat exchange between water and air, heat exchange between water and the person, heat exchange between water and the bathtub, heat of hot water from the faucet and heat of water flow outside. According to thermology, we conduct the equation of water temperature in the bathtub. The initial-boundary value condition could be conducted according to Fourier theorem, vaporization equation[ ].

Since the equation is a relatively complex 3D partial differential equation, we use matlab to solve it. We build a geometry as the bathtub. By finite element method, the geometry discrete into several parts. The solution of each part is considered to be a relatively precise value solution.

3.2 Heat Equation

In this part, we use physical theorem to conduct the heat conduction equation in the bathtub.

Figure 1 shows thermal analysis of water in bathtub.

First,we assume that the heat change of all water in a tiny time Δt from t_1 to t_2 is ΔQ. As shown in Figure 1, water in the bathtub have heat exchange with air, the person, and the bathtub. In addition, heat changes when the hot water flow in and the excess water flow out. Thus ΔQ is divided into five parts, i.e.

$$\Delta Q = \Delta Q_1 + \Delta Q_2 - \Delta Q_3 - \Delta Q_4 \tag{1}$$

in which ΔQ_1 is heat transferred from air, person and bathtub, ΔQ_2 is the heat of the hot water from the faucet in Δt, ΔQ_3 is the heat of the water flow away in Δt, ΔQ_4 is the heat loss of vaporization.

Totally, internal energy of water changes while heat changes. Then the temperature changes with

$$Q = \iiint\limits_{\Omega} \rho C_w (u_f - u_i) dxdydz \tag{2}$$

, in which Ω is the region of water, ρ is the density of water, u_f is the final temperature when t=t_2, u_i is the initial temperature when t=t_1.

Considering the right side of equation 1. Fourier theorem indicates that,

$$dQ = -k\frac{\vartheta u}{\vartheta n} dSdt \tag{3}$$

holds for the heat flow through surface with square of dS, as shown in Figure 2. In the equation, k is thermal conductivity, $\frac{\vartheta u}{\vartheta n}$ is the derivative of u along the normal line. Thus, after integration of dQ by time and space

$$\Delta Q_1 = int_{t_1}^{t_2}[int_\Gamma k\frac{\vartheta u}{\vartheta n} dS]dt \tag{4}$$

in which Γ is the boundary of Ω. The heat flow into the water by thermal conduction could be calculated by the equation.

ΔQ_2 is determined by the internal energy of hot water from the faucet in Δt. And ΔQ_2 can be calculated by $\Delta Q_2 = \Delta m C u_h$. Since the hot water is constant, $\Delta m = v\Delta t$.

It is obvious that the excess water is as the same value of hot water in any time period. Thus ΔQ_3 can be calculated by $\Delta Q_3 = \Delta m C_w u$. Also, $\Delta m = v\Delta t$. Δv is the weight of hot water each second, Δm is the weight of hot water in Δt.

According to thermal theorem, heat loss in vaporization is proportional to the weight of vapor water. $\Delta Q_4 = C_v \Delta m_v$. Δm_v is the weight of evaporation water in Δt, ΔC_v is evaporation heat of water.

Since water could be considered as manifold, according to Green formula,

(5)

.

Combining the equations above,

(6)

. In this heat equation, u_h and v_h are two variables that control the strategy.

Now discuss g, evaporation rate per square. As Figure 3 shows, water evaporate into air and air humidity increases, until it saturate. According to empirical formula in [], we know that

$$g = \frac{(25+19v)(x_s - x)}{3600} \tag{7}$$

in which v is the velocity of air, x_s is humidity ratio in saturated air, x is humidity ratio in air. Considering the humidity ratio in the air, it indicates

$$x = x_0 + \int \frac{Ag}{\rho V_a} dt \tag{8}$$

in which, x_0 is the initial humidity ratio in the air, ρ is density of water, V_a is the total air volume of surrounding air.

Combing the two equations and differentiating, we get a ordinary partial equation

$$\frac{3600}{25+19v}\frac{dg}{dt} = \frac{A}{\rho V_a} \tag{9}$$

Solving the equation, we get the solution

$$g = \frac{(25+19v)(x_s - x_0)}{3600} e^{-\lambda t}, \lambda = \frac{A(25+19v)}{3600\rho V_a} \tag{10}$$

3.3 Initial-Boundary Value Condition

Boundary value condition is quite essential in partial differential equations. In our model, water is not a regular research object. Thus boundary value condition is complex.

The boundary of water consists of four parts: water-air boundary, water-person boundary, water-bathtub boundary, and the square for faucet. We denote them as Γ_1, Γ_2, Γ_3, Γ_4 respectively.

Since person is temperature-constant, the boundary value condition of Γ_1 and Γ_3 could be considered as the Dirichlet value condition. That is

$$u = u_i, (x, y, z) \in \Gamma_i, i = 2.4 \tag{11}$$

In the equation, u_2 is the surface temperature of person. We assume it is constant. u_4 is the temperature of hot water.

The boundary value condition of Γ_1 and Γ_3 could be considered as the third boundary value condition approximately. That is

$$k\frac{\vartheta u}{\vartheta n} + \gamma_i u = \gamma_i u_i, (x, y, z) \in \Gamma_i \tag{12}$$

. In the equation, i=1,3, and u_i is the temperature of contact material, γ_i is the heat exchange coefficient of Γ_i.

In the equation above, u_1, u_3 are temperatures of air and bathtub, they are time-dependant functions. It takes several steps to calculate them.

Consider the temperature of air, u_1. Heat change of air consists of two ways, heat conduction and evaporation.

In our model, we assume the temperature of water is equilibrium throughout the bathtub initially.

3.4 PDE Solution

Since our equations are complex and the region is quite irregular, it's impossible to get a regular solution. Finite element method is normally adopted to deal with this problem.

We use Partial Differential Equation Toolbox in matlab to solve our equations. Partial Differential Equation Toolbox makes use of finite element method to discrete irregular geometry and calculate a relatively precise solution. get solution of temperature corresponding to different u_h and v_h. By comparing temperature distribution, we choose proper parameters as the best strategy.

First, we build a 3D geometry model to simulate a proper-size bathtub, as shown in Figure 2. Matlab divided the geometry into several elements.

Then we input the heat equation and initial-boundary value conditions. Boundary value conditions are different in three parts.

4 Results and Analysis

Our strategy focus on two variables, v_h and u_h. We change the value of the two parameters to find the best strategy. We test pairs of data

	v_h	u_h
1		5
2		1
3		3
4		4

5 Sensitivity Analysis

5.1 Parameters of Bathtub

- We change the shape of the bathtub from square to sphere and elliptical with the same strategy. The temperature graphic shows that sphere is the most proper shape for keeping water temperature. Square performs the worst on keeping temperature. However, temperature of square bathtub is more evenly than others.
- We change the volume of the bathtub without changing its proportion. The result shows that with the same v_h,

5.2 Parameters of Person

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-
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5.3 Effects of Motion

5.4 Effects of Bubble Bath Additive

6 Strengths and Weaknesses

Strengths

- Our model make fully use of the theory of multi-layer networks so that it quantizes the relation accurately and reasonably.
- Our model excellently proves the interaction among these factors:leave probability, promotion probability and productivity.
- The network we built include both microcosmic part and macrocosmic part, and they react to each other.
- Our model proves the effect of time.

Weaknesses

- We assume that the water is still. In fact, streams flow and fluid dynamic is better taken into consideration. The actual situation is much complicated.
- The model of the person should be more realistic. In fact, model of human body includes hands, legs or other parts, and the surface of human skin isn't even.
- The condensation of water should be taken into consideration. Water that evaporate become water vapour, and some will condense into water on the bathtub.

References

[1]