Human Capital Model

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Contents

1 Introduction

Considering the shortage of the talent, it's essential for companys to retain good people and make them well-trained. However, current situation is not satisfactory while many talents always tend to get a good job via job-hopping, causing orgnizational churn in employees who are closely connected to them. In order to simulate this process and improve it, we build a human capital model based on Social Network Analysis and Markov process.

$$Q = \iiint\limits_{\Omega} \rho C_w(u_f - u_i)dxdydz$$

$$\begin{aligned}
\int_{t_1}^{t_2} \iint\limits_{\Gamma} k\frac{\partial u}{\partial n}dsdt &= \iiint\limits_{\Omega} \rho C_w(u_f - u_i)dxdydz \\
&= \int_{t_1}^{t_2} \iiint\limits_{\Omega} [\frac{\partial}{\partial x}(k\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial u}{\partial z})]dxdydx \\
&= \iiint\limits_{\Omega} \rho C_w(\int_{t_1}^{t_2} \frac{\partial u}{\partial t}dt)dxdydz
\end{aligned}$$

$$\rho C_w\frac{\partial u}{\partial t} = k \triangle u + \frac{C_w V_h(u_h - u)}{V} - \frac{C_v g}{V}$$

1.1 Restatement of the Problem

We are required to build a mathematical model to solve the problem of heating water in bathtub. Thus we have some subproblems:

- Build a basic model that demonstrate the change of the temperature in the bathtub in space and time without other intervention.
- Figure out the influence of parameters such as the shape of the bathtub or the motions made by the person or so.
- Propose a strategy to keep the temperature as even as possible.

In the first step, we build the simplest model. The shape of the bathtub is a cuboid, and the person will stay still. In the second step, the person move slowly and the shape of the bathtub changes. In the third step, we develop the conclusion of the strategy.

1.2 Literature Review

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2 Terminology

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3 Assumptions and Justifications

- **assumption 1** If an employee has probability to promote, he won't churn.

  The possibility of the unforeseen accidents, which could force an employee to leave his position,is neglected.Human nature, an employee will stay at his position to chase for higher level.

- **assumption 2** For a vacancy,if there exists an employee measures up to it already,ICM won't recruit for it.

  Since recruiting good people is difficult, time consuming and expensive according to issue 5,it is wasteful to recruit for a position if an employee can promote to it.

- **assumption 3** Demotion won't occur.

- **assumption 4** Administrative clerk won't promote or be transferred.

- **assumption 5** For the promotion probability and organization change, the other factors effects the churn probability is invariable.

  Though churn derives from varieties of reasons and they are actually lacking of known conditions and data to estimate them, we have to regard it as stable in our model.

- **assumption 6** Each division or office have at least one middle manager or senior manager.

4 Human Capital Model

4.1 Model Overview

Partial differential equations are widely used in natural science, engineering and economical management. It is natural to use partial differential equation to solve this physical problem.

In our model, five parts accounts for heat transfer of water: heat exchange between water and air, heat exchange between water and the person, heat exchange between water and the bathtub, heat of hot water from the faucet and heat of water flow outside. According to thermology, we conduct the equation of water temperature in the bathtub. The initial-boundary value condition could be conducted according to Fourier theorem, vaporization equation[ ].

Since the equation is a relatively complex 3D partial differential equation, we use matlab to solve it. We build a geometry as the bathtub. By finite element

Constant	Description
k	Thermal conductivity of water
ρ	Density of water
γ_1	Heat exchange coefficient between water and air
γ_2	Heat exchange coefficient between water and person
γ_3	Heat exchange coefficient between water and bathtub
C_w	Specific heat capacity of water
V	Total volume of water
x_s	Humidity ratio in saturated air
x_0	Initial humidity ratio in the air
A	Square of water surface
V_a	Volume of surrounding air
C_v	Evaporation heat of water
u_2	Temperature of person's surface
Variable	**Description**
u	Temperature of water
u_1	Temperature of air
u_3	Temperature of bathtub

method, the geometry discrete into several parts. The solution of each part is considered to be a relatively precise value solution.

Definitions of symbols employed in this paper are listed in **Table 1**.

4.2 Heat Equation

In this part, we use physical theorem to conduct the heat conduction equation in the bathtub.

Figure 1 shows thermal analysis of water in bathtub.

First,we assume that the heat change of all water in a tiny time Δt is ΔQ. As shown in Figure 1, water in the bathtub have heat exchange with air, the person, and the bathtub. In addition, heat changes when the hot water flow in and the excess water flow out. Thus ΔQ is divided into five parts, i.e.

$$\Delta Q = \Delta Q_1 + \Delta Q_2 - \Delta Q_3 - \Delta Q_4 \tag{1}$$

in which ΔQ_1 is heat transferred from air, person and bathtub, ΔQ_2 is the heat of the hot water from the faucet in Δt, ΔQ_3 is the heat of the water flow away in Δt, ΔQ_4 is the heat loss of vaporization.

Totally, internal energy of water changes while heat changes. Then the temperature changes with

$$1 \tag{2}$$

Considering the right side of equation 1. Fourier theorem indicates that,

$$dQ = -k\frac{\vartheta u}{\vartheta n}dSdt \tag{3}$$

holds for the heat flow along a surface with square of dS. In the equation, k is thermal conductivity, $\frac{\vartheta u}{\vartheta n}$ is the derivative of u along the normal line. Thus, after integration of dQ by time and space

$$\Delta Q_1 = int_{t_1}^{t_2}[int_\Gamma k\frac{\vartheta u}{\vartheta n}dS]dt \tag{4}$$

The heat flow into the water by thermal conduction could be calculated by this equation.

ΔQ_2 is determined by the internal energy of hot water from the faucet in Δt. And ΔQ_2 can be calculated by $\Delta Q_2 = \Delta mCu_h$. Since the hot water is constant, $\Delta m = v\Delta t$.

It is obvious that the excess water is as the same value of hot water in any time period. Thus ΔQ_3 can be calculated by $\Delta Q_3 = \Delta mCu$. Also, $\Delta m = v\Delta t$.

According to thermal theorem, heat loss in vaporization is proportional to the weight of vapor water. $\Delta Q_4 = C_v\Delta m_v$.

Since water could be considered as manifold, according to Green formula,

$$2 \tag{5}$$

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Combining the equations above,

$$3 \tag{6}$$

. Some parameters would be discussed in the following sections.

4.3 Initial-Boundary Value Condition

Boundary value condition is quite essential in partial differential equations. In our model, water is not a regular research object. Thus boundary value condition is complex.

The boundary of water consists of three parts: water-air boundary, water-person boundary, and water-bathtub boundary. We denote them as Γ_1, Γ_2, Γ_3 respectively.

Since person is temperature-constant, the boundary value condition of Γ_1 and Γ_3 could be considered as the Dirichlet value condition. That is

$$u = u_2, (x, y, z) \in \Gamma_2 \tag{7}$$

.

The boundary value condition of Γ_1 and Γ_3 could be considered as the third boundary value condition approximately. That is

$$k\frac{\vartheta u}{\vartheta n} + \gamma_i u = \gamma_i u_i, (x, y, z) \in \Gamma_i \tag{8}$$

. In the equation, i=1,3, and u_i is the temperature of contact material, γ_i is the heat exchange coefficient of Γ_i.

In our model, the initial value condition is the initial temperature of water. We assume the temperature is equilibrium throughout the bathtub.

4.4 PDE Solve

5 Results and Analysis

5.1 Analysis for Task 2

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5.2 Analysis for Task 3

We assume that the company offers training programs for its employees monthly and newly hired employees start to get their salaries next month after they enter the company. With these two assumption, results can be drawn according to our model through simulation.

Budget can be divided into threee parts: salary budget, training budget and recruiting budget. The budget requirement predicted for next two years is listed in the table below in terms of σ.

Total Budget	Salary Budget	Training Budget	Recruiting Budget
1170.8σ	951.387σ	164.423σ	55.08σ

5.3 Analysis for Task 4

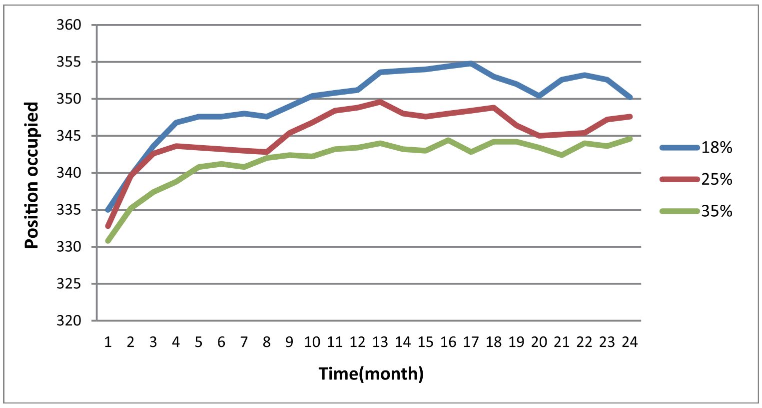

Figure 1: Status of positions

To analyze the status of positions under different churn rate, we use our model to simulate dynamic processes with these churn rate constraints. We execute our program 100 times for each churn rate and average the predicted values. Figure 1 shows the averaged results our model predicted. Under all of these three conditions, the number of employees in the company keeps rising. The higher the churn rate, the lower the final full rate the company reaches after two years. But ICM can sustain its 80% for positions even if the churn rate goes to 35% according to our model's precition.

The churn rate effect the budget of the company as well. The three parts of the bugdet behave differently when churn rate increases. The calculated budget is shown in Figure 2 and Figure 3. Each data point in three charts is an averaged result of 10 predictions and a linear trendline is added to each chart. It is clear

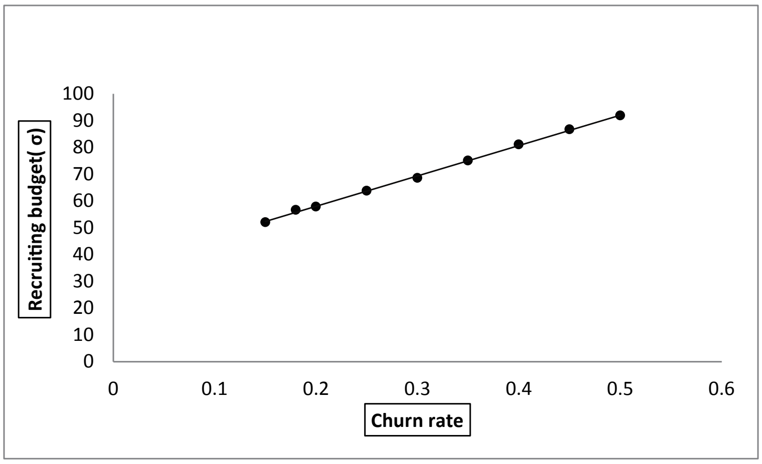

Figure 2: Recruiting budget

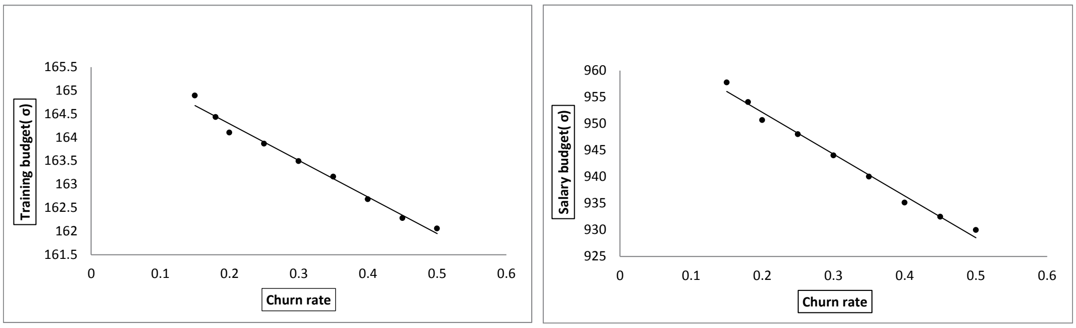

Figure 3: Training budget and salary budget

that recruiting budget showed in Figure 2 is likely to be proportional to the churn rate while salary budget and training budget showed in Figure 3 are likely to be inversely proportional to the churn rate.

To maintain enough employees, the company has to spend more on recruiting. So high turnover rate directly increase the recruiting budget. High turnover rate's effect on training budget and salary budget is more complex. On the one hand, when churn rate goes up, vacancies in the middle level keeps rising due to long recruiting time and low promote rate. On the other hand, the vacancies in lower level remains low because of the short recruiting time. So the full rate of the company decreases when churn rate rises. Since training budget and salary budget are closely related to full rate, both of them decrease when turnover rate goes up.

5.4 Analysis for Task 5

We apply following changes to our model to simulate the required process:

- Change the churn rate of junior managers and experienced supervisors to 30%
- Prohit external recruiting
- Promoting only qualified employees

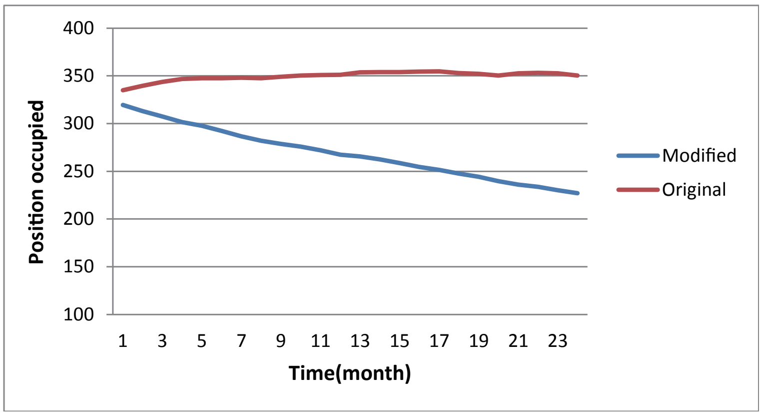

Figure 4: Status of position

Level of Position	Modified	Original
Senior manager/Executive	5.6	8.4
Junior manager/Executive	9.0	18.4
Experienced supervisor	7.4	23.0
Inexperienced supervisor	9.2	23.2
Experienced employee	72.6	107.4
Inexperienced employee	99.6	149.6
Administrative clerk	24.0	24.0

Table 1: Status of position

The result of simulation is shown in Figure 4 and Table 1. All the data shown is an average of ten predictions. While the number of positions occupied remains stable with origial conditions, it drops remarkably with modified conditions.

We list specific data of each level in Table 1. In the modified case, the numbers of employees are lower than original case especially the those of middle levels. Since there is no external recruiting in modified case, it is obivious that the full rate will decrease due to employees' leave. Although some qualified employees can be promoted into higher level, the high churn rate of the middle level and difficulty of satisfying the promotion conditions make the numbers of middle level employees relatively low. The situation given in that task 5 will cause unrecoverable damage to ICM's HR health. With the full rate of middle level employees lower than 50%, the HR structure is broken into fragments and the company won't be able to function normally.

6 Advice for HR

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6.1 Incentive Mechanism

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6.2 Matching Employees to the Right Position

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7 Team Science

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8 Sensitivity Analysis

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9 Strengths and Weaknesses

Strengths

- Our model make fully use of the theory of multilayer networks so that it quantizes the relation accurately and reasonablly.
- Our model exellently proves the interaction among these factors:leave probability, promotion probability and productivity.
- The network we built include both microcosmic part and macrocosmic part, and they react to each other.
- Our model proves the effection of time.

Weaknesses

- We assume that the water is still. In fact, streams flow and fluid dynamic is better taken into consideration. The actual situation is much complicated.
- The model of the person should be more realistic. In fact, model of human body includes hands, legs or other parts, and the surface of human skin isn't even.
- The condensation of water should be taken into consideration. Water that evaporate become water vapour, and some will condense into water on the bathtub.

10 Conclusions

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References

[1]

11 Non-technical explanation

Users,
If you'd like to use the bathtub, it's better to acknowledge that:

- Obviously, the water exchange thermal energy directly with the air. That's why you can see water vapour float above the water.
- On the other hand, the bathtub is cooler than the water on most occasions. It's reasonable that the bathtub absorbs heat from the water. However, the bathtub conduct heat easily, while in this way the air's temperature rises.

From statement above, you may understand our bathtub can't stay in a even temperature evenly. Consequently we have some suggestions for you, if you'd like to enjoy more in your bath :

- To keep the water warm and relax yourself, you may turn the faucet on. Water-in in high temperature loses it's energy quickly, and water-in in low temperature is unable to warm your surrounding. It's best that you set the speed of the water-in at xxx, and temperature at yyy.
- Further more, your movement effects the water. It would be great if you stay still, but it doesn't count too much. For a better temperature-keeping situation, you may add some bubble bath additive to decrease the temperature loss.