第 6 考场使用！请确认你所在 6 考场！

北京邮电大学 2019——2020 学年第二学期

Discrete Mathematics – Final Exam 6

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 6 月 23 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	8	8	4	
得分															
阅卷教师															

1. [5 points]

   a) Which of these sentences are propositions? What are the truth values of those that are propositions?

      i) Make sure that you use the correct test paper. __________

      ii) 2 + 8 =10. __________

      iii) Scientific cooperation is the only way that the world will defeat COVID-19. __________

      iv) 120 is a perfect square. __________

      v) Birds can fly, only if pigs can fly. __________

   b) Let L(x, y) be the statement “x loves y,” and H(x,y) be the statement “ x hates y”, where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements (1),(2),(3). And translate each of these (4),(5) nested quantifications into a statement.

      （1） Everybody loves somebody or hates somebody

      （2） There is somebody whom everybody loves and there is no one whom everybody hates.

      （3） Nobody loves everybody.

      (4) $\exists x \forall y (L(x,y) \wedge H(y,x))$

      (5) $\forall x \exists y L(x,y) \rightarrow \forall x \exists y H(x,y)$

姓名：　　线　　班内序号：　　订　　装　　班级：

2. [10 points] Show that $t \vee ((r \rightarrow p) \wedge (r \rightarrow \neg q))$ and $(p \rightarrow q) \rightarrow (r \rightarrow t)$ are logically equivalent.

3. [10 points] Find the principal disjunctive normal form of (a) and (b).

(a) $(\neg p \vee \neg q) \rightarrow (\neg p \rightarrow \neg q)$

(b) $(p \rightarrow (q \wedge r)) \wedge (\neg p \leftrightarrow (\neg q \wedge \neg r))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $(\forall x P(x) \vee \exists y R(y)) \rightarrow \forall x F(x)$

(b) $\exists x (\neg \exists y\, P(x,y)) \rightarrow (\exists z Q(z) \vee R(x))$

5. [10 points] Show that the premises "There is someone in this class who has been to Paris," and " Everyone who goes to Paris visits the Louvre" imply the conclusion "Someone in this class has visited the Louvre."

6. [5 points] Prove that the equation $2x^2 + y^2 = 11$ has positive integer solutions and explain the proof method you use.

7. [6 points]Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Find $A^{[3]}$.

8. [6 points]Let f and g be functions from {1, 2, 3, 4} to {a, b, c, d} and from {a, b, c, d} to {1, 2, 3, 4}, respectively, with f (1) = d, f (2) = c, f (3) = b, and f (4) = a, and g(a) = 2, g(b) = 1, g(c) = 3, and g(d) = 2.

a) Is *f* one-to-one? Is *g* one-to-one?

b) Is *f* onto? Is g onto?

c) Does either *f* or *g* have an inverse? If so, find this inverse.

9. [6 points]Use Fermat's little theorem to evaluate 9^{20010} mod 17.

10. [6 points] Find all solutions, if any, to the system of congruences $x \equiv 3(mod\ 7), x \equiv 2(mod\ 5)$.

11. [6 points]List these functions so that each function is big-O of the next function in the list: $(\log n)^2$, $n^3/1000000$, $n^{1/3}$, $100n + 1001$, 3^n, $n!$, $2^n n^2$.

12. [8 points]Suppose that the only paper money consists of 3-dollar bills and 10-dollar bills. Show that any dollar amount greater than 17 dollars could be made from a combination of these bills.

13. [8 points] (a) Find the number of solutions to $x+y+z=47$, where x, y, and z are nonnegative integers. (b) Answer part (a), but assume that $x \geqslant 7$ and $y \geqslant 15$.

14. [4 points] Suppose that $m_1, m_2, \ldots, m_n$ are pairwise relatively prime moduli and let m be their product. By the Chinese remainder theorem, we can show that an integer a with $0 \leqslant a < m$ can be uniquely represented by the n-tuple consisting of its remainders upon division by m_i, $i = 1, 2, \ldots, n$. That is, we can uniquely represent a by (a **mod** m_1, a **mod** m_2, . . . , a **mod** m_n). For example: we have the following representations, obtained by finding the remainder of each integer which is the nonnegative integers less than 12 when it is divided by 3 and by 4:

    0 = (0, 0) 4 = (1, 0) 8 = (2, 0)
    1 = (1, 1) 5 = (2, 1) 9 = (0, 1)
    2 = (2, 2) 6 = (0, 2) 10 = (1, 2)
    3 = (0, 3) 7 = (1, 3) 11 = (2, 3).

    Suppose that performing arithmetic with integers less than 5.

    a) Use these pairs to find the sum of 4 and 6.
    b) Use these pairs to find the product of 2 and 5

第 9 考场使用！请确认你所在第 9 考场！

北京邮电大学 2019——2020 学年第二学期

Discrete Mathematics – Final Exam 9

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 6 月 23 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	6	10	4	
得分															
阅卷教师															

姓名： 班内序号： 班级：

装 订 线

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) $x + y = 100$. _________

ii) 80 is a perfect square. ________

iii) If you use a wrong test paper, then you will get an invalid score. ________

iv) Birds can fly, unless pigs can not fly. ________

v) To eliminate poverty in all poor counties and regions of China by 2020.

____

b) Let L(x, y) be the statement "x loves y," and H(x,y) be the statement " x hates y", where the domain for both x and y consists of all people in the world. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

（1） $\neg\forall y\, \exists x\, H(y,x)$

（2） $\forall x\, \exists y L(x,y) \rightarrow \forall x\, \exists y H(x,y)$

（3） Everybody loves somebody and hates somebody

(4) Nobody loves everybody.

(5) There is somebody whom everybody loves and there is no one whom everybody hates.

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2. [10 points] Show that $t \vee ((r \rightarrow w) \wedge (r \rightarrow \neg s))$ and $(w \rightarrow s) \rightarrow (r \rightarrow t)$ are logically equivalent.

3. [10 points] Find the principal disjunctive normal form of (a) and (b).

(a) $(\neg p \vee \neg q) \wedge (p \rightarrow \neg q)$

(b) $(p \rightarrow (q \vee r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $(\forall x P(x) \wedge \exists y Q(y)) \rightarrow \forall z W(z)$

(b) $\neg \exists x \exists y\, Q(x,y) \rightarrow (\exists z F(z) \vee R(x))$

5. [10 points] Show that the premises "There is someone in this class who has been to Guangdong," and " Everyone who goes to Guangdong visits Shenzhen" imply the conclusion "Someone in this class has visited Shenzhen."

6. [5 points] Prove or disprove that the equation $2x^3 + y^2 = 16$ has positive integer solution and explain the proof method you use.

7. [6 points]Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Find $A^{[3]}$.

8. [6 points]Let f and g be functions from {1, 2, 3, 4} to {a, b, c, d} and from {a, b, c, d} to {1, 2, 3, 4}, respectively, with f (1) = d, f (2) = c, f (3) = a, and f (4) = b, and g(a) = 2, g(b) = 1, g(c) = 3, and g(d) = 2.

a) Is *f* one-to-one? Is *g* one-to-one?

b) Is *f* onto? Is g onto?

c) Does either *f* or *g* have an inverse? If so, find this inverse.

9. [6 points]Use Fermat's little theorem to evaluate 9^{20000} mod 19.

10. [6 points] Find all solutions, if any, to the system of congruences $x \equiv 5 (mod\ 7), x \equiv 4 (mod\ 5)$.

11. [6 points]List these functions so that each function is big-O of the next function in the list: $(\log n)^3$, $n^3/1000000$, $n^{1/2}$, $100n + 101$, 3^n, $n!$, $2^n n^2$

12. [6 points] Prove that the distributive law $A_1 \cap (A_2 \cup \cdots \cup A_n) = (A_1 \cap A_2) \cup \cdots \cup (A_1 \cap A_n)$ is true for all $n \geq 3$.

第 13 考场使用！请确认你所在第 13 考场！

北京邮电大学 2019——2020 学年第二学期

Discrete Mathematics – Final Exam 13

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 6 月 23 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	8	8	4	
得分															
阅卷教师															

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) x + y =100. __________

ii) 80 is a perfect square. _________

iii) If you use a wrong test paper, then you will get an invalid score. _________

iv) Birds can fly, unless pigs can not fly. _________

v) To eliminate poverty in all poor counties and regions of China by 2020.

____

b) Let L(x, y) be the statement “x loves y,” and H(x,y) be the statement “ x hates y”, where the domain for both x and y consists of all people in the world. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

（1） $\forall y\,(\neg\exists x\, H(y,x))$

（2） $\forall x\, \exists y L(x,y) \rightarrow \forall x\, \exists y H(x,y)$

（3） Everybody loves somebody and hates somebody

(4) Nobody loves everybody.

(5) There is somebody whom everybody loves and there is no one whom everybody hates.

姓名：　线　班内序号：　订　班级：　装

2. [10 points] Show that $t\vee((r\rightarrow w)\wedge(r\rightarrow\neg s))$ and $(w\rightarrow s)\rightarrow(r\rightarrow t)$ are logically equivalent.

3. [10 points] Find the principal disjunctive normal form of (a) and (b).

(a) $(\neg s \vee \neg t) \rightarrow (s \leftrightarrow \neg t)$

(b) $(p\rightarrow (q \vee r)) \wedge (\neg p\rightarrow (\neg q \wedge \neg r))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $(\forall xP(x) \wedge \exists yQ(y)) \rightarrow \forall zW(z)$

(b) $\neg\exists x \exists y\, Q(x,y) \rightarrow (\exists zF(z) \vee R(x))$

5. [10 points] Show that the premises "There is someone in this class who has been to Guangdong," and " Everyone who goes to Guangdong visits Shenzhen" imply the conclusion "Someone in this class has visited Shenzhen."

6. [5 points] Prove that the equation $2x^3 + y^2 = 17$ has positive integer solution and explain the proof method you use.

7. [6 points] Find the Boolean product of A and B, where

$$A=\begin{bmatrix}1&0&1&0\\1&1&0&1\\0&0&1&1\end{bmatrix} \text{ and } B=\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\\0&0&1\end{bmatrix}.$$

8. [6 points]

a) Prove or disprove: if A, B and C are sets, then $(A - C) - (B - C) = A - B$.

b) Give an example of a function f: $\mathbf{Z} \rightarrow \mathbf{N}$ that is both 1-1 and onto.

9. [6 points]Use Fermat's little theorem to evaluate 9^{20022} mod 13.

10. [6 points] Find all solutions, if any, to the system of congruences $x \equiv 3(mod\ 5), x \equiv 4(mod\ 7)$..

11. [6 points]List these functions so that each function is big-O of the next function in the list: $(100\log n)^2$, $n^2/10000$,$n^{1/2}$,$200 n^2 +100n + 1001$, 9^n, n^n, 2^n+c.

12. [8 points] Prove that the distributive law $B\cap(A_1 \cup\cdots\cup A_n) = (A_1\cap B) \cup\cdots\cup(A_n \cap B)$ is true for all $n\geq 2$

13. [8 points] How many ways can be made by distribute hands of 7 cards each to 3 players from a standard deck of 54 cards?

14. [4 points] INDISTINGUISHABLE OBJECTS AND INDISTINGUISHABLE BOXES Some counting problems can be solved by determining the number of ways to distribute indistinguishable objects into indistinguishable boxes. We illustrate this principle with an example.

Example：How many ways are there to pack six copies of the same book into four identical boxes, where a box can contain as many as six books?

Solution:We will enumerate all ways to pack the books. For each way to pack the books, we will list the number of books in the box with the largest number of books, followed by the numbers of books in each box containing at least one book, in order of decreasing number of books in a box. The ways we can pack the books are

6
5, 1
4, 2
4, 1, 1
3, 3
3, 2, 1
3, 1, 1, 1
2, 2, 2
2, 2, 1, 1.

For example, 4, 1, 1 indicates that one box contains four books, a second box contains a single book, and a third box contains a single book (and the fourth box is empty). We conclude that there are nine allowable ways to pack the books, because we have listed them all.

Try to find how many ways there are to pack seven copies of the same book into three identical boxes, where a box can contain as many as seven books?

班级： 班内序号： 姓名：

装 订 线

第 24 考场使用！请确认你所在 24 考场！

北京邮电大学 2019——2020 学年第二学期

Discrete Mathematics – Final Exam 24

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 6 月 23 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	8	8	4	
得分															
阅卷教师															

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) 160 is a perfect square. ________

ii) Keep quite during test. ________

iii) Birds can fly if and only if pigs can fly. ________

iv) $K=FC^2$. ________

v) 15+15=30 ________

b) Let L(x, y) be the statement "x loves y," and H(x,y) be the statement " x hates y", where the domain for both x and y consists of all people in the world. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

(1) $\forall y\, \exists x\, H(x,y)$

(2) $\forall x\, \exists y L(x,y) \wedge \forall x\, \exists y H(x,y)$

(3) Somebody is loved by everybody.

(4) There is somebody whom everybody loves or there is no one whom everybody hates.

(5) Everybody hates somebody and loves somebody.

2. [10 points] Prove or disprove that $p \vee ((r \to q) \wedge (r \to \neg s))$ and $(s \vee \neg q) \to (r \to p)$ are logically equivalent.

3. [10 points] Find the principal conjunctive normal form of (a) and (b).

(a) $(\neg p \vee \neg q) \leftrightarrow (\neg p \to q)$

(b) $(x \to (y \vee z)) \wedge (\neg x \to (\neg y \wedge \neg z))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $\forall x P(x) \to (\exists y Q(y) \wedge \forall z W(z))$

(b) $\neg \exists s \exists t P(s,t) \to (\exists z Q(z) \to \neg R(t))$

5. [10 points] Show that the premises "There is someone in this class who has been to Russia," and "Everyone who goes to Russia visits the Red square." imply the conclusion "Someone in this class has visited the Red square."

6. [5 points] Prove that the equation $2x^2 + y^2 = 14$ has no nonnegative integers solutions and explain the proof method you use.

7. [6 points] Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

.

8. [6 points]

a) Prove or disprove: if A, B and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$.

b) Give an example of a function $f: \mathbf{R} \to \mathbf{N}$ that is onto $\mathbf{N}$ and not 1-1.

9. [6 points]

(1) Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for 116cents.

(2) Show that $(x^2 + xy + x \log y)^2$ is $O(x^4y^2)$ by directly finding the constant of k, C.

10. [6 points] Show that among any group of nine (not necessarily consecutive) integers, there are two with the same remainder when divided by 8.

11. [6 points] There are some candies, if use the box which can contains 25 candies, there are 3 candies left, if use the box which can contains 36 candies, there are 5 candies left, at least how many candies are there?

12. [8 points] Find the smallest positive integer n_0 such that $2^{n_0} > n_0^2$. and prove $2^n > n^2$ for all n $\geq n_0$.

13. [8 points]

(a) How many different strings can be made by reordering the letters of the word BETWEENNESS?

(b)How many bit strings of length 11 have more 0's than 1's?

14. [4 points] The **Vigenère cipher** is a block cipher, with a key that is a string of letters with numerical equivalents $k_1k_2 \ldots k_m$, where $k_i \in \mathbf{Z}_{26}$ for $i = 1, 2, \ldots, m$. Suppose that the numerical equivalents of the letters of a plaintext block are $p_1p_2 \ldots p_m$.The corresponding numerical ciphertext block is $(p_1 + k_1)$ **mod** 26 $(p_2 + k_2)$ **mod** 26 . . . $(p_m + k_m)$ **mod** 26. Finally,we translate back to letters. For example, suppose that the key string is RED, with numerical equivalents 17 4 3. Then, the plaintext ORANGE, with numerical equivalents 14 17 00 13 06 04, is encrypted by first splitting it into two blocks 14 17 00 and 13 06 04. Then, in each block we shift the first letter by 17, the second by 4, and the third by 3. We obtain 5 21 03 and 04 10 07. The cipherext is FVDEKH.

A 00	B 01	C 02	D 03	E 04	F 05
G 06	H 07	I 08	J 09	K 10	L 11
M 12	N 13	O 14	P 15	Q 16	R 17
S 18	T 19	U 20	V 21	W 22	X 23
Y 24	Z 25				

Use the Vigenère cipher with key PINK to encrypt the message VIRUSATTCKAGAIN.
The ciphertext HISOPWQCGCAN was produced by encrypting a plaintext message using the Vigenère cipher with key PINK. What is the plaintext message?

姓名: 线

班内序号: 订

班级: 装

北京邮电大学 2019-—2020 学年第二学期补考

Discrete Mathematics – Supplementary Examination

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 8 月 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	6	10	4	
得分															
阅卷教师															

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) The Medal of the Republic was conferred on renowned respiratory disease expert Zhong Nanshan in this month. ________

ii) The area of logic that deals with propositions is called the propositional calculus. ________

iii) $F=ma^2$. ________

iv) 2+2=3 unless 1+1=2. ________

v) Working together to defeat the COVID-19 outbreak. ________

b) Let S(x, y) be the statement "x+ y=0," and M(x,y) be the statement " x *y=0", where the domain for both x and y are real number. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

(1) $\neg\forall x\ \exists y\ M(x,y)$

(2) $\exists x\ \forall y S(x,y) \rightarrow \exists x\ \forall y M(x,y)$

(3) For every real number x there is a real number y such that M(x,y) or for any real number x and y , S(x,y).

(4) There is a real number x such that for every real number y, S(x,y).

(5) There is a real number y such that for every real number x, M(x,y) or S(x,y).

2. [10 points] Determine whether $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are logically equivalent.

3. [10 points] Find the principal conjunctive normal form of (a) and (b).

(a) $(p \wedge \neg q) \to (p \leftrightarrow q)$

(b) $(s \to (t \to w)) \wedge (\neg s \to (t \vee \neg w))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $\exists x Q(x) \vee (\forall y P(y) \to \exists z R(z))$

(b) $\forall y\, S(x,y) \to (\neg \exists z W(z) \wedge \exists x T(x))$

5. [10 points] Show that the premises "If you send me a bag of cookies, then I will finish homework on time," "If you do not send me a bag of cookies, then I will go out to buy food," and "If I go out to buy food, then I will go to the cinema" lead to the conclusion "If I do not finish homework on time, then I will go to the cinema."

6. [5 points] Prove that the product of two of the numbers $6^{510020} - 85^{29001} + 3^{310707}$, $70^{91212} - 98^{812399} + 23^{82001}$, and $24^{449223} - 58^{69192} + 7^{27901777}$ is nonnegative and explain the proof method you use.

7. [6 points] Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

8. [6 points]

a) a) Prove or disprove: if A, B, C and D are sets, then $A \cup (B \cap C \cap D) = (A \cup B \cup C) \cap D$.

b) Give an example of a function $f: \mathbf{R} \to \mathbf{N}$ that is onto $\mathbf{N}$ and not 1-1.

9. [6 points] Use the definition of "f (x) is O(g(x))" to show that $\sum_1^n (i^k + i)$ is $O(n^{k+1})$, where k is positive integer, and find the constant of k, C.

10. [6 points] There are 12 signs of the zodiac. How many people are needed to guarantee that at least six of these people have the same sign?

11. [6 points] Find all solutions, if any, to the system of congruences:

$$3x \equiv 5 (mod\ 7)$$
$$4x \equiv 7 (mod\ 11).$$

12. [6 points] Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

13. [10 points]

(a) How many different strings can be made by reordering the letters of the word *GOOGOL*?

(b) Find the number of solutions to $x + y + z = 32$, where x, y, and z are nonnegative integers.

14. [4 points] A **routing transit number (RTN)** is a bank code used in the United States which appears on the bottom of checks.The most common form of an RTN has nine digits, where the last digit is a check digit. If $d_1d_2 \ldots d_9$ is a valid RTN, the congruence $3(d_1 + d_4 + d_7) + 7(d_2 + d_5 + d_8) + (d_3 + d_6 + d_9) \equiv 0 \pmod{10}$ must hold.

(a) Show that if $d_1d_2 \ldots d_9$ is a valid RTN, then

$d_9 = 7(d_1 + d_4 + d_7) + 3(d_2 + d_5 + d_8) + 9(d_3 + d_6)$ **mod** 10.

(b) Furthermore, use this formula to find the check digit that follows the eight digits 11100002 in a valid RTN.