

High-Performance Decimal Floating-Point Arithmetic: Algorithms and Implementation in C++

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This article presents a comprehensive, portable C++ implementation of decimal floating-point arithmetic conforming to IEEE 754-2019 standards[8]. Our system offers three IEEE 754-compliant types, and three additional types that prioritize performance over strict standard adherence, providing flexible options for various computational needs. We describe in detail the Decimal system architecture, its standard library, and usage guidelines. The implementation incorporates novel algorithms for key operations, significantly improving performance in common use cases. Rigorous testing results demonstrate the system's correctness and IEEE 754 compliance. Performance benchmarks show competitive or superior results compared to existing implementations. This work addresses the growing demand for precise decimal arithmetic in financial, scientific, and engineering applications, offering a robust, efficient solution for C++ developers.

CCS Concepts: • **General and reference** → **Design; Performance**; • **Mathematics of computing** → **Mathematical software performance**; • **Software and its engineering** → *Software architectures*.

Additional Key Words and Phrases: C++, Decimal Floating Point, Object Oriented, Special Functions, System Architecture

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1 INTRODUCTION

Decimal floating-point arithmetic is crucial for many applications[4], particularly in financial and scientific computing. While several C++ decimal floating-point packages exist, they often lack IEEE 754[8] conformance, interoperability with the C++ Standard Template Library (STL), or both, and have limited portability. This paper presents a novel decimal system that addresses these limitations and advances decimal floating-point technology. Our system is standalone, relies on a minimal subset of the C++ STL, and has been tested on a wide range of devices, from S390X mainframes to AVR boards. It provides seamless interoperability with existing C++ standard types and full IEEE 754 conformance, features not collectively offered by any known package. This work significantly enhances the toolset available for high-precision decimal computations across diverse computing environments.

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Parameter	decimal32	decimal64	decimal128
Storage Width	32	64	128
Precision (decimal digits)	7	16	34
Max Exponent	96	384	6144
Exponent Bias	101	398	6176
Sign Width	1	1	1
Combination Field Width	11	13	17
Significand Continuation Width	20	50	110

Table 1. Decimal Floating-Point Format Parameters

The three so-called fast types (i.e. `decimalXX_fast`) have the same values of precision, range, and exponent as their analogous conformant type, but require more space.

2.3 System Architecture

The decimal system architecture is robust and flexible. Each type is implemented completely independently of each other, but they share many of the same function implementations from the STL.

2.4 Using the System

Every effort was made during design and implementation to ensure that using the decimal system was straightforward and intuitive. The library contains zero dependencies, and uses only a small subsection of the C++ STL. The required language standard for the library is C++14.

```

1 #include <boost/decimal.hpp>
2 #include <iostream>
3 #include <iomanip>
4
5 int main()
6 {
7     using namespace boost::decimal;
8
9     constexpr decimal32 val_1 {100};           // Construction from an
        integer
10    constexpr decimal32 val_2 {10, 1};         // Construction from an
        integer and exponent
11    constexpr decimal32 val_3 {1, 2, false};    // Construction from an
        integer, exponent, and sign
12
13    std::cout << "Val_1: " << val_1 << '\n'
14              << "Val_2: " << val_2 << '\n'
15              << "Val_3: " << val_3 << '\n';
16
17    if (val_1 == val_2 && val_2 == val_3 && val_1 == val_3)
18    {
19        std::cout << "All equal values" << std::endl;
20    }
21

```

```

22     constexpr decimal64 val_4 {decimal64{2, -1} + decimal64{1, -1}};
23     constexpr double float_val_4 {0.2 + 0.1};
24     const decimal64 val_5 { float_val_4 }; // Explicit Conversion from
        double
25
26     std::cout << std::setprecision(17) << "Val_4: " << val_4 << '\n'
27             << "Float: " << float_val_4 << '\n'
28             << "Val_5: " << val_5 << '\n';
29
30     if (val_4 == val_5)
31     {
32         std::cout << "Floats are equal" << std::endl;
33     }
34     else
35     {
36         std::cout << "Floats are not equal" << std::endl;
37     }
38
39     return 0;
40 }

```

Listing 1. Basic Usage

In Listing 1 we show how to construct the type and that it works just like a built-in floating point type. The following is an example that leverages one of decimal-floating points main strengths and one that we expect many people will use.

```

1  #include <boost/decimal.hpp>
2  #include <iostream>
3  #include <cassert>
4
5  int main()
6  {
7      using namespace boost::decimal;
8
9      decimal64 val {0.25}; // Construction from a double (not recommended
        but explicit construction is allowed)
10
11     char buffer[256];
12     auto r_to = to_chars(buffer, buffer + sizeof(buffer) - 1, val);
13     assert(r_to); // checks std::errc()
14     *r_to.ptr = '\0';
15
16     decimal64 return_value;
17     auto r_from = from_chars(buffer, buffer + std::strlen(buffer),
        return_value);
18     assert(r_from);
19
20     assert(val == return_value);
21

```

```

22     std::cout << " Initial Value: " << val << '\n'
23         << "Returned Value: " << return_value << std::endl;
24
25     return 0;
26 }

```

Listing 2. Basic Usage

In Listing 2 we show how to serialize and parse numbers. These techniques and functions are an extension of Boost.Charconv[3].

3 IMPLEMENTATION DETAILS

3.1 IEEE 754 Conformant Decimal Types

The decimal system provides three IEEE-754 compliant types: decimal32, decimal64, and decimal128. Each of these are constructed using a single unsigned integer of the same width to hold the bits. For example decimal32 consists of a single `std::uint32_t`. decimal128 uses a custom implementation of a 128-bit unsigned integer for portability reasons rather than relying on the existence of unsigned `__int128`. The discussion of implementing big integers is outside the scope of the paper, but papers and implementations can be found in numerous places[3][12][10].

The decoding step of decimal32 is somewhat involved because the values of the significand and exponent depend on the bits in the combination field. Our first case is where the bits in the combination field are in the form 00XXX, 01XXX, and 10XXX.

	Comb.	Exponent	Significand
1 //	00	TTT (00)eeeeee	(0TTT)[ssssssssss][ssssssssss]
2 // s	01	TTT (01)eeeeee	(0TTT)[ssssssssss][ssssssssss]
3 // s	10	TTT (10)eeeeee	(0TTT)[ssssssssss][ssssssssss]

Listing 3. Combination Field Pattern 1

In Listing 3 we can see that with these combination field bit patterns the first two bits of the combination field are appended to the front of the exponent, the next three bits are appended to the front of the significand. The second case is where the bits in the combination field are in the form 1100X, 1101X, and 1110X.

	Comb.	Exponent	Significand
1 //	1100	T (00)eeeeee	(100T)[ssssssssss][ssssssssss]
2 // s	1101	T (01)eeeeee	(100T)[ssssssssss][ssssssssss]
3 // s	1110	T (10)eeeeee	(100T)[ssssssssss][ssssssssss]

Listing 4. Combination Field Pattern 2

In Listing 4 we can see that with these combination field bit patterns there is now an implied leading bit to the significand, followed by 00T bits. Bits 2 and 3 are now what represents the leading bits of the exponent as opposed to 0 and one like in Listing 3.

The final case is where the bits in the combination field are in the form 1111X which is used for non-finite numbers:

	Comb.	Exponent	Significand
1 //	1111	0 eeeeeee	[ssssssssss][ssssssssss] = inf
2 // s	1111	1 eeeeeee	[ssssssssss][ssssssssss] = qnan
3 // s	1111	1 1 eeeee	[ssssssssss][ssssssssss] = snan

Listing 5. Combination Field Pattern 3

Since this combination field pattern is only for non-finite numbers it makes the implementation of `isnan`, `isinf`, etc. straightforward. We can also see the signbit remains unused so we can represent signed infinities. One can also use the remaining exponent and significand bits to provide a payload to NaN analogous to the use of `std::nan[6]`.

Now that we have seen what each bit pattern of the combination field corresponds to; the full implementation of decoding the significand then becomes:

```

1 constexpr auto decimal32::full_significand() const noexcept ->
  significand_type
2 {
3     significand_type significand {};
4
5     if ((bits_ & detail::d32_comb_11_mask) == detail::d32_comb_11_mask)
6     {
7         // Only need the one bit of T because the other 3 are implied
8         significand = (bits_ & detail::d32_comb_11_significand_bits) ==
          detail::d32_comb_11_significand_bits ?
9         static_cast<std::uint32_t>(0b1001'0000000000'0000000000) :
10        static_cast<std::uint32_t>(0b1000'0000000000'0000000000);
11    }
12    else
13    {
14        // Last three bits in the combination field, so we need to shift
15        // past the exp field
16        // which is next
17        significand |= (bits_ & detail::
18          d32_comb_00_01_10_significand_bits) >> detail::d32_exponent_bits;
19    }
20
21    significand |= (bits_ & detail::d32_significand_mask);
22
23    return significand;
24 }
```

Listing 6. Basic Usage

Encoding the value is a similar, but more complex process in that we need to work backwards through the steps to figure out what the value of the combination field needs to be, and then we use a series of masks to write the value.

3.2 IEEE 754 Fast Types

Now that we have discussed the three IEEE-754 conformant types we will turn our attention to much more performant, but non-compliant types. The library offers: `decimal32_fast`, `decimal64_fast`, and `decimal128_fast`. These types are similar to how instead of `std::uint32_t` you can use `std::uint_fast32_t` which on x86_64 platforms generally aliases to `std::uint64_t`. This is the classic optimization of trading space for time. Again we will focus on the 32-bit type for clarity and simplicity. Unlike `decimal32` consisting of a data `std::uint32_t`, `decimal32_fast` actually contains its internal state in a structure.

```

1 class decimal32_fast final
```

```

2 {
3 public:
4     using significand_type = std::uint_fast32_t;
5     using exponent_type = std::uint_fast8_t;
6     using biased_exponent_type = std::int_fast32_t;
7
8 private:
9     // In regular decimal32 we have to decode the 24 bits of the
10    // significand and the 8 bits of the exp
11    // Here we just use them directly at the cost of at least 2 extra
12    // bytes of internal state
13    // since the fast integer types will be at least 32 and 8 bits
14    // respectively
15
16    significand_type significand_ {};
17    exponent_type exponent_ {};
18    bool sign_ {};
19
20    // Continued
21 };

```

Listing 7. Internal state of decimal32_fast

In Listing 7 we see that now instead of having to encode and decode the number every time we perform an operation we can now just access the value directly.

3.3 Standard Library

3.3.1 Special Functions. The implementation of transcendental, and C++17 mathematical special functions[5][9] extensively uses Páde Approximations and Remez Polynomials.

4 RESULTS

4.1 Testing and Precision

4.2 Performance

As this system is implemented in software it will never be as performant as binary floating point is on computing systems with floating-point hardware. For example the table below shows the runtime difference for comparison operations between float, double, and the types provided by this system:

Type	Runtime (μ s)	Ratio to double
float	8,587	1.376
double	6,240	1.000
decimal32	275,597	44.166
decimal64	296,929	47.587
decimal32_fast	99,664	15.972
decimal64_fast	102,132	16.367

Table 2. Comparison of runtime and ratio for comparison operations

The gap in performance increases even further for the multiplication operation which is typically only a few cycles in hardware[1]:

Type	Runtime (μ s)	Ratio to double
float	1,646	0.957
double	1,720	1.000
decimal32	313,219	182.104
decimal64	583,818	339.429
decimal32_fast	86,093	50.054
decimal64_fast	333,582	193.943

Table 3. Comparison of runtime and ratio for multiplication operations

For floating-point operations that are implemented in software like parsing and serializing numbers the performance gap is quite smaller:

Type	Runtime (μ s)	Ratio to double
float	235,816	0.953
double	247,307	1.000
decimal32	366,682	1.483
decimal64	485,965	1.965

Table 4. Comparison of runtime and ratio for from_chars

Type	Runtime (μ s)	Ratio to double
float	316,300	1.040
double	304,272	1.000
decimal32	406,053	1.335
decimal64	678,451	2.230

Table 5. Comparison of runtime and ratio for to_chars

5 CONCLUSION AND OUTLOOK

The portable C++ decimal system has been presented. The system allows for users to easily employ decimal-floating point numbers in their existing code bases. For the first time a complete and interoperable system has been fully provided.

During the course of our research and implementation we have limited ourselves to 32, 64, and 128-bit types. Further research can be conducted into making higher precision types as the properties of such numbers are specified in IEEE 754. We could also expand our range of compatibility to allow the types to be massively parallelized such as providing support for CUDA devices.

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