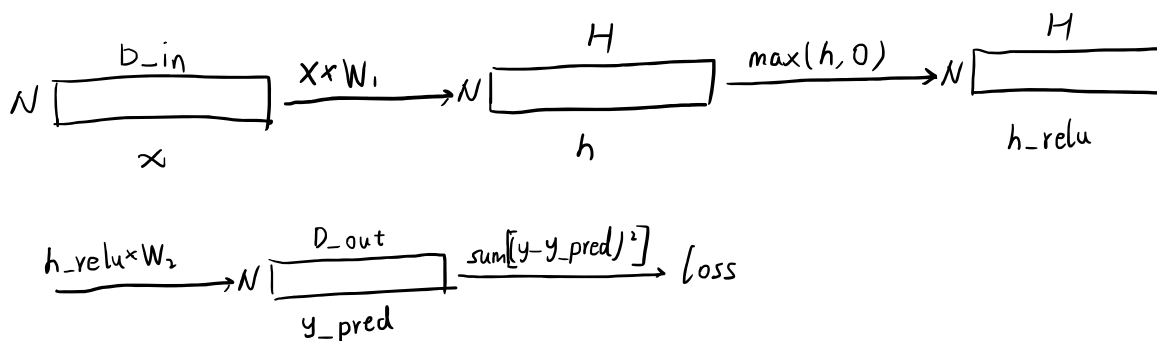


正向传播



反向传播 (不妨假设 $N=1$) 约定: "·" 表示对位相乘, "×" 表示矩阵乘法, "d" 表示 (偏) 微分

$$d\text{loss} = d(\sum (y - y_pred)^2) = \sum [2(y_pred - y) \cdot d(y_pred)] = 2(y_pred - y) \times d(y_pred)^T$$

$$= 2(y_pred - y) \times d(W_2^T \times h_relu^T)$$

记 $2(y_pred - y) = \alpha$, $h_relu = \beta$. 则 α, β 均为行向量.

$$d\text{loss} = \alpha \times d(W_2^T) \times \beta^T = \sum [(\alpha^T \times \beta) \cdot d(W_2^T)] = \sum [\beta^T \times \alpha] \cdot d(W_2)$$

故 W_2 的梯度即为 $\beta^T \times \alpha = h_relu^T \times 2(y_pred - y)$

$$d\text{loss} = \underbrace{\alpha}_{\alpha} \times \underbrace{d(W_2^T)}_{d(W_2^T)} \times \underbrace{\beta^T}_{\beta^T} = \left(\underbrace{\beta^T \times \alpha}_{\beta^T \times \alpha} \cdot \underbrace{d(W_2)}_{d(W_2)} \right) \cdot \text{sum}()$$

同理有

$$d\text{loss} = 2(y_pred - y) \times d(y_pred^T) = 2(y_pred - y) \times W_2^T \times d(h_relu^T)$$

$$= 2(y_pred - y) \times W_2^T \times \text{diag}(h > 0) \times d(h^T) = 2(y_pred - y) \times W_2^T \times \text{diag}(h > 0) \times d(W_1^T) \times x^T$$

同理可得 W_1 的梯度为 $x^T \times 2(y_pred - y) \times W_2^T \times \text{diag}(h > 0) = x^T \times [2(y_pred - y) \times W_2^T \cdot (h > 0)]$

故令 $W_1 -= \text{grad_}W_1 \cdot \text{learning_rate}$ 即完成一次梯度下降过程

$W_2 -= \text{grad_}W_2 \cdot \text{learning_rate}$