Evolution of a globular cluster

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ABSTRACT

The evolution of a globular cluster is considered as the N-body problem. It is believed that the motion of stars obeys the laws of classical mechanics and the interaction of stars is described by the Newtonian law of gravity. The paper considers the evolution of globular clusters of 500, 1000 and 14000 stars. It is shown that during the evolution the rate of evaporation of stars increases. It is also shown that the evaporation of a globular cluster of 500 stars takes at least 600 million years. The evaporation of a cluster of 14,000 stars requires already the cosmological time.

Key words. .

1. Introduction

Globular stellar cluster is a spherical collection of stars that orbits the Galaxy center, as a satellite. Other galaxies also include the globular clusters. Usually, the globular cluster consists of about 10^4 to 10^6 stars and the density in its center may reach $100-1000$ stars per cubic parsec. The nearest globular cluster M4 is located in Scorpius constellation at the distance of 7200 light years (2.2 kiloparsec), contains several tens of thousand stars (Marks & Kroupa 2010) and has the visual stellar magnitude 5.6^m.

The origin of the globular clusters and the details of their evolution are not absolutely clear yet. For example, it is unknown whether there is a black hole in the center of a globular cluster. It is only assumed. And if so, then is it really an intermediate mass black hole ($\sim 10^3 \div 10^5 M_\odot$)? To answer this question, it is necessary to carry out a numerical simulation of the internal dynamics (Meylan & Heggie 1997) of a globular cluster both with a black hole in its center and with its absence there.

Currently, it is also unclear whether there is an evolutionary connection between the dwarf galaxies and globular clusters. The assumption is that the globular cluster may be the innermost part of the dwarf elliptical galaxy, which has been earlier merged by a giant spiral galaxy. The numerical simulation of the internal dynamics of both objects could clarify this issue.

Numerical simulation of the dynamics of a globular star cluster has been carried out earlier (Spitzer & Harm 1958; Piet Hut & Djorgovski 1992; Joshi, Rasio & Portegiesi 2000; Takahashi, Portegies & Simon 2000; Ernsti et al. 2007; Komatsu, Kiwata & Kimura 2010; Trenti & van der Marel 2013; Rodriguez et al. 2016; Madrid et al. 2017; Ryabova et al. 2018). The formation of close binaries discussed in Zhou & Hua (1992) The dynamics of rotating clusters has been considered in (Fiestas, Spurzem & Kim 2007).The rotating globular clusters with primodial binaries has been considered in Chatterjee, Fregeau & Rasio (2009).

The dynamics of the globular cluster under certain assumptions is considered in the paper.

2. Equations of motion and computation technique

2.1. Equations of motion

We used the standard equations of motion for N-body problem:

$$\frac{d\mathbf{v}_i}{dt} = \sum_{j\neq i}^{N} Gm_j \cdot \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}, \tag{1}$$

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \tag{2}$$

where m_i, $\mathbf{v}_i$ and $\mathbf{r}_i$ are the mass, velocity and radius-vector of i-th body, respectively and G is the gravitational constant.

As it is well known, the system (1)-(2) can be solved analytically (explicitly) for two bodies only (Landau & Lifshitz (1976)). For the motion of three bodies, the periodic solutions can be obtained only for a set of special cases (Suvakov & Dmitrasinovic (2013)). For more number of bodies, the system (1)-(2) can be solved only by the numerical integration methods.

2.2. Initial distributions

We consider the initially virialized system, so that the modulus of its potential energy is twice the value of the kinetic energy. The full system energy is, certainly, negative.

The stars are considered to be uniformly distributed by mass in the range from $1M_\odot$ to $2M_\odot$, i.e. between $2\cdot 10^{30}$ kg and $4\cdot 10^{30}$ kg. When setting the initial conditions, the mass of each particular star is selected in the specified interval by a random number generator.

The distribution of stars along the radial coordinate has been set in two different ways. In the first case, the stars have been uniformly distributed (i.e., with a constant spatial density) inside a sphere of radius R, which, in turn, has been chosen so that the density of the stars would correspond to the one in the real globular cluster. In the second case, the density of stars increases towards the center of the cluster, so that in the center it is 5 times greater than at the edge of the cluster. The mean density in this case is also corresponds to the real globular cluster. After all the masses and coordinates have been fixed we can get the full gravitational potential energy

$$\Pi = -\frac{1}{2} \cdot \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{3}$$

of the cluster.

The velocity distribution of stars has been defined in two alternative ways too. In the first case, the components of the velocities have been set randomly from zero to some maximal value, and then adjusted so that the total kinetic energy of the stars satisfied the virial theorem. In the second case, the velocity components have also been random, but the radial part of each velocity has been set to zero, so that finally all the velocities were directed tangentially and satisfied the virial theorem.

We consider a globular cluster that does not initially rotate as a whole, so its total angular momentum should be close to zero. After setting the initial conditions according to the procedure described above, it really turned out that the total angular momentum of the cluster does not exceed twice the average value of the angular momentum of an individual star.

2.3. CUDA technology

In the computation process it is necessary to perform the same operations for each body. In regard with that, the idea of parallel computing arises. One possible option to realize that is to apply NVIDIA CUDA technology (CUDA homepage 2019).

The distribution process of the computation is determined by the architecture of video cards, which support the foregoing technology. Video cards contain a large number of multiprocessors, on which the threads are run parallel in order to perform general purpose computations.

From the programming point, one needs to write a program that will be executed on each thread. Then the developer must choose the size of the block of threads so that the video card resources are used as efficiently as possible. It is desirable that the maximum possible number of threads on a video card be an integer multiple of the number of threads in one block.

CUDA technology includes the software tools to optimize the interaction of different levels of video card memory and computer RAM. The synchronization of parallel processes is included in CUDA as well.

2.4. Laws of conservation

The conservation laws are not satisfied automatically and we need to apply a special procedure to count them. This procedure can be launched one time per approximately 10 time steps.

The conservation laws for energy and impulse are independent, so if we satisfied one of them it does not garantee that we satisfied the other. We will have to satisfy both, independently of each other.

To correct the total energy one needs to know the initial energy of the system and energy after several time steps. After that one can calculate the correction coefficient and multiply all velocities by this coefficient so that the total energy remains the same as it was at the initial time moment. Certainly, this correction coefficient is very close to unity. After that one should correct the total impulse. Initially we've constructed the system with zero total momentum. This means that the center of mass of the system must always remain at rest. In fact, after a few steps, the center of mass moves at a certain speed. To satisfy the law of momentum conservation we subtract this velocity from the velocities of all bodies, so that after subtracting the center of mass will remain at rest. However, the impulse correction slightly changes the energy of the system. Therefore, both correction procedures have to be applied alternately 2-3 times. Such a process turns out to be convergent.

As it turned out, the law of conservation of angular momentum does not require correction because it is satisfied automatically. The total angular momentum of the system, initially small, decreases at each time step and tends to zero.

2.5. Close passage and collision of stars

Close convergence and collision of stars determine many processes in the internal dynamics of a globular cluster. To take these processes into account we are to define the radii of the stars. We believe that the radii are uniformly distributed between $0.5R_\odot$ and $2R_\odot$, i.e approximately between $4{\cdot}10^5$ and $1.5{\cdot}10^6$ km. Each particular value is defined by the random number generator.

The close passage means that the distance between two stars first decreases and then increases, but during all the time of the motion the minimal distance between the centers of the pair is greater than the sum of their radii. We consider the stars as the solid balls with uniform density distribution inside, so the close passage can be counted by a standard integration technique (decreasing the integration step, for example). It also means that we do not consider any hydrodynamical effects, the change of the shape of the stars, the redistribution of mass inside them and so on,but we believe that the general form of cluster evolution remains about the same.

The collision of stars means that the distance between the centers of two stars became less than the sum of their radii. These stars are assumed to form the new star, whose mass and volume are equal to the sum of the masses and volumes of the collided stars, respectively. The volume of a new star defines its new radius, which can be used further to take correctly into account its close passage or collision with another star. The stars are considered to be the solid balls, so that the collision is represented as an absolutely inelastic interaction of two bodies. In that case the total impulse of the system is conserved, but the part of the mechanical energy being lost according to the standard equations:

$$(m_i + m_j)\,\mathbf{v} = m_i\mathbf{v}_i + m_j\mathbf{v}_j, \tag{4}$$

$$\frac{(m_i + m_j)\,\mathbf{v}^2}{2} = \frac{m_i\mathbf{v}_i^2}{2} + \frac{m_j\mathbf{v}_j^2}{2} - Q, \tag{5}$$

where Q is the energy loss.

We can consider only one collision event at a time step. More than one simultaneous events can be counted only at the consecutive time steps. Thus, we cannot directly consider two pairs of simultaneous collisions, triple collisions, collisions of type three plus two and very rare quadruple collisions. Those events really happen, but they are counted in a number of consecutive time steps, one collision per step.

2.6. Evaporation of stars

Some fast stars may leave globular cluster and this process should be counted. The star is considered to be evaporated if it has retired to a distance of three times the initial radius of the cluster and at the same time its total energy (potential plus kinetic) is positive.

The evaporated star will never come back to the cluster. It carries away positive energy. This means that the total energy of the remaining stars decreases and, therefore, the cluster is compressed, the stars become closer and the average velocity of the stars increases.

2.7. Time step selection

The time step should be chosen so that the movement of the fastest body is correctly taken into account. To do this, we first find the body with the greatest velocity v_{max}. Then we find two bodies, the distance l_{min} between which is minimal. When determining this distance, the amount of additional calculations will be small, since the distance matrix is recalculated at each step, and when one is to select the time step, this matrix is already known. One only needs to find the minimal element. The time interval during which the fastest body will cover the shortest distance is $\Delta t_0 = l_{min}/v_{max}$. In our computation, the time step is equal to one hundredth of this value: $\Delta t = \Delta t_0/100$.

3. Computation results

3.1. Evaporation process

The model evolution of globular cluster starts from the state when the system is already virialized. The total energy of 500 star system with uniform density distribution (the tangent velocities case is also included here) is approximately $E_{tot\ 1} \approx -7.5 \cdot 10^{38}\,\mathrm{J} = -7.5 \cdot 10^{45}\,\mathrm{erg}$. The case with higher density in the center has the total energy $E_{tot\ 2} \approx -9.3 \cdot 10^{38}\,\mathrm{J} = -9.3 \cdot 10^{45}\,\mathrm{erg}$. This difference is clear because the higher density in the central region implies the greater modulus of the gravitational potential energy.

The initial radius of the cluster is $R = 6 \cdot 10^{16}$ m ≈ 2 ps, so the mean density of the stars is close to the one in the real astronomical object.

A typical process of evaporation of stars during the globular cluster evolution is shown in Fig. 1. As it follows from the Figure, the process takes about 700 million years. At the beginning, the curve goes horizontally for a while and this is clear, since the first evaporated stars need time to move away from the center of the cluster by a distance of three radii. Subsequently the evaporation is accelerated over time. After half the population is evaporated this acceleration is so high that the right side of the curve goes almost vertical and at the selected scale we cannot reveal the details of the final stage of the process.

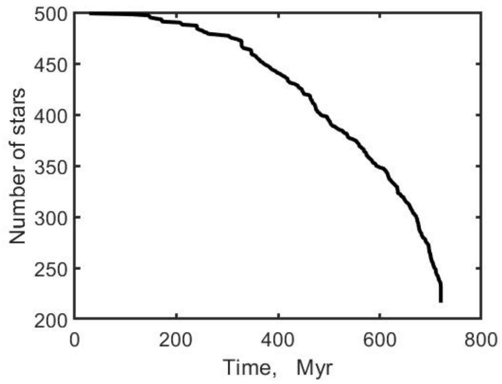

Fig. 1. Evaporation of stars in globular cluster. The cluster initially consists of 500 stars. The plot shows the number of stars remaining in the globular cluster in the process of its evolution. The horizontal axis indicates the time in the units of mega years.

This fact can also be understood from the general physics consideration. The evaporated star, as stated above, carries positive energy and is far from the center of the cluster. This means that the total energy of the remaining stars decreased in comparison to the original value. According to the virial theorem, this means that the remaining stars must now be in average closer to each other, and their average velocities should become greater. At higher energies and velocities, the processes go faster and, accordingly, the evaporation also occurs faster.

We can correctly track the evaporation of about half of the stars in the globular cluster. But, we can present the distribution of the stars along the radial coordinate just before the final moment. The fact is that the remaining half of the stars actually also "evaporated", but have not yet reached the distance of three cluster radius (they are still flying toward it), therefore this half is not taken into account as evaporated. Nevertheless, this moment approximately corresponds to the complete decay of the globular cluster.

Other evolutionary tracks for different initial conditions are presented in Fig. 2. As it follows from the Figure all the trajectories look very similar: they start from the horizontal part and then fall down with a gradually increased slope. The slope becomes almost vertical after evaporation of about half the stars.

The initial conditions, albeit weakly, affect the time of evolution of the system. The panels in Fig. 2 are marked with numbers that correspond to the different ways of setting the initial conditions: 1 – uniform distribution, arbitrary directions of velocities, 2 – uniform distribution, tangential velocities, and 3 – star density increases towards the center of the cluster, arbitrary directions of velocities. A cluster with uneven distribution of star density evolves a little faster, but in general the picture is similar for all clusters and the total evolutionary time is 700-800 million years, as it is presented in the Figure.

The typical evaporation of mass against the time coordinate is presented in Fig. 3. The horizontal axis represents the time in mega years and the vertical axis is the mass of the evaporated star in the units of Solar mass ($M_\odot = 1.99 \cdot 10^{30}$ kg $= 1.99 \cdot 10^{33}$ g). The linear trend is also presented on the chart. As in Figs. 1, 2, approxi-

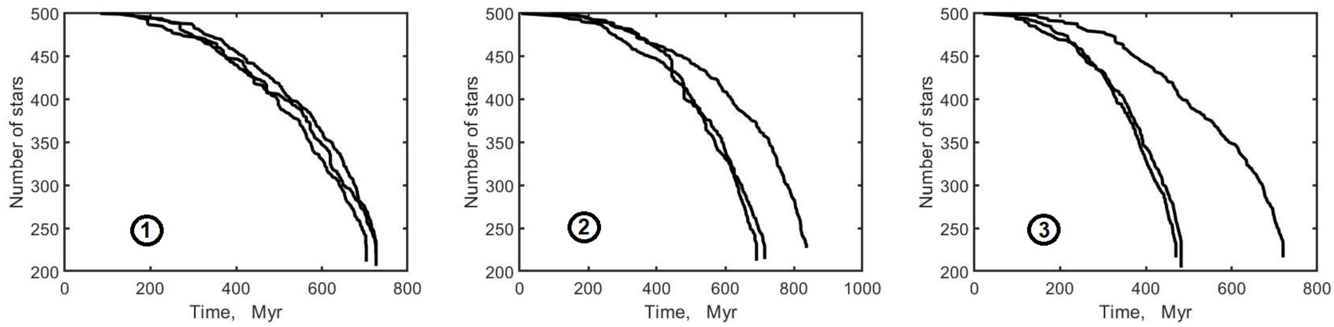

Fig. 2. Evaporation of stars in globular cluster. The panels show the number of stars remaining in the globular cluster in the process of its evolution. The horizontal axis indicates the time measured in megayears. The numbers 1, 2 and 3 refer to the cases of even distribution, tangent velocities and uneven distribution, respectively. The curve, shown in Fig. 1 is also presented for comparison.

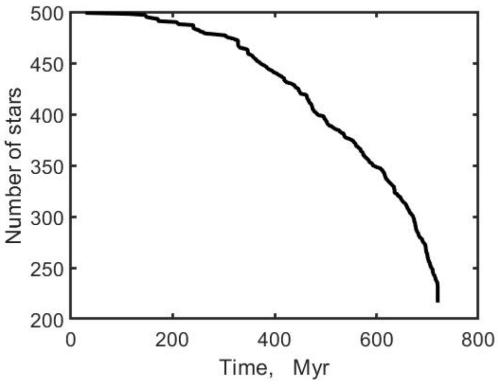

Fig. 3. Typical evaporation of mass against the time coordinate. The horizontal axis indicates the time in the units of megayears. The vertical axis represents the mass of the evaporated star in the units of Solar mass. The linear trend has a positive slope.

mately half the stellar population of the cluster is taken into account here.

Despite the large scatter of the points one can see even with the naked eye that this trend has a positive slope. In other words, this means that the light stars evaporate on average earlier than the heavy ones. And this is in agreement with the laws of general physics, more precisely thermodynamics. Since all stars in the equilibrium system have the same kinetic energy on average, the lighter stars must also have the higher velocities. Therefore, these stars have the greater probability to be evaporated, since the second cosmic velocity does not depend on the mass of the star.

Three more examples of mass evaporation for various initial conditions are presented in Fig. 4. The numbers 1, 2 and 3 in the panels have the same sense as in Fig. 2

Note that among the evaporated stars there are also those whose mass exceeds $2M_\odot$. Such stars can appear only as a result of the merge of several stars of lower mass, but these stars can also leave the cluster. Usually it happens at the final stage of evolution.

It is also noticeable from Figs. 3-4, that the frequency of star evaporation increases with time.

The evolution track and evaporation of mass for 1000-star cluster with even initial distribution of stars is shown on the left panel in Fig. 5. The initial distribution density of stars is set the same as in the previous case. The right panel demonstrates the evaporation of mass along with a linear trend. The Figure in general looks very similar to Figs. 1–4 for 500-star cluster except the evolution time, which increased here to 1.1 billion year.

The first 120 million years of evolution for the cluster of 14000 stars with even initial distribution of density is shown in Fig. 6. The density of stars is chosen the same as in previous cases. The left panel represents the evaporation of stars. This part of the curve can be approximated by a straight line with negative slope. This case consumes a lot of computational resources (even with CUDA technology), but taking into account the general similarity of evolutionary curves, we can conclude that it takes at least 6 billion years to evaporate such a cluster, and this time is comparable to the age of the Universe.

The right panel in Fig. 6 presents a general view of the cluster (its ''photograph'') after 120 million years of evolution. The star in the top left corner has not yet evaporated, but it is very close to this. The evaporated stars are outside of the view field.

3.2. Radial density distribution

In the process of globular cluster evaporation, the distribution of stars along the radial coordinate changes. The initial distributions have already been discussed earlier.

The typical distribution of stars along the radial coordinate for even initial distribution at the different moments of evolution is shown on the left panel in Fig. 7. The dashed curve refers to the initial time moment. The solid curve with dots marks the middle of evolution and the solid curve with crosses marks the final stage of evolution. The distribution of spatial density presented on the right panel in Fig. 7, where the same notation is used. The radial distance on both panels is expressed in the units of the initial radius of the cluster.

As it is clearly visible on the right panel, the uniform distribution of the spatial density of the stars actually experiences the significant statistical fluctuations. If there were no these fluctuations, then the distribution would have looked like a horizontal segment. The radial density in this case would be described by a quadratic function. It is for these reasons that the uniform distribution is the most interesting for the results demonstration.

As it is clear from the Figure, the radial distribution has a maximum, but its position depends on the evolution time. At the beginning of evolution, the maximum shifts

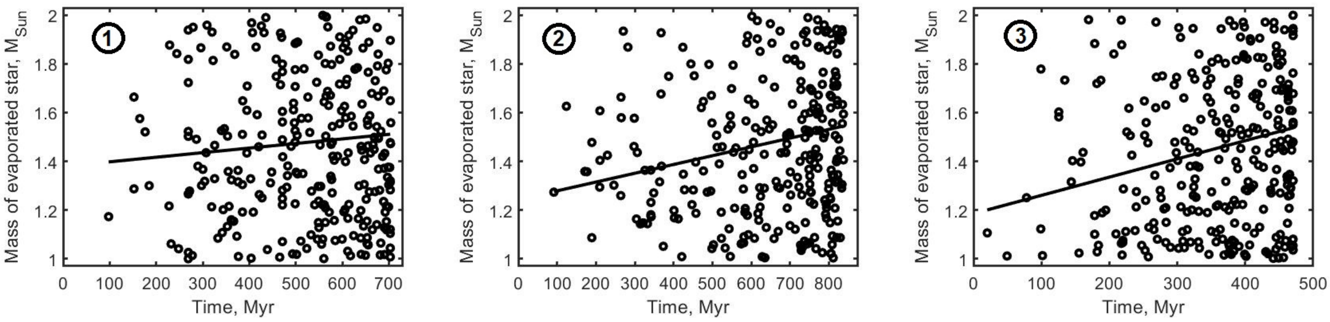

Fig. 4. Evaporation of mass in globular cluster. The panels show the mass of evaporated star against the time axis along with a linear trend. The horizontal axis indicates the time measured in megayears. The numbers 1, 2 and 3 refer to the cases of even distribution, tangent velocities and uneven distribution, respectively.

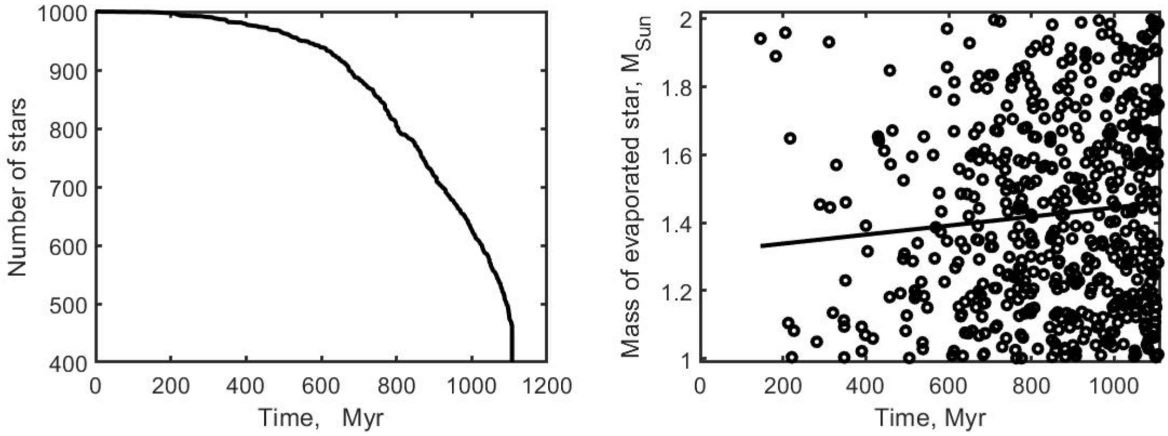

Fig. 5. The left panel presents the evaporation of 1000-star globular cluster. The right panel shows the evaporation of mass in the same cluster. The horizontal axis indicates the time in megayears.

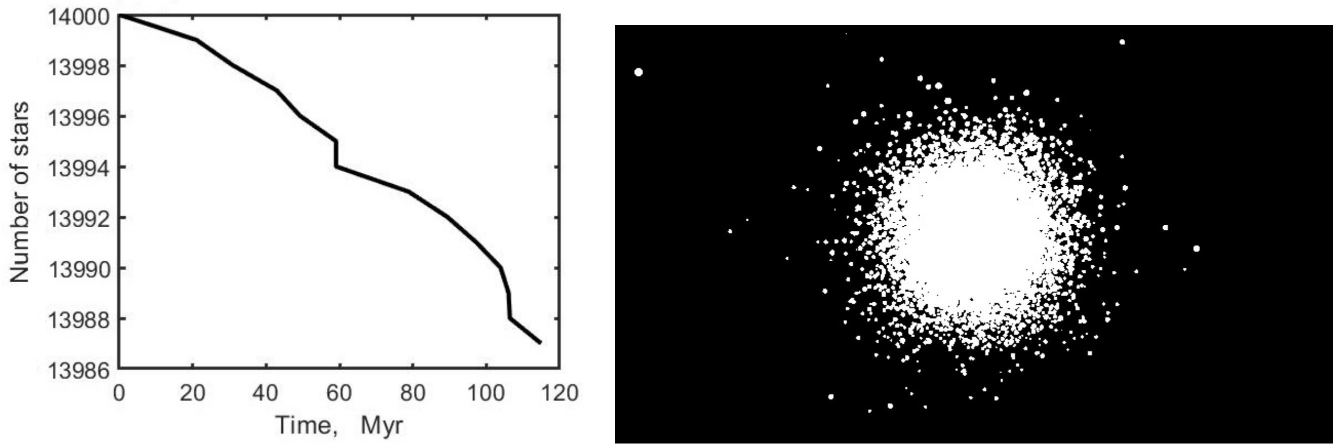

Fig. 6. The left panel presents the evaporation of 14000-star globular cluster during 120 million years. The horizontal axis indicates the time in megayears. The right panel a general view of the cluster at the same moment.

toward the center of the cluster, then returns to its previous state (this moment is not shown in Fig. 7), and then it is shifted even further from the center. This shift occurs due to the fact that the stars begin to evaporate and, therefore, their average distance from the center increases.The same statement is also true for the spatial density distribution of stars except the initial moment.

Similar distributions for other initial conditions (i.e. not only for constant density of stars) basically do not differ from those shown in Fig. 7. With increasing the time, the maximum density of stars also moves away from the center of the cluster. The same conclusion remains valid for the clusters of 1000 and 14000 stars, regardless of the initial conditions.

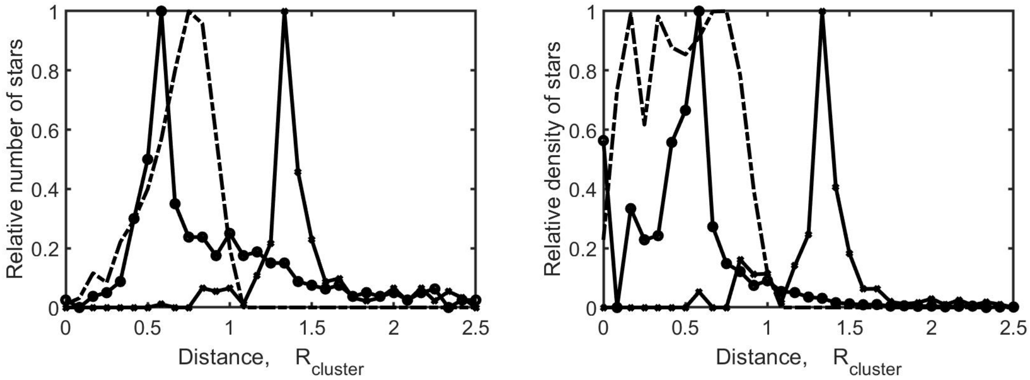

Fig. 7. Typical distribution of stars in the process of globular cluster evolution. The left panel demonstrates distribution of stars along the radial coordinate. The right panel represents distribution of the density of stars along the radial coordinate. The distance is expressed in the units of initial radius of a cluster. The distribution is indicated in relative units.

4. Discussion and conclusions

The main conclusion that follows from the computation is that the evaporation of a globular cluster, moving only according to the law of Newtonian gravity, takes a long time. The evolution of a cluster of 500 stars takes about 700 million years, and the evaporation of a cluster of 14000 stars requires time comparable to the cosmological one.

The time of evaporation of a globular cluster is not a linear function of the number of stars in it. However, taking this into account and extrapolating the results obtained, we can conclude that the evaporation time of the cluster, consisting of 140-160 thousand stars, will be 12-15 billion years. This time coincides with the age of the Universe.

Numerical simulation also confirms the fact that light stars are more likely to evaporate than the heavy ones. All the trends in Figs. 3-5 demonstrate the positive slope.

As another task it would be interesting to trace the energy of evaporating stars and its dependence on time. From general physical considerations it is clear that this energy should increase over time, but the problem requires numerical simulation.

As a generalization, it would be interesting to consider a problem where not only gravitational forces are taken into account.

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