

# STAT 435 HW1

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1.

a)

Taking a parametric approach will have following pros:

- Does not need a lot of data
- Simplifies the problem because it is generally much easier to estimate a set of parameters and cons:
- The model we choose will usually not match the true unknown form of  $f$ , and if the chosen model is too far from the true  $f$ , then our estimate will be poor.

Taking a nonparametric approach will have following pros:

- Avoid unnecessary assumptions about the functional form of  $f$  will have the potential to accurately fit a wider range of possible shapes for  $f$ .

and cons:

- A very large number of observations is required in order to obtain an accurate estimate for  $f$ .

b)

For parametric approach, I would say when we have a small number of observations to work with, such as getting survey on people's blood pressure and hours of physical exercises they do each week. And we know that having more time to exercises will result in a lower blood pressure as a matter of fact. Hence we can make assumptions to  $f$  in this case to be a linear model:

$$\text{blood pressure} \approx \beta_0 + \beta_1 \times \text{physical exercises}$$

c)

For non-parametric approach, I would say when we have a lot of data to work with, we can use this method to do the same prediction as part b).

2.

a)

In this case, I would expect the inflexible methods to perform better. Since sample size is small, there won't be enough data for flexible methods such as deep learning and etc. Also, the number of predictors are large, hence it would be a good practice to use OLS so that we have more interpretability. Hence inflexible methods tends to perform better.

b)

In this case, I would expect the inflexible method to perform better. Since the

The risk of over-fitting is very high and any patterns picked up by flexible methods are more likely to be mere noise. Therefore an inflexible method should be preferred.

c)

Inflexible methods have a lot of trouble picking up non-linear relationships, so we should prefer a flexible method.

d)

In this case, I would expect inflexible method to perform better. Higher variance will tend to introduce noise. The high variance of error terms means that the sample will have a lot of noise in the relationship. Therefore we should prefer an inflexible method that is less likely to over-fit to this noise.

3.

a)

It is a regression problem. And the goal is prediction, where  $n = 50, p = 8$ .

b)

It is a classification problem. And the goal is inference, where  $n = 50, p = 6$ .

4.

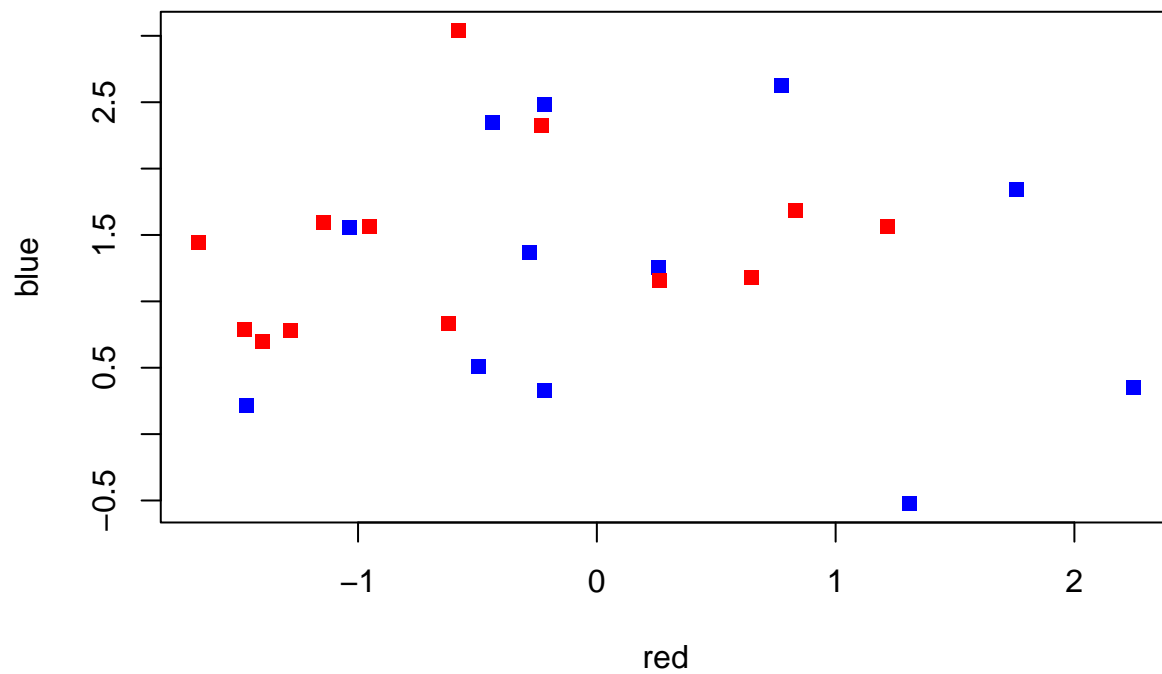
a)

5.

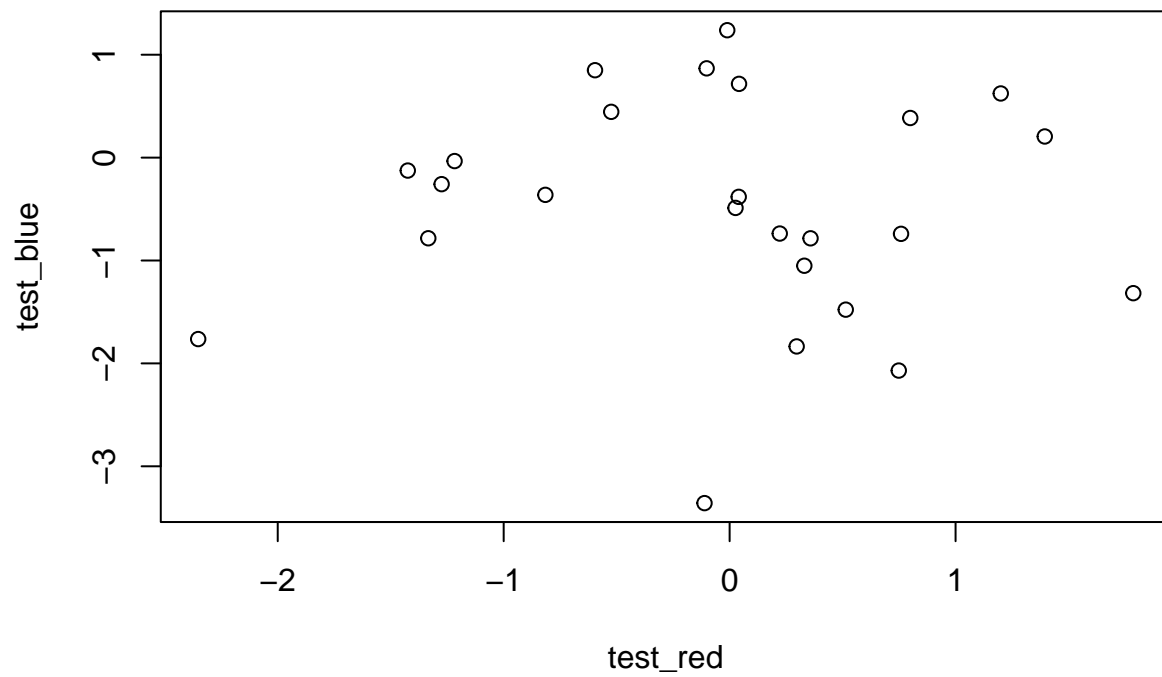
a)

```
n <- 25
red <- rnorm(n, 0, 1)
blue <- rnorm(n, 1.5, 1)
df_train <- data.frame("red" = red, "blue" = blue)

plot(df_train,
     pch = 15,
     col = c("red", "blue"))
```



```
# Generating test set  
test_red <- rnorm(n, 0, 1)  
test_blue <- rnorm(n, 0, 1.5)  
df_test <- data.frame("test_red" = test_red, "test_blue" = test_blue)  
  
plot(df_test)
```



6.

a)

7.

```
library(ISLR2)
```

a)

```
data <- Boston  
head(data)
```

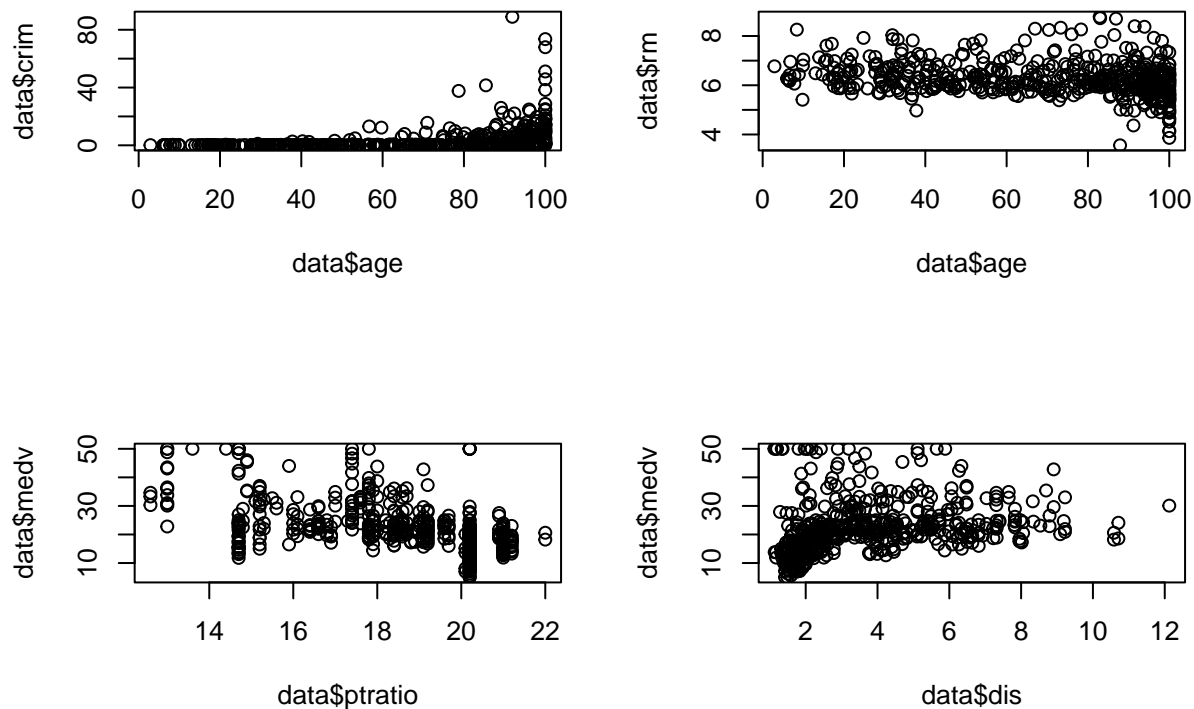
```
##      crim zn  indus chas   nox    rm  age    dis rad tax ptratio lstat medv  
## 1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900  1 296    15.3  4.98 24.0  
## 2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671  2 242    17.8  9.14 21.6  
## 3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671  2 242    17.8  4.03 34.7  
## 4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622  3 222    18.7  2.94 33.4  
## 5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622  3 222    18.7  5.33 36.2  
## 6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622  3 222    18.7  5.21 28.7
```

```
# Get number of rows  
row_number <- nrow(data)  
  
# Get number of columns  
col_number <- ncol(data)
```

And there are 13 columns, and 506 rows. Number of rows represent the number of observations in our dataset. And number of columns represent the number of features/predictors each observations have we have.

b)

```
par(mfrow = c(2,2))  
plot(data$age, data$crim) # check and see if crime rate is related to age of the units build prior to 1940  
plot(data$age, data$rm) # check if number of rooms is related to age of the units build prior to 1940  
plot(data$ptratio, data$medv) # check if the number of students per teacher has an effect on median value of property  
plot(data$dis, data$medv) # check if the distance to employment centers has an effect on median value of property
```



My

findings are:

- newly built units tends to have lower per capita crime rate by town
- newly build units tend to have more rooms compare to old units
- the higher student per teacher ratio, the lower their median house prices tend to be
- People lives close to the business center of Boston tend to have lower median house prices

c.

From my finding in part b. I believe that there's relationship between proportion of owner-occupied units built prior to 1940 and per capita crime rate.

And the relationship is exponential, as the proportion of old houses grow, the number of per capita crime rate grows exponentially.

d.

e.

```
sum(data$chas)
```

```
## [1] 35
```

35 suburbs in this data set bound the Charles river

f.

```
mean(data$ptratio)
```

```
## [1] 18.45553
```

```
sd(data$ptratio)
```

```
## [1] 2.164946
```

Mean of pupil-teacher ratio is 18.45553 and standard deviation of pupil-teacher ratio is 2.164946.

g.

```
highest_medv <- sqldf("SELECT * FROM data
                       WHERE medv =(SELECT MAX(medv) FROM data)")
highest_medv
```

| ##    | crim    | zn | indus | chas | nox    | rm    | age   | dis    | rad | tax | ptratio | lstat | medv |
|-------|---------|----|-------|------|--------|-------|-------|--------|-----|-----|---------|-------|------|
| ## 1  | 1.46336 | 0  | 19.58 | 0    | 0.6050 | 7.489 | 90.8  | 1.9709 | 5   | 403 | 14.7    | 1.73  | 50   |
| ## 2  | 1.83377 | 0  | 19.58 | 1    | 0.6050 | 7.802 | 98.2  | 2.0407 | 5   | 403 | 14.7    | 1.92  | 50   |
| ## 3  | 1.51902 | 0  | 19.58 | 1    | 0.6050 | 8.375 | 93.9  | 2.1620 | 5   | 403 | 14.7    | 3.32  | 50   |
| ## 4  | 2.01019 | 0  | 19.58 | 0    | 0.6050 | 7.929 | 96.2  | 2.0459 | 5   | 403 | 14.7    | 3.70  | 50   |
| ## 5  | 0.05602 | 0  | 2.46  | 0    | 0.4880 | 7.831 | 53.6  | 3.1992 | 3   | 193 | 17.8    | 4.45  | 50   |
| ## 6  | 0.01381 | 80 | 0.46  | 0    | 0.4220 | 7.875 | 32.0  | 5.6484 | 4   | 255 | 14.4    | 2.97  | 50   |
| ## 7  | 0.02009 | 95 | 2.68  | 0    | 0.4161 | 8.034 | 31.9  | 5.1180 | 4   | 224 | 14.7    | 2.88  | 50   |
| ## 8  | 0.52693 | 0  | 6.20  | 0    | 0.5040 | 8.725 | 83.0  | 2.8944 | 8   | 307 | 17.4    | 4.63  | 50   |
| ## 9  | 0.61154 | 20 | 3.97  | 0    | 0.6470 | 8.704 | 86.9  | 1.8010 | 5   | 264 | 13.0    | 5.12  | 50   |
| ## 10 | 0.57834 | 20 | 3.97  | 0    | 0.5750 | 8.297 | 67.0  | 2.4216 | 5   | 264 | 13.0    | 7.44  | 50   |
| ## 11 | 0.01501 | 90 | 1.21  | 1    | 0.4010 | 7.923 | 24.8  | 5.8850 | 1   | 198 | 13.6    | 3.16  | 50   |
| ## 12 | 4.89822 | 0  | 18.10 | 0    | 0.6310 | 4.970 | 100.0 | 1.3325 | 24  | 666 | 20.2    | 3.26  | 50   |
| ## 13 | 5.66998 | 0  | 18.10 | 1    | 0.6310 | 6.683 | 96.8  | 1.3567 | 24  | 666 | 20.2    | 3.73  | 50   |
| ## 14 | 6.53876 | 0  | 18.10 | 1    | 0.6310 | 7.016 | 97.5  | 1.2024 | 24  | 666 | 20.2    | 2.96  | 50   |
| ## 15 | 9.23230 | 0  | 18.10 | 0    | 0.6310 | 6.216 | 100.0 | 1.1691 | 24  | 666 | 20.2    | 9.53  | 50   |
| ## 16 | 8.26725 | 0  | 18.10 | 1    | 0.6680 | 5.875 | 89.6  | 1.1296 | 24  | 666 | 20.2    | 8.88  | 50   |

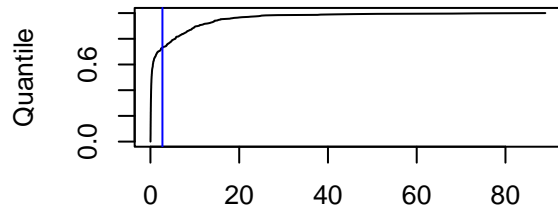
```
n <- row_number
par(mfrow = c(2,2))
plot(sort(data$crim), (1:n - 1)/(n - 1), type="l",
     xlab = "Per capita crime rate by town",
     ylab = "Quantile")
abline(v=mean(highest_medv$crim), col="blue")

plot(sort(data$zn), (1:n - 1)/(n - 1), type="l",
     xlab = "proportion of residential land zoned for lots over 25,000 sq.ft",
     ylab = "Quantile")
abline(v=mean(highest_medv$zn), col="blue")

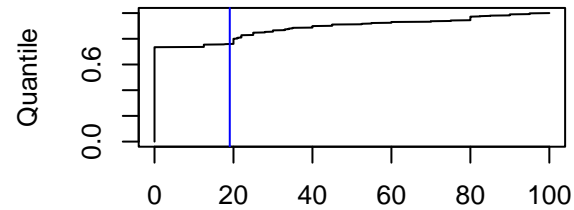
plot(sort(data$indus), (1:n - 1)/(n - 1), type="l",
```

```
xlab = "proportion of non-retail business acres per town",
ylab = "Quantile")
abline(v=mean(highest_medv$indus), col="blue")
```

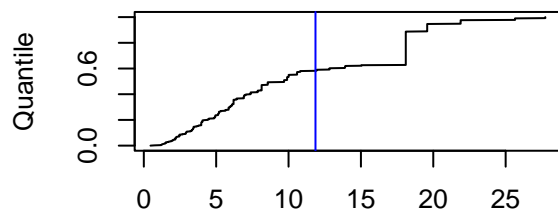
```
plot(sort(data$chas), (1:n - 1)/(n - 1), type="l",
xlab = "Charles River dummy variable",
ylab = "Quantile")
abline(v=mean(highest_medv$chas), col="blue")
```



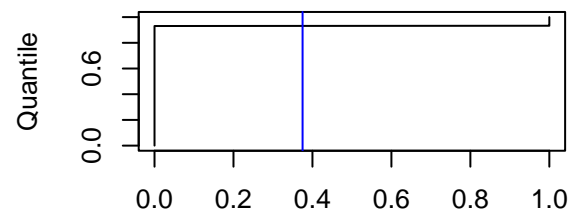
Per capita crime rate by town



proportion of residential land zoned for lots over 25,00



proportion of non-retail business acres per town



Charles River dummy variable

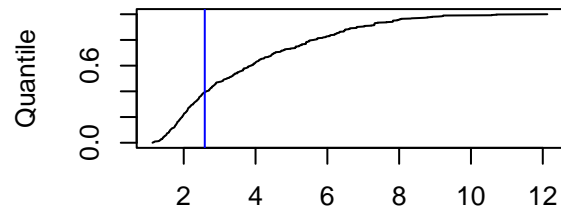
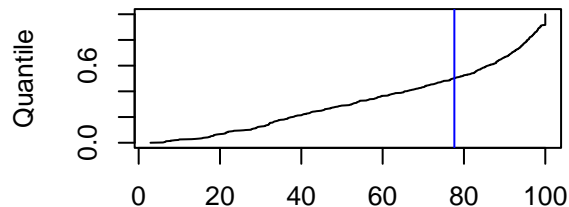
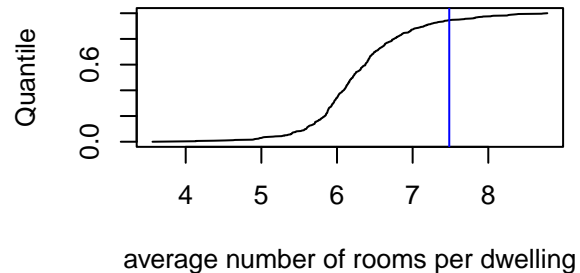
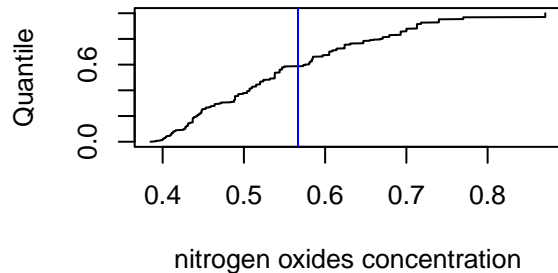
```
par(mfrow = c(2,2))
```

```
plot(sort(data$nox), (1:n - 1)/(n - 1), type="l",
xlab = "nitrogen oxides concentration",
ylab = "Quantile")
abline(v=mean(highest_medv$nox), col="blue")
```

```
plot(sort(data$rm), (1:n - 1)/(n - 1), type="l",
xlab = "average number of rooms per dwelling",
ylab = "Quantile")
abline(v=mean(highest_medv$rm), col="blue")
```

```
plot(sort(data$age), (1:n - 1)/(n - 1), type="l",
xlab = "proportion of owner-occupied units built prior to 1940",
ylab = "Quantile")
abline(v=mean(highest_medv$age), col="blue")
```

```
plot(sort(data$dis), (1:n - 1)/(n - 1), type="l",
     xlab = "weighted mean of distances to five Boston employment centres",
     ylab = "Quantile")
abline(v=mean(highest_medv$dis), col="blue")
```



proportion of owner-occupied units built prior to 1980

```
par(mfrow = c(2,2))

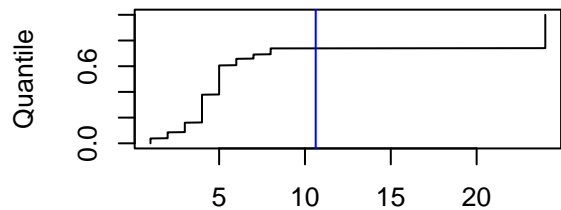
plot(sort(data$rad), (1:n - 1)/(n - 1), type="l",
     xlab = "index of accessibility to radial highways",
     ylab = "Quantile")
abline(v=mean(highest_medv$rad), col="blue")

plot(sort(data$tax), (1:n - 1)/(n - 1), type="l",
     xlab = "full-value property-tax rate per $10,000",
     ylab = "Quantile")
abline(v=mean(highest_medv$tax), col="blue")

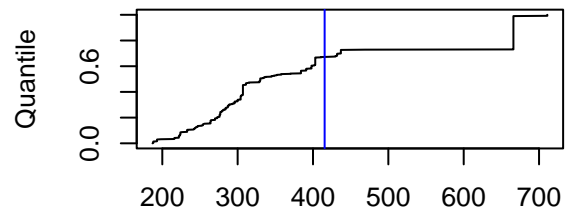
plot(sort(data$prratio), (1:n - 1)/(n - 1), type="l",
     xlab = "pupil-teacher ratio by town",
     ylab = "Quantile")
abline(v=mean(highest_medv$prratio), col="blue")

plot(sort(data$lstat), (1:n - 1)/(n - 1), type="l",
     xlab = "lower status of the population",
     ylab = "Quantile")
abline(v=mean(highest_medv$lstat), col="blue")
```

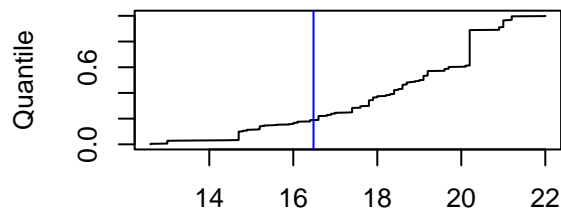




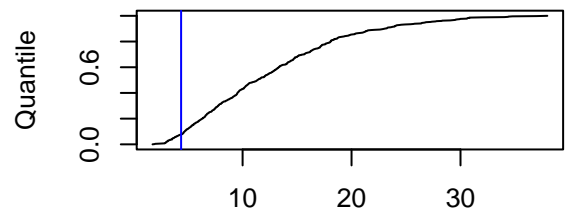
index of accessibility to radial highways



full-value property-tax rate per \$10,000



pupil-teacher ratio by town



lower status of the population

```
summary(Boston)
```

```
##      crim              zn          indus          chas
## Min.   : 0.00632   Min.   : 0.00   Min.   : 0.46   Min.   :0.00000
## 1st Qu.: 0.08205   1st Qu.: 0.00   1st Qu.: 5.19   1st Qu.:0.00000
## Median : 0.25651   Median : 0.00   Median : 9.69   Median :0.00000
## Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917
## 3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000
## Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000
##      nox              rm          age          dis
## Min.   :0.3850   Min.   :3.561   Min.   : 2.90   Min.   : 1.130
## 1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100
## Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207
## Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795
## 3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188
## Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127
##      rad              tax          ptratio          lstat
## Min.   : 1.000   Min.   :187.0   Min.   :12.60   Min.   : 1.73
## 1st Qu.: 4.000   1st Qu.:279.0   1st Qu.:17.40   1st Qu.: 6.95
## Median : 5.000   Median :330.0   Median :19.05   Median :11.36
## Mean   : 9.549   Mean   :408.2   Mean   :18.46   Mean   :12.65
## 3rd Qu.:24.000   3rd Qu.:666.0   3rd Qu.:20.20   3rd Qu.:16.95
## Max.   :24.000   Max.   :711.0   Max.   :22.00   Max.   :37.97
##      medv
## Min.   : 5.00
## 1st Qu.:17.02
## Median :21.20
## Mean   :22.53
## 3rd Qu.:25.00
## Max.   :50.00
```

As we can see from above quantile graphs, we can see that suburbs with highest median value of owner-occupied homes(in blue line) tends to:

- have low per capita crime rate by town
- have smaller proportion of residential land
- at 50th percentile for proportion of non-retail business acres per town
- at around 50th percentile for nitrogen oxides concentration
- have more rooms
- at around 50th percentile for proportion of owner-occupied units built prior to 1940
- closer to five Boston employment centers
- between median and 3rd quantile in terms of accessibility to radial highways
- at around 50th percentile for full-value property-tax rate per \$10,000.
- have smaller pupil-teacher ratio by town
- have smaller percent of lower status of the population

h.

```
sqldf("SELECT COUNT(*) AS more_than_six FROM Boston
      WHERE rm > 6")
```

```
##  more_than_six
## 1             333
```

```
sqldf("SELECT * FROM Boston
      WHERE rm > 8")
```

```
##      crim zn indus chas      nox      rm age      dis rad tax ptratio lstat medv
## 1  0.12083  0  2.89    0 0.4450 8.069 76.0 3.4952  2 276    18.0  4.21 38.7
## 2  1.51902  0 19.58    1 0.6050 8.375 93.9 2.1620  5 403    14.7  3.32 50.0
## 3  0.02009 95  2.68    0 0.4161 8.034 31.9 5.1180  4 224    14.7  2.88 50.0
## 4  0.31533  0  6.20    0 0.5040 8.266 78.3 2.8944  8 307    17.4  4.14 44.8
## 5  0.52693  0  6.20    0 0.5040 8.725 83.0 2.8944  8 307    17.4  4.63 50.0
## 6  0.38214  0  6.20    0 0.5040 8.040 86.5 3.2157  8 307    17.4  3.13 37.6
## 7  0.57529  0  6.20    0 0.5070 8.337 73.3 3.8384  8 307    17.4  2.47 41.7
## 8  0.33147  0  6.20    0 0.5070 8.247 70.4 3.6519  8 307    17.4  3.95 48.3
## 9  0.36894 22  5.86    0 0.4310 8.259  8.4 8.9067  7 330    19.1  3.54 42.8
## 10 0.61154 20  3.97    0 0.6470 8.704 86.9 1.8010  5 264    13.0  5.12 50.0
## 11 0.52014 20  3.97    0 0.6470 8.398 91.5 2.2885  5 264    13.0  5.91 48.8
## 12 0.57834 20  3.97    0 0.5750 8.297 67.0 2.4216  5 264    13.0  7.44 50.0
## 13 3.47428  0 18.10    1 0.7180 8.780 82.9 1.9047 24 666    20.2  5.29 21.9
```

It tends to have lower crime rate.