Computer Vision Systems Programming VO

Models vs. Algorithms

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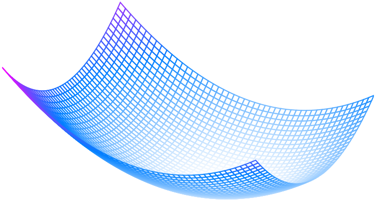
Topics

Models vs. algorithms

Approaching computer vision problems

Numerical optimization



Solving CV Problems

CV is about infering some world state $\mathbf{w}$ from measurements $\mathbf{x}$

$\mathbf{x}$ is a feature vector extracted from images

$\mathbf{w} \in \mathbb{R}^m$: **regression** problem (e.g. stereo)

$\mathbf{w} \in \mathbb{N}^m$: **classification** problem (e.g. face recognition)

Solving CV Problems
How Not To Do It

Don't think in terms of **algorithms**

- First I'll convert my image to grayscale
- Then I'll threshold it at some value
- Then I'll clean up holes via morphology

Such solutions

- Rely on many magic numbers that all interact
- Do not encode uncertainty about $\mathbf{w}$

Solving CV Problems

How To Do It

Think about how your data came into being

Use this information to **model** the relationship between $\mathbf{x}$ and $\mathbf{w}$

Ideally use probabilistic models as inference is ill-posed

“Say **what** you want to do, not **how** you are going to to it”
$\sim$ A. Fitzgibbon

On this basis, CV is about

- Defining a **model** that relates $\mathbf{x}$ and $\mathbf{w}$
- **Learning** its parameters $\boldsymbol{\theta}$ from **training samples**
- Using an **inference algorithm** that returns $\Pr(\mathbf{w}|\mathbf{x}, \boldsymbol{\theta})$

Solving CV Problems
Example: Background Subtraction

We want to segment images in foreground and background

- Based on the perceived brightness of objects
- In scenes with a constant background
- On a per-pixel basis

Image from Prince 2012

Therefore we have $w \in \{0, 1\}$ (classification problem)

Given our problem, we model

$$\begin{aligned}
\Pr(x|w=0) &= \underset{x}{\mathrm{Norm}}(\mu, \sigma^2) \\
\Pr(x|w=1) &= \underset{x}{\mathrm{Uniform}}(0, 255) \\
\Pr(w) &= \underset{w}{\mathrm{Bern}}(\lambda)
\end{aligned}$$

We assume $\lambda = 0.5$

And learn $\boldsymbol{\theta} = (\mu, \sigma)$ from training samples $\{x_i\}_{i=1}^n$

- Using the maximum likelihood method
- Assuming that the training samples are independent

We thus learn

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg\max_{\boldsymbol{\theta}} \Pr(x_1, \ldots, x_n | \boldsymbol{\theta}) \\ &= \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{n} \Pr(x_i | \boldsymbol{\theta}) \end{aligned}$$

Continuing from before

$$
\begin{aligned}
\hat{\boldsymbol{\theta}} &= \underset{\boldsymbol{\theta}}{\arg\max} \prod_{i=1}^{n} \underset{x_i}{\mathrm{Norm}}(\mu, \sigma^2) \\
&= \underset{\boldsymbol{\theta}}{\arg\max} \sum_{i=1}^{n} \log \underset{x_i}{\mathrm{Norm}}(\mu, \sigma^2) \\
&= \underset{\boldsymbol{\theta}}{\arg\max} \left(-0.5n \log \sigma^2 - 0.5n \log(2\pi) - 0.5 \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2} \right)
\end{aligned}
$$

If we differentiate and equate to zero, we obtain

$$\begin{aligned}\hat{\mu} &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ \hat{\sigma}^2 &= \frac{1}{n}\sum_{i=1}^{n} (x_i - \hat{\mu})^2\end{aligned}$$

We have derived **algorithms** for finding our model parameters

Models vs. Algorithms

Models describe the relationship between $\mathbf{x}$ and $\mathbf{w}$

Algorithms are recipes for finding $\boldsymbol{\theta}$

Model	Algorithm
Gaussian	Mean & variance
Linear models*	Linear least squares
Structure from motion*	Bundle adjustment
Rigid motion between point sets*	ICP

* Assuming zero-mean Gaussian noise

Inference

In the example case we have modeled $\Pr(\mathbf{x}|\mathbf{w})$

- Such models are called **generative**
- In this case, we perform inference using Bayes' rule

$$\Pr(\mathbf{w}|\mathbf{x}) = \frac{\Pr(\mathbf{x}|\mathbf{w})\Pr(\mathbf{w})}{\Pr(\mathbf{x})}$$

We can also model $\Pr(\mathbf{w}|\mathbf{x})$ directly

- Such models are called **discriminative**
- In this case, we can perform inference directly

Assume $x = 100, \mu = 95, \sigma = 2$

In this case, we obtain

- $\Pr(w = 0) = \Pr(w = 1) = 0.5$
- $\Pr(x = 100|w = 0) = \mathrm{Norm}_{100}(90, 2^2) \approx 0.009$
- $\Pr(x = 100) = \sum_{i \in \{0,1\}} \Pr(x = 100|w = i)\Pr(w = i) \approx .007$
- $\Pr(w|x = 100) \approx (0.69, 0.31)$

Therefore we classify the pixel as background

Complex Models

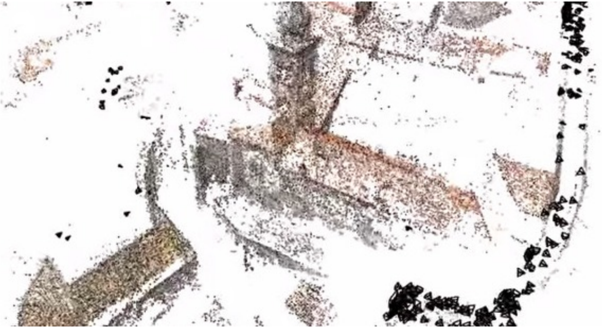


Image from https://www.youtube.com/watch?v=sQegEro5Bfo

Optimization

Learning $\boldsymbol{\theta}$ is an **optimization problem**

- We seek $\hat{\boldsymbol{\theta}}$ that maximizes some objective function $f(\boldsymbol{\theta})$
- E.g. before we sought $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \Pr(x_1, \ldots, x_n | \boldsymbol{\theta})$

There are algorithms available for finding $\boldsymbol{\theta}$ of many models

- But this is not always the case

Therefore, a brief intro on optimization ...

Optimization

We seek $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} f(\boldsymbol{\theta}) = \arg\min_{\boldsymbol{\theta}} -f(\boldsymbol{\theta})$

- Sometimes $\hat{\boldsymbol{\theta}}$ can be found analytically (like before)
- Often we have to resort to iterative methods

Iterative methods

- Start with an estimate $\boldsymbol{\theta}^{[0]}$
- Improve this estimate iteratively using local information
- Until no more improvement can be made

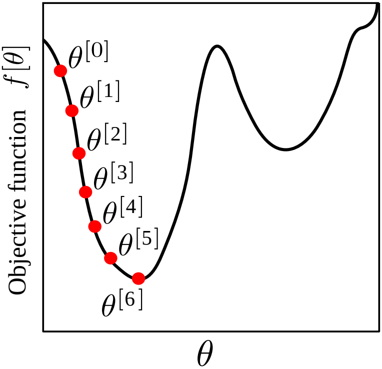


Image adapted from Prince 2012

This means we need a good initial estimate $\boldsymbol{\theta}^{[0]}$

- Otherwise we might get stuck in a **local minimum**
- Unless f is **convex**

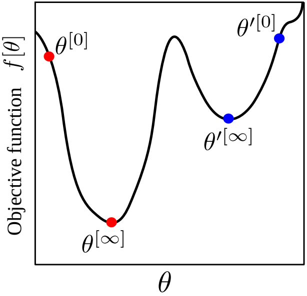


Image from Prince 2012

Iterative methods work by alternatively

- Choosing a search direction using local information
- Searching the minimum along that direction in some interval
- Updating $\boldsymbol{\theta}$ according to the found minimum

Local information from first and second order derivatives

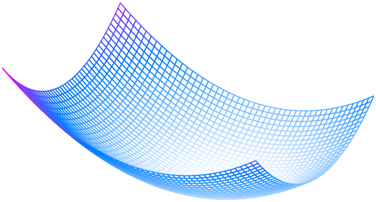
Many different optimization algorithms

- Steepest decent, Newton, Levenberg-Marquardt, BFGS ...
- Some can be tested at `https://awful.codeplex.com/`

Optimization

Iterative Methods – Simple Example

We seek the minimum of $f(\boldsymbol{\theta}) = (\theta_1 - 2)^2 + (\theta_2 - 1)^2$



Finding the minimum with Python and SciPy

```
def f(x):
  return np.power(x[0]-2,2) + np.power(x[1]-1,2)

scipy.optimize.minimize(f, [5, 5]) # returns [2, 1]
```

Finding the minimum with Matlab

```
fminunc(@(x)(x(1)-2)^2 + (x(2)-1)^2, [5, 5]) % returns [2, 1]
```

Summary

We have seen how (not) to approach CV problems

- See Prince 2012 for more information on CV models

Optimization is fundamental in vision problems

- We briefly discussed why, and how optimization works

Literature

Books for further information on these topics

- Prince 2012 (`http://computervisionmodels.com/`)
- Bishop et al. 2006
- Boyd and Vandenberghe 2009
- Wright and Nocedal 1999

Bibliography

Bishop, Christopher et al. (2006). **Pattern Recognition and Machine Learning**. Springer.

Boyd, Stephen and Lieven Vandenberghe (2009). **Convex Optimization**. Cambridge University Press.

Prince, S.J.D. (2012). **Computer Vision: Models Learning and Inference**. Cambridge University Press.

Wright, SJ and J Nocedal (1999). **Numerical optimization**. Springer.