A Soft Introduction to Deep Learning

Deep Ray

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- You will NOT be able to design a self-driving car
- You will NOT be able to solve PDEs using Al
- You will NOT be able to predict the stock-market

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- You will understand the fundamentals of training networks
- You will get the courage to experiment and explore
- You might get ideas to use Deep Learning solve your problems















































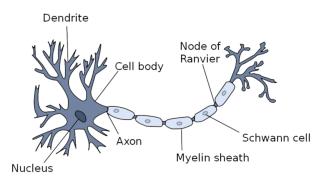


How did we get here?

1940 1950 1960 1970 1980 1990 2000 2010

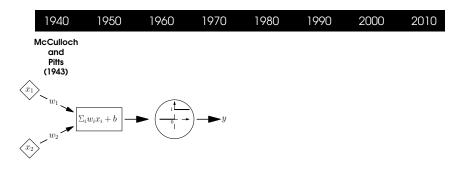
McCulloch and Pitts (1943)

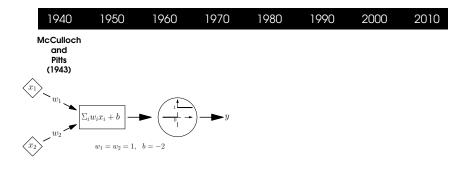
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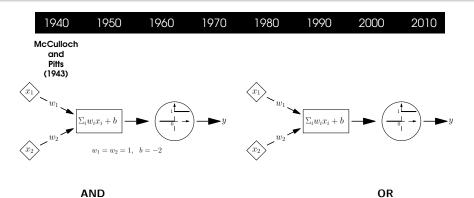
Biological neuron

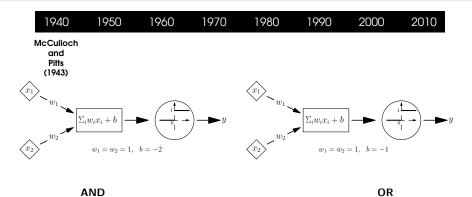
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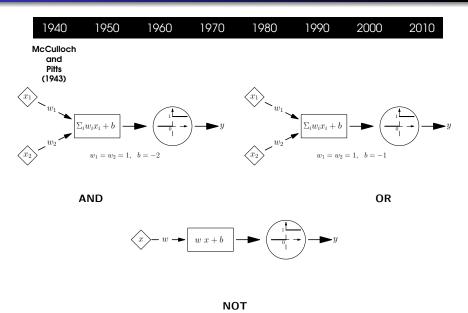


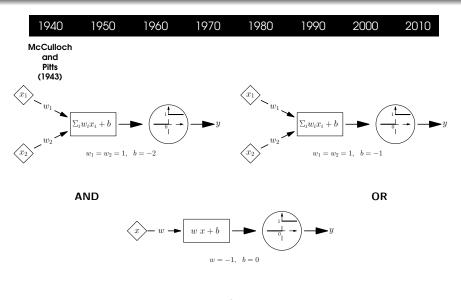


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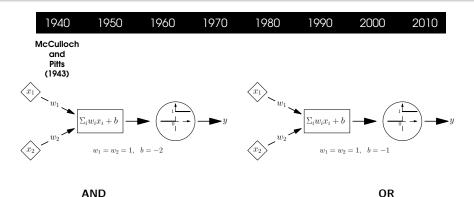








Intro to DL

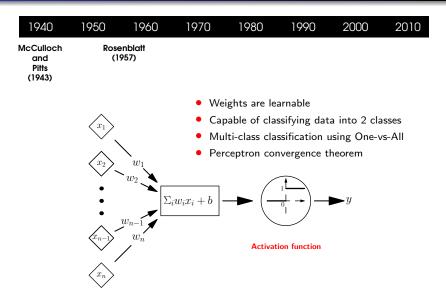


 $\begin{array}{c}
x \\
\hline
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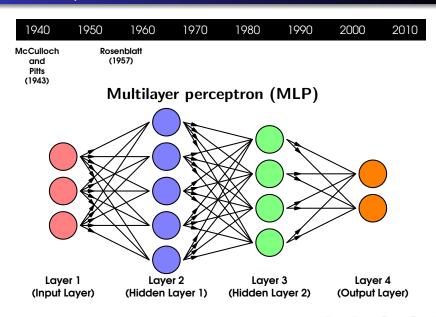
Weights are adjustable but not learned!

NOT

The Perceptron



The Perceptron



The Perceptron: Criticism

1940	1950	1960	1970	1980	1990	2000	2010
McCulloch and Pitts (1943)		enblatt 957)	Minsky and Papert (1969)				

Perceptron: An Introduction to Computational Geometry

- Detailed mathematical analysis of perceptrons
- Limitations of perceptrons for e.g. XOR problem
- Claimed issues exist for other variants

INP	TU	OUTPUT		
X_1	X_2	OUTFUT		
0	0	0		
0	1	1		
1	0	1		
1	1	0		

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"The perceptron has shown itself worthy of study despite (and even because of!) its severe limitations. It has many features to attract attention: its linearity; its intriguing learning theorem; its clear paradigmatic simplicity as a kind of parallel computation. There is no reason to suppose that any of these virtues carry over to the many-layered version. Nevertheless, we consider it to be an important research problem to elucidate (or reject) our intuitive judgement that the extension to multilayer systems is sterile."

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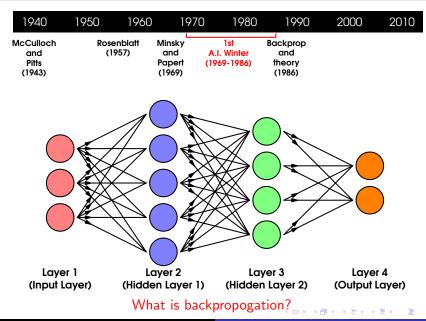
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Revival with Backpropagation



- Learning representations by back-propagating errors by Rumelhart, Hinton and Williams (1986)
- Other advancements by Bengio, Lecun and others ...
- Efficient evaluation of gradients
- Universal Function Approximation theorem for MLPs by Cybenko (1989)
- Theoretical investigation by Barron, Pinkus, Mhaskar ...

Revival with Backpropagation



8 / 29 D. Ray Intro to DL

Second freeze and resurgence



Issues with back-propagation

- Not enough labelled data
- Learning time scales badly (exponentially) with multiple layers
- Deep networks can have several local minima

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Resurgence

- Availability of large data sets
- GPUs and other computational advancements
- Better training algorithms

Components of MLPs

Components of MLPs

- Depth Vs width
- Data sets
- Activation functions
- Loss/cost functions
- Initialization
- Stochasticity and mini-batches
- Overfitting and underfitting
- Optimizers and learning-rate
- Hyper-parameter tuning

Depth Vs Width

- Several partial results exist about the width and depth
- Need $O(N_{inp} + N_{out} + M)$ parameters to represent a dataset of size M
- There is a gap between theory and training
- Going "deeper and narrow" gives better results than staying "shallow but wider"
- Need to find optimal structure based on application

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Training, validation and testing

Three types of data sets:

• **Training data:** Used to optimize the weights and biases.

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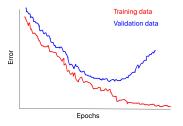
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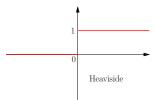


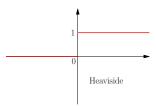
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Scaling and pre-processing the input data – important for generalization.

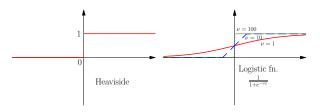
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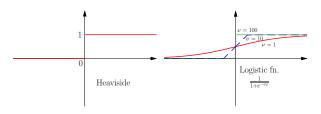




- McCullock-Pitts neuron
- Zero gradient bad for backpropagation
- Not used anymore

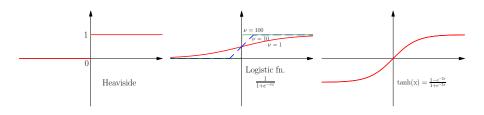


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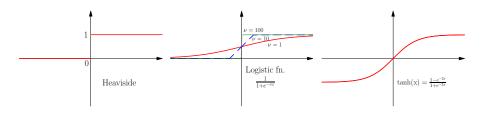
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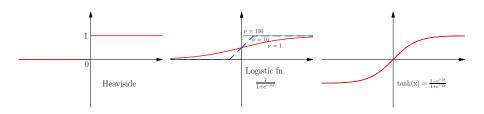
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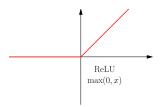


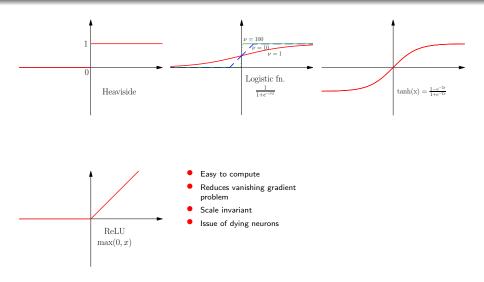
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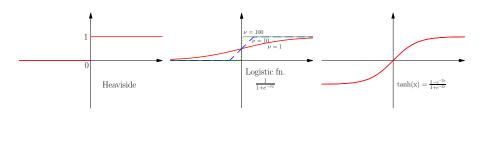
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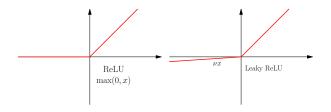
- Symmetric unlike Logisitic func.
- Smooth
- Vanishing gradients away from 0.

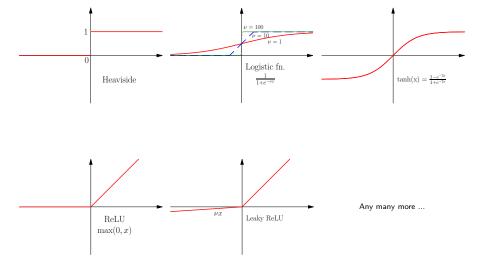












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Loss/cost function

Regression problem

- Mean squared error
- Mean L1 error
- Mean absolute error
- ...

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Classification problem

• Use softmax output function

$$\hat{Y}^{(k)} = \frac{e^{\hat{Y}^{(k)}}}{\sum_{j} e^{\hat{Y}^{(j)}}} \quad \in \quad [0,1] \quad \longrightarrow \quad \text{probabilities/classification}$$

Cross-entropy loss function

$$C = -\sum_{i=1}^{\mathsf{M}} \sum_{j} Y_i^{(j)} \log \left(\hat{Y}_i^{(j)} \right)$$

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- Do not initialize w to be zero leads to linear model
- · Initialize randomly using normal distribution etc
- Exploding gradients avoided using heuristic scaling depending on activation function – minimize variance of the weights
 - For ReLU

$$w^l = \operatorname{nrand}(N_l \ , \ N_{l-1}).\sqrt{\frac{2}{N_{l-1}}}$$

For tanh (Xavier initialization)

$$w^l = \operatorname{nrand}(N_l \ , \ N_{l-1}).\sqrt{\frac{1}{N_{l-1}}}$$



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Randomly shuffle data \longrightarrow introduces stochasticity – speeds-up convergence

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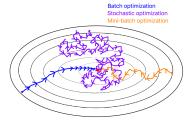
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We complete **1 epoch** when we have finished striding over the whole dataset – approx. M/m optimizations steps.

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- Train longer
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- Taking more data won't help!

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- Regularization
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Regularization

Improves generalization error

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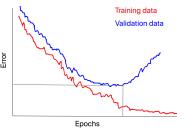
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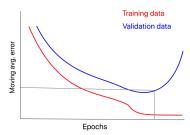
 \bullet Early stopping of training after M succesive increases in validation error



Improves generalization error

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$$\overline{E_n} = \frac{1}{n} \sum_{i=1}^n E_i$$

Improves generalization error

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- ullet Early stopping of training after M succesive increases in validation error not recommended
- Penalizing the cost function

$$\widetilde{C}(Y, \hat{Y}; W, b) = C(Y, \hat{Y}; W, b) + \alpha\Omega(W)$$

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b not regularized as

- ▶ b easier to fit
- W model variable interactions
- regularizing b can cause severe underfitting

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- $lackbox{ }\Omega(W)=\|W\|_2^2 \ \longrightarrow {\sf drives} {\sf weights} {\sf closer} {\sf to} {\sf zero}$
- $\Omega(W) = ||W||_1 \longrightarrow \text{induces sparsity}$

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- Dropout randomly kill hidden neuron outputs while training (only)

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Several optimizers are available (a nice summary available here)

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Estimate the first and second moments of gradient $\ensuremath{g_t}$

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t, \qquad v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

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$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t$$

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Learning-rate η needs to be chosen. Can be adapted if needed

$$\eta_t = \frac{\eta_0}{1 + \gamma t}, \quad \text{or} \quad \eta_t = 0.9^t \eta_0, \quad \text{or} \quad \dots$$

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Hyper-parameter tuning: where the most time is spent!

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- Learning-rate
- Choice of regularization and associated parameter
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- Number of training epochs
- Number of retrains (with different initialiations)

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Several strategies proposed. For instance random coarse to fine search

