Total number of printed pages–5

52 (2) MATH-II

2017

MATHEMATICS – II

Paper : 2·1

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give very short answer : *(any eight)* $1\times8=8$

(i) What is a partially ordering relation ?

(ii) Define a spanning subgraph.

(iii) What are predicates ?

(iv) State the pigeonhole principle.

(v) What do you mean by a bounded lattice ?

(vi) What is a eigenvalue of a matrix ?

(vii) Give an example of bijective function.

Contd.

(viii) Define a bipartite graph.

(ix) What is contradiction?

(x) Define basis of a vector space.

2. Give answer : ***(any six)*** $3 \times 6 = 18$

(i) Prove that in a graph the number of odd vertices is always even.

(ii) Determine the eigenvalues of the following matrix

$$\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

(iii) For real-valued functions $f : A \to B$, $g : B \to C$ and $h : C \to D$, prove that $h \circ (g \circ f) = (h \circ g) \circ f$.

(iv) In how many ways 8 men and 6 women are made to sit around a circular table such that no two women are allowed to sit next to each other?

(v) Obtain the *dnf* of the following expression :

$$P \to ((p \to q)) \wedge \sim (\sim q \vee \sim p)$$

(vi) Show that the following expression is a tautology

$$(p \wedge q) \to (p \vee q)$$

(vii) Determine the number of sides of a regular polygon with 27 diagonals.

3. Answer the following : ***(any five)*** $4 \times 5 = 20$

(i) Test the consistency of the following system of equations

$6x + 20y - 6z = 3$; $2x + 6y + 11 = 0$; $6y - 18z + 1 = 0$. If consistent, find the solution.

(ii) State and prove the inclusion-exclusion principle for three sets *A*, *B*, and *C*.

(iii) Show that intersection of two equivalence relation is an equivalence relation.

(iv) Apply mathematical induction to prove that, $10^{2n-1} + 1$ is divisible by 11.

(v) In a class of 30 students, 12 like tea, 6 like tea but not coffee. Find the number of students who like tea and coffee. Also find out how many like coffee but not tea.

(vi) State and prove the Cayley-Hamilton theorem and verify it for the following matrix :

$$\begin{pmatrix} 2 & -2 \\ 3 & 4 \end{pmatrix}$$

4. Answer the following : 3

(a) Prove that $\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$, for $n \geq 0$

Or

(b) Verify the logical equivalence of the following :

$$(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$$

(c) Express the vector (1, 5, 4) as a linear combination of vectors (1, −2, 3) and (2,1,−1) in $V_3(\mathbb{R})$. 3

Or

(d) Obtain the principal *cnf* of the following expression

$$(\sim p \to r) \wedge (q \leftrightarrow p)$$

(e) Prove that a square matrix A is invertible if and only if $|A| \neq 0$. 4

Or

(f) Simplify the following Boolean expression

$$A.(B+C)(A+B+C)$$

(g) For the propositions p and q, obtain the truth table of the following : 4

(i) $p \wedge q$

(ii) $p \vee q$

(iii) $p \to q$

(iv) $p \leftrightarrow q$

Or

(h) Prove that if R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation on A.