Total number of printed pages–7

52 (2) MATH-II 2·1

2018

MATHEMATICS-II

Paper : 2·1

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Solve the following questions : $1\times10=10$

(i) Find the trace of A^2, where

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}.$$

(ii) What is the sum of degrees of vertices of $K_{5,3}$?

(iii) What is the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 4 & 9 & 0 \end{bmatrix} ?$$

(iv) How many relations can be defined from set A to set B where $|A| = 3, |B| = 4$?

(v) Let p and q are any two statements. Then construct the truth table for $p \wedge q$.

(vi) Define reflexive function.

(vii) What do you mean by Tautology?

(viii) Let $A = \{x | x \in \mathbb{N} \text{ and } 1 \leq x \leq 5\}$. Then find $|P(A)|$ where $P(A)$ is the power set of A.

(ix) Define Symmetric function.

(x) Define 'eigenvalue' of a square matrix.

2. Solve the following problems : $2 \times 5 = 10$

(i) Define transitive relation with an example.

(ii) Draw the graph $K_3 + K_2$.

(iii) What do mean by negation of a statement and construct the truth table for '$\sim p$' where p is a statement?

(iv) Find $|A|$, where $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$.

(v) What is the value of $A.\,adj A$ where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} ?$$

3. Solve ***any three*** of the following questions : $3 \times 3 = 9$

(i) By elementary row transformation find the inverse of the matrix A, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}.$$

(ii) Prove that the intersection of any two subspace W_1 and W_1 of the vectorspace $V(E)$ is again a subspace of $V(F)$.

(iii) Prove that for any two sets A and B

(i) $A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$.

(iv) Solve the following system of equations by matrix method :

$$5x + 7y + 2 = 0$$

$$4x + 6y + 3 = 0$$

(v) Define union and intersection of two graphs and draw the following graph—

$$C_4 \cup K_2$$

4. Solve ***any five*** of the following questions : $5 \times 5 = 25$

(i) Prove that for any three sets A, B, C

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(ii) Construct the truth table for

$$\sim (p \vee q) \vee (\sim p \wedge \sim q)$$

(iii) Prove that a complete graph K_n with n vertices contains $\frac{n(n+1)}{2}$ number of edges.

(iv) Find A^{-1}, where $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

(v) The union of two subspaces of a vectorspace $V(F)$ is a subspace if and only if one is contained in the other.

(vi) Prove Handshaking theorem.

(vii) Define the following :

(a) Hamiltonian path

(b) Trail

(c) Circuit

(d) Connected graph

(e) Complete graph.

5. Find the eigenvalues and eigenvectors of the following matrix : 6

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -6 & 1 & 3 \\ -12 & -2 & 8 \end{bmatrix}.$$

OR

What do you mean by spanning tree ? Prove that a tree with n vertices contains $n-1$ number of edges. Also find how many components are there in a tree ? 1+4+1=6
