Quadcopter System Modeling and Autopilot Synthesis

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Abstract **- The developments in applied mathematics and computational capabilities facilitate the design and implementation of control. In addition, huge developments in nanotechnology and its availability attract many of the researchers towards embedded systems especially the embedded flight control. Among the real applications are the unmanned air vehicles (UAV), which is the state of art in the last few years especially the four rotors vertical take-off and landing (VTOL) aircraft known as the quadcopter, due to their maneuverability, ease of design and control. Although it remains a complete nonlinear system, this paper manipulate with mathematical representation of the quadcopter and modelling of the intended system. A linearization of the obtained mathematical model has been achieved via algebraic manipulation, the next objective for this paper is the autopilot design using with justification against previous work concerning the performance requirements of time responses and flight path characteristics. So a PID controller has been designed. Also, a FUZZY logic controller has been established, the evaluation of the obtained controllers and the original one with the nonlinear system has been achieved. The evaluation results reveals that the designed PID controller has the best performance and less control effort compared to the original and designed fuzzy controller.**

I. INTRODUCTION

Unmanned aerial vehicle is the center of attention nowadays especially quadcopter, also known as Quadrotor helicopter, quadcopter is a multirotorhelicopter that is lifted and propelled by four rotors. Quadcopter are classified as rotorcraft, as opposed to fixed-wing aircraft, because their lift is generated by a set of rotors (vertically oriented propellers). Unlike most helicopters, quadcopter use 2 sets of identical fixed pitched propellers, 2 clockwise (CW) and 2 counter-clockwise (CCW). These use variation of RPM to control lift and torque. Control of vehicle motion is achieved by altering the rotation rate of one or more rotor discs, thereby changing its torque load and thrust/lift characteristics, it has a very high maneuverability over both helicopter and normal aircraft.

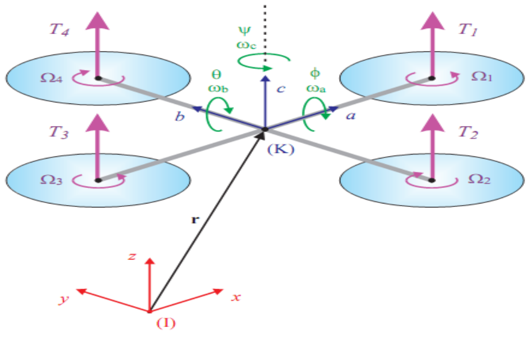

Figure 1: Quadcopter movement

More recently quadcopter designs have become popular in unmanned aerial vehicle (UAV) research. These vehicles use an electronic control system and electronic sensors to stabilize the aircraft. With their small size and agile manoeuvrability, these quadcopter can be flown indoors as well as outdoors.

Even though there are a lot of different topics about the quadcopter structure, but most of the publications have focused on the control algorithm [1]. It can be stated that most of the articles propose a complex control algorithm or compare the performance of few of them. The most important techniques and the respective publications are now presented:

The first control is done using Lyapunov Theory [2, 3, 4, 5]. According to this technique, it is possible to ensure, under certain condition, the stability of the quadcopter.

The second control is provided by PD2 feedback [6, 7,8]. The strength of the PD^2 feedback is the exponential convergence property mainly due to the compensation of the Coriolis and gyroscopic terms.

The third control uses adaptive techniques [9,10]. These methods provide good performance with parametric uncertainties and unmodeled dynamics.

The fourth control is based on Linear Quadratic Regulator (LQR) [6, 11]. The main advantage of this technique is that the optimal input signal turns out to be obtainable from full state feedback (by solving the Ricatti equation). On the other hand the analytical solution to the Ricatti equation is difficult to compute.

The fifth control is done with backstepping control [12, 13,14]. In the respective publications the convergence of the quadcopter internal states is guaranteed, but a lot of computation is required.

The sixth control is based on visual feedback. The camera used for this purpose can be mounted on-board [15, 16, 17] (fixed on the helicopter) or off board [18, 19] (fixed on the ground).

The main objective of the present paper is to design two controllers on a granted quadcopter Matlab Simulink model [2].

- A classical PID controller that sustain the stability of the quadcopter.
- An advanced controller (FUZZY controller).
- Then comparison between the designed controllers and the original one has been presented.

II. QUADCOPTER MODEL AND LINEARIZATION

The model developed in this thesis assumes the following [1]:

• The structure is supposed rigid.
• The structure is supposed symmetrical.
• The center of gravity and the body fixed frame origin are assumed to coincide.
• The propellers are supposed rigid.
• Thrust and drag are proportional to the square of propeller's speed.

The rotation dynamics of the quadcopter is modelled using Euler-Lagrange Formalism. Let us consider earth fixed frame E and body fixed frame B, as seen in Fig.2. The airframe orientation in space is given by a rotation R from B to E, where R belong to SO3 is the rotation matrix.

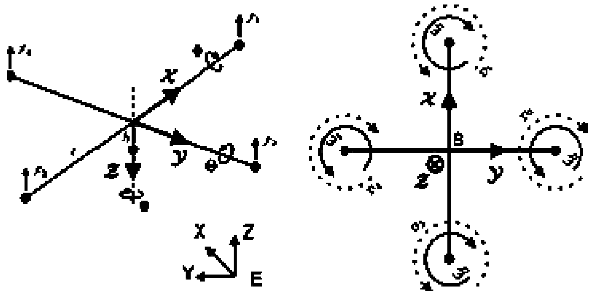

Figure 2: coordinatesystem

The modelling of the quadcopter differs from fixed wing vehicle in the fact that we are not making the rotational transformations in the same order to go from the earth to body axes. Indeed, the most practical way is to carry out the final rotation of the earth to body transformation along the thrust direction [20]. Thus, the body to earth transformation, is done by the following direction cosine matrix:

$$R_{zxy} = \begin{bmatrix} \sin\theta\sin\phi\sin\psi+\cos\psi\cos\theta & \sin\phi\sin\theta\cos\psi-\cos\theta\sin\psi & \sin\theta\cos\phi \\ \cos\phi\sin\psi & \cos\psi\cos\phi & -\sin\phi \\ \cos\theta\sin\psi\sin\phi-\sin\theta\cos\psi & \cos\theta\sin\phi\cos\psi+\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix} \quad (1)$$

Where ϕ,θ,ψ : roll, pitch, yaw angle as shown in Fig.2.

The equation of motion including thrust force, hub force, drag moment, rolling moment, pitching moment, yawing moment and forces along X, Y, Z axis can be summarized as follows.

$$\begin{cases} I_{xx}\ddot{j}=\dot{\theta}\dot{\psi}\left(I_{yy}-I_{zz}\right)+J_r\dot{\theta}\Omega_r+l\left(-T_2+T_4\right)-h\sum\limits_{i=1}^{4}H_{yi}+(-1)^{i+1}\sum\limits_{i=1}^{4}R_{mxi} \\ I_{yy}\ddot{\theta}=\dot{j}\dot{\psi}\left(I_{zz}-I_{xx}\right)+J_r\dot{j}\Omega_r+l\left(T_1-T_3\right)+h\sum\limits_{i=1}^{4}H_{xi}+(-1)^{i+1}\sum\limits_{i=1}^{4}R_{myi} \\ I_{zz}\ddot{\psi}=\dot{j}\dot{\theta}\left(I_{xx}-I_{yy}\right)+J_r\Omega_r+(-1)^{i}\sum\limits_{i=1}^{4}Q_i+l\left(H_{x2}-H_{x4}\right)+l\left(-H_{y1}+H_{y3}\right) \\ m\ddot{z}=mg-(cjc\psi)\sum\limits_{i=1}^{4}T_i \\ m\ddot{x}=\left(c\psi s\theta cj+s\psi sj\right)\sum\limits_{i=1}^{4}T_i-\sum\limits_{i=1}^{4}H_{xi}-\frac{1}{2}C_xA_c\rho\dot{x}\left|\dot{x}\right| \\ m\ddot{y}=\left(s\psi s\theta cj-c\psi sj\right)\sum\limits_{i=1}^{4}T_i-\sum\limits_{i=1}^{4}H_{yi}-\frac{1}{2}C_yA_c\rho\dot{y}\left|\dot{y}\right| \end{cases} \quad (2)$$

Where Ixx , Iyy , IzzBody inertia moment,Ω_r: rotor speed, J_r: rotor inertia moment. σ : solidity ratio , λ : inflow ratio, a : lift slope , Y : induced velocity , μ : rotor advance ratio , ρ : air density ,T: motor thrust ,H: hub forces, c: cosine, s: sine

A simulation program using Matlab m files and Simulink file has been established for modeling the underlying system [2] as shown in fig.3.

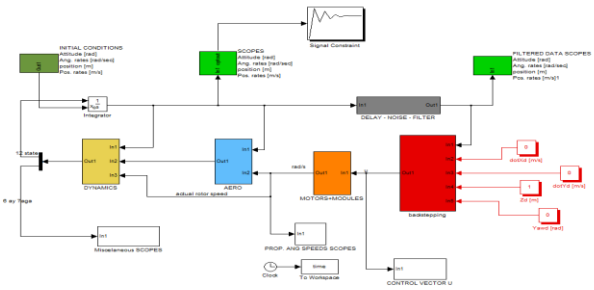

Figure 3: quadcopter Matlab Simulink model

2.1 Quadcopter linearization.

The quadcopter dynamics must be linearized to provide an easy inversemodel which can be implemented in the control algorithms.so equation (2) can be rearranged concerning to the following consideration.[1]

The angular contributes are quite complex because several variables havebeen taken into account. Most of those come from cross coupling of angularspeeds (gyroscopic effects and Coriolis-centripetal form). Since the motion of the quadcopter can be assumed close to the hovering condition, smallangular changes occur (especially for roll and pitch) It follows that theseterms can be simplified because smaller than the main ones, and also one can neglect these gyroscopic effects and thus remove the cross coupling due to this near hovering position.

The whole control algorithm is used to give the right signals to the propellers. Since they are four, no more than four variables can be controlled in the loop. From the beginning of the project, it has been decided to stabilize attitude (Euler angles) and height. According to this choice, the equations which describe the X and Y position have been deleted. And the model is rewritten as follows.

$$\begin{cases} \ddot{\phi} = \dfrac{U_2}{I_{xx}} \\ \ddot{\theta} = \dfrac{U_3}{I_{xx}} \\ \ddot{\psi} = \dfrac{U_4}{I_{zz}} \end{cases} \qquad (3)$$

III. Design of PID controller

PID technique represents the basics of control, PID is often chosen because of its Simple structure, Good performance and Tuning even without a specific model of the controlled system [21].

If a mathematicalmodel of the plant can be derived, then it is possible to apply various designtechniques for determining parameters of the controller that will meet the transient andsteady-state specifications of the closed-loop system. However, if the plant is so complicatedthat its mathematical model cannot be easily obtained, then an analytical orcomputational approach to the design of a PID controller is not possible. Then we mustresort to experimental approaches to the tuning of PID controllers. Fig. 4 shows the control loop.

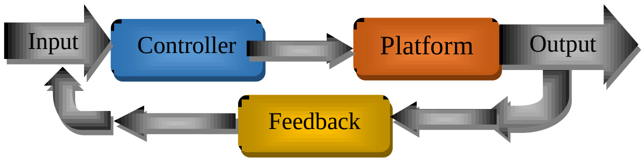

Figure 4: closed loop control loop

The closed loop system including the designed controller and the linearized airframe has the following step response as shown in fig. 5

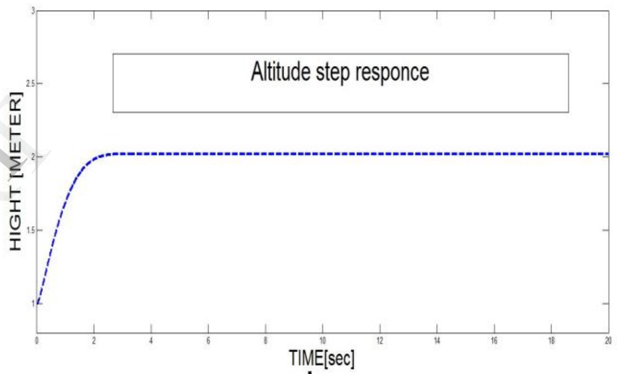

Figure 5: Altitude step response

Table 1: Altitude Response

Controller	Parameters		
	Settling time	*Max. overshoot*	*Steady state error*
PID on linear platform	2 sec	0.015 radian	0.01 radian

IV. Fuzzy Controller Design and Tuning

Fuzzy logic is a convenient way to map an input space to an output space [22]. Between the input and the output there is a black box that does the work [23]. What could go in the black box ? any number of things: fuzzy systems, linear systems, expert systems, neural networks, differential equations…. etc.Clearly the list could go on and on. Of the dozens of ways to make the black box work, it turns out that fuzzy is often the very best way.

The fuzzy logic system block diagram is shown in Fig 6. It consists mainly of four basic blocks: the fuzzification interface, the inference engine (mechanism), the rule-base, and the defuzzification interface, [24].

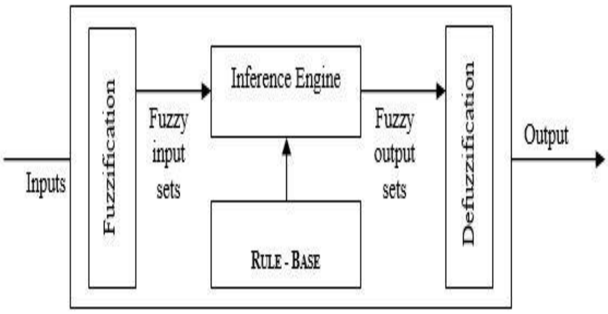

Figure 6: Fuzzy control loop

The rule –base is provided by experts or can be extracted from numerical data, fuzzificationmaps crisp input numbers (controller input) into fuzzy input sets (information) that can be used to activate rules,Inference engine maps fuzzy input sets into fuzzy output sets,defuzzification maps output sets into crispoutput numbers, which correspond to control activities.

According to past experience on the underlying system, a FUZZY logic controller is designed that is consists of:

- Five membership function.
- Three of them is triangular and the two trapezoidal as shown in fig.7
- The input to the fuzzy system is the error signal and its derivative.
- The output of each channel is the control signal to the motors.

For example in the following figures 7, 8 shows the membership function of the pitch channel input error signal and output signal.

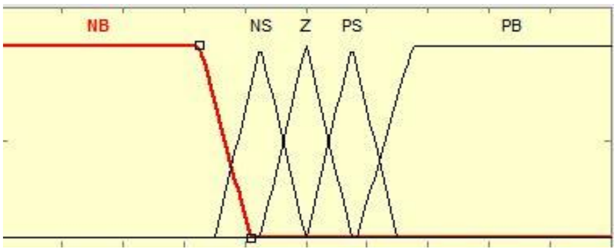

Figure 7: Pitch error membership function

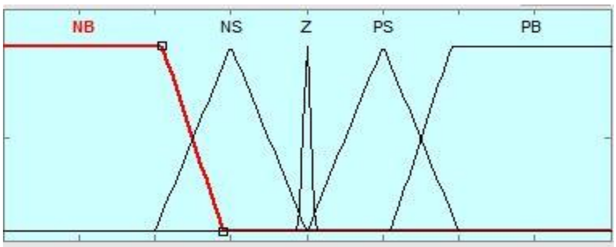

Figure 8: Output membership function

The input to the FUZZY controller is the error and the derivative of the error, so according to the inputs and the membership function a twenty five rule is written as shown in fig.9. For the rules of the pitch channel.

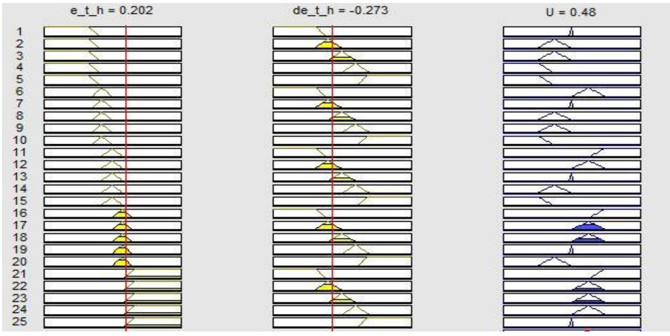

Figure 9: Pitch rules

The surface shown in figure.10 shows the surface of the pitch channel which indicates a smooth surface indicating a smooth changing from one rule to another that yields to a suitable controller. As shown in fig.10.

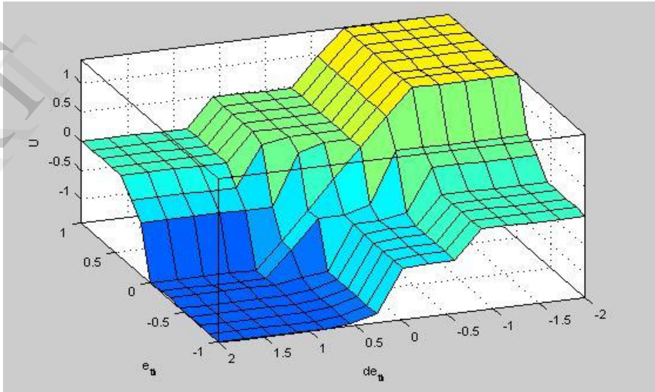

Figure 10: Pitch control surface

V. EVALUATION OF THE DESIGNED CONTROLLER ON THE NONLINEAR PLATFORM.

In order to test the performance of the designed controllers a test procedure is chosen so that the initial condition for the pitch, roll and yaw angle is 0.3 radian=17 [deg.] and the height is 1[m.], the controller is supposed to make the 3 angles reach 0 radian and the height to reach 3 meter .

The following figures shows the response of the system, the blue line indicates the designed PID controller, the red indicates the designed FUZZY controller and the black one indicates the original controller.

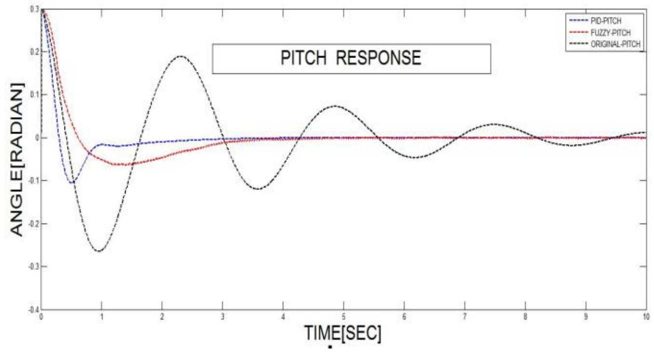

Figure 11: Pitch response.

Table 2: Pitch Response

Controller	Parameters		
	Settling time	***Max. overshoot***	***Steady state error***
DESIGNED PID	12 sec	0radian	0 radian
DESIGNED FUZZY	3.2 sec	0.068 radian	0 radian
ORIGINAL CONTROLLER	15 sec	0.245 radian	0 radian

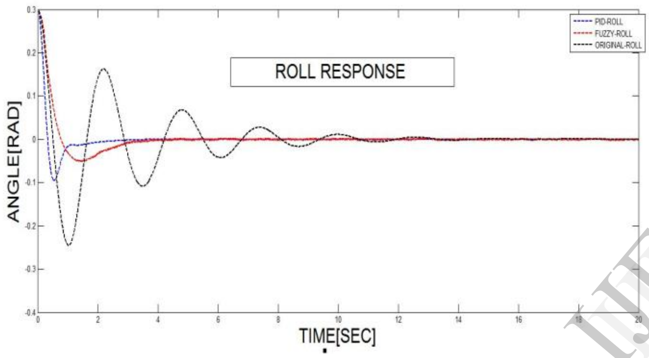

Figure 12: Roll response

Table 3: roll Response

Controller	Parameters		
	Settling time	***Max. overshoot***	***Steady state error***
DESIGNED PID	1.8 sec	0.09 radian	0 radian
DESIGNED FUZZY	3.8 sec	0.051 radian	0 radian
ORIGINAL CONTROLLER	14 sec	0.2 radian	0 radian

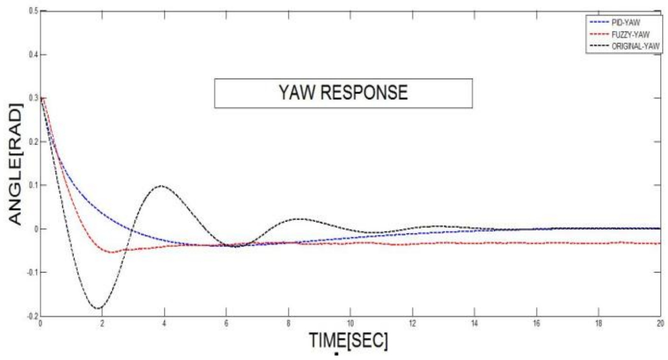

Figure 13: Yaw

Table 4: Yaw Response

Controller	Parameters		
	Settling time	***Max. overshoot***	***Steady state error***
DESIGNED PID	12 sec	0 radian	0 radian
DESIGNED FUZZY	6 sec	0.05 radian	0.04 radian
ORIGINAL CONTROLLER	12.5 sec	0.19 radian	0 radian

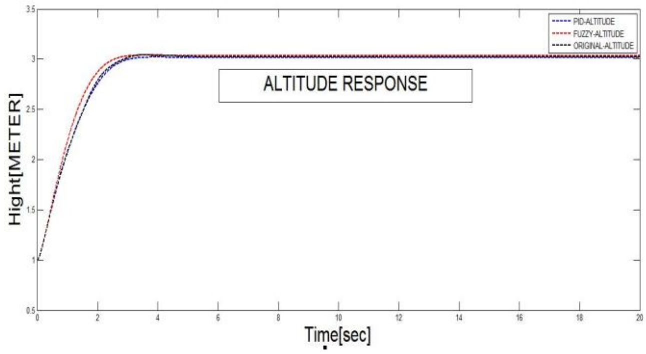

Figure 14: Altitude response

Table 5: Altitude Response

Controller	Parameters		
	Settling time	***Max. overshoot***	***Steady state error***
DESIGNED PID	3.6 sec	0 meter	0 radian
DESIGNED FUZZY	3.5 sec	0 meter	0.04 radian
ORIGINAL CONTROLLER	4 sec	0.05 meter	0 radian

As shown from figures and tables the designed PID controller gives a very stable and good performance in both attitude and altitude control than FUZZY controller and original controller, in settling time ,steady state error, max overshoot and the response of the PID system is faster than the other two controller.

And from the point concerning the controller control effort as an example the yaw controller as shown in Table6.

Parameter	Controller		
	DESIGNED PID	DESIGNED FUZZY	ORIGINAL model
VARIANCE	7.7682e-07	2.1523e-06	1.4003e-06

The control effort from the designed PID controller is smaller than the other two controllers and that's shown from calculating the variance of the control signal from each controller.

VI. CONCLUSION

The paper presented the modelling of the intended system concerning the reference frames, coordinates' transformations and equations of motion. This model is built in the form of modules assigned to each process within the Quadcopter system. Then, it is programmed within MATLAB environment.The simulation is conducted with different engagement scenarios and different sources of uncertainties. The designedautopilots proved its robustness, but the PID controller shows better in performance in both attitude and altitude than fuzzy and the original controller, even the control effort of the designed PID controller is less than that of the FUZZY and the original controller and much simpler so the PID controller is more than enough for this platform.

VII. FUTURE WORK

Implementation of the designed PID controller on an embedded system and place it on a real quadcopter platform to stabilize its attitude performance.

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