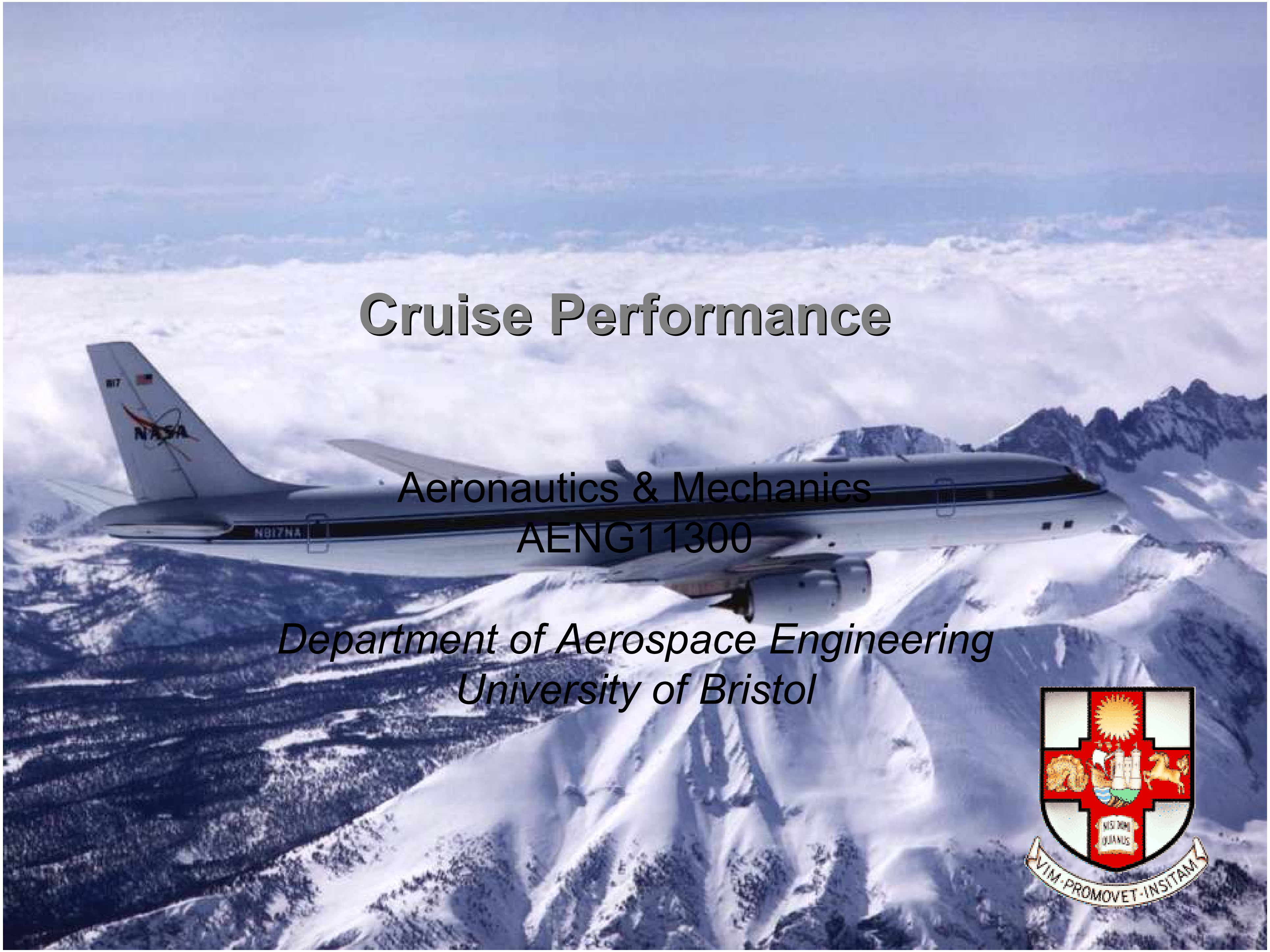


Cruise Performance

Aeronautics & Mechanics
AENG11300

Department of Aerospace Engineering
University of Bristol

Engine Characteristics

- two main classes of engines:
  1. ‘constant-power’ engines where **shaft horsepower** output is roughly constant with speed
     - piston engines & turboprops
     - ie propeller-driven
  2. ‘constant-thrust’ engines where thrust output is roughly constant with speed
     - turbojets & turbofans
     - ie jet-driven
- *a gross simplification – but useful for preliminary performance work*
- **all** produce thrust by imparting an increase in velocity to the air passing through the engine

Thrust Generation

- basic thrust equation
  - Newton's 2nd Law → *force = rate of change of momentum*

$$\boxed{T = \dot{m}\left(V_j - V\right)}$$

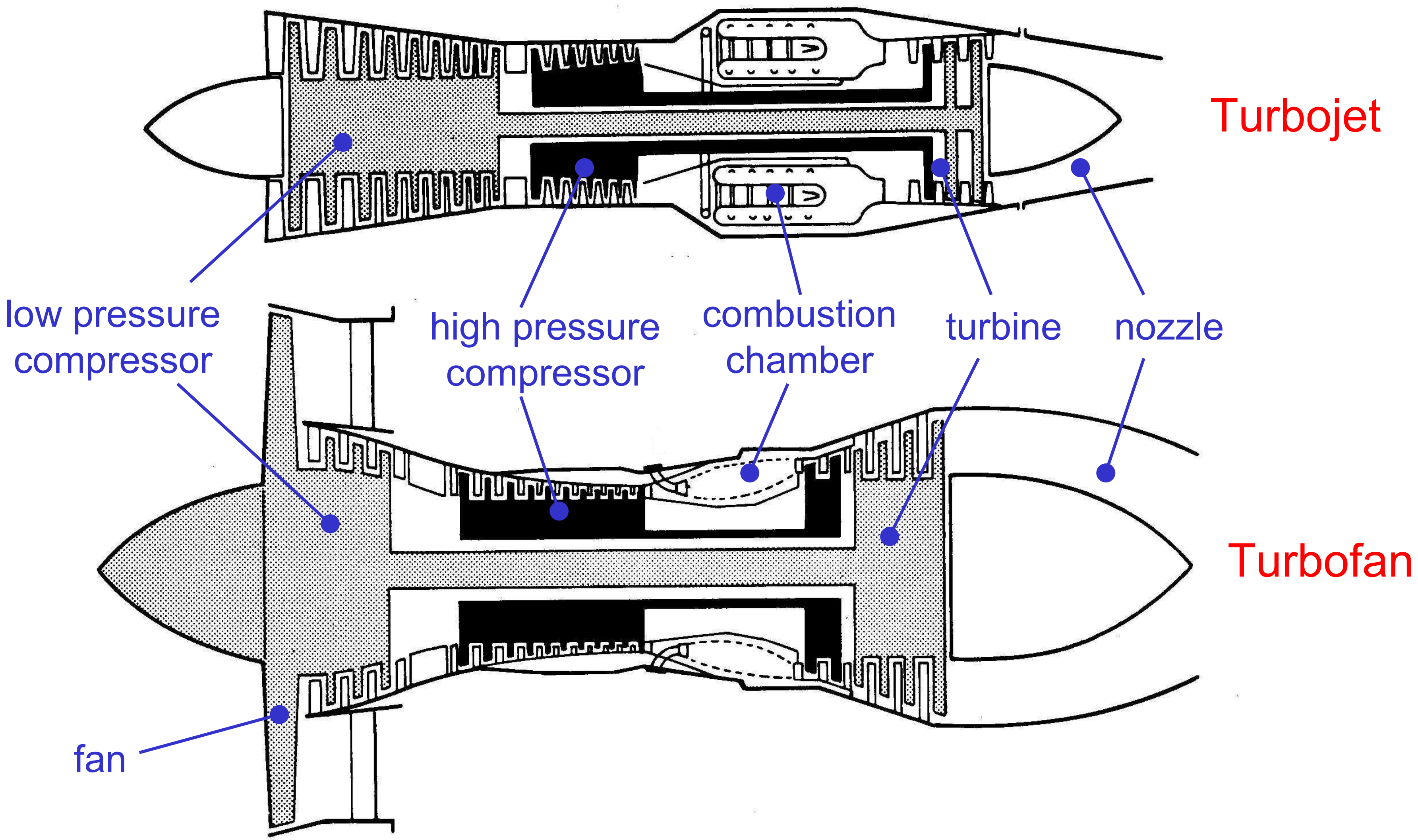
$\dot{m}$ = mass flow rate (kg/s)
V_j = jet efflux velocity

- fuel consumption related to power 'wasted' in the jet

$$\boxed{P_{lost} = \tfrac{1}{2}\dot{m}\left(V_j - V\right)^2}$$

- propeller = high mass flow but low 'jet' velocity V_j
  - low fuel consumption
  - rapid loss of thrust with forward speed
- jet = low mass flow but high jet velocity V_j
  - high fuel consumption
  - little loss of thrust with forward speed

Turbojet and Turbofan Engines



Turbojet and Turbofan Engines

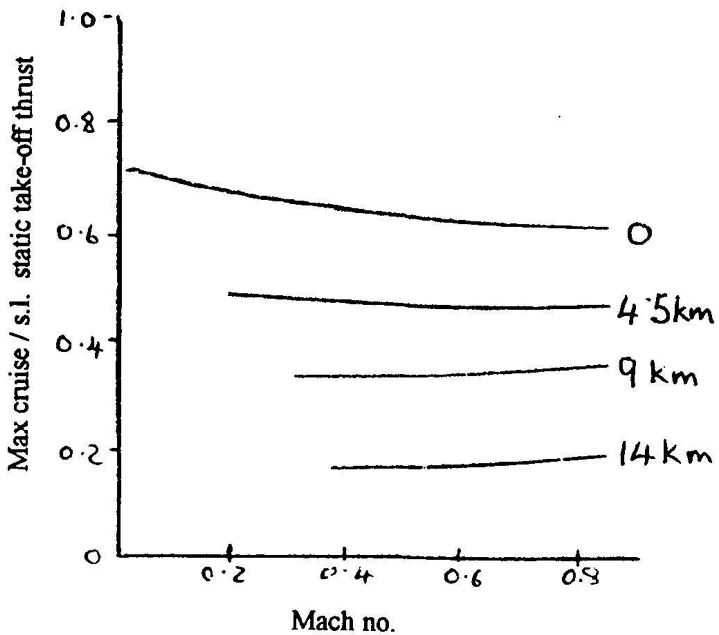


Figure 5 Turbojet max. cruise thrust characteristic, based on;
Pratt & Whitney JT4A-3 turbojet engine
(s. level static thrust at take-off 70,300N

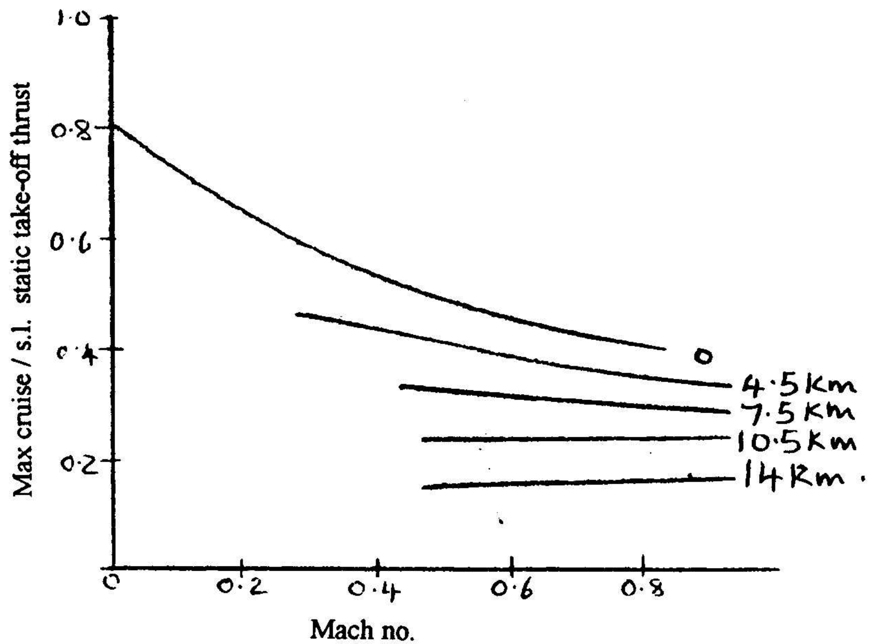
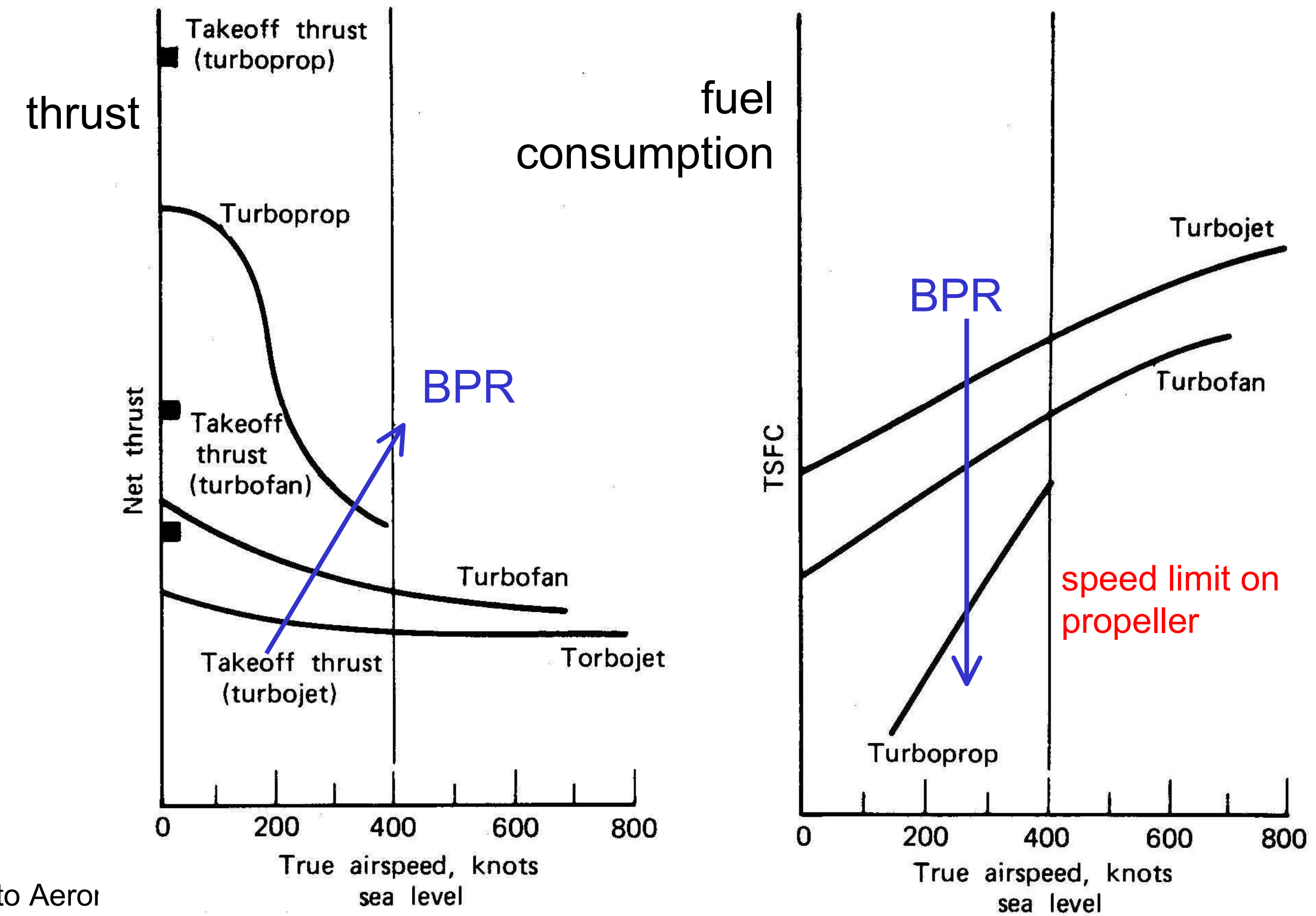


Figure 6 Turbofan max. cruise thrust characteristic, based on;
Pratt & Whitney PW 4056 turbofan engine
(s. level static thrust at take-off 248,000N

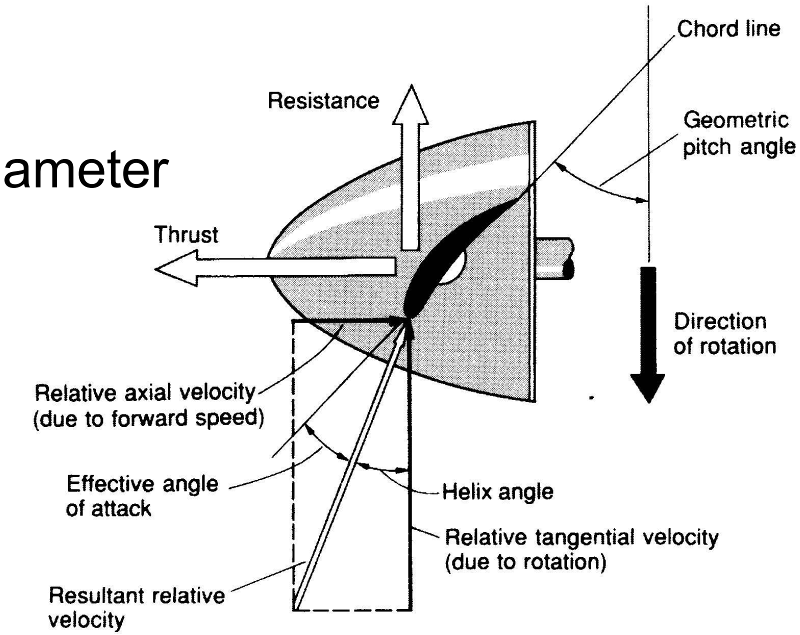
Turbojet vs Turboprop Engines

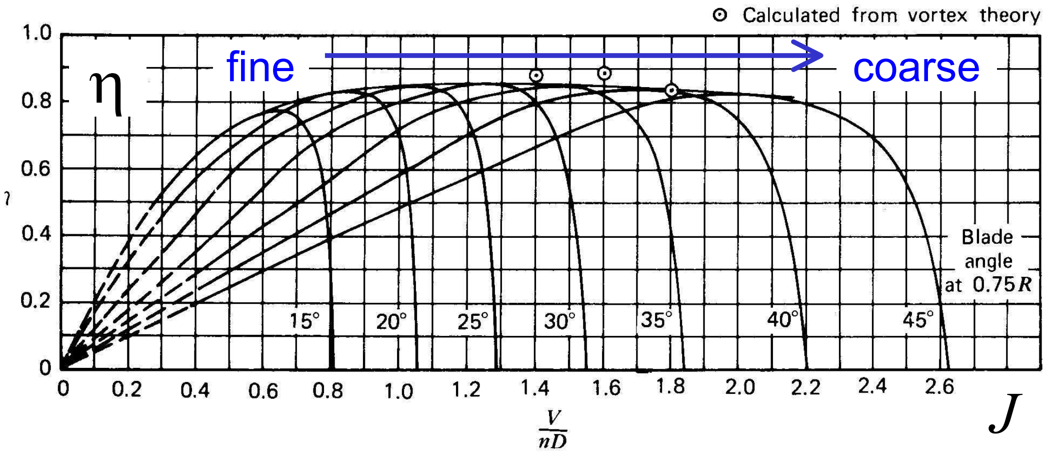
- characteristics determined by the **bypass ratio** (BPR)
  - ratio of air passing through fan (or propeller) to air passing through engine core



Propeller Efficiency

- propeller acts as rotating wing
  - 'direction of flight' determined by **advance ratio** $J = V/nD$
  - where n is RPM and D is propeller diameter
- local angle of attack governed by advance ratio and **blade pitch** angle setting
  - blade **twist** (washout) needed
- efficiency η peaks in narrow speed range
  - effect of stall
- use **variable-pitch** prop
  - fine pitch at low speed
  - coarse pitch at high speed

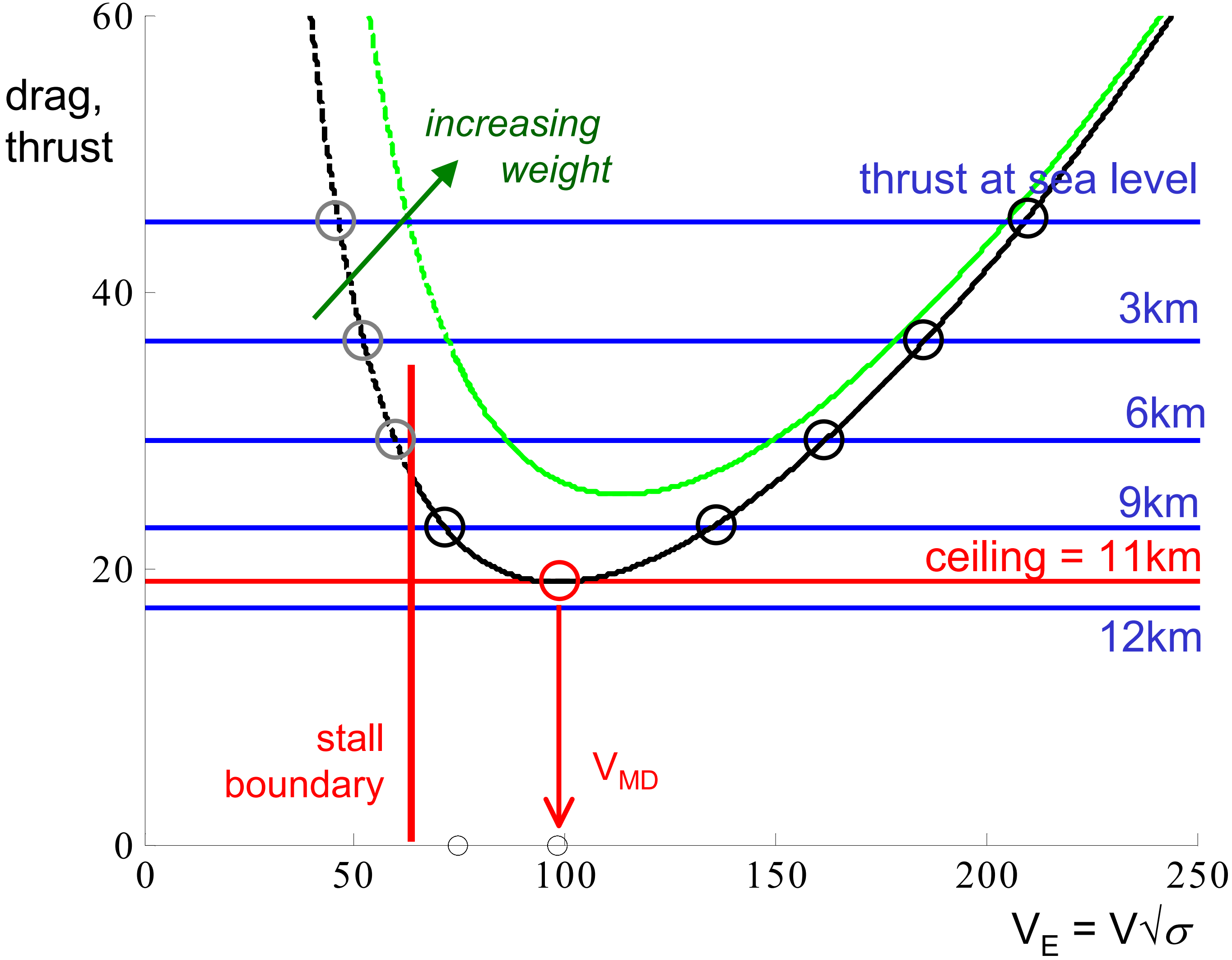




Basic Cruise Performance

- Hunter flypast

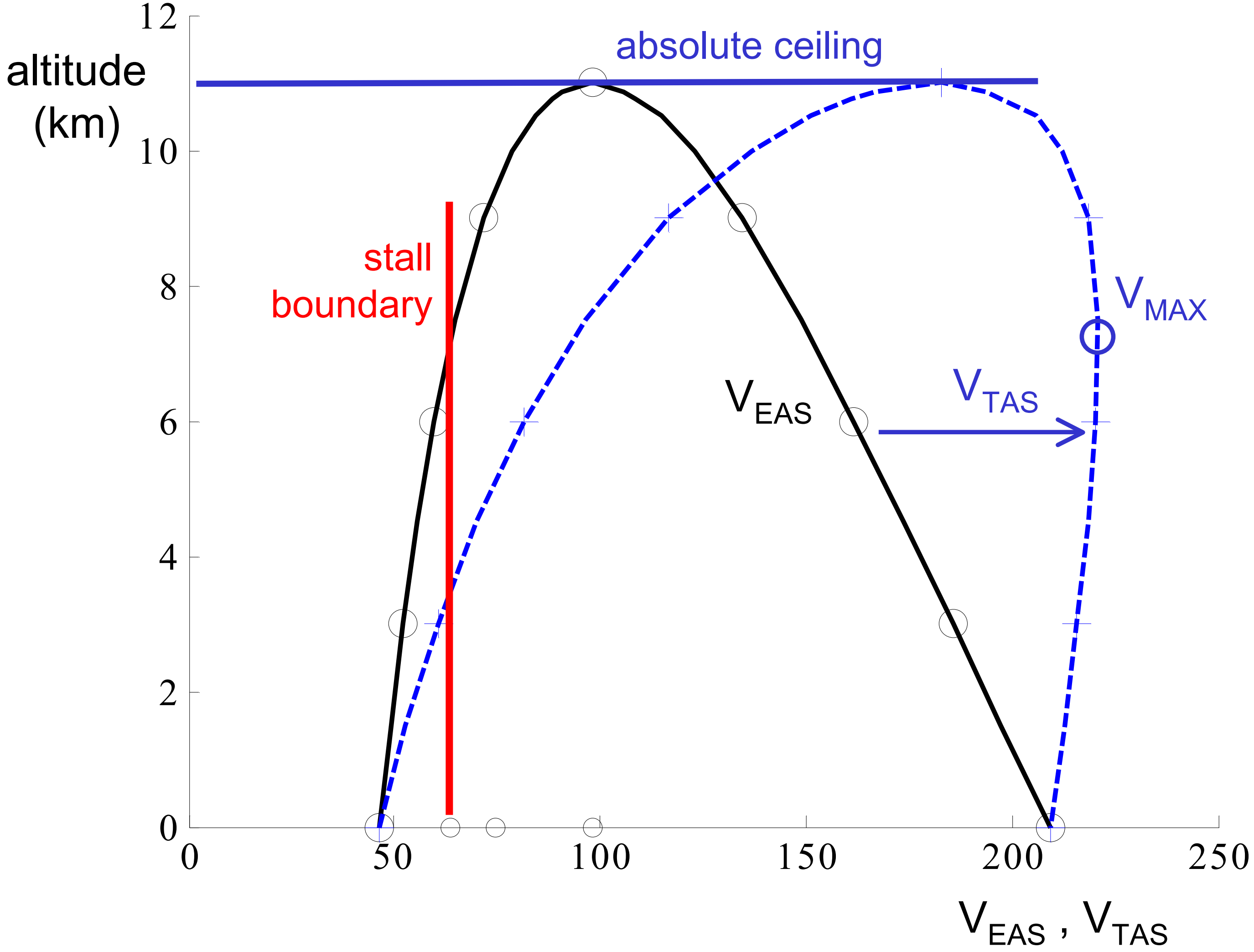
Jet Aircraft in Cruise



Features of Jet Cruise Diagram

- *simplified* representation of thrust $\boxed{T = kT_0\sigma^{0.7}}$
  - **throttle setting** k
  - thrust at sea level T_0 (independent of speed)
  - density ratio σ (to power *0.7* below 11km, to power *1.0* above)
- maximum and minimum speed at each altitude for $T = D$
  - lower speed may be unattainable at low altitude due to stall
  - upper speed is practical **cruise speed**
  - between upper and lower speed aircraft will accelerate or climb unless throttle setting is reduced
- at one particular height there is just sufficient height to cruise at one speed only
  - this is the **absolute ceiling** for the throttle setting used
  - achieved at minimum drag speed
- increasing weight reduces cruise speed and lowers ceiling

Jet Aircraft Cruise Speed



Features of Jet Cruise Speed

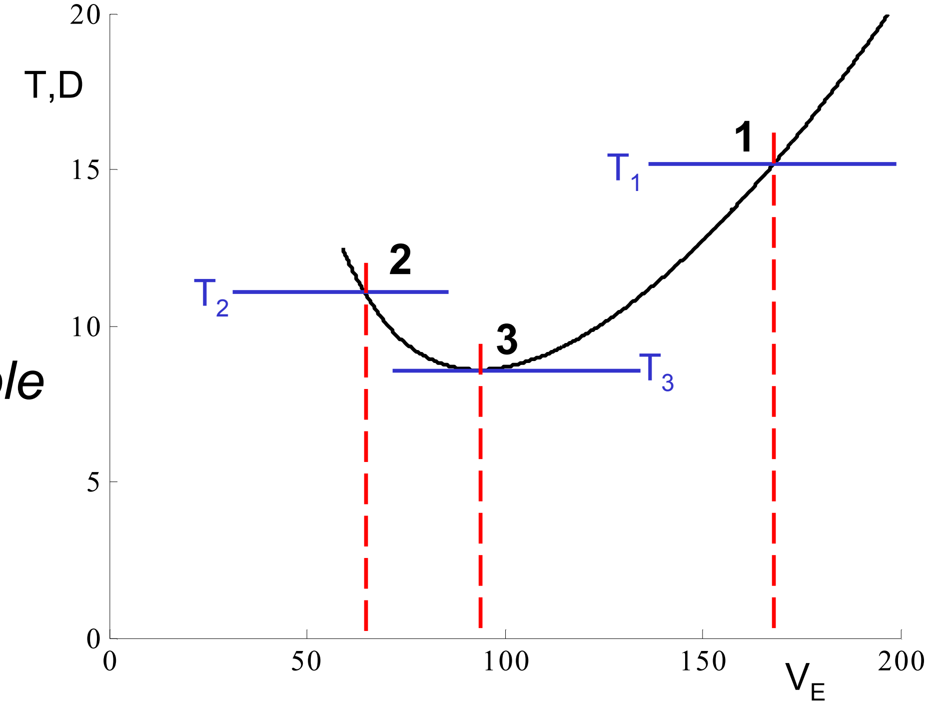
- cruise speed in EAS *reduces* steadily as altitude increases
- maximum cruise speed in TAS *increases* with altitude
  - up to a maximum V_{max} before the absolute ceiling is reached
- demonstrates some advantages of cruise at high altitude
  - maximum cruise speed in TAS (ie ground speed) similar to (or greater than) speed at sea level
  - thrust at maximum cruise speed reduces with altitude
    $\rightarrow$ fuel consumption reduces with altitude
- minimum fuel consumption at minimum drag speed
  - work done = thrust $\times$ distance
  - in theory should be unaffected by altitude (since D_{min} constant), but

  1. at low altitudes engine would need to be throttled back
     $\rightarrow$ reduced thermodynamic efficiency & hence increased fuel burn
  2. cruise speed in TAS for minimum drag increases with altitude

Speed Stability in Cruise

- consider aircraft with throttle adjusted to cruise at points 1, 2 and 3
  - what is effect of *small* fluctuations in velocity (eg due to gusts) ??

1. speed increase
   = increase in drag
   - aircraft decelerates = *stable*
2. speed increase
   = reduction in drag
   - aircraft accelerates = *unstable*
3. speed increase
   = no change in drag
   - aircraft is *neutrally stable*

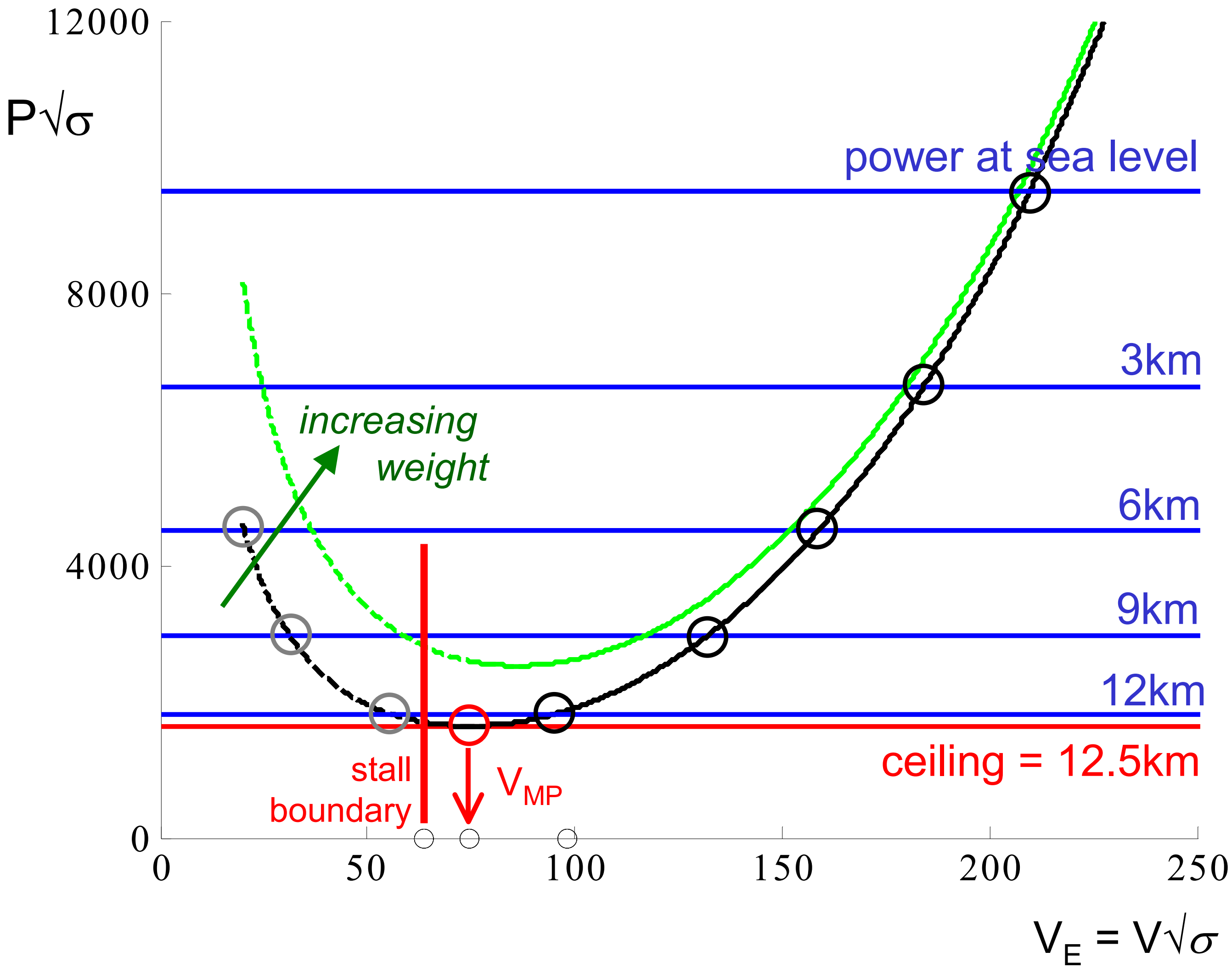


- in case 2 pilot must continually adjust throttle to maintain speed
  - flight on ‘**backside of drag curve**’ – rather unsafe!

Speed Stability at Ceiling

- **absolute ceiling** is an unstable condition to maintain
  - maximum thrust setting at minimum drag speed
    $\rightarrow$ *any* change in speed will *increase* drag above available thrust and hence cause aircraft to descend
- excess thrust and hence rate of climb drop to zero as ceiling is approached
  - absolute ceiling cannot be established in reasonable time!
- **service ceiling** is a practical alternative definition of maximum **operating altitude**
  - at the service ceiling the aircraft still has a small specified rate of climb
  - defined as 2.5 m/s for jet aircraft and 0.5 m/s for propeller-driven aircraft

Propeller-Driven Aircraft in Cruise



Range and Endurance – Jet Aircraft

- Proteus

Range and Endurance

- **endurance** - the time an aircraft can remain in flight
  - important for surveillance type missions
- **range** – the horizontal distance that an aircraft can cover:

1. **Safe Range** – the maximum distance between two airfields for which an aircraft can fly a safe and reliably regular service with a specified payload
   - rather lengthy calculation of full mission profile (take-off/climb/ cruise/descent/landing, headwinds, diversion allowance etc)
   - therefore simplified measures of range used in project work …
2. **Still Air Range** (SAR)
   - take-off with full fuel, climb to cruise altitude, cruise until all fuel expended (!)
3. **Gross Still Air Range** (GSAR)
   - begin at selected altitude with full fuel, cruise until all fuel has gone
   - approximate factor between GSAR and Safe Range known

Breguet Range Equation – Jet Aircraft

- actually derived by Coffin in the 1920's
  - compact and simple way of calculating GSAR
- **"rate at which fuel is burnt**
  = rate at which aircraft weight is reduced"
- define **thrust specific fuel consumption** (sfc or TSFC) as
  - ***f*** **= mass of fuel burnt per unit of thrust per second**
  - consistent units are $kg/N.s$
  - but often given in terms of $kg/N.hr$ so don't forget to convert!
- for thrust T and weight W the basic Breguet equation is

$$\boxed{\frac{dW}{dt} = -fgT}$$

  - a differential equation (in units of N/s)

Integration of Breguet Equation - Endurance

- rearranging and substituting $T = D$ and $W = L$

$$dt = -\frac{dW}{fgT}$$

$$T = \frac{D}{L}W = \frac{C_D}{C_L}W$$

$$\Rightarrow \quad \boxed{dt = -\frac{1}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that C_L/C_D and f remain *constant*
- can then integrate from start weight W_1 to end weight W_2 to obtain the endurance E

$$\int \frac{dx}{x} = \ln x \quad \Rightarrow \quad \boxed{E = t_2 - t_1 = t_{12} = \frac{1}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

Integration of Breguet Equation - Range

- increment in distance dS at velocity V is given by

$$\boxed{dS = Vdt = -\frac{V}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that *true air speed* V remains *constant*
- can then integrate from start weight W_1 to end weight W_2 to obtain the range R

$$\boxed{R = S_2 - S_1 = \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

Implications of Assumptions (1)

- is it justifiable to assume C_L/C_D, f and V_{TAS} to be constant?
  - only in particular circumstances…
- as fuel is burnt weight W and hence required lift L reduces
- if V_{TAS} held constant then either C_L and/or σ must reduce
- constant $C_L/C_D \rightarrow$ constant C_L (= constant incidence α)
  $\rightarrow$ therefore σ must decrease $\equiv$ **altitude must increase**
- constant C_L/C_D implies constant C_D
  $\rightarrow$ therefore drag D decreases in proportion to σ
- since $T = D$, thrust must also decrease with altitude while constant sfc f implies **constant throttle setting**
  $\rightarrow$ approximately true for turbojet in **stratosphere** (above ~ 11km), where $\boxed{T \approx kT_0\sigma}$

$$\boxed{L = W = \tfrac{1}{2}\rho_0 \sigma V^2 S C_L}$$

Cruise-Climb

- this cruise case often referred to as a **cruise-climb**
  - usually precluded by air traffic control restrictions!
- note that below 11km (in the **troposphere**) temperature falls with altitude

  $\rightarrow$ thrust falls off less rapidly $\rightarrow$ need to back-off on throttle to maintain V and C_L constant, hence sfc would worsen

maximise cruise speed – ie cruise at high altitude

$$\boxed{R = \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

a measure of *thermodynamic efficiency* – ie minimise fuel consumption f

a measure of *aerodynamic efficiency* – ie minimise drag D

a measure of *structural efficiency* – ie minimise fixed weight W_2

Constant Altitude Cruise

- as before

$$\boxed{dS = Vdt = -\frac{V}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- but now assume σ and hence ρ are constant, but V varies
- substitute velocity equation

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}} \Rightarrow \boxed{dS = -\frac{1}{fg}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\frac{dW}{W^{1/2}}}$$

and integrate

$$\int \frac{dx}{x^{1/2}} = 2x^{1/2} \Rightarrow \boxed{R = S_2 - S_1 = \sqrt{\frac{8}{\rho S}}\frac{1}{fg}\frac{C_L^{1/2}}{C_D}\left(W_1^{1/2} - W_2^{1/2}\right)}$$

Implications of Assumptions (2)

- height (hence density ρ) and C_L held constant
- as fuel is burnt weight W and hence required lift L reduces
  $\rightarrow$ V_{TAS} must also reduce
  $\rightarrow$ range is less than for cruise-climb for same C_L/C_D
- since C_D is constant, drag D and hence thrust T also reduce
  $\rightarrow$ therefore throttle setting must be reduced progressively during the cruise
  $\rightarrow$ therefore some variation in sfc f will occur
- need to use average value of f, or treat as a series of shorter steps
  – if each step flown at an increased height an approximation to a cruise-climb profile can be obtained

Maximum Range – Cruise-Climb (1)

- maximum range depends on variation of thrust with height

$$\boxed{R = \frac{1}{fg}\left(V\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- start with basic $T = T_0\sigma$ with fixed throttle
  $\rightarrow$ sfc f and velocity V constant
  - but altitude (and hence σ) undefined
- for maximum range we require $V(C_L/C_D)$ to be a maximum
- use the thrust/drag relation to eliminate σ

$$T = T_0\sigma = D = C_D \tfrac{1}{2}\rho_0\sigma V^2 S \quad \Rightarrow \quad V = \sqrt{\frac{T_0}{\tfrac{1}{2}\rho_0 S C_D}}$$

and hence $\boxed{V\frac{C_L}{C_D} = \sqrt{\frac{2T_0}{\rho_0 S}}\frac{C_L}{C_D^{3/2}}}$ $\rightarrow$ therefore need to find minimum $C_D^{3/2}/C_L$

Maximum Range – Cruise-Climb (2)

$$\frac{C_D^{3/2}}{C_L} = \frac{\left(C_{D0} + KC_L^2\right)^{3/2}}{C_L}$$

$$d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$$

$$\Rightarrow \frac{d\left(C_D^{3/2}/C_L\right)}{dC_L} = \frac{C_L \frac{3}{2}\left(C_{D0} + KC_L^2\right)^{1/2} 2KC_L - \left(C_{D0} + KC_L^2\right)^{3/2}}{C_L^2}$$

- so at the minimum point $C_{D0} = 2KC_L^2$

$$\Rightarrow \boxed{C_{D_{\max R}} = \frac{3}{2}C_{D0}\,, \quad C_{L_{\max R}} = \sqrt{C_{D0}/2K}}$$

cruise-climb with $T = T_0\sigma$

- cruise conditions fixed by thrust
  - C_L and C_D obtained from above

    $\rightarrow$ substitute with T_0 into $V\dfrac{C_L}{C_D} = \sqrt{\dfrac{2T_0}{\rho_0 S}\dfrac{C_L}{C_D^{3/2}}}$
  - then substitute into Breguet Equation to find range

Maximum Range – Cruise-Climb (3)

- alternatively specify start altitude (and hence σ_1) and let thrust vary as required
  $\rightarrow$ sfc f and velocity V constant

$$\boxed{R = \frac{1}{fg}\left(V\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- for maximum range we still require $V(C_L/C_D)$ to be a maximum
- since σ is not a variable, we can simply substitute the speed equation for V

$$V = \sqrt{\frac{L}{\frac{1}{2}\sigma\rho_0 S C_L}} \quad \Rightarrow \quad \boxed{V\frac{C_L}{C_D} = \sqrt{\frac{W_1}{\frac{1}{2}\sigma_1\rho_0 S}}\,\frac{C_L^{1/2}}{C_D}}$$

$\rightarrow$ therefore need to find minimum $C_D/C_L^{1/2}$

Maximum Range – Cruise-Climb (4)

$$\frac{C_D}{C_L^{1/2}} = \frac{C_{D0} + KC_L^2}{C_L^{1/2}} = \frac{C_{D0}}{C_L^{1/2}} + KC_L^{3/2}$$

$$\Rightarrow \quad \frac{d\left(C_D / C_L^{1/2}\right)}{dC_L} = -\frac{1}{2}\frac{C_{D0}}{C_L^{3/2}} + \frac{3}{2}C_L^{1/2}$$

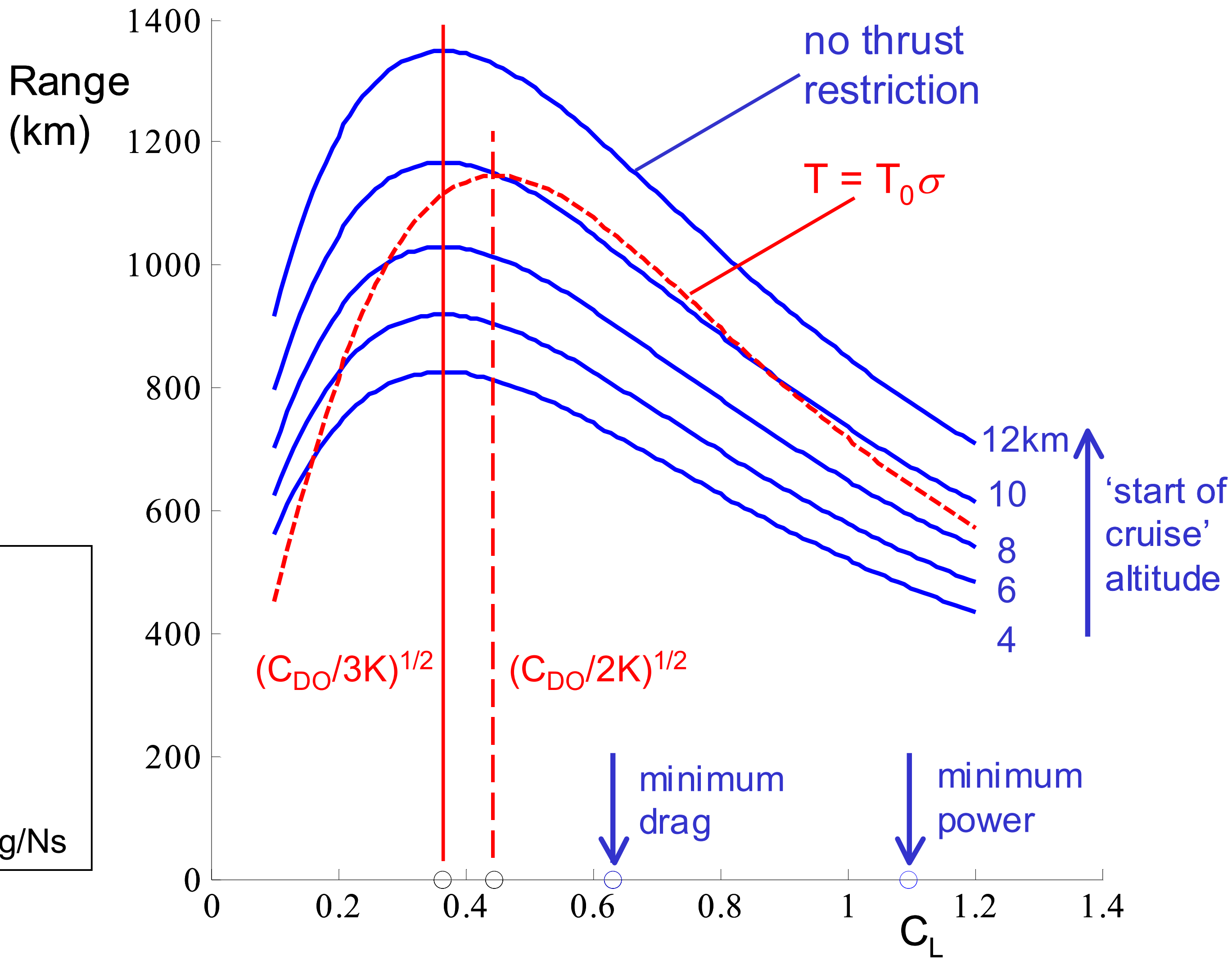
- so at the minimum point $C_{D0} = 3KC_L^2$

$$\Rightarrow \quad \boxed{C_{D_{\max R}} = \frac{4}{3}C_{D0}\,, \quad C_{L_{\max R}} = \sqrt{C_{D0}/3K}}$$

cruise-climb with unrestricted thrust T

- cruise conditions fixed by start altitude and weight
  - C_L and C_D obtained from above
    $\rightarrow$ substitute with W_1 and σ_1 into
    $$V\frac{C_L}{C_D} = \sqrt{\frac{W_1}{\frac{1}{2}\sigma_1\rho_0 S}}\frac{C_L^{1/2}}{C_D}$$
  - then substitute into Breguet Equation to find range

Jet Cruise-Climb Range



W_1 =100 kN
W_2 = 60 kN
S = 50 m^2
C_{D0} = 0.02
K = 0.05
T_0 = 20 kN
f = 0.0001 kg/Ns

Maximum Range – Constant Altitude (1)

- start with (initial) thrust
  $T_1 = T_0\sigma$ as before
  - altitude (and hence σ) undefined

$$R = \sqrt{\frac{8}{\rho S}}\,\frac{1}{fg}\,\frac{C_L^{1/2}}{C_D}\left(W_1^{1/2} - W_2^{1/2}\right)$$

- use the thrust/drag relation to eliminate ρ

$$\rho = \rho_0\sigma = \rho_0\frac{T_1}{T_0} = \frac{\rho_0}{T_0}D_1 = \frac{\rho_0}{T_0}\frac{D_1}{L_1}W_1 = \frac{\rho_0}{T_0}\frac{C_D}{C_L}W_1$$

$$\Rightarrow \quad \boxed{R = \sqrt{\frac{8T_0}{\rho_0 S W_1}}\,\frac{1}{fg}\,\frac{C_L}{C_D^{3/2}}\left(W_1^{1/2} - W_2^{1/2}\right)}$$

- for maximum range require $C_D^{3/2}/C_L$ to be a minimum
  $\rightarrow$ *same as cruise-climb result !*

Maximum Range – Constant Altitude (2)

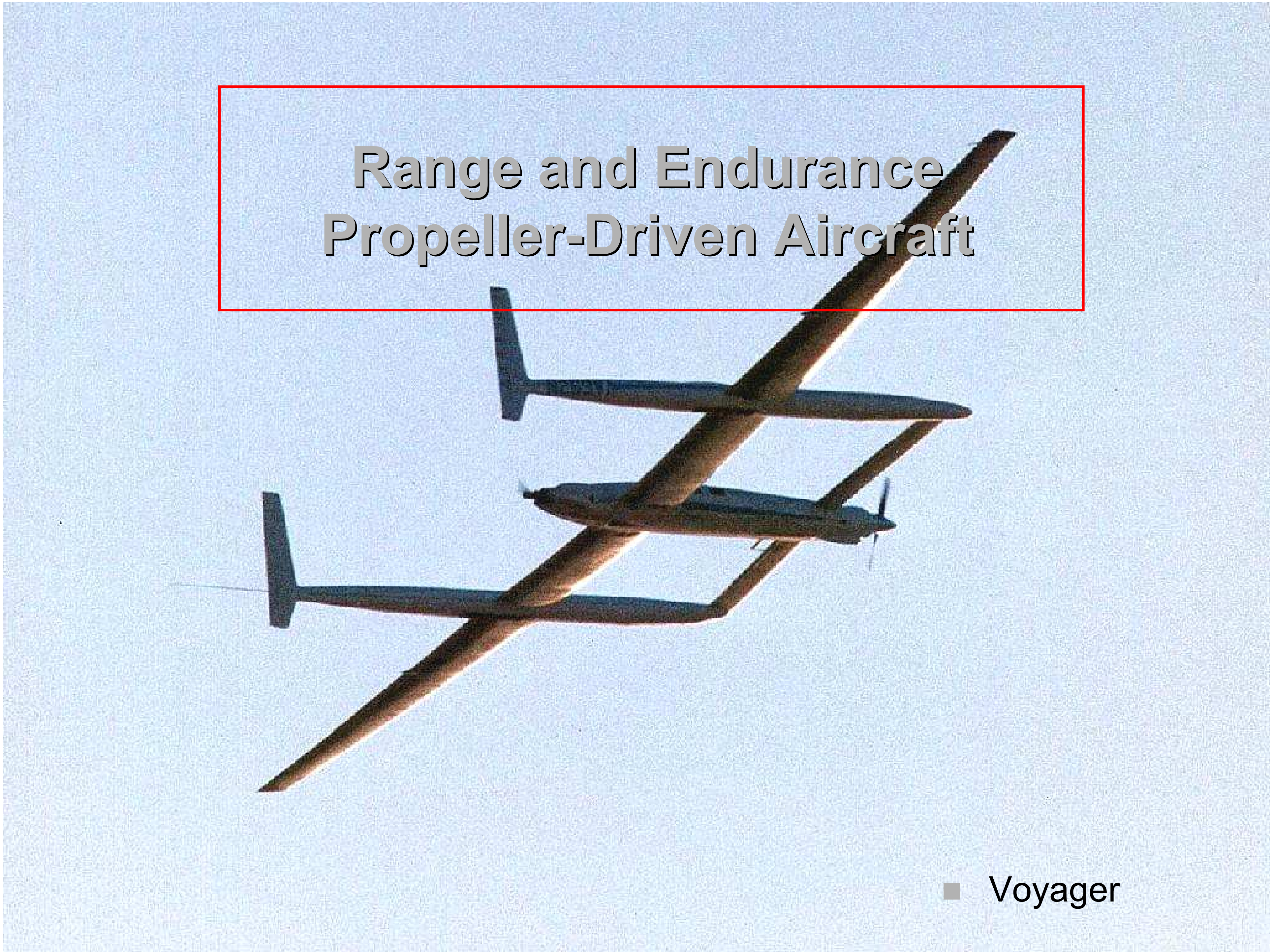
- alternatively specify cruise altitude (and hence σ) and let thrust vary as required
  - altitude (and hence ρ) are constant

$$\boxed{R = \sqrt{\frac{8}{\rho S}}\frac{1}{fg}\frac{C_L^{1/2}}{C_D}\left(W_1^{1/2} - W_2^{1/2}\right)}$$

- for maximum range require $C_D / C_L^{\ 1/2}$ to be a minimum

  $\rightarrow$ *again the same as cruise-climb result !*

Range and Endurance
Propeller-Driven Aircraft



- Voyager

Breguet Range Equation – Propeller-Driven Aircraft

- **"rate at which fuel is burnt**
  = rate at which aircraft weight is reduced"
- define **specific fuel consumption** as
  - f **= mass of fuel burnt per unit of power per second**
  - consistent units are $kg/W.s$
  - but often given in terms of $kg/kW.hr$ so don't forget to convert!
- for power P and weight W the basic Breguet equation is

$$\boxed{\frac{dW}{dt} = -fgP}$$

  - a differential equation (in units of N/s)

Integration of Breguet Equation - Endurance

- power delivered by propeller
  - where η = propeller efficiency

$$\boxed{\eta P = DV}$$

$$\Rightarrow \quad \frac{dW}{dt} = -fg\frac{DV}{\eta} \quad \Rightarrow \quad \boxed{dt = -\frac{\eta}{fgV}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that C_L/C_D , f and V remain *constant*
  - same as jet cruise-climb
- integrate as before from W_1 to W_2 to obtain the endurance E

$$\boxed{E_{prop} = t_2 - t_1 = t_{12} = \frac{\eta}{fg}\frac{1}{V}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

Maximum Endurance

- for maximum range we require $(C_L/C_D)/V$ to be a maximum
  - or VC_D/C_L to be a minimum

$$E_{prop} = \frac{\eta}{fg}\left(\frac{1}{V}\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)$$

- expanding this term we obtain

$$V\frac{C_D}{C_L} = \frac{VD}{L} = \frac{VD}{W}$$

- $D \times V$ is the power required to overcome drag
  $\rightarrow$ **maximum endurance** achieved at **minimum power speed** for propeller-driven aircraft
  - very much slower than for jet aircraft
  - compare with glider performance

Integration of Breguet Equation - Range

- increment in distance dS at velocity V is given by

$$\boxed{dS = Vdt = -\frac{\eta}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- can then integrate from start weight W_1 to end weight W_2 to obtain the range R

$$\boxed{R_{prop} = S_2 - S_1 = \frac{\eta}{fg}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- **maximum range** achieved at **minimum drag speed** for propeller-driven aircraft
  - much slower than for jet aircraft