Recursion

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What is Recursion?

Recursion means for a function or method to call itself.

A typical example is computing factorials:

```
n! = n * (n-1)!    when n>0
0! = 1
```

n factorial is (recursively) defined using n-1 factorial.

```
n! = n*(n-1)!
          (n-1)! = (n-1)*(n-2)!
                             .
                             .
                             2! = 2 * 1!
                                       1! = 1*0!
                                              0! = 1
```

Recursive factorial(n)

We can write a function that computes factorials by calling itself to compute factorial of a smaller number:

```
def factorial(n: int):
    if n <= 1:
        return 1
    return n * factorial(n-1)
```

Suppose we call this function to compute factorial(4). What statements will be executed?

factorial(n) execution trace

```
long result = factorial( 4 );
```

call

```
factorial( 4 ) {
    return 4 * factorial( 3 );
}
```

call

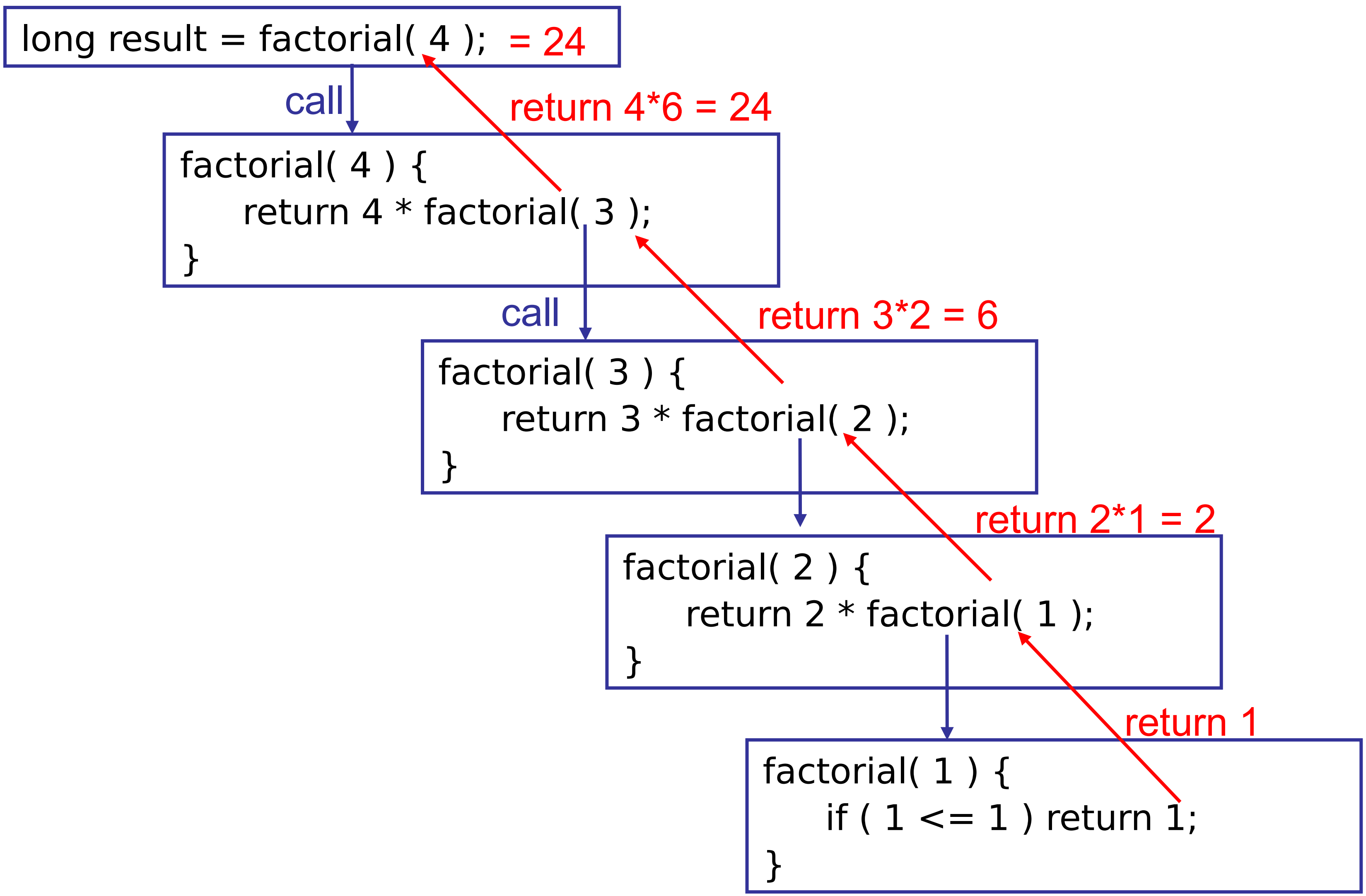
```
factorial( 3 ) {
    return 3 * factorial( 2 );
}
```

call

```
factorial( 2 ) {
    return 2 * factorial( 1 );
}
```

```
factorial( 1 ) {
    if ( 1 <= 1 ) return 1;
}
```

factorial(n) return trace



Recursion Must Eventually Stop

- Recursion must guarantee to stop eventually (no infinite calls)
- Recursion should not change any state variable that other levels of recursion will use, except by design.

```
def factorial( n ):
    if n <= 1:   return 1
    return n * factorial(n-1)
```

The test n <= 1 guarantees that factorial( ) will eventual stop using recursion.

Wrong:

```
def factorial( n ):
    if n == 1:   return 1
    return n * factorial(n-1)
```

What happens if factorial(0) is called?

Base Case

The case where recursion stops is called the **base case**.

factorial(n):   base case is n == 1

but you should also test for n < 1

Recursive Sum

`long sum(int n)` - compute sum of 1 to n

Recursion: n + { sum of 1 to n-1 }

Code for recursive sum

Complete this code

```
def sum(n):
    """Return sum of 1 + ... + n."""
    # what is the base case?
    if n <= 0:
        return 0
    # what is the recursive step?
    return n + sum(n-1)
```

Designing Recursion

1) Discover a **pattern** for recursion:
   - solve a small problem by hand
   - observe how you break down the problem
2) Determine the **base case(s)** when recursion stops.
3) Termination criteria: what can you test to **guarantee** recursion will stop?
4) Construct an expression for the recursive step.

Designing Recursion Example

`sum(n) = 1 + 2 + 3 + ... + n`

1) Discover a pattern for recursion:

   - sum(n) = (1 + 2 + ...+ n-1) + n = sum(n-1) + n

2) **base case**: sum(n) = 0 for any n <= 0.
   Note: to *guarantee* recursion will always stop we need to consider case n < 0, too! Not just n == 0.
   If n < 0 either throw exception or return 0.

3) Guarantee Termination? Yes - each time we reduce the value of the parameter (n) by 1,
   so eventually we must have n <= 0.

Does Recursion Provide Insight?

For some problems, recursion makes the solution **easier** to **understand** and **implement**.

Recursion provides *insight* into the solution.

For other problems, it provides no insight.

Only use recursion when it makes the problem easier to understand or solve.

Famous examples: Quicksort algorithm. Knight's tour.

Does Recursive Sum Offer Insight?

```
sum(n) = n + sum(n-1)    if n > 0
```

My opinion: No.

1 + 2 + ... + n ==> looks like iteration (a loop)

Sum Elements in a List

What is wrong with this code?

```
def sum( lst ):
    if not lst:  # empty list
        return 0
    last_element = lst.pop()
    return last_element + sum(lst)
```

It computes the correct result, but is a poor design.

>> We should not modify the `lst` parameter.

Helper Function

Sometimes you need a "helper function" with extra parameters in order to perform recursion.

To sum a list, without modifying the list, use a helper function that has a param for the last element to sum.

```
def sum(lst):  return sumTo(lst, len(lst)-1)
```

```
def sumTo(lst, last_index):
  """sum elements 0 up to last_index."""
  if last_index < 0:
      return 0      # nothing to sum
  return lst[last_index] \
                   + sumTo(lst, last_index-1)
```

Learn more about Helper Functions

Big Java, Chapter 13 (*Recursion*) has a section on helper methods.

Recursion uses more memory

- We can easily sum 1 to 10,000,000 using a loop.
  but recursive sum will fail with "out of memory" error.

- Why?
  - each function call creates a ***stack frame*** to store information about the invocation (parameters, local vars, saved register values) and a return value.
  - The stack frames consume memory.
  - Eventually, recursive calls may fill all the stack space.

- For the curious: read about "tail recursion"
  - avoids creating stack frames in special cases

Backtracking

- In some problems, an attempt to find a solution using recursion fails.
- You have to "undo" or "backtrack" some recursive steps and try a different solution.

References

Recursion in Python on RealPython.com

`https://realpython.com/python-recursion/`

`http://codingbat.com` - programming problems using recursion. Recursion-1 set is easy, Recursion-2 is more challenging & use backtracking. Only available for Java.

Big Java, Chapter 13 *Recursion.*