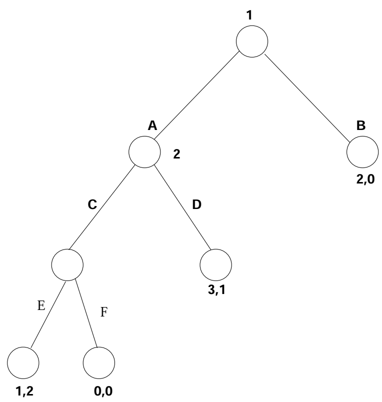
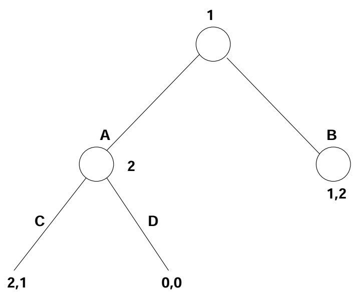
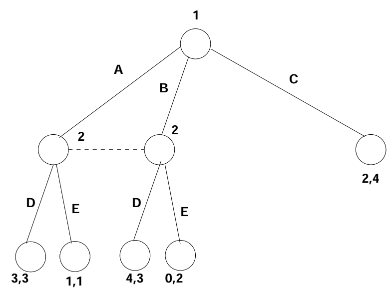
Problem set 6
MTL 763 (Introduction to Game Theory)

1. Consider a Centipede Game where there are two players, 1 and 2. The players each start with 1 dollar in front of them. They alternate saying "stop" or "continue," starting with player 1. When a player says "continue," 1 dollar is taken by a referee from her pile and 2 dollars are put in her opponent's pile. As soon as either player says "stop," play is terminated, and each player receives the money currently in her pile. Alternatively, play stops if both players' piles reach 100 dollars. Represent this game using game tree representation. Find subgame perfect Nash equilibrium using backward induction procedure.

2. For games shown in Figures 7,8, and 9, write down the terminal histories, proper subhistories, information sets, strategic form game representation.







3. Consider the following game: There are 2 steps. In Step 1 Player 1 chooses between throwing 1 unit of his own payoff (strategy T) or not (strategy N). Observing his action in Step 2 they simultaneously play the following game:

P1 \ P2	L	R
U	(4, 2)	(1, 1)
D	(1, 1)	(2,4)

Represent this scenario as an extensive form game and find all the Nash equilibria and subgame perfect Nash equilibria of the game.

4. Player 1 and Player 2 play the following two step game: In the first step they play the matching pennies game where they simultaneously choose between H and T each. If there is a match Player 1 is the winner and they play the following simultaneous game in the second step:

P1 \ P2	L	R
U	(5, 1)	(0, 0)
D	(0, 0)	(3,1)

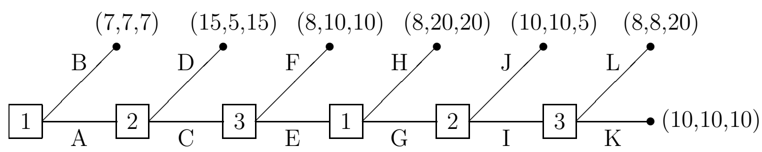
If there is no match Player 2 is the winner and they play the following simultaneous game in the second step:

P1 \ P2	L	R
U	(1, 5)	(0, 0)
D	(0, 0)	(1,3)

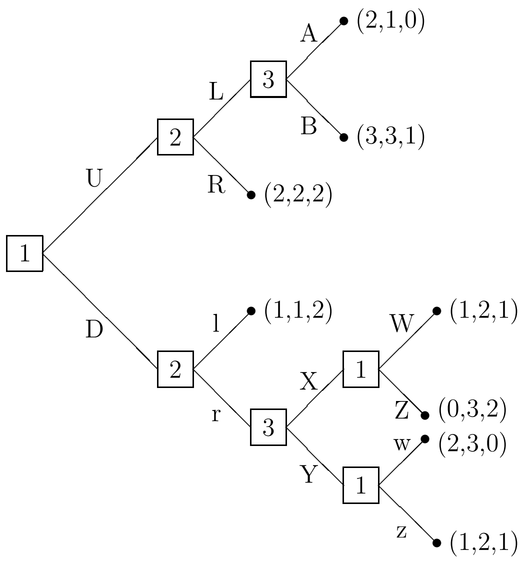
Represent this scenario as an extensive form game and find all the Nash equilibria and subgame perfect Nash equilibria of the game.

5. Solve the following games with backwards induction. Give the equilibrium strategies as well as equilibrium payoffs.

(a)



(b)



6. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the Councils other two members are Mr. Jones and Mrs.

Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either for or against the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally without mentioning who voted what. If the secretary reports that both slips say for, then the tax in-crease is implemented and the game is over. If both vote against, then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one for and one against, then Ms. Tuttle has to vote. If she votes for, then the tax increase is implemented, and if she votes against, then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Joness payoff is 1. As for Ms. Tuttle, she prefers to have a tax in-creasebelieving that it will provide the funds to improve the citys schoolsbut would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her for vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted).

i. Write down the extensive form of the game composed of Ms. Tuttle, Mr.Jones, and Mrs. Doubtfire using a game tree.
ii. List out strategies and information sets of all the players.
iii. Find all the Nash equilibria in corresponding simultaneous form game. Are they sub-game perfect Nash equilibria?

7. Apple decides to develop the new iPhone (whatever the new name is, 5s, 6, ∞ etc.) with radically new software which allows for faster and better applications (apps). These apps are, however, still not developed by app developers. If Apple does not develop the new iPhone, then all companies make zero profit in this emerging market. If, instead, the new iPhone is introduced, then company 1 (the leader in the app industry) gets to decide whether to develop apps or not that are compatible with the new iPhones software. If company 1 decides to develop the app, the followers (firm 2 (F2) and firm 3 (F3)) simultaneously decide whether to develop apps (D) or not develop (ND) and then all the companies receive payoffs given by following matrix

F2 \ F3	D	ND
D	(10, 4, 4, 4)	(6, 2, 2, 0)
ND	(6, 2, 0, 2)	(-4, -2, 0, 0)

If company 1 decides not to develop the app, the followers (F2 and F3) simultaneously decide whether to develop apps (D) or not develop (ND) and then all the companies receive payoffs given by following matrix

F2 \ F3	D	ND
D	(6, 0, 2, 2)	(-4, 0, -2, 0)
ND	(-4, 0, 0, -2)	(-6, 0, 0, 0)

The first two components in above matrices represent the payoffs of apple and company 1 (leader firm), respectively. (i) Represent above situation using a game tree, (ii) Find all the sub-game perfect Nash equilibria of the game.