Problem set 5
MTL 763 (Introduction to Game Theory)

Q.1 Consider a network game with $n = 3$ or $n = 4$, where the utility function of player i at network g is given by

$$u_i(g) = R_i(g) - c \cdot d_i(g),$$

where $R_i(g)$ is the number of other players that can be reached from player i at network g, and $d_i(g)$ is the degree (number of neighbors) of node i. What are the pairwise stable networks and efficient networks for all values of c.

Q.2 Consider a network game where the value of network is the total number edges which is divided equally among the players. What are the pairwise stable and efficient networks?

Q.3 Construct a network game which has Nash stable, pairwise stable, pairwise Nash stable, and efficient networks.

Q.4 Consider a set of all feasible 4 nodes networks given by Figure 1 What are the

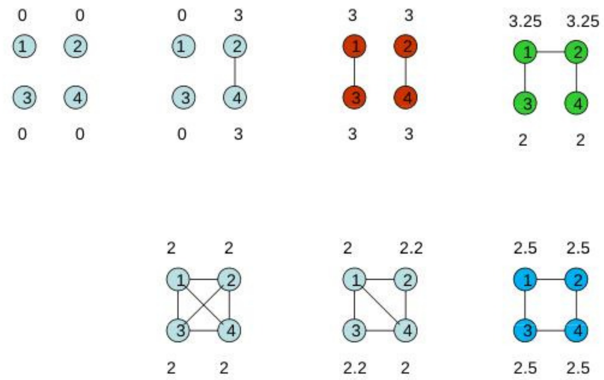

Figure 1: Networks in a Four Person Society.

pairwise stable, Pareto efficient, and efficient networks?

Q.5 Construct a network game possibly with 3 or 4 nodes which has unique pairwise stable which can be reached via improving deviations.

Q.6 Consider the symmetric connections model with three players

- What are the Nash stable and pairwise Nash stable networks for all values of c?
- Show that the strategy profiles corresponding to a Nash stable network are Nash equilibrium in link-announcement game.

Q.7 Consider a 4 link network in a 4-player network game where the degree of each node is 2. List out the strategy profiles in a link announcement game which generate this network.

Q.8 Consider an n-player network formation game. The cost incurred by a player from direct link is $\alpha > 0$. Players also incur cost due to the distance from other nodes. Then, the cost incurred by player i at network g is given by

$$c_i(g) = \alpha \cdot K_i(g) + \sum_{j=1}^{n} d_{i,j}(g),$$

where $K_i(g)$ denote the number of direct links connected to node i and $d_{i,j}(g)$ denote the shortest distance between i and j at network g.

(a) Find all values of α for which complete network is pairwise stable.
(b) Find all values of α for which star network is pairwise stable.
(c) For all values of α find the PoA in case of 3 players.