Problem set 4
MTL 763 (Introduction to Game Theory)

We consider the examples of two types of network routing games, where $i)$ there are infinitely many players each controlling a negligible amount of traffic, $ii)$ there are finitely many players controlling a non-negligible amount of traffic.

Q.1 In the Pigou example discussed in the class, consider the following combination of the costs for upper and lower edge:

(i) $c_u(x) = x$, $c_l(x) = x$.

(ii) $c_u(x) = \frac{1}{2}$, $c_l(x) = \frac{1}{2}$.

(iii) $c_u(x) = 1$, $c_l(x) = x$.

(iv) $c_u(x) = 1$, $c_l(x) = 1$.

Find the equilibrium and optimal flows, and PoA of the game for all the cases.

Q.2 Consider a network with three directed paths that can be used by large population in order to reach destination t from source s. The paths are as follows:
$i)$ $s \xrightarrow{x} v \xrightarrow{1} t$,
$ii)$ $s \xrightarrow{x} t$,
$iii)$ $s \xrightarrow{1} w \xrightarrow{x} t$.
The number above the arrow represent the cost of the corresponding edge. Find the equilibrium and optimal flows, and PoA of the game.

Q.3 Consider the Braess's networks. The costs for the different edges are given below:
$i)$ $s \xrightarrow{2x+3} v$, $ii)$ $v \xrightarrow{x} t$
$iii)$ $s \xrightarrow{x} w$, $iv)$ $w \xrightarrow{3x+4} t$ $v)$ $v \xrightarrow{0} w$.
Find the equilibrium and optimal flows, and PoA of the game.

Q.4 Consider the network shown in Figure 1. We assume that there are two players both with source s and sink t. Player 1 and player 2 want to route 1 and 2 units of traffic respectively on a particular path. Show that there is no equilibrium flow in this network game.

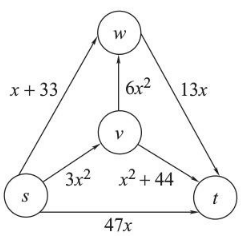

Figure 1: Bi-directed triangle network.