

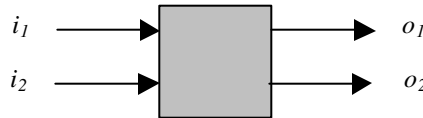
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## Problem H

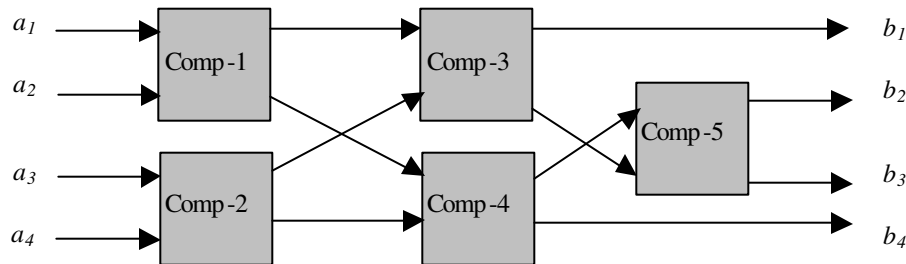
### Professor Monotonic's Networks

Input: sort.in

Professor Monotonic has been experimenting with comparison networks, each of which includes a number of two-input, two-output comparators. A comparator, as illustrated below, will compare the values on its inputs,  $i_1$  and  $i_2$ , and place them on the outputs,  $o_1$  and  $o_2$ , so that  $o_1 \leq o_2$  regardless of the relationship between the input values.



A comparison network has  $n$  inputs  $a_1, a_2, \dots, a_n$  and  $n$  outputs  $b_1, b_2, \dots, b_n$ . Each of the two inputs to a comparator is either connected to one of the network's  $n$  inputs or connected to the output of another comparator. Each of the two outputs from a comparator is either connected to one of the network's  $n$  outputs or is connected to the input of another comparator. A graph of the interconnections of comparators must be acyclic. The illustration below shows a comparison network with four inputs, four outputs, and five comparators.



In operation, the network's inputs are applied and the comparators perform their functions. Of course a comparator cannot operate until both of its inputs are available. Assuming a comparator requires one unit of time to operate, this sample network will require three units of time to produce its outputs. Comp-1 and Comp-2 operate in parallel, as do Comp-3 and Comp-4. Comp-5 cannot operate until Comp-3 and Comp-4 have completed their work.

Professor Monotonic needs help in determining which proposed comparison networks are also sorting networks, and how long they will take to perform their task. A sorting network is a comparison network for which the outputs are monotonically increasing regardless of the input values. The example above is a sorting network, since for all possible input values the output values will have the relation  $b_1 \leq b_2 \leq b_3 \leq b_4$ .

### Input

The professor will provide a description of each comparison network to be examined. Each description will begin with a line containing values for  $n$  (the number of inputs) and  $k$  (the number of comparators). These values satisfy  $1 \leq n \leq 12$  and  $0 \leq k \leq 150$ . This is followed by zero or more non-empty lines, each containing at most 15 pairs of comparator inputs. The source of the input to each comparator is given by a pair of integers  $i$  and  $j$ . Each of these specifies either the subscript of a network input that is input to the comparator (that is,  $a_i$  or  $a_j$ ), or the corresponding output of a preceding comparator.

The outputs of a comparator are numbered the same as its inputs (in other words, if the comparator's inputs are  $i$  and  $j$ , the corresponding outputs are also labeled  $i$  and  $j$ ). The order in which these pairs appear is significant, and affects the order in which the comparators operate. If two pairs contain an integer in common, the order of the

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corresponding comparators in the network is determined by the order of the pairs in the list. For example, consider the input data for the example shown:

```
4 5
1 2 3 4 1 3
2 4 2 3
```

This indicates there will be four input values and five comparators in the network. The first comparator (Comp-1) will receive its input values from network inputs  $a_1$  and  $a_2$ . The second comparator (Comp-2) will receive its input values from network inputs  $a_3$  and  $a_4$ . The third comparator (Comp-3) will receive its first input from the first output of Comp-1, and will receive its second input from the first output of Comp-2. Similarly, the fourth comparator (Comp-4) will receive its first input from the second output of Comp-1, and will receive its second input from the second output of Comp-2. Finally, the fifth comparator (Comp-5) will receive its first input from the first output of Comp-4, and will receive its second input from the second output of Comp-3. The outputs  $b_1, b_2, \dots, b_n$  are taken from the first output of Comp-3, the first output of Comp-5, the second output of Comp-5, and the second output of Comp-4, respectively.

A pair of zeros will follow the input data for the last network.

## Output

For each input case, display the case number (cases are numbered sequentially starting with 1), an indication of whether the network is a sorting network or not, and the number of time units required for the network to operate (regardless of whether it is a sorting network or not).

### Sample Input

```
4 5
1 2 3 4 1 3
2 4 2 3
8 0
3 3
1 2 2 3 1 2
0 0
```

### Output for the Sample Input

```
Case 1 is a sorting network and operates in 3 time units.
Case 2 is not a sorting network and operates in 0 time units.
Case 3 is a sorting network and operates in 3 time units.
```