

## 108 Maximum Sum

### Background

A problem that is simple to solve in one dimension is often much more difficult to solve in more than one dimension. Consider satisfying a boolean expression in conjunctive normal form in which each conjunct consists of exactly 3 disjuncts. This problem (3-SAT) is NP-complete. The problem 2-SAT is solved quite efficiently, however. In contrast, some problems belong to the same complexity class regardless of the dimensionality of the problem.

### The Problem

Given a 2-dimensional array of positive and negative integers, find the sub-rectangle with the largest sum. The sum of a rectangle is the sum of all the elements in that rectangle. In this problem the sub-rectangle with the largest sum is referred to as the *maximal sub-rectangle*. A sub-rectangle is any contiguous sub-array of size  $1 \times 1$  or greater located within the whole array. As an example, the maximal sub-rectangle of the array:

0	-2	-7	0
9	2	-6	2
-4	1	-4	1
-1	8	0	-2

is in the lower-left-hand corner:

9	2
-4	1
-1	8

and has the sum of 15.

### Input and Output

The input consists of an  $N \times N$  array of integers. The input begins with a single positive integer  $N$  on a line by itself indicating the size of the square two dimensional array. This is followed by  $N^2$  integers separated by white-space (newlines and spaces). These  $N^2$  integers make up the array in row-major order (i.e., all numbers on the first row, left-to-right, then all numbers on the second row, left-to-right, etc.).  $N$  may be as large as 100. The numbers in the array will be in the range  $[-127, 127]$ .

The output is the sum of the maximal sub-rectangle.

### Sample Input

```
4
0 -2 -7 0 9 2 -6 2
-4 1 -4 1 -1
8 0 -2
```

### Sample Output

```
15
```