

# Problem H

## Prefix Codes

Input: Standard Input  
Output: Standard Output

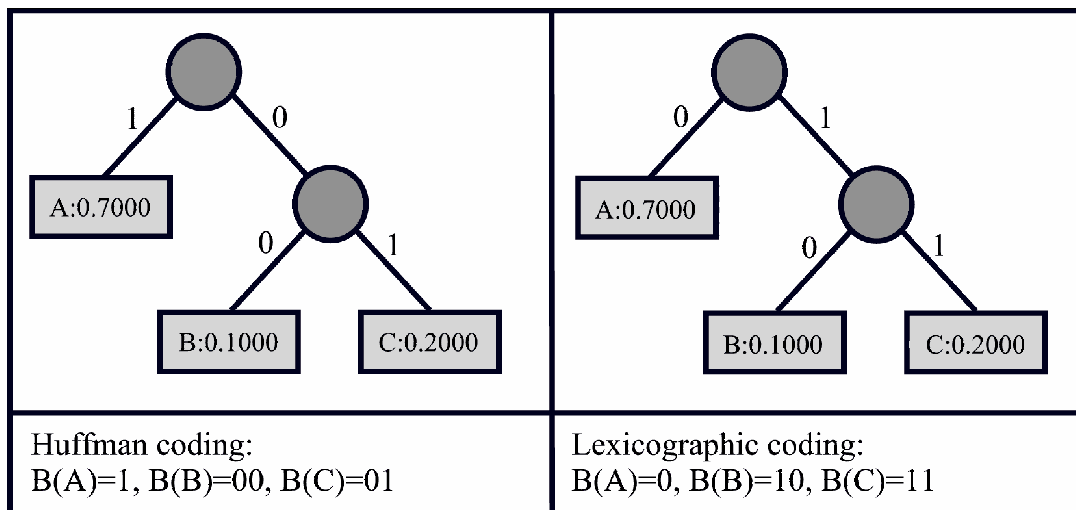
Given an alphabet  $S$ , and a probability  $\text{Prob}(a)$  for each  $a \in S$ , a binary prefix code represents each  $a$  in  $S$  as a bit string  $B(a)$ , such that  $B(a_1)$  is not a prefix of  $B(a_2)$  for any  $a_1 \neq a_2$  in  $S$ .

Huffman's algorithm constructs a binary prefix code by pairing the two least probable elements of  $S$ ,  $a_0$  and  $a_1$ .  $a_0$  and  $a_1$  are given codes with a common (as yet to be determined) prefix  $p$  and differ only in their last bit:  $B(a_0) = p0$  while  $B(a_1) = p1$ .  $a_0$  and  $a_1$  are removed from  $S$  and replaced by a new element  $b$  with  $\text{Prob}(b) = \text{Prob}(a_0) + \text{Prob}(a_1)$ .  $b$  is an imaginary element standing for both  $a_0$  and  $a_1$ . The Huffman code is computed for this reduced  $S$ , and  $p$  is set equal to  $B(b)$ . This reduction of the problem continues until  $S$  contains one element  $a$  represented by the empty string; that is, when  $S = \{a\}$ ,  $B(a) = \epsilon$ .

Huffman's code is optimal in that there is no other prefix code with a shorter average length defined as

$$\sum_{a \in S} \text{Prob}(a) \times |B(a)|$$

One problem with Huffman codes is that they don't necessarily preserve any ordering that the elements may have. For example, suppose  $S = \{A, B, C\}$  and  $\text{Prob}(A) = 0.7$ ,  $\text{Prob}(B) = 0.1$ ,  $\text{Prob}(C) = 0.2$ . A Huffman code for  $S$  is  $B(A) = 1$ ,  $B(B) = 00$ ,  $B(C) = 01$ . The lexicographic ordering of these strings is  $B(B)$ ,  $B(C)$ ,  $B(A)$  [i.e. 00,01,1], so the coding does not preserve the original order  $A$ ,  $B$ ,  $C$ . Therefore, algorithms like binary search might not work as expected on Huffman-coded data.



Given an ordered set  $S$  and  $\text{Prob}$ , you are to compute an ordered prefix code - one whose lexicographic order preserves the order of  $S$ .

## Input

Input consists of several data sets. Each set begins with  $0 < n \leq 100$ , the number of elements in  $S$ .  $n$  lines follow; the  $i$ -th line gives the probability of  $a_i$ , the  $i$ -th element of  $S$ . Each probability is given as **0.dddd** (that is, with exactly four decimal digits). The probabilities sum to **1.0000** exactly. A line containing **0** follows the last data set.

## Output

For each data set, compute an optimal ordered binary prefix code for  $S$ . The output should consist of one line giving the average code length, followed by  $n$  lines, with the  $i$ -th line giving the code for the  $i$ -th element of  $S$ . If you have solved the problem, these  $n$  lines will be in lexicographic order. If there are many optimal solutions, choose any one.

Output an empty line between cases.

### Sample Input

```
3
0.7000
0.1000
0.2000
3
0.7000
0.2000
0.1000
0
```

### Output for Sample Input

```
1.3000
0
10
11
1.3000
0
10
11
```