Furuta Pendulum

by

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Introduction:

For this project the object was to create a furuta pendulum, an inverted pendulum that relies on sensors and feedback loops to keep a pendulum that is suspended in the air upright. This project is great for controls systems to understand topics like: state space equations, transfer functions, Euler-Lagrange equations, sensors, feedback loops, etc.

Modeling:

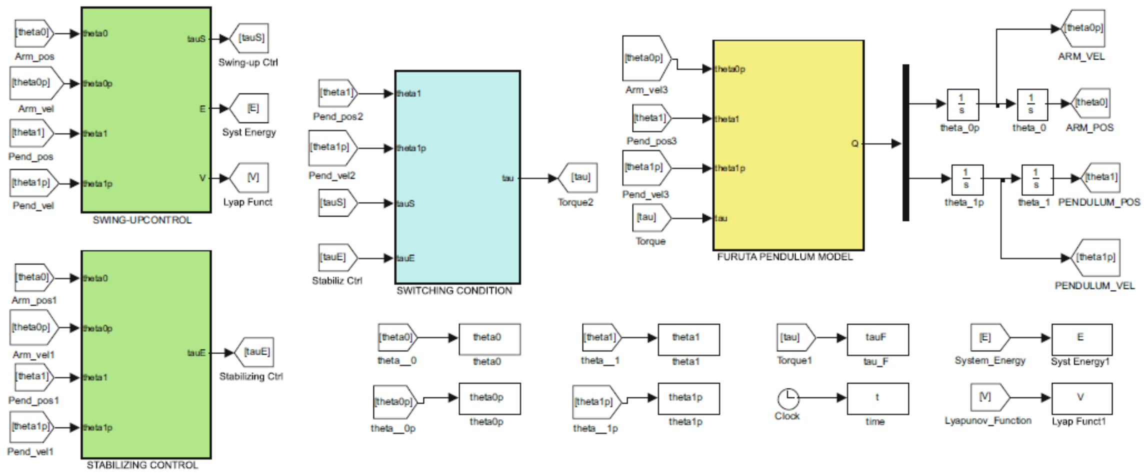
Modeling for this project started with creating a mathematical model which is based on the Euler Lagrange theorem. The standard Euler-Lagrange theorem is shown below.

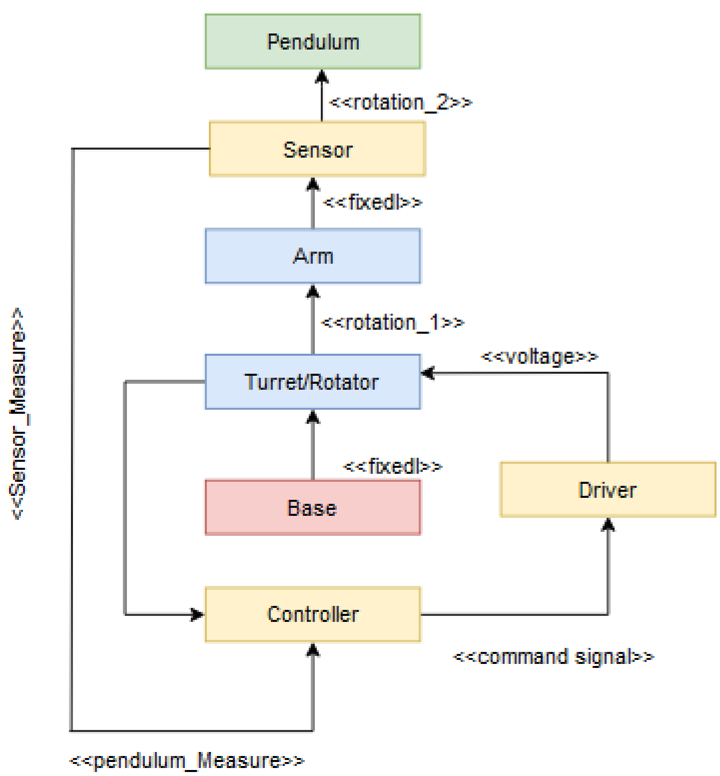
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_0} - \frac{\partial L}{\partial \theta_0} = \tau,$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = 0.$$

By applying boundary conditions and stating the parameters of the variables, a full mathematical model can be constructed which takes into account things like state space functions of the pendulum and the arm

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = F, \tag{15.9}$$
$$M(q) = \begin{bmatrix} I_0 + m_1(L_0^2 + l_1^2\sin^2(\theta_1)) & m_1 l_1 L_0 \cos(\theta_1) \\ m_1 l_1 L_0 \cos(\theta_1) & J_1 + m_1 l_1^2 \end{bmatrix},$$
$$C(q,\dot{q}) = \begin{bmatrix} \frac{1}{2}m_1 l_1^2 \dot{\theta}_1 \sin(2\theta_1) & -m_1 l_1 L_0 \dot{\theta}_1 \sin(\theta_1) + \frac{1}{2}m_1 l_1^2 \dot{\theta}_0 \sin(2\theta_1) \\ -\frac{1}{2}m_1 l_1^2 \dot{\theta}_0 \sin(2\theta_1) & 0 \end{bmatrix},$$
$$g(q) = \begin{bmatrix} 0 \\ -m_1 l_1 g \sin(\theta_1) \end{bmatrix}, \quad F = \begin{bmatrix} \tau \\ 0 \end{bmatrix}, \quad q = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}.$$

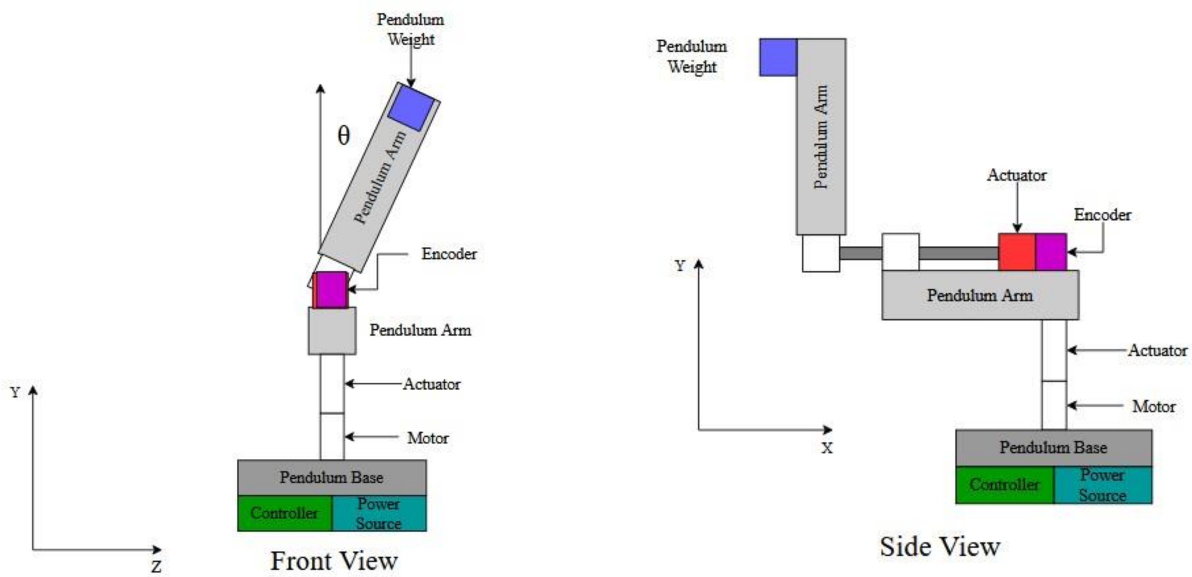
The equations shown come from chapter 15 in automatic controls with experiments.





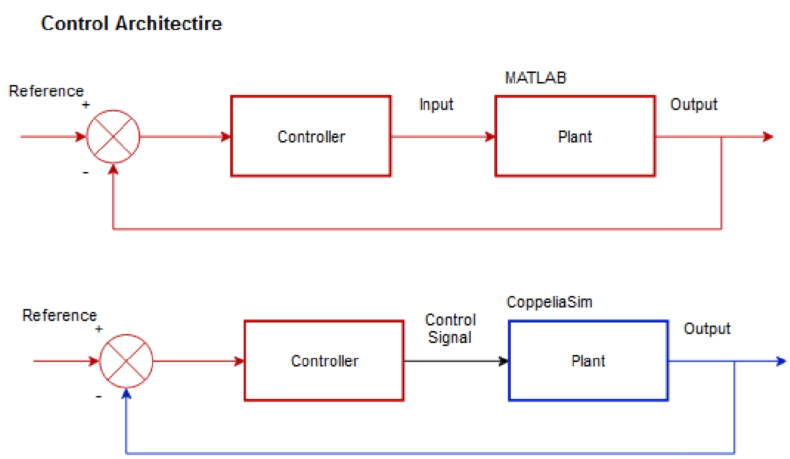
Sensor Calibration:

Operational Viewpoint:



This operational viewpoint shows the object hierarchy and how all the components interact with each other. The Logical and Functional Viewpoint also show how

Controller Design and Simulations:



This controller design shows how the feedback loop and sensor controls the model and simulations. As a note, the red lines and components represent the communication in MATLAB while the blue represents the CoppeliaSim.

Mathematical Model and Equations Continued:

$$y_p = l\cos(\theta) \qquad x_p = x - l\cos(\theta)$$

$$\dot{x}_p = \dot{x} - l\dot{\theta}\cos(\theta) \qquad \dot{y}_p = -l\dot{\theta}\sin(\theta)$$

$$L = T - V$$

$$L = \frac{(m+m)\dot{x}^2}{2} + \frac{ml^2\theta^2}{2} - ml\dot{\theta}\dot{x}cos(\theta) - mglcos(\theta)$$

$$T = \frac{m\dot{x}^2}{2} + \frac{m(\dot{x}_p^2 + y_p^2)}{2}$$

$$T = \frac{m\dot{x}^2}{2} + \frac{m[\dot{x}^2 - 2l\dot{\theta}\dot{x}cos(\theta) + l^2\dot{\theta}^2]}{2}$$

$$T = \frac{(m+m)\dot{x}^2}{2} + \frac{ml^2\dot{\theta}^2}{2} - ml\dot{\theta}\dot{x}\cos(\theta)$$

$$V = mgy_p$$

$$V = mglcos(\theta)$$

$$Qi = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}i}\right) - \left(\frac{\partial L}{\partial qi}\right)$$