MTH 251: Week 2 lab write up

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February 2, 2022

Lab activity 1.2.4

Find the difference quotient of $f(x)$ when $f(x) = x^3$.
We proceed as demonstrated in the lab manual; assuming that $h \neq 0$ we have

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^3 - x^3}{h} \\
&= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
&= \frac{3x^2h + 2xh^2 + h^3}{h} \\
&= \frac{h(3x^2 + 2xh + h^2)}{h} \\
&= 3x^2 + 2xh + h^2
\end{aligned}$$

Lab activity 2.3.4

Use the definition of the derivative to find $f'(x)$ when $f(x) = x^{\frac{1}{4}}$.
Using the definition of the derivative, we have

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{(x+h)^{1/4} - x^{1/4}}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^{1/4} - x^{1/4}}{h} \cdot \frac{((x+h)^{1/4} + x^{1/4})((x+h)^{1/2} + x^{1/2})}{((x+h)^{1/4} + x^{1/4})((x+h)^{1/2} + x^{1/2})} \\
&= \lim_{h\to 0} \frac{(x+h) - x}{h((x+h)^{1/4} + x^{1/4})((x+h)^{1/2} + x^{1/2})} \\
&= \lim_{h\to 0} \frac{1}{((x+h)^{1/4} + x^{1/4})((x+h)^{1/2} + x^{1/2})} \\
&= \frac{1}{(x^{1/4} + x^{1/4})(x^{1/2} + x^{1/2})} \\
&= \frac{1}{(2x^{1/4})(2x^{1/2})} \\
&= \frac{1}{4x^{3/4}} \\
&= \frac{1}{4}x^{-3/4}
\end{aligned}$$

Note: the key observation here is that

$$\begin{aligned}
a^4 - b^4 &= (a^2 - b^2)(a^2 + b^2) \\
&= (a-b)(a+b)(a^2 + b^2),
\end{aligned}$$

with

$$a = (x+h)^{1/4}, \qquad b = x^{1/4},$$

which allowed us to rationalize the denominator.