Today's class? → Count the no. of iterations.

- TC & SC ✗
- Asymptotic Analysis ✗
- Big O notation ✗
- TLE. ✗

TC → 2

Quiz 1:

Sum of N natural no's.

$$1+2+3+4+5+ \cdots + N = \frac{N(N+1)}{2}$$

Quiz 2: $[3, 10] \Rightarrow \underbrace{3, 4, 5, 6, 7, 8, 9, 10}$

[Inclusive

(Exclusive

$[a, b]$	$[a, b)$	(a, b)
$b-a+1$	$b-a$	$b-a-1$

$16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ → /2

$16 \leftarrow 8 \leftarrow 4 \leftarrow 2 \leftarrow 1$ ← ×2

Quiz 3:

$$\underbrace{N \rightarrow N/2 \rightarrow N/4 \cdots \rightarrow 1}_{\boxed{\log_2 N}}$$

A.P : Arithmetic Progression

Series: 4 7 10 13 16 19 22 ...

$\underbrace{4\quad 7}_{3}\ \underbrace{\quad 10}_{3}\ \underbrace{\quad 13}_{3}\ \underbrace{\quad 16}_{3}\ \underbrace{\quad 19}_{3}\ \underbrace{\quad 22}_{3}$

In general

1	2	3	4		N
a	$a+d$	$a+2d$	$a+3d$	...	$a+(N-1)d$

$$\boxed{\text{Sum of an AP} = \frac{n}{2}\left[2a+(n-1)d\right]}$$

First term $= a$

Common diff $= d$

$$\boxed{\log_a a^x = x}$$

GP : Geometric Progression

eg 3 6 12 24 48 ...

$\underbrace{3\quad 6}_{2}\ \underbrace{\quad 12}_{2}\ \underbrace{\quad 24}_{2}\ \underbrace{\quad 48}_{2}$

General

	2	3	4		N
a	ar	ar^2	ar^3	...	ar^{N-1}

Sum of 1st N terms of a GP : $a\left[\frac{r^n-1}{r-1}\right]$; $r \neq 1$ → $r>1$

$a\left[\frac{1-r^n}{1-r}\right]$ → $r<1$

$a \rightarrow$ first term

$r \rightarrow$ common ratio

$N \rightarrow$ no of terms

Q

```
void fun(int N){
    S=0
    f(i=1; i<=N; i++){
        S=S+1;
    }
    return S;
}
```

i: 1, 2, 3 --- N

i : [1, N]

$\boxed{\#\ \text{iterations} = N}$

Q

```
void func(int N, int M){
    f(i=1; i<=N; i++){
        if(i%2==0){
            print(i);
        }
    }
    f(i=1; i<=M; i++){
        if(i%2==0){
            print(i);
        }
    }
}
```

i: 1, 2, 3 ---- N

i: [1, N]

N

i=1, 2 --- M

i: [1, M]

M

$\boxed{\text{Total itr} = N+M}$

Q Quiz

```
int func(int N) {
    s = 0
    for (i=1; i<=N; i = i+2) {
        s = s + i;
    }
}
```

i: 1, 3, 5, 7, 9 ... N

odd no's

[1 ... N]

N : 10 → $\underbrace{1, 3, 5, 7, 9}$ $10/2 = 5$ $\frac{10+1}{2} = 5$

$\frac{N+1}{2}$

N = 7 → $\underbrace{1, 3, 5, 7}$ $7/2 = 3$ ✗

→ $\frac{7+1}{2} = 4$ ✓

Q

```
int func(int N) {
    S = 0
    f(i= 0; i<=100; i++) {
        S= S + i + i^2
    }
    return S;
}
```

$i: \underbrace{0, 1, 2 \dots 100}$

$i: [0, 100]$

$\# it = 100 - 0 + 1 = 11$

Q

```
void func(N) {
    f(i=1; i * i <= N; i++) {
        S= S + i^2
    }
    return S;
}
```

$i= \underbrace{1, 2, \dots \sqrt{N}}$

$i^2 < N$

$i < \sqrt{N}$

$i: [1, \sqrt{N}]$ $[a, b)$

$\boxed{\# it \rightarrow \sqrt{N}}$

Q

```
void func(N) {
    i = N;
    while (i > 1) {
        i = i/2;
    }
}
```

Initial : $i \rightarrow N$

iterations	i
1	$N/2 \rightarrow N/2^1$
2	$N/4 \rightarrow N/2^2$
3	$N/8 \rightarrow N/2^3$
⋮	⋮
K	$1 \rightarrow N/2^K$

Assume → After K iterations
$i \rightarrow 1$

$$N/2^K = 1 \Rightarrow N = 2^K$$

$2^K = N$ take $\log_2$ on both sides!

$$\log_2 2^K = \log_2 N$$

#it → $\boxed{K = \log_2 N}$

$\boxed{\log_a a^n = n}$

$/3 \rightarrow \log_3 N$

Q

```
void func (N)
    s = 0
    f (i = 0; i < N; i = i * 2) {
        s = s + i;
    }
}
```

initial i → 0

Iter	i
1	0
2	0
3	0
⋮	
∞	0

Q

```
void func (N) {
    s = 0
    f (i = 1; i < N; i = i * 2) {
        s = s + i;
    }
}
```

i → 1

iter	i
1	$2 \to 2^1$
2	$4 \to 2^2$
3	$8 \to 2^3$
4	$16 \to 2^4$
K	$N = 2^K$

i/N

STOP!

$N = 2^K$

$2^K = N$

$\boxed{K = \log_2 N}$

Q

```
void func(N) {
    f(i=1; i<=10; i++) {
        f(j=1; j<=N; j++) {
            print(..);
        }
    }
}
```

i	j [1,N]	# it
1	[1, N]	N
2	[1, N]	N
3		
4		
:		
10	[1, N]	N

Total # it → 10N

$\boxed{\# \text{ it} \rightarrow 10N}$

Q

```
void func(N) {
    f(i=1; i<=N; i++) {
        f(j=1; j<=N; j++) {
            print(i*j);
        }
    }
}
```

i	j [1,N]	# it
1	[1, N]	N
2	[1, N]	+ N
:		+
N	[1, N]	N

Total # it → N × N

→ N^2

Q

```
void func(N) {
    f(i=0; i<N; i++) {
        f(j=0; j<=i; j++) {
            print(i+j);
        }
    }
}
```

i	j [0,i]	# it
0	[0,0]	1
1	[0,1]	+ 2
2	[0,2]	+ 3
3	[0,3]	+ 4
⋮		⋮
N-1	[0,N-1]	+

Total it $\rightarrow$ $1+2+3+\ldots+N$

$$\boxed{\#\,it \rightarrow \frac{N(N+1)}{2}}$$

Q

```
void func(N) {
    f(i=1; i<=N; i++) {
        f(j=1; j<=N; j=j*2) {
            print(i+j);
        }
    }
}
```

i	j	# it
1	[1,N]	$\log_2 N$
2	[1,N]	+ $\log_2 N$
3		+
4		
5		
⋮		⋮
N		+ $\log_2 N$

Total #it $\rightarrow$ $N \log_2 N$

$\#\,it \rightarrow N \log_2 N$

Q

```
void func(N) {
    for(i=1; i<=2^N; i++){
        print(i);
    }
}
```

$i: 1, 2, 3 \ldots 2^N$

$\boxed{\#\text{it} \longrightarrow 2^N}$

Q

```
void func(N) {
    for(i=1; i<=N; i++){
        for(j=1; j<=2^i; j++){
            print(i*j);
        }
    }
}
```

i	$j: [1, 2^i]$	#it
1	$[1, 2^1]$	2^1
2	$[1, 2^2]$	$+ 2^2$
3	$[1, 2^3]$	$+ 2^3$
⋮	⋮	⋮
N	$[1, 2^N]$	$+ 2^N$

Total #it $\longrightarrow$

$2^1 + 2^2 + 2^3 \ldots + 2^N$

$a \to 2$
$r \to 2$
$n \to N$

Sum of GP: $\dfrac{a[r^n - 1]}{r-1} = \dfrac{2[2^N - 1]}{2-1}$

$$\boxed{2(2^N - 1)}$$

Q

```
f(i=N; i>0; i=i/2){
    f(j=1; j<=i; j++){
        print(i*j);
    }
}
```

	i	j[1,i]	# it	
1	N	$[1,N]$	N	$N/2^0$
2	$N/2$	$[1,N/2]$	$+\ N/2$	$N/2^1$
3	$N/4$	$[1,N/4]$	$+\ N/4$	$N/2^2$
4	$N/8$	$[1,N/8]$	$+\ N/8$	
⋮			⋮	
$\log_2 N$	$\to 1$	$[1,1]$	1	$N/2^{\log_2 N}$

Total # it $\to$

$$N + \left(\frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + \frac{N}{2^{\log_2 N}}\right)$$

$$N + \underbrace{N\underbrace{\left[\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \frac{1}{2^i} \dots + \frac{1}{2^{\log_2 N}}\right]}_{GP}}$$

$a = 1/2$

$r = 1/2$

$n = \log_2 N$

$$a\left[\frac{1-r^n}{1-r}\right]$$

$$\cancel{\frac{1}{2}}\left[\frac{1-\left(\frac{1}{2}\right)^{\log_2 n}}{\cancel{1-\frac{1}{2}}}\right]$$

$$N + \cancel{N}\left(\frac{N-1}{\cancel{N}}\right)$$

$$N + N - 1$$

$$= 2N - 1\ !$$

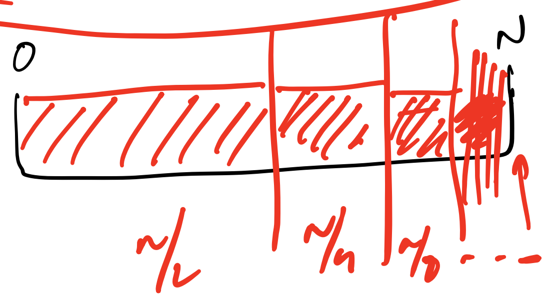
$$\boxed{2^{\log_2 N} = N}$$

$$1-\left(\frac{1}{2}\right)^{\log_2 N}$$

$$1-\frac{1}{2^{\log_2 N}}$$

$$= 1-\frac{1}{N} = \left(\frac{N-1}{N}\right)$$

(*) $$N + \underbrace{\frac{N}{2}+\frac{N}{4}+(-)\cdots - 1}_{N-1}$$



$N + N - 1$

$2N-1$

Compare

$$\boxed{\log N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < 2^N} \; N!$$

How to write Big O? $\longrightarrow$ Next

1) Calc. # iterations based on Input
2) Neglect lower order terms.
3) Neglect the constant coefficients!

$$f(N) = 10N^2 + \cancel{100N} + \cancel{10^4 \cdot N^0}$$

$$10N^2 \longrightarrow N^2$$

$$O(N^2)$$

Eg $f(N) = 4 \cdot N^2 + 3N + 6$

$$4 \cdot N^2 \longrightarrow N^2$$

$$\longrightarrow O(N^2)$$

$N = 1000$

$$4N^2 + 3N + 6$$

$$4 \times 10^6 \qquad 3000 \qquad 6$$

$$4003006$$

$$f(N) = 4N\log N + \underbrace{3N\sqrt{N}}_{} + 10^6$$

$$\cancel{3}N\sqrt{N}$$

$$\boxed{O(N\sqrt{N})}$$

$$f(N) = 10^6 N^0$$

$$N^0 \rightarrow \boxed{O(1)}$$

$N = 1 \rightarrow 10^6$

$N = 100 \rightarrow 10^6$

$N = 10^{12} \rightarrow 10^6$