Solutions

Test 1.1 LED Circuit

Solution: (c) **E** due to a negative charge decreases as $1/R^2$ and points **towards** the charge.

Test 1.2 Electric Force

Solution: (b) The electric force for two charges of the same polarity is **repulsive** and varies as $1/R^2$.

Test 1.3 Steady Current

Solution: (d) A steady current induces **both** an electric field and a magnetic field.

Test 1.4 Electric and Magnetic Fields

Solution: (d) A time-varying electric field induces a time-varying magnetic field, and **vice versa**.

Test 1.5 Acoustic Wave

Solution: (c)

$$\lambda = \frac{\upsilon}{f} = \frac{1.4 \times 10^3}{2.8 \times 10^3} = 0.5 \text{ m}.$$

Test 1.6 Lead/Lag

Solution: (c) Red wave reaches its max *after* the blue wave does. Hence, red wave lags the blue wave.

The phase lag corresponds to 1/8 of a period T. Since a period corresponds to 360°, the lag is

$$\frac{360}{8} = 45°.$$

Test 1.7 Time Shift

Solution: (a)

$$\Delta t = \frac{\Delta\phi}{2\pi} \times T = \frac{\pi/4}{2\pi} \times 16 = 2 \text{ s}.$$

Test 1.8 Traveling Wave

Solution: (a)

$$0.6 = 3e^{-0.4x}$$

$$\ln\left(\frac{0.6}{3}\right) = -0.4x \quad \Rightarrow \quad x \approx 4 \text{ m}.$$

Test 1.9 Traveling Wave

Solution: (a)

$$0.8 = 4e^{-0.4x}$$

$$\ln\left(\frac{0.8}{4}\right) = -0.4x \quad \Rightarrow \quad x \approx 4 \text{ m}.$$

Test 1.10 EM Spectrum

Solution: (c) 0.4 μm (blue) – 0.7 μm (red).

Test 1.11 Mobile Phone Bands

Solution: (c) Mobile phone operates in UHF and SHF bands.

Test 1.12 Complex Numbers

Solution: (d)

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{6 - j8}{3 + j4} = \frac{10e^{-j53^\circ}}{5e^{j53^\circ}} = 2e^{-j106^\circ}.$$

Test 1.13 Complex Numbers

Solution: (c)

$$\begin{aligned}\mathbf{V}_1\mathbf{V}_2^* &= (6-j8)(3+j4)^* \\ &= 10e^{-j53^\circ} \times (5e^{j53^\circ})^* \\ &= 50e^{-j106^\circ}.\end{aligned}$$

Test 1.14 Complex Numbers

Solution: (a)

$$\begin{aligned}\mathbf{V}_1\mathbf{V}_2^* &= (6-j8)(3+j4) \\ &= 10e^{-j53^\circ} \times (5e^{j53^\circ}) \\ &= 50.\end{aligned}$$

Test 1.15 Complex Algebra

Solution: (b)

$$\begin{aligned}\mathbf{z} &= 2e^{-j0.5} \\ \ln\mathbf{z} &= \ln(2e^{-j0.5}) \\ &= \ln 2 + \ln(e^{-j0.5}) \\ &= 0.69 - j0.5.\end{aligned}$$

Test 1.16 Phasors

Solution: (a)

$$\begin{aligned}\upsilon(t) &= 10\sin(\omega t + 45^\circ) \\ &= 10\cos(90^\circ - \omega t - 45^\circ) \\ &= 10\cos(-\omega t + 45^\circ) \\ &= 10\cos(\omega t - 45^\circ).\end{aligned}$$

Hence, $\widetilde{V} = 10e^{-j45^\circ}$.

Test 1.17 Phasors

Solution: (b)

$$
\begin{aligned}
\upsilon(t) &= -4\cos(\omega t - 30^\circ) \\
&= 4\cos(\omega t - 30^\circ + 180^\circ) \\
&= 4\cos(\omega t + 150^\circ).
\end{aligned}
$$

Hence, $\widetilde{V} = 4e^{j150^\circ}$.

Test 1.18 Phasors

Solution: (c)

$$
\begin{aligned}
\upsilon(t) &= \mathfrak{Re}[\widetilde{V} e^{j\omega t}] \\
&= \mathfrak{Re}[-5e^{j30^\circ} e^{j\omega t}] \\
&= \mathfrak{Re}[5e^{-j180^\circ} e^{j30^\circ} e^{j\omega t}] \\
&= 5\cos(\omega t - 150^\circ).
\end{aligned}
$$

Test 1.19 Phasors

Solution: (d)

$$
\begin{aligned}
\upsilon(t) &= \mathfrak{Re}[\widetilde{V} e^{j377t}] \\
&= \mathfrak{Re}[3e^{-j30^\circ} e^{j377t}] \\
&= 3\cos(377t - 30^\circ) \\
&= 3\sin(90^\circ - 377t + 30^\circ) \\
&= 3\sin(120^\circ - 377t) \\
&= -3\sin(377t - 120^\circ).
\end{aligned}
$$

Test 1.20 Traveling Wave

Solution: (b)

$$
8\pi = \frac{2\pi}{\lambda} \quad \rightarrow \quad \lambda = 0.25 \text{ m}.
$$

Test 3.1 Commutative Vector Operations

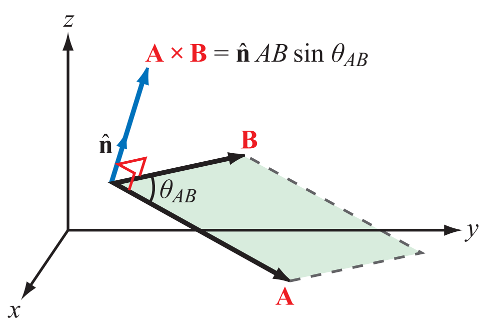
Solution: (b) For two vectors $\mathbf{A}$ and $\mathbf{B}$, an operation is commutative if interchanging $\mathbf{A}$ and $\mathbf{B}$ results in the same outcome.

Since $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$, it follows that the *dot product is commutative*.

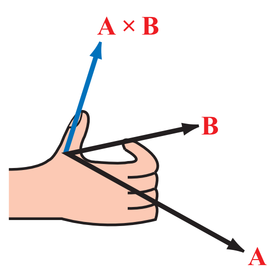
However, because $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$, the *cross product is anticommutative*.

Test 3.2 Cross Product

Solution: (c) The cross product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ is illustrated in Fig. 3-6: the direction of $\mathbf{C}$ is orthogonal to the plane containing $\mathbf{A}$ and $\mathbf{B}$, in accordance with the right-hand rule.



(a) Cross product



(b) Right-hand rule

Test 3.3 Meaningful Products

Solution: (d) Neither option (a) nor option (b) is a meaningful statement because $\mathbf{B} \cdot \mathbf{C}$ is a scalar and it is not meaningful to perform a dot product or a cross product between a vector $\mathbf{A}$ and a scalar.

Option (c) represents a simple product between two vectors, namely $\mathbf{A}$ and $(\mathbf{B} \times \mathbf{C})$, which is inapplicable.

Option (d) represents a simple product between a vector $\mathbf{A}$ and a scalar $(\mathbf{B} \cdot \mathbf{C})$, which is applicable.

Hence, the answer is (d).

Test 3.4 Differential Length

Solution: **(c)** Each component of $d\mathbf{l}$ should be a measure of distance. Terms containing $r\,dr$ or $r\,dz$ represent area, not distance, so they should be eliminated. Also, a term containing neither r nor z (as in $\hat{\boldsymbol{\phi}}\,d\phi$), is equally unacceptable.

The definition given by (c) is the only one in which all three components are a measure of distance.

Test 3.5 Angle between Vectors

Solution: **(c)** By definition, angle θ_{AB} between vectors $\mathbf{A}$ and $\mathbf{B}$ lies in the range between 0 and 180°, which eliminates answers (c) and (d). Also,

$$\begin{aligned}
\theta_{AB} &= \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|A|\,|B|}\right) \\
&= \cos^{-1}\left(\frac{(\hat{\mathbf{x}}3 - \hat{\mathbf{z}}4) \cdot (\hat{\mathbf{z}}2)}{\sqrt{3^2 + 4^2}\sqrt{2^2}}\right) \\
&= \cos^{-1}\left(\frac{-8}{10}\right) = 143.13^\circ.
\end{aligned}$$

Test 3.6 Gradient and Curl Operators

Solution: **(a)** According to the definitions given in Sections 3-5 and 3-6, the gradient operates on scalar fields and produces vector fields, and the curl operates on vector fields and produces vector fields.

Test 3.7 Directional Derivative

Solution: **(d)** From Eq. (3.75),

$$\frac{dV}{dl} = \nabla V \cdot \hat{\mathbf{a}}_l.$$

Here, $\hat{\mathbf{a}}_\ell = \hat{\mathbf{z}}$. Hence,

$$\begin{aligned}
\nabla V &= \hat{\mathbf{x}}\,\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial V}{\partial z} \\
&= \hat{\mathbf{x}}\,\frac{\partial}{\partial x}(x^2 y - 2z^2) + \hat{\mathbf{y}}\,\frac{\partial}{\partial y}(x^2 y - 2z^2) + \hat{\mathbf{z}}\,\frac{\partial}{\partial z}(x^2 y - 2z^2) \\
&= \hat{\mathbf{x}}\,2xy + \hat{\mathbf{y}}\,x^2 - \hat{\mathbf{z}}\,4z. \\
\nabla V \cdot \hat{\mathbf{z}} &= -4z.
\end{aligned}$$

At $P[(1,2,3)$,

$$\left(\frac{dV}{dl}\right)\bigg|_{(1,2,3)} = -4 \times 3 = -12.$$

Test 3.8 Directional Derivative

Solution: **(a)** From Eq. (3.75),

$$\frac{dV}{dl} = \nabla V \cdot \hat{\mathbf{a}}_\ell.$$

Here, $\hat{\mathbf{a}}_l = \hat{\boldsymbol{\phi}}$. Hence,

$$\begin{aligned}
\nabla V &= \hat{\mathbf{r}}\,\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\,\frac{1}{r}\,\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\,\frac{\partial V}{\partial z} \\
&= \hat{\mathbf{r}}\,\frac{\partial}{\partial r}(5e^{-2r}\sin\phi) + \hat{\boldsymbol{\phi}}\,\frac{1}{r}\,\frac{\partial}{\partial \phi}(5e^{-2r}\sin\phi) + \hat{\mathbf{z}}\,\frac{\partial}{\partial z}(5e^{-2r}\sin\phi) \\
&= -\hat{\mathbf{r}}\,10e^{-2r}\sin\phi + \hat{\boldsymbol{\phi}}\,\frac{5e^{-2r}}{r}\cos\phi,
\end{aligned}$$

and

$$\nabla V \cdot \hat{\boldsymbol{\phi}} = \frac{5}{r}\,e^{-2r}\cos\phi.$$

At $P = (0.5, \pi/4, 2)$:

$$\left(\frac{dV}{dl}\right)_{(0.5,\pi/4,2)} = \frac{5}{0.5}\,e^{-1}\cos(\pi/4) = 2.6.$$

Test 3.9 Divergence

Solution: (c)

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$= \frac{\partial}{\partial x}(-2xy) + \frac{\partial}{\partial y}(2y^2) = -2y + 4y = 2y.$$

At $P = (1, 2)$,

$$(\nabla \cdot \mathbf{A})|_{(1,2)} = 2 \times 2 = 4.$$

Test 3.10 Flux Out of a Cube

Solution: (d) Since $\mathbf{A}$ is along $\hat{\mathbf{x}}$, we need to compute only the outward flux through the sides at $x = 10$ and $x = -10$:

$$\oint_S \mathbf{A} \cdot d\mathbf{s} = \int_{\substack{y=-10 \\ z=-10}}^{\substack{z=10 \\ y=10}} \mathbf{A} \cdot d\mathbf{s} \Bigg|_{x=10} + \int_{\substack{y=-10 \\ z=-10}}^{\substack{z=10 \\ y=10}} \mathbf{A} \cdot d\mathbf{s} \Bigg|_{x=-10}.$$

At $x = 10$, the outward direction is along $\hat{\mathbf{x}}$. Hence,

$$d\mathbf{s} = \hat{\mathbf{x}}\, dy\, dz \qquad @\ x = 10.$$

At $x = -10$, the outward direction is along $-\hat{\mathbf{x}}$. Hence,

$$d\mathbf{s} = -\hat{\mathbf{x}}\, dy\, dz \qquad @\ x = -10,$$

$$\oint_S \mathbf{A} \cdot d\mathbf{s} = \int_{z=-10}^{z=10} \int_{y=-10}^{y=10} \hat{\mathbf{x}}\, x \cdot \hat{\mathbf{x}}\, dy\, dz \Bigg|_{x=10} + \int_{z=-10}^{z=10} \int_{y=-10}^{y=10} \hat{\mathbf{x}}\, x \cdot (-\hat{\mathbf{x}})\, dy\, dz \Bigg|_{x=-10}$$
$$= 10\, y|_{-10}^{10}\, z|_{-10}^{10} + 10\, y|_{-10}^{10}\, z|_{-10}^{10}$$
$$= 4000 + 4000 = 8000.$$

Hence, the correct answer is (c).

Test 3.11 Flux Out of a Cube

Solution: (b) Since $\mathbf{A}$ is along $\hat{\mathbf{x}}$, we need to compute only the outward flux through the sides at $x = 10$ and $x = -10$:

$$\oint_S \mathbf{A} \cdot d\mathbf{s} = \int_{\substack{y=-10 \\ z=-10}}^{\substack{z=10 \\ y=10}} \mathbf{A} \cdot d\mathbf{s} \Bigg|_{x=10} + \int_{\substack{y=-10 \\ z=-10}}^{\substack{z=10 \\ y=10}} \mathbf{A} \cdot d\mathbf{s} \Bigg|_{x=-10}.$$

At $x = 10$, the outward direction is along $\hat{\mathbf{x}}$. Hence,

$$d\mathbf{s} = \hat{\mathbf{x}}\,dy\,dz \qquad @\ x = 10.$$

At $x = -10$, the outward direction is along $-\hat{\mathbf{x}}$. Hence,

$$d\mathbf{s} = -\hat{\mathbf{x}}\,dy\,dz \qquad @\ x = -10,$$

$$\begin{aligned}
\oint_S \mathbf{A}\cdot d\mathbf{s} &= \int_{z=-10}^{10}\int_{y=-10}^{10} (\hat{\mathbf{x}}\,xy^2)\cdot\hat{\mathbf{x}}\,dy\,dz\Big|_{x=10} + \int_{z=-10}^{10}\int_{y=-10}^{10} (\hat{\mathbf{x}}\,xy^2)\cdot(-\hat{\mathbf{x}})\,dy\,dz\Big|_{x=-10} \\
&= \frac{10y^3}{3}\Big|_{-10}^{10} z\Big|_{-10}^{10} + \frac{10y^3}{3}\Big|_{-10}^{10} z\Big|_{-10}^{10} \\
&= \frac{8}{3}\times 10^5.
\end{aligned}$$

Test 3.12 Conservation Vector

Solution: **(b)** A field is conservative if its curl is zero: $\nabla\times\mathbf{A} = 0$.

Test 3.13 Divergence

Solution: **(a)** Since $\nabla\cdot\mathbf{E} < 0$, the small volume appears like a *sink* into which the field lines (flux) converge.

Test 3.14 Divergence Theorem

Solution: **(c)** The divergence theorem states that the divergence $\nabla\cdot\mathbf{A}$, integrated over a volume $\mathcal{V}$, is equal to the flux $\mathbf{E}\cdot d\mathbf{s}$ flowing out of the surface S bounding $\mathcal{V}$:

$$\int_{\mathcal{V}} \nabla\cdot\mathbf{A}\,d\mathcal{V} = \oint_S \mathbf{A}\cdot d\mathbf{s}.$$

Test 3.15 Stokes's Theorem

Solution: **(d)** Stokes's theorem relates the curl, $\nabla\times\mathbf{B}$, across a surface S to the line integral of $\mathbf{B}$ along the contour C bounding S.

$$\int_S (\nabla\times\mathbf{B})\cdot d\mathbf{s} = \oint_C \mathbf{B}\cdot d\boldsymbol{\ell}.$$

Test 3.16 Divergence

Solution: (c)

$$\begin{aligned}\nabla \cdot \mathbf{A} &= \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \\ &= \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial x}(xy^2) = 2xy - 2yx = 0.\end{aligned}$$

Test 3.17 Divergence

Solution: (a)

$$\begin{aligned}\nabla \cdot \mathbf{A} &= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} \\ &= \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\cos\phi}{r}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\frac{\sin\phi}{r^2}\right) \\ &= -\frac{\cos\phi}{r^3} + \frac{\cos\phi}{r^3} = 0.\end{aligned}$$

Test 3.18 Laplacian

Solution: (d)

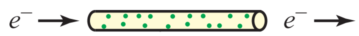
$$\begin{aligned}\nabla^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\ &= \frac{\partial^2}{\partial x^2}(x^2y + y^2z + y^2x) + \frac{\partial^2}{\partial y^2}(x^2y + y^2z + y^2x) + \frac{\partial^2}{\partial z^2}(x^2y + y^2z + y^2x) \\ &= 2y + 2(z + x) = 2(x + y + z).\end{aligned}$$

Test 3.19 Arrow Representation

Solution: (b) $\mathbf{E}$ points along $\hat{\mathbf{y}}$ when x is in the right-hand side of the x–y plane and points along $-\hat{\mathbf{y}}$ when x is in the left-hand side.

Test 4.1 Static Conditions

Solution: **(c)** **E** is induced by electric charge, so if the distribution of ρ_v within the specified volume is constant with time, then **E** is static. If electric charge enters an elemental volume at the same rate as the volume loses charge, then both ρ_v and **J** remain constant, and consequently both **E** and **H** are static fields. An example of **J** = constant is a dc current, wherein the same number of electrons enter an elemental cross section as those that leave it.



Test 4.2 Static and Dynamic Conditions

Solution: **(d)** Under dynamic conditions ($\frac{\partial \mathbf{E}}{\partial t} \neq 0$ and $\frac{\partial \mathbf{H}}{\partial t} \neq 0$), **E** and **H** are a coupled pair; **E** induces **H** and vice versa. However, if ρ_v and **J** are non–time-varying, then **E** and **H** become decoupled and non–time-varying.

Test 4.3 Electric Charge

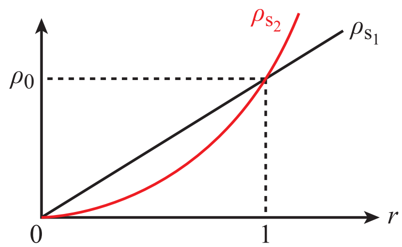
Solution: **(b)**

$$
\begin{aligned}
Q_1 &= \int_S \rho_{s_1}\, ds \\
&= \int_0^{2\pi} \int_0^1 (\rho_0 r) r\, dr\, d\phi \\
&= 2\pi\rho_0 \left. \frac{r^3}{3} \right|_0^1 = \frac{2\pi}{3}\, \rho_0 . \\
Q_2 &= \int_S \rho_{s_2}\, ds \\
&= \int_0^{2\pi} \int_0^1 (\rho_0 r^2) r\, dr\, d\phi \\
&= 2\pi\rho_0 \left. \frac{r^4}{4} \right|_0^1 = \frac{2\pi}{4}\, \rho_0 .
\end{aligned}
$$

Hence,

$$
\frac{Q_1}{Q_2} = \frac{4}{3}\, .
$$

Even though ρ_{s_2} increases quadratically, its value is smaller than ρ_{s_1} for $0 \leq r < 1$:



Test 4.4 Electric Charge

Solution: (a)

$$Q_1 = \int_S \rho_{s_1}\, ds$$
$$= \int_0^{2\pi}\int_0^1 (\rho_0 r) r\, dr\, d\phi$$
$$= 2\pi\rho_0 \left.\frac{r^3}{3}\right|_0^1 = \frac{2\pi}{3}\,\rho_0.$$
$$Q_2 = \int_S \rho_{s_2}\, ds$$
$$= \int_0^{2\pi}\int_0^{a_2} (\rho_0 r^2) r\, dr\, d\phi$$
$$= 2\pi\rho_0 \left.\frac{r^4}{4}\right|_0^{a_2} = \frac{2\pi}{4}\,\rho_0 a^4.$$

For $Q_1 = Q_2$,

$$\frac{2\pi}{3}\,\rho_0 = \frac{2\pi}{4}\,\rho_0 a_2^4$$

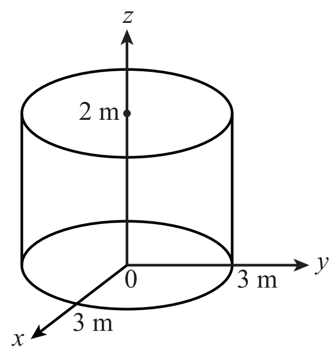
and

$$a_2 = \left(\frac{4}{3}\right)^{1/4} = 1.075.$$

Test 4.5 Electric Charge

Solution: (d) For the cylinder shown in the figure, application of Eq. (4.5) gives

$$Q = \int_{z=0}^{2}\int_{\phi=0}^{2\pi}\int_{r=0}^{3} 20rz\, r\, dr\, d\phi\, dz$$
$$= \left(\frac{10}{3} r^3 \phi z^2\right)\Bigg|_{r=0}^{3}\Bigg|_{\phi=0}^{2\pi}\Bigg|_{z=0}^{2} = 720\pi \text{ (mC)} = 2.26 \text{ C}.$$



Test 4.6 Electric Charge

Solution: (b)

$$Q = \int_{-5}^{5} \rho_l \, dy = \int_{-5}^{5} 12y^2 \, dy = \left. \frac{12y^3}{3} \right|_{-5}^{5} = 1000 \text{ mC} = 1 \text{ C}.$$

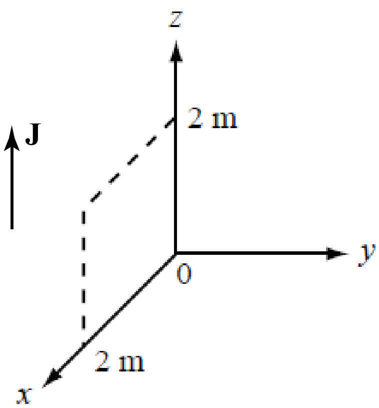
Test 4.7 Electric Charge

Solution: (b)

$$\begin{aligned} Q &= \int \rho_s \, ds \\ &= \int_{r=0}^{3} \int_{\phi=0}^{2\pi} 2e^{-r} r \, dr \, d\phi \\ &= 2 \times 2\pi \int_{0}^{3} re^{-r} \, dr \\ &= 4\pi[-re^{-r} - e^{-r}]\big|_0^3 \\ &= 4\pi[-3e^{-3} - e^{-3} + 1] \\ &= 10.06 \text{ C}. \end{aligned}$$

Test 4.8 Electric Current

Solution: (a) The square resides in the x–z plane and $\mathbf{J}$ is along $\hat{\mathbf{z}}$, so no current flows through the square.



Test 4.9 Electric Field

Solution: **(c)** This problem is the same as Example 4-4 in the book except that the charge density is negative in polarity. Hence, **E** points downward along $-\hat{\mathbf{z}}$.

Test 4.10 Electric Field

Solution: **(b)** The electric field is zero at the origin, so its direction is irrelevant.

Test 4.11 Electric Field

Solution: **(d)** From Example 4-6,

$$\mathbf{E} = \hat{\mathbf{r}}\,\frac{\rho_\ell}{2\pi\varepsilon_0 r}\,.$$

Here, ρ_ℓ is negative because the charges are electrons. Hence, the direction of **E** is along $-\hat{\mathbf{r}}$.

Test 4.12 Electric Potential

Solution: **(a)** As noted in the text, the term *voltage* is short for *voltage potential* and synonymous with electric potential.

Test 4.13 Electric Flux Density

Solution: **(c)** Using $\nabla \cdot \mathbf{D} = \rho_\text{v}$ leads to

$$\begin{aligned}\rho_\text{v} = \nabla \cdot \mathbf{D} &= \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z \\ &= \frac{\partial}{\partial x}(xz^2) = z^2.\end{aligned}$$

At $(0,0,2)$,

$$\rho_\text{v} = 2^2 = 4 \text{ (C/m}^3\text{)}.$$

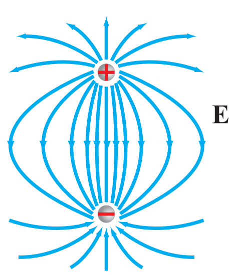
Test 4.14 Electric Flux Density

Solution: **(b)** From $\nabla \cdot \mathbf{D} = \rho_\text{v}$ and the expression for the divergence in spherical coordinates (given on the book's inside back cover),

$$\begin{aligned}\rho_\text{v} &= \nabla \cdot \mathbf{D} \\ &= \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 D_R) + \frac{1}{R \sin\theta} \frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{R\sin\theta} \frac{\partial D_\phi}{\partial \phi} \\ &= \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 \cdot 5R^2) \\ &= \frac{1}{R^2} \frac{\partial}{\partial R}(5R^4) \\ &= \frac{1}{R^2}(20R^3) \\ &= 20R \\ &= 20 \times 2 = 40 \text{ (C/m}^3\text{) at } R = 2 \text{ m}.\end{aligned}$$

Test 4.15 Electric Field

Solution: **(c)**



The electric field pattern of the dipole is such that along the x–y plane, **E** points along $-\hat{\mathbf{z}}$.

Test 4.16 Electrical Conductivity

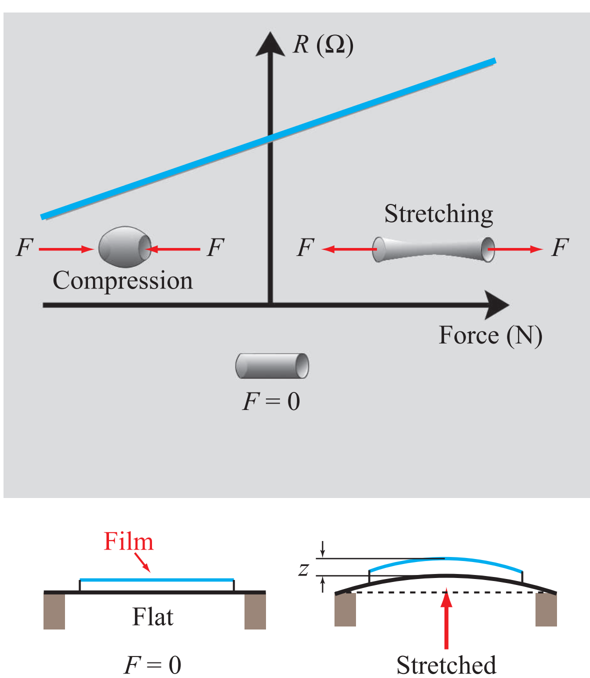
Solution: **(a)** From Table 4-1 in the book,

$$\sigma_c = 5.8 \times 10^7 \text{ (S/m) for copper, and}$$
$$\sigma_m = 10^{-15} \text{ (S/m) for mica.}$$

Hence
$$\frac{\sigma_c}{\sigma_m} \approx 10^{23}.$$

Test 4.17 Piezoresistor

Solution: **(d)**



According to Tech Brief 7, the resistance R of a piezoresistor is related to the force

F applied to it by

$$R = R_0 \left(1 + \frac{\alpha F}{A_0}\right).$$

It is used to detect pressure, as in a touchscreen.

Test 4.18 Voltage Breakdown

Solution: **(b)** At dielectric breakdown,

$$V = E_{\text{ds}} d = 3 \times 10^6 \times 600 = 1.8 \text{ GV}.$$

Test 4.19 Capacitance

Solution: **(d)** For the inner cylinder,

$$\begin{aligned} C_1 = \varepsilon_1 \frac{A_1}{d_1} &= 8\varepsilon_0 \frac{\pi r_1^2}{d_1} \\ &= 8\varepsilon_0 \frac{\pi (2 \times 10^{-3})^2}{2 \times 10^{-2}} = 16\pi\varepsilon_0 \times 10^{-4} \text{ F}. \end{aligned}$$

For the outer cylinder,

$$\begin{aligned} C_2 = \varepsilon_2 \frac{A_2}{d_2} &= 2\varepsilon_0 \frac{\pi r_2^2 - r_1^2}{d_1} \\ &= 2\varepsilon_0 \frac{\pi (8^2 - 2^2) \times 10^{-6}}{2 \times 10^{-2}} = 60\pi\varepsilon_0 \times 10^{-4} \text{ F}. \end{aligned}$$

Combined capacitance of two in-parallel capacitors is:

$$C = C_1 + C_2 = 76\pi\varepsilon_0 \times 10^{-4} = 7.6\pi\varepsilon_0 \text{ mF}.$$

Test 4.20 Supercapacitor

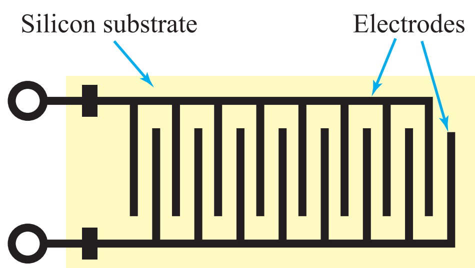
Solution: **(a)** The advantage of a supercapacitor over a traditional capacitor is in the supercapacitor's higher energy storage capacity per unit weight, but a supercapacitor has slower charge and discharge rates.

Test 4.21 Supercapacitor

Solution: **(b)** A supercapacitor cannot store as much energy as an equal-weight battery can, but it can charge and discharge faster than a battery.

Test 4.22 Humidity Sensor

Solution: **(c)** The capacitance of any two-conductor structure depends on the geometry of the structure and the permittivity of the material between the conducting elements. A humidity sensor uses an interdigital structure with a substrate material selected because of the high sensitivity of its permittivity ε to humidity.

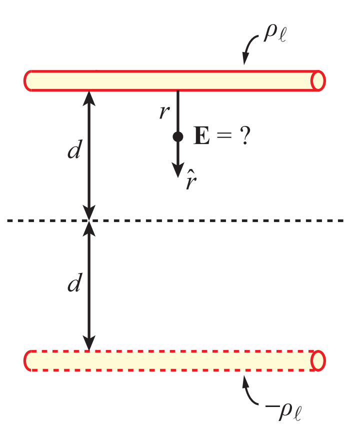


Test 4.23 Image Method

Solution: **(c)** From Eq. (4.33), **E** due to the line in the absence of the conducting surface is

$$\mathbf{E}_1 = \frac{\hat{\mathbf{r}}\rho_\ell}{2\pi\varepsilon_0 r}\,.$$

The image method allows us to remove the conducting surface and to replace it with a line with charge density $-\rho_\ell$ at a distance $2d$ from the original line:



The total field due to the two lines is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\hat{\mathbf{r}}\rho_\ell}{2\pi\varepsilon_0 r} + \frac{-\hat{\mathbf{r}}(-\rho_\ell)}{2\pi\varepsilon_0(2d-r)} = \frac{\hat{\mathbf{r}}\rho_\ell}{2\pi\varepsilon_0}\left[\frac{1}{r} + \frac{1}{2d-r}\right].$$

Test 4.24 Electrical Energy

Solution: **(d)** With $E = E_{\rm ds} = 200 \times 10^6$ V/m in the space between the conducting plates, the stored energy is

$$\begin{aligned}
W_{\rm e} &= \frac{1}{2}\,\varepsilon E_{\rm ds}^2 \mathcal{V} \\
&= \frac{1}{2} \times 6\varepsilon_0 \times (2\times 10^8)^2 \times (2\times 10^{-2})^2 \times 10^{-3} \\
&= 4.8\varepsilon_0 \times 10^{10} \\
&= 4.8 \times 10^{10} \times 8.85 \times 10^{-12} = 0.425 \text{ J.}
\end{aligned}$$