## Test 9.1 Dipole Efficiency

**Solution: (c)**

$$\lambda = \frac{c}{f} = \frac{3\times 10^8 \text{ m/s}}{10^7 \text{ Hz}} = 30 \text{ m}.$$

As

$$\frac{l}{\lambda} = \frac{2 \text{ m}}{30 \text{ m}} = 6.7\times 10^{-2},$$

this antenna is a short (Hertzian) dipole. Thus, from respectively Eqs. (9.35), (9.32), and (9.31),

$$R_{\text{rad}} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(6.7\times 10^{-2}\right)^2 = 3.5 \qquad (\Omega),$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f\mu_{\text{c}}}{\sigma_{\text{c}}}}$$

$$= \frac{2 \text{ m}}{2\pi(10^{-3} \text{ m})}\sqrt{\frac{\pi(10^7 \text{ Hz})(4\pi\times 10^{-7} \text{ H/m})}{5.8\times 10^7 \text{ S/m}}} = 262 \qquad (\text{m}\Omega),$$

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}}+R_{\text{loss}}} = \frac{3500 \text{ m}\Omega}{3500 \text{ m}\Omega + 262 \text{ m}\Omega} = 93\%.$$

## Test 9.2 Antenna Gain

**Solution: (b)**

$$G = \xi D,$$

so

$$G\,(\text{dB}) = \xi\,(\text{dB}) + D\,(\text{dB}) = 10\log 0.8 + 8.0 = -0.97 + 8.0 = 7.03 \text{ dB}.$$

## Test 9.3 Antenna Directivity

**Solution: (a)**

A circular lobe means that $\beta_{xz} = \beta_{yz} = 6^\circ = 0.105$ rad. Using Eq. (9.26), we have

$$D = \frac{4\pi}{\beta_{xz}\beta_{yz}} = \frac{4\pi}{(0.105)^2} = 1.15\times 10^3.$$

In dB,

$$D(\text{dB}) = 10\log D = 10\log(1.15\times 10^3) = 30.59 \text{ dB}.$$

## Test 9.4 Radiated Power

**Solution: (d)**

At 150 MHz,

$$\lambda = \frac{3\times 10^8}{1.5\times 10^8} = 2 \text{ m}.$$

Hence, the dipole is $\lambda/2$ in length, in which case we can use Eq. (9.46) to calculate $P_{\text{rad}}$:

$$P_{\text{rad}} = 36.6 I_0^2 = 36.6\times 5^2 = 915 \text{ W}.$$

## Test 9.5 Dipole Effective Area

**Solution: (c)**

At $f = 75$ MHz,

$$\lambda = \frac{c}{f} = \frac{3\times 10^8 \text{ m/s}}{75\times 10^6 \text{ Hz}} = 4 \text{ m}.$$

From Eq. (9.47), a half wave dipole has a directivity of $D = 1.64$. From Eq. (9.64),

$$A_{\text{e}} = \frac{\lambda^2 D}{4\pi} = \frac{4 \text{ m}^2\times 1.64}{4\pi} = 2.09 \text{ m}^2.$$

## Test 9.6 Communication Link

**Solution: (c)**

At $f = 3$ GHz,

$$\lambda = \frac{c}{f} = \frac{3\times 10^8 \text{ m/s}}{3\times 10^9 \text{ Hz}} = 0.10 \text{ m}.$$

Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$\begin{aligned} P_{\rm t} &= P_{\rm rec}\,\frac{\lambda^2 R^2}{\xi_{\rm t}\xi_{\rm r}A_{\rm t}A_{\rm r}} \\ &= 10^{-8}\,\frac{(0.100\ \text{m})^2\left(40\times 10^3\ \text{m}\right)^2}{1\times 1\times(\pi\,(1\ \text{m})^2)(\pi\,(1\ \text{m})^2)} \\ &= 1.62\times 10^{-2}\ \text{W} \\ &= 16.2\ \text{mW}. \end{aligned}$$

## Test 9.7 Communication Link

**Solution: (b)**

At $f = 100$ MHz,

$$\lambda = \frac{c}{f} = \frac{3\times 10^8\ \text{m/s}}{100\times 10^6\ \text{Hz}} = 3\ \text{m},$$

for which a half wave dipole, or larger antenna, is very reasonable to construct. Assuming the TV transmitter to have a vertical half wave dipole, its gain in the direction of the home would be $G_{\rm t} = 1.64$. The home antenna has a gain of $G_{\rm r} = 3$ dB $= 2$. From the Friis transmission formula (Eq. (9.75)):

$$\begin{aligned} P_{\rm rec} = P_{\rm t}\frac{\lambda^2 G_{\rm r}G_{\rm t}}{(4\pi)^2 R^2} = 10^3\,\frac{(3\ \text{m})^2\times 1.64\times 2}{(4\pi)^2\,(30\times 10^3\ \text{m})^2} &= 2.1\times 10^{-7}\ \text{W} \\ &= 0.21\ \mu\text{W}. \end{aligned}$$

## Test 9.8 Link Power Budget

**Solution: (a)**

$G_{\rm t} = 20$ dB $= 100$, $G_{\rm r} = 23$ dB $= 200$, and $\lambda = c/f = 3$ cm.

$$P_{\rm rec} = P_{\rm t}G_{\rm t}G_{\rm r}\left(\frac{\lambda}{4\pi R}\right)^2 = 10\times 100\times 200\times\left(\frac{3\times 10^{-2}}{4\pi\times 2\times 10^4}\right)^2 = 2.85\times 10^{-9}\ \text{W}.$$

## Test 9.9 Antenna Directivity

**Solution: (d)**

From Eqs. (9.94a), (9.94b), and (9.96),

$$\beta_{yz} = \beta_{xz} = 0.88\,\frac{\lambda}{l_x} = \frac{0.88 \times 3 \times 10^{-2}}{2} = 1.32 \times 10^{-2} \text{ rad} = 0.75^\circ,$$

$$D = \frac{4\pi}{\beta_{xz}\beta_{yz}} = \frac{4\pi}{(1.32 \times 10^{-2})^2} = 7.21 \times 10^4 = 48.6 \text{ dB}.$$

## Test 9.10 Reciprocity

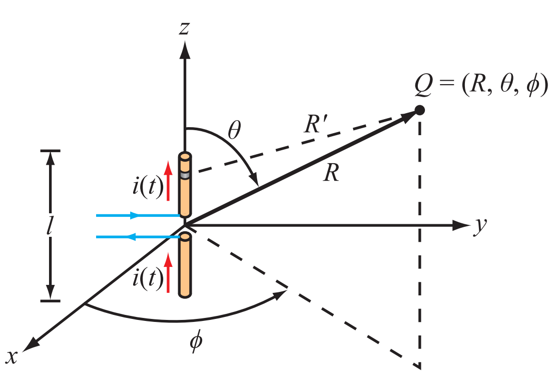
**Solution: (b)**

The radiation pattern of an antenna defines the directional distribution of power radiated or received by an antenna. For a reciprocal antenna, the radiation pattern is the same for transmission and for reception.

## Test 9.11 Short Dipole

**Solution: (c)**

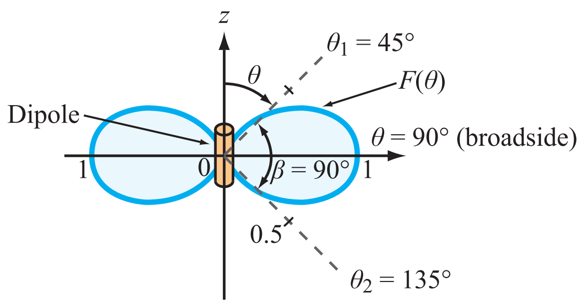
In order to (1) approximate the current distribution along the length of the antenna as constant and (2) approximate the ranges $R'$ between all locations along the length of the antenna to the observation point as the same as $R$, the distance between the center of the antenna and the observation point, $l$ should be much shorter than $R$, which in practice is set at $l \lesssim \lambda/50$.



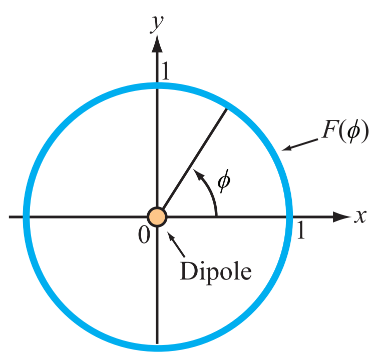

## Test 9.12 Short Dipole Radiation Pattern

**Solution: (a)**

The azimuth pattern in the $x$–$y$ plane is circular.




(a) Elevation pattern



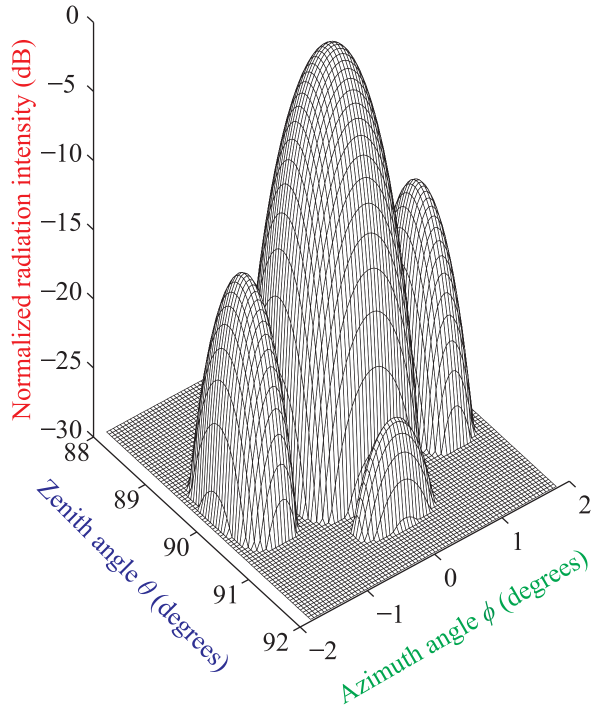

(b) Azimuth pattern

## Test 9.13 Antenna Pattern

**Solution: (d)**

The antenna radiation pattern represents the radiation intensity radiated by the antenna along a given direction, normalized to the maximum radiation intensity radiated along all possible directions. Hence, the maximum value of the antenna pattern $F$ is 1, or 0 in dB, since

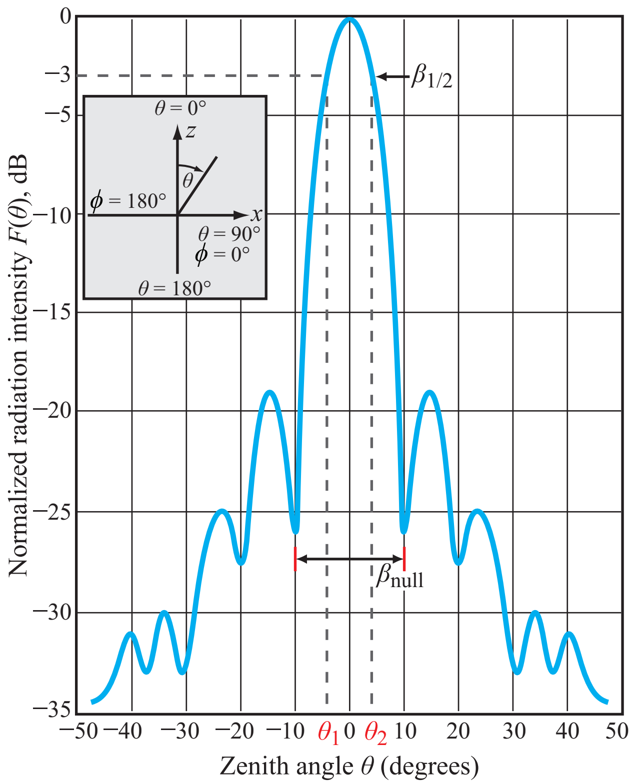
$$10 \log 1 = 0 \text{ dB}.$$




## Test 9.14 Half-Power Beamwidth

**Solution: (b)**

$\beta_{1/2}$ is the angular distance between $\theta_2$ and $\theta_1$, corresponding to the $-3$ dB points on the radiation pattern.




## Test 9.15 Antenna Gain and Directivity

**Solution: (d)**

$G = \xi D$, so $G$ is indeed related to $D$, but $D$ is based on only the antenna pattern relative to that of an isotropic antenna, so it does not account for ohmic losses. The radiation efficiency is given by

$$\xi = \frac{P_{\text{rad}}}{P_{\text{t}}}.$$

For a lossless antenna $P_{\text{rad}} = P_{\text{t}}$ and $\xi = 1$, but for a real antenna $\xi < 1$.

## Test 9.16 Directivity Variation with Frequency

**Solution: (a)**

$$D \approx \frac{4\pi A}{\lambda^2} = \frac{4\pi A f^2}{c^2},$$

so doubling $f$ increases $D$ by a factor of 4, or equivalently, by 6 dB.

### Test 9.17 Communication Receiver

**Solution: (c)**

$$S_n = \frac{P_{rec}}{P_n}$$

and

$$P_n = KT_{sys}B.$$

Hence,

$$S_n = \frac{P_{rec}}{KT_{sys}B},$$

so $S_n \sim 1/(T_{sys}B)$.

### Test 9.18 Eye's Beamwidth

**Solution: (d)**

$$\beta \approx \frac{\lambda}{d} = \frac{0.7 \times 10^{-6}}{4 \times 10^{-3}} = 1.75 \times 10^{-4} \text{ rad}$$
$$\approx 0.01^\circ.$$

### Test 9.19 Frequency Scanning

**Solution: (a)**

The antenna array pattern is determined by the pattern of the individual elements and by the phase and amplitude distributions enacted on the input signal when the array is operated in the transmit mode or on the received channels when the array is operated in the receive mode. Thus, the beamsteering operation is the same for transmission and for reception (so long as the array has no one-directional elements such as isolators).

### Test 9.20 Frequency Scanning

**Solution: (b)**

By changing the signal frequency from $f_0$ to $(f_0 + \Delta f)$, the incremental phase delay $\delta$ changes from 0 to $2n_0\pi(\Delta f/f_0)$, where $n_0$ is a positive integer (see Eq. (9.125)). Hence, adjusting the frequency leads to a shift in beam direction, so electronic scanning is realized through *frequency scanning*.