Test 6.1 Faraday's Law

Solution: (d) V_{emf} is induced whenever the magnetic flux crossing through the loop's surface is changing with time, which can occur if **R** is changing in time, the loop is rotating, or both.

Test 6.2 Lenz's Law

Solution: (c) Lenz's law states that the current in the loop is always in a direction that opposes the change of magnetic flux that produced I. Hence, Lenz's law determines the direction of I.

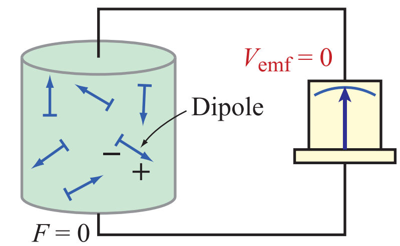
Test 6.3 Ideal Transformer

Solution: (b)

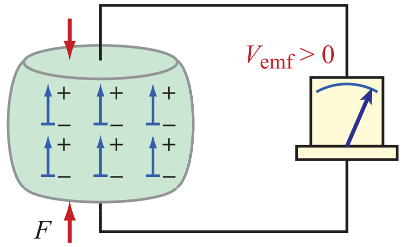
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = 10, \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = 0.1,$$
$$P_1 = I_1 V_1 = P_2 = I_2 V_2.$$

Test 6.4 Piezoelectricity

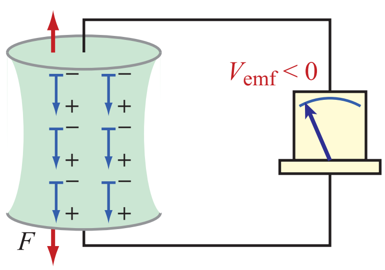
Solution: (a) Piezoelectric crystals induce an electromotive force V_{emf} when compressed or stretched. They are used in speakers, microphones, and ultrasound transducers.



(a) No force



(b) Compressed crystal



(c) Stretched crystal

Test 6.5 TV Antenna

Solution: (b) A TV loop antenna has one turn. At maximum orientation, $\Phi = BA$ for a loop of area A and a uniform magnetic field with magnitude $B = |\mathbf{B}|$. Since $f = 300$ MHz, we can express B as

$$B = B_0 \cos \omega t$$

with $\omega = 2\pi \times 300 \times 10^6$ rad/s. Hence,

$$V_{\text{emf}} = -N \frac{d\Phi}{dt} = -A \frac{d}{dt}(B_0 \cos \omega t) = AB_0 \omega \sin(\omega t).$$

V_{emf} is maximum when $\sin(\omega t) = 1$. Hence,

$$15 \times 10^{-3} = AB_0\omega = 0.02 \times B_0 \times 6\pi \times 10^8,$$

which yields $B_0 = 0.4$ (nT).

Test 6.6 Square Loop

Solution: **(c)** The **B** field due to the wire enters half the loop along $-\hat{\mathbf{x}}$ (into the page) and exists the other half along $\hat{\mathbf{x}}$ (out of the page). Hence, the net magnetic flux is zero, and

$$V_{\text{emf}} = 0.$$

Test 6.7 EM Generator

Solution: **(d)** Since **B** is parallel to the plane containing the loop surface (namely the $\hat{\mathbf{z}}$ direction), no magnetic flux enters the loop surface. The loop should cross **B** lines to generate emf. Hence, an easy solution is to change **B** so it points in a direction in the x–y plane.

Test 6.8 EM Generator

Solution: **(a)** From Eq. (6.38), the sinusoidal voltage generated by the a-c generator is $V_{\text{emf}} = A\omega B_0 \sin(\omega t + C_0) = V_0 \sin(\omega t + C_0)$. Hence,

$$V_0 = A\omega B_0 = 0.1 \times \frac{2\pi \times 1{,}800}{60} \times 1.6 = 30.16 \qquad \text{(V)},$$

$$I = \frac{V_0}{R} = \frac{30.16}{150} = 0.2 \qquad \text{(A)}.$$

Test 6.9 EM Potentials

Solution: **(b)** The phasor equivalent for **H** is

$$\widetilde{\mathbf{H}} = \hat{\mathbf{x}}\, 5e^{j0.2\pi y},$$

and the corresponding electric field phasor is, from Eq. (6.86),

$$\widetilde{\mathbf{E}} = \frac{1}{j\omega\varepsilon} \nabla \times \widetilde{\mathbf{H}}$$

$$= \frac{1}{j2\pi \times 10^7 \times 9\varepsilon_0} \left(\hat{\mathbf{y}} \frac{\partial \widetilde{H}_x}{\partial z} - \hat{\mathbf{z}} \frac{\partial \widetilde{H}_x}{\partial y} \right)$$

$$= \frac{1}{j18\pi\varepsilon_0 \times 10^7} \left(-\hat{\mathbf{z}} \frac{\partial}{\partial y} (5e^{j0.2\pi y}) \right)$$

$$= -\hat{\mathbf{z}}630e^{j0.2\pi y} \text{ (V/m)}.$$

The time-domain $\mathbf{E}$ field is

$$\mathbf{E} = -\hat{\mathbf{z}}630\cos(2\pi \times 10^7 t + 0.2\pi y) \text{ (V/m)}.$$

Hence, the correct solution is (b).

Test 6.10 EM Potentials

Solution: (c) Converting to phasor form, the electric field is given by

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}4e^{-jz} - j\hat{\mathbf{y}}3e^{-jz} \qquad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\widetilde{\mathbf{H}}(z) &= \frac{1}{-j\omega\mu} \nabla \times \widetilde{\mathbf{E}} \\
&= \frac{1}{-j\omega\mu} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4e^{-jz} & -j3e^{-jz} & 0 \end{vmatrix} \\
&= \frac{1}{-j\omega\mu} (\hat{\mathbf{x}}3e^{-jz} - \hat{\mathbf{y}}j4e^{-jz}) \\
&= \frac{j}{3 \times 10^8 \times 4\pi \times 10^{-7}} (\hat{\mathbf{x}}3 - \hat{\mathbf{y}}j4)e^{-jz} \\
&\approx j\hat{\mathbf{x}}8.0e - jz + \hat{\mathbf{y}}10.6e - jz \qquad \text{(mA/m)}.
\end{aligned}$$

Converting back to instantaneous values, this is

$$\mathbf{H}(t,z) = -\hat{\mathbf{x}}8.0\sin\left(6 \times 10^8 t - z\right) + \hat{\mathbf{y}}10.6\cos\left(6 \times 10^8 t - z\right) \qquad \text{(mA/m)}.$$

Test 6.11 Lenz's Law

Solution: (a) Lenz's law states that the induced current in the loop should be in the direction that would resist the "change" in magnetic flux. When the loop is moving away from the wire, the flux decreases, so the current direction should be such that the magnetic flux it creates is in the same direction as the flux due to the wire. The converse is true for when the loop is moving towards the wire.

Hence, the correct answer is (a).

Test 6.12 Charge Dissipation

Solution: (d) The relaxation time constant is

$$\tau_r = \frac{\varepsilon}{\sigma} = \frac{20\varepsilon_0}{\sigma} = \frac{20 \times 8.85 \times 10^{-12}}{3 \times 10^4} = 5.9 \times 10^{-15} \text{ s.}$$

Setting $e^{-t/\tau_r}/10^{-3}$ leads to

$$t = 4.07 \times 10^{-14} \approx 40 \text{ fs} \quad \text{(almost instantaneously!).}$$

Test 6.13 Boundary Conditions

Solution: (a) As explained in Section 6-8, the boundary conditions derived for electrostatic and magnetostatic fields remain valid under time-varying conditions.