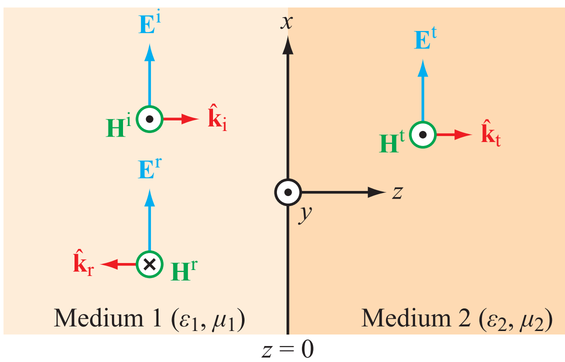
Test 8.1 Normal Incidence

Solution: (c)

The boundary conditions for both **E** and **H**, which state that the *total* fields have to be equal at the boundary, are needed. The total field in Medium 1 is the sum of the fields of the incident and reflected waves:

$$\widetilde{\mathbf{E}}_1(0) = \widetilde{\mathbf{E}}_2(0) \quad \text{or} \quad E_0^{\mathrm{i}} + E_0^{\mathrm{r}} = E_0^{\mathrm{t}},$$

$$\widetilde{\mathbf{H}}_1(0) = \widetilde{\mathbf{H}}_2(0) \quad \text{or} \quad \frac{E_0^{\mathrm{i}}}{\eta_1} - \frac{E_0^{\mathrm{r}}}{\eta_1} = \frac{E_0^{\mathrm{t}}}{\eta_2}.$$



Note that to satisfy the relationship between the directions of **E**, **H**, and $\hat{\mathbf{k}}$, the reflected wave's **H** field points along $-\hat{\mathbf{y}}$, whereas those of the incident and transmitted waves are along $+\hat{\mathbf{y}}$. That's the reason why E_0^{r}/η_1 has a negative sign.

Test 8.2 Oblique Incidence

Solution: (d)

The phase-matching condition is also known as Snell's laws (Eqs. 8.55 and 8.56):

$$\theta_{\mathrm{r}} = \theta_{\mathrm{i}},$$

$$\frac{\sin\theta_{\mathrm{t}}}{\sin\theta_{\mathrm{i}}} = \frac{k_1}{k_2} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\sqrt{\mu_2\varepsilon_2}} = \frac{n_1}{n_2}.$$

When Snell's laws are used in conjunction with the conditions mandating that at the boundary the total electric and magnetic fields in Madium 1 (incident plus reflected) have to be equal to the transmitted fields, we can obtain the fields of the reflected and transmitted waves.

Test 8.3

Solution: (b)

At $f = 5$ GHz,

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{c}{f\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{3 \times 10^8}{5 \times 10^9 \sqrt{16}} = 0.015 \text{ m} = 1.5 \text{ cm}.$$

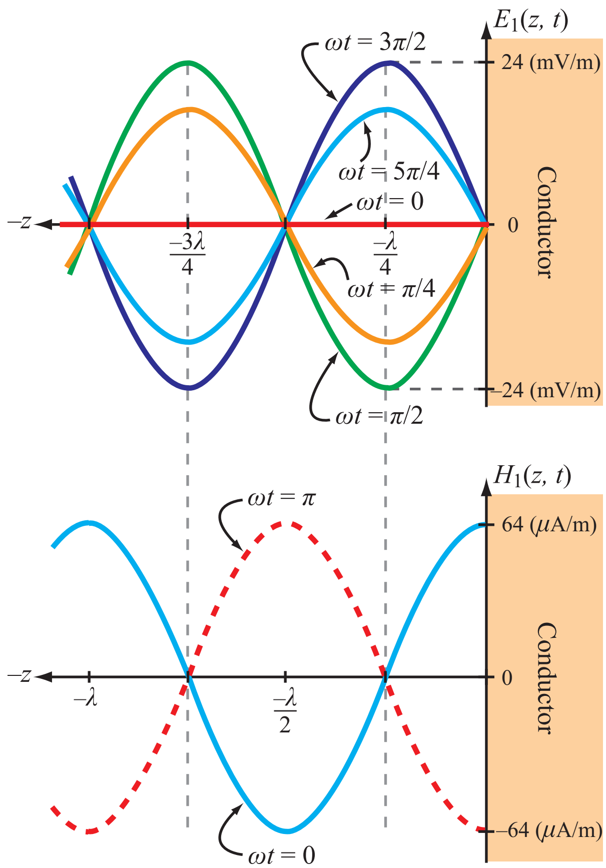
For the radome to be "transparent" to the radar signal, d should be a multiple of $\lambda/2 = 1.5/2 = 0.75$ cm.

Hence, we choose

$$d = \frac{3\lambda}{2} = 2.25 \text{ cm}.$$

Test 8.4 Normal Incidence on Conductor

Solution: (a)



Since the reflection coefficient at surface of a good conductor is $\Gamma = -1$, the total field at the boundary is

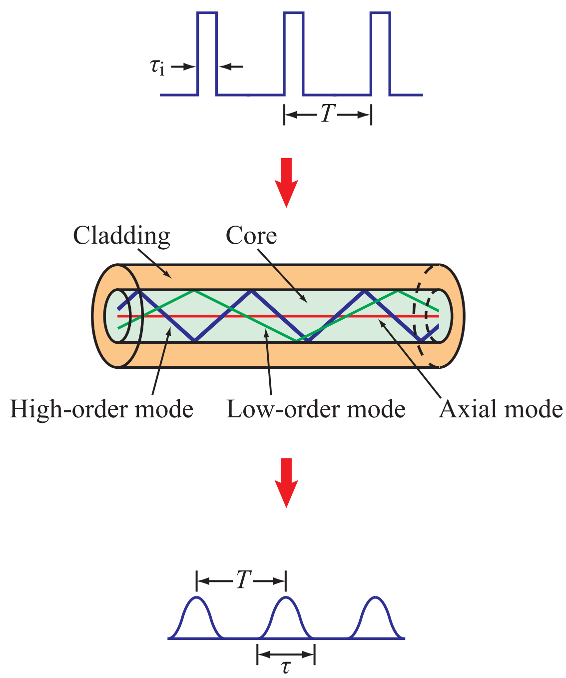
$$\mathbf{E}_1 = E^{\mathrm{i}} + \Gamma E^{\mathrm{i}} = E^{\mathrm{i}}(1+\Gamma) = E^{\mathrm{i}}(1-1) = 0.$$

The field in the air medium repeats every $\lambda/2$ and its first maximum occurs at

$$\frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times 10^8}{4\times 7.5\times 10^9} = 0.01 \text{ m} = 1 \text{ cm}.$$

Test 8.5 Modal Dispersion

Solution: (c)



Modal dispersion refers to different modes zigzagging at different angles, as a result of which they have different transit times between the two ends.

Test 8.6 Plane of Incidence

Solution: (b)

The plane of incidence is defined as the plane containing the direction of propagation $\hat{\mathbf{k}}_i$ and the normal to the boundary, which in this case is the $\hat{\mathbf{z}}$ direction.

Test 8.7 Brewster Angle

Solution: (a)

For parallel polarization, the Brewster angle occurs at

$$\theta_{B\parallel} = \sin^{-1}\sqrt{\frac{1}{1+(\varepsilon_1/\varepsilon_2)}} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad (\text{for } \mu_1 = \mu_2),$$

which is valid for nonmagnetic materials. However, perpendicular polarization, occurrence of the Brewster angle requires that

$$\sin\theta_{B\perp} = \sqrt{\frac{1-(\mu_1\varepsilon_2/\mu_2\varepsilon_1)}{1-(\mu_1/\mu_2)^2}}$$

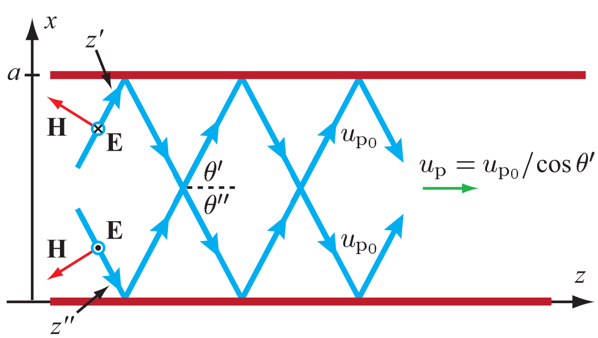
be satisfied. But if we set $\mu_1 = \mu_2$, we end up with a nonphysical solution.

Hence, the Brewster angle occurs only for parallel polarization.

Test 8.8 Rectangular Waveguide

Solution: (d)

In the figure, the arrow indicates that the cutoff frequency for TE_{10} mode is 5 GHz. This means that waves can propagate through the waveguide only if their frequencies are higher than 5 GHz. At 5 GHz, the zigzag angle θ'_{10} is 90°, which means the wave oscillates up and down between the conducting boundaries, but does not advance along z.



Test 8.9 Resonant Cavity

Solution: (b)

$$\Delta f = \frac{f_{\text{resonant}}}{Q} = \frac{10^{10}}{10^4} = 10^6 \text{ Hz} = 1 \text{ MHz}.$$

Test 8.10 Wave Power

Solution: (c)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_\mathrm{r}}} = \frac{120\pi}{5} = 24\pi \quad (\Omega).$$

From Eqs. (8.8a) and (8.9),

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{24\pi - 120\pi}{24\pi + 120\pi} = \frac{-96}{144} = -0.67,$$
$$\tau = 1 + \Gamma = 1 - 0.67 = 0.33.$$

$$S^\mathrm{i}_\mathrm{av} = \frac{|E^\mathrm{i}_0|^2}{2\eta_0} = \frac{1600}{2 \times 120\pi} = 2.08 \text{ W/m}^2,$$
$$S^\mathrm{t}_\mathrm{av} = |\tau|^2 \frac{|E^\mathrm{i}_0|^2}{2\eta_2} = |\tau|^2 \frac{\eta_1}{\eta_2} S^\mathrm{i}_\mathrm{av} = (0.33)^2 \times \frac{120\pi}{24\pi} \times 2.08 = 1.12 \text{ W/m}^2.$$

Test 8.11 Minima and Maxima

Solution: (d)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{120\pi}{6} = 20\pi \quad (\Omega),$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71.$$

Hence, $|\Gamma| = 0.71$ and $\theta_\mathrm{r} = 180^\circ$.

In medium 1 (air),

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{25 \times 10^6} = 12 \text{ m}.$$

From Eqs. (8.16) and (8.17),

$$l_\mathrm{max} = \frac{\theta_\mathrm{r}\lambda_1}{4\pi} = \frac{\pi \times 12}{4\pi} = 3 \text{ m},$$
$$l_\mathrm{min} = l_\mathrm{max} - \frac{\lambda_1}{4} = 3 - 3 = 0 \text{ m (at the boundary)}.$$

Test 8.12 Light Color

Solution: (a)

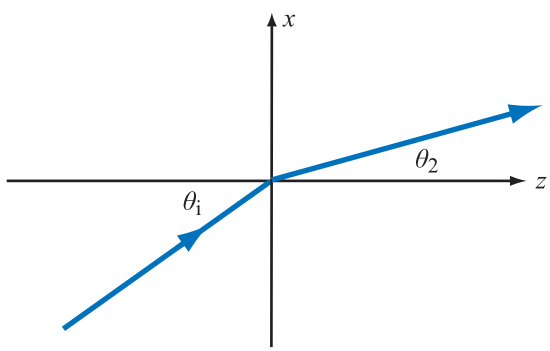
In the glass,

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{0.61}{\sqrt{1.68}} = 0.47\ \mu\text{m}.$$

The light would appear blue.

Test 8.13 Incidence Angle

Solution: (b)



From the exponential of the given expression, it is clear that the wave direction of travel is in the x–z plane. By comparison with the expressions in Eq. (8.48a) for perpendicular polarization or Eq. (8.65a) for parallel polarization, both of which have the same phase factor, we conclude that:

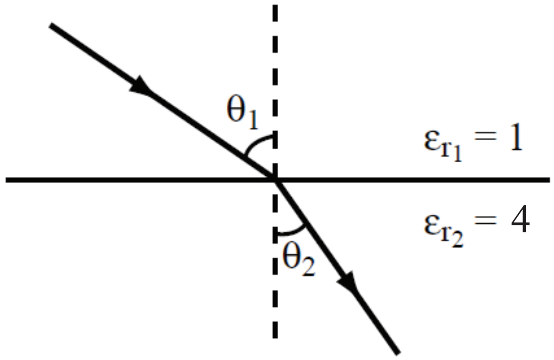
$$k_1 \sin\theta_i = 4,$$
$$k_1 \cos\theta_i = 6.$$

Hence,

$$\theta_i = \tan^{-1}(2/3) = 33.7^\circ.$$

Test 8.14 Refraction Angle

Solution: (d)



For nonmagnetic materials, Eq. (8.72) gives

$$\theta_1 = \theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon 1}} = \tan^{-1} 2 = 63.44^\circ.$$

But

$$\sin\theta_2 = \frac{\sin\theta_1}{\sqrt{\varepsilon_{r_2}}} = \frac{\sin\theta_1}{2} = \frac{\sin 63.44^\circ}{3} = 0.45,$$

or $\theta_2 = 26.57^\circ$.

Test 8.15 Waveguide

Solution: (a)

Comparison of the given expression with Eq. (8.110a) reveals that

$$\frac{m\pi}{a} = 20\pi, \qquad \text{hence } m = 1$$
$$\frac{n\pi}{b} = 100\pi, \qquad \text{hence } n = 3.$$

Mode is TE_{13}.