Chapter 1

Problem 1.1 The floor indicator for an elevator is what type of signal?

(a) Analog and continuous-time
(b) Analog and discrete-time
(c) Digital and continuous-time
(d) Digital and discrete-time

Solution: $\boxed{\text{C}}$ The floor number is an integer, but it varies with continuous time.

Problem 1.2 If $x(t)$ is 2 seconds long, how long is $y(t) = x(t/2)$?

(a) 2 seconds
(b) 1 second
(c) 4 seconds
(d) 1/2 second

Solution: $\boxed{\text{C}}$ See Section 1-2.2. Note that $y(4) = x(2)$ and $y(0) = x(0)$.

Problem 1.3 $x(t) = 5\cos(t/2 + 3)$ is a pure cosine $5\cos(t/2)$ delayed by:

(a) 3
(b) –3
(c) 6
(d) –6

Solution: $\boxed{\text{D}}$ $5\cos((t-(-6))/2) = 5\cos(t/2+3) = x(t)$. Note that the delay is -6, not 6.

Problem 1.4 If $x(t)$ is a sinusoid with a nonzero frequency f_0 Hz, $y(t) = x(3t)$ is a sinusoid with frequency (in Hz):

(a) $3f_0$
(b) $f_0/3$
(c) f_0
(d) 0

Solution: $\boxed{\text{A}}$ If $x(t) = A\cos(2\pi f_0 t + \theta)$,
$y(t) = A\cos(2\pi f_0(3t) + \theta) = A\cos(2\pi(3f_0)t + \theta)$.

Problem 1.5 Given an expression for $x(t)$, to obtain an expression for $y(t) = x(at - b)$, where $a > 1$ and $b > 0$, you should:

(a) Shift right by b, then compress by a factor of a
(b) Shift right by b, then expand by a factor of a
(c) Shift left by b, then compress by a factor of a
(d) Shift left by b, then expand by a factor of a

Solution: $\boxed{\text{A}}$ See Section 1-2.4.

Problem 1.6 The even part of $\sqrt{2}\cos(t-45^\circ)$ is (Appendix C may be useful):

(a) $\cos(t)$
(b) $2\cos(t)$
(c) $\sin(t)$
(d) $2\sin(t)$

Solution: $\boxed{\text{A}}$. $\sqrt{2}\cos(t-45^\circ) = \sqrt{2}\cos(t)\cos(45^\circ) + \sqrt{2}\sin(t)\sin(45^\circ)$. The even part is $\sqrt{2}\cos(t)\cos(45^\circ) = \cos(t)$. Note that (c) and (d) are odd, so they couldn't be right.

Problem 1.7 The period of $4\cos(2t-1)$ is:

(a) $1/2$
(b) 2
(c) π
(d) $1/\pi$

Solution: $\boxed{\text{C}}$ From Section 1-3.4, the period of $A\cos(\omega_0 t+\theta)$ is $2\pi/\omega_0$. Here, $2\pi/2 = \pi$.

Problem 1.8 The period of $3\cos(2\pi t+1) + 5\cos(6\pi t+2)$ is:

(a) 1
(b) 3
(c) $1/3$
(d) $1/6$

Solution: $\boxed{\text{A}}$ The period of $3\cos(2\pi t+1)$ is $\frac{2\pi}{2\pi} = 1$. The period of $5\cos(6\pi t+2)$ is $\frac{2\pi}{6\pi} = \frac{1}{3}$. The 2nd term repeats 3 times while the first term repeats once, so the period of the combined expression is 1.

Problem 1.9 A ramp $r(t)$ is related to a step $u(t)$ by:

(a) $r(t) = t\,u(t)$
(b) $\frac{dr}{dt} = u(t)$
(c) $r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$
(d) All of the above

Solution: $\boxed{\text{D}}$ See Eq. (1.18).

Problem 1.10 A triangle with height 1 centered at $t=0$, extending from $t=-1$ to $t=1$, can be represented using ramps $r(t)$ as:

(a) $r(t+1) - r(t-1)$
(b) $r(t+1) - 2r(t-1)$
(c) $r(t+1) - 2r(t)$
(d) $r(t+1) - 2r(t) + r(t-1)$

Solution: $\boxed{\text{D}}$ $-2r(t)$ levels off the ramp and starts it going down and $r(t-1)$ levels it off.

Problem 1.11 An impulse $\delta(t)$ is related to a step $u(t)$ by:

(a) $\delta(t)$ is $\frac{1}{2\epsilon}(u(t+\epsilon)-u(t-\epsilon))$ as $\epsilon \to 0$

(b) $\delta(t) = \frac{du}{dt}$

(c) $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$

(d) All of the above

Solution: $\boxed{\text{D}}$ See Eq. (1.23) and Eq. (1.24) and Fig. 1-19(b).

Problem 1.12 $\int_{-\infty}^{\infty} t\delta(2t-4)\, dt =$?

(a) 1

(b) 2

(c) 4

(d) 1/2

(e) 1/4

Solution: $\boxed{\text{A}}$ From Eq. (1.30), $\delta(2t-4) = \delta(2(t-2)) = \frac{1}{2}\delta(t-2)$. From Eq. (1.29),

$$\int_{-\infty}^{\infty} t\delta(2t-4)\, dt = \int_{-\infty}^{\infty} \frac{t}{2}\delta(t-2)\, dt = \int_{-\infty}^{\infty} \frac{2}{2}\delta(t-2)\, dt = 1.$$

Problem 1.13 $x(t) = e^t\, u(-t)$ does which of these:

(a) Blow up as $t \to -\infty$

(b) Blow up as $t \to \infty$

(c) Decay to 0 as $t \to -\infty$

(d) Decay to 0 as $t \to \infty$

Solution: $\boxed{\text{C}}$ See Fig. 1-22(a). e^t blows up as $t \to \infty$, but decays to 0 as $t \to -\infty$

Problem 1.14 The energy of $x(t) = r(t) - r(t-1) - u(t)$ is:

(a) 1

(b) 1/2

(c) 1/3

(d) 0

Solution: $\boxed{\text{C}}$ $x(t) = t$ for $0 < t < 1$ and 0 elsewhere, since $r(t-1)$ levels off the initial ramp and $u(t-1)$ drops the result to 0. So the energy of $x(t)$ is

$$\int_0^1 t^2\, dt = \frac{1}{3}.$$

Problem 1.15 The average power of $4\cos(3t-1)$ is:

(a) 16

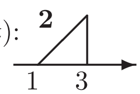
(b) 8

(c) 4

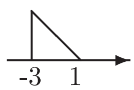
(d) 0

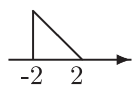
Solution: $\boxed{\text{B}}$ From Eq. (1.38) the average power of a sinusoid with a nonzero frequency is half the square of its amplitude. Here $(4)^2/2 = 8$.

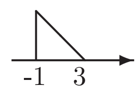
Problem 1.16 See the plots below.

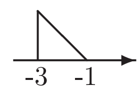
If $x(t)$: 

then which plot represents $y(t) = x(-t/2 + 2)$?

(a): 

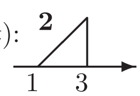
(b): 

(c): 

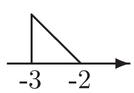
(d): 

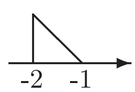
Solution: $\boxed{\text{B}}$ $x(-t/2 + 2) = x(\frac{-1}{2}(t-4))$.
Expand by 2 and reverse, then delay by 4.

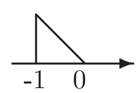
Problem 1.17 See the plots below.

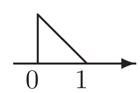
If $x(t)$: 

then which plot represents $y(t) = x(-2t + 1)$?

(a): 

(b): 

(c): 

(d): 

Solution: $\boxed{\text{C}}$ Shift then scale: Advance by 1, then compress by 2, then reverse.

Problem 1.18 The energy of the signal $x(t) = 5t^2,\ \ 0 < t < 2$ with $x(t) = 0$ otherwise, is:

(a) 25

(b) 50

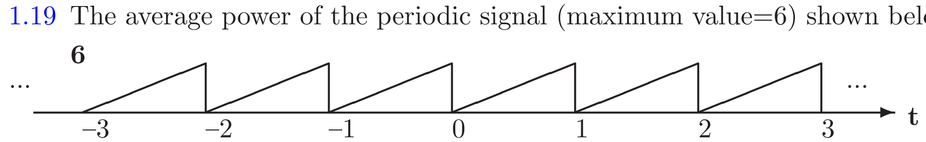
(c) 100

(d) 160

Solution: $\boxed{\text{D}}$ Energy of $x(t)$ is

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_0^2 (5t^2)^2\, dt = \int_0^2 25t^4\, dt = 5t^5|_0^2 = 160.$$

Problem 1.19 The average power of the periodic signal (maximum value=6) shown below is:



(a) 10

(b) 12

(c) 36

(d) 72

Solution: $\boxed{\text{B}}$ Energy of one period is

$$\int_0^1 |6t|^2 \, dt = 36 \int_0^1 t^2 \, dt = 12.$$

Average Power $= \frac{\text{Energy}}{\text{Period}} = \frac{12}{1} = 12.$

Problem 1.20 An LTI system has an impulse response $h(t) = e^{-|t|}$. The system is:

(a) BIBO stable but not causal

(b) Causal but not BIBO stable

(c) Stable and causal

(d) Neither stable nor causal

Solution: $\boxed{\text{A}}$ The system is stable because

$$\int_{-\infty}^{\infty} |h(t)| \, dt = 2 \int_0^{\infty} e^{-t} \, dt = 2 < \infty,$$

but not causal because $h(t) \neq 0$ for $t < 0$.

Chapter 2

Problem 2.1 The system

$$\frac{d^2y}{dt^2} + 3t\frac{dy}{dt} + y(t) = x(t)$$

is:

(a) Linear

(b) Time-invariant

(c) Linear time-invariant

(d) Neither linear nor time-invariant

Solution: $\boxed{\text{A}}$ The system is not time-invariant due to the time-varying coefficient $(3t)$.

Problem 2.2 The system

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y(t) = x(t)$$

is:

(a) Linear

(b) Time-invariant

(c) Linear time-invariant

(d) Neither linear nor time-invariant

Solution: $\boxed{\text{C}}$ This is a linear constant-coefficient differential equation (LCCDE), so the system is LTI.

Problem 2.3 The significance of a system with zero initial conditions being LTI is:

(a) If the input is $x(t)$ and the impulse response is $h(t)$, then the output $y(t)$ is $y(t) = h(t) * x(t)$.

(b) The response to an eternal sinusoid is an eternal sinusoid at the same frequency.

(c) The response to a linear combination of inputs is the same linear combination of the outputs due to each input acting alone.

(d) All of the above.

Solution: $\boxed{\text{D}}$ All of these properties.

Problem 2.4 The response of an RC circuit with RC = 2 s to a rectangular pulse of duration 1 μs and height 10^6 V for $t > 1$ μs is:

(a) $e^{-t}\, u(t)$

(b) $\frac{1}{2}e^{-t/2}\, u(t)$

(c) $2e^{-2t}\, u(t)$

(d) $10^6 e^{-t/2}\, u(t)$

Solution: $\boxed{\text{B}}$ From Eq. (2.17), the impulse response is

$$h(t) = \frac{1}{RC}e^{-t/(RC)}\, u(t) = \frac{1}{2}e^{-t/2}\, u(t).$$

The short pulse acts as an impulse with area $(10^6)1$ μs $= 1$ V-s.

Problem 2.5 $u(t) - u(t-1) \rightarrow \boxed{h(t) = \delta(t) - \delta(t-1)} \rightarrow y(t) = ?$

(a) $u(t) - u(t-2)$

(b) $u(t) - 2u(t-1) + u(t-2)$

(c) $u(t)$

(d) 0

Solution: $\boxed{\text{B}}$ From entry #6 of Table 2-1, $x(t) * \delta(t-T) = x(t-T)$.
So $y(t) = (u(t) - u(t-1)) - (u(t-1) - u(t-2)) = u(t) - 2u(t-1) + u(t-2)$.

Problem 2.6 The step response of an LTI system is triangle $y_{\text{step}}(t) = r(t) - 2r(t-1) + r(t-2)$. Its impulse response $h(t)$ is

(a) $r(t) - 2r(t-1) + r(t-2)$

(b) $u(t) - 2u(t-1) + u(t-2)$

(c) $\delta(t) - 2\delta(t-1) + \delta(t-2)$

(d) $\delta(t) - 2r(t-1) + r(t-2)$

Solution: $\boxed{\text{B}}$ From Eq. (2.22), $h(t) = dy_{\text{step}}/dt$. From Eq. (1.18), $dr/dt = u(t)$.

Problem 2.7 $e^{-4t}\, u(t) \rightarrow \boxed{h(t) = e^{-3t}\, u(t)} \rightarrow ?$ (Initial conditions are zero.)

(a) $e^{-3t}\,u(t) + e^{-4t}\,u(t)$

(b) $e^{-3t}\,u(t) - e^{-4t}\,u(t)$

(c) $e^{-4t}\,u(t) - e^{-3t}\,u(t)$

(d) $3e^{-3t}\,u(t) + 4e^{-4t}\,u(t)$

Solution: $\boxed{\text{B}}$ $y(t) = h(t) * x(t)$. Compute this directly or use entry #3 in Table 2-2.

Problem 2.8 $e^{-3t}\,u(t) \rightarrow \boxed{h(t) = e^{-3t}\,u(t)} \rightarrow ?$ (Initial conditions are zero.)

(a) $te^{-3t}\,u(t)$

(b) $(t+1)e^{-3t}\,u(t)$

(c) $(t+3)e^{-3t}\,u(t)$

(d) $9e^{-3t}\,u(t)$

Solution: $\boxed{\text{A}}$ $y(t) = h(t) * x(t)$. Compute this directly or use entry #4 in Table 2-2.

Problem 2.9 $e^{-t}\,u(t) \rightarrow \boxed{h(t) = u(t-1)} \rightarrow ?$ (Initial conditions are zero.)

(a) $e^{-t}\,u(t)$

(b) $e^{-(t-1)}u(t-1)$

(c) $(e^{-t} - 1)\,u(t)$

(d) $(1 - e^{-(t-1)})u(t-1)$

Solution: $\boxed{\text{D}}$ $y(t) = h(t) * x(t)$. Compute this directly or use entries #5 and #9 in Table 2-1. Then

$$x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{0}^{t} e^{-\tau}\,d\tau = (1 - e^{-t})\,u(t).$$

Then delay this by 1.

Problem 2.10 An LTI system with impulse response $h(t) = e^{j3t}\,u(t)$ is best described as:

(a) BIBO stable

(b) Marginally stable

(c) Unstable

(d) OBIB stable

Solution: $\boxed{\text{B}}$ An LTI system with impulse response $h(t) = e^{(\alpha+j\beta)t}\,u(t)$ is BIBO stable if and only if $\alpha < 0$. Here $\alpha = 0$, so the system is marginally stable. See Section 2-6.4.

Problem 2.11 The response of an uncharged RC circuit with RC=1s to input $\sqrt{2}\cos(t)$ is:

(a) $\cos(t + 45^\circ)$

(b) $\cos(t - 45^\circ)$

(c) $2\cos(t + 135^\circ)$

(d) $2\cos(t - 135^\circ)$

Solution: $\boxed{\text{B}}$ From Eq. (2.106), the frequency response function is $\mathbf{H}(\omega) = 1/(j\omega + 1)$. Here $\omega = 1$ (from $\sqrt{2}\cos(t)$), so

$$\mathbf{H}(1) = \frac{1}{1 + j1} = \frac{1}{\sqrt{2}e^{j45^\circ}}.$$

The $\sqrt{2}$ factors cancel.

Problem 2.12 $5\cos(t) \longrightarrow \boxed{\frac{dy}{dt} + y(t) = \frac{dx}{dt} - x(t)} \longrightarrow ?$ (Initial conditions are zero.)

(a) $5\cos(t + 45^\circ)$
(b) $5\cos(t - 45^\circ)$
(c) $5\sin(t)$
(d) $-5\sin(t)$

Solution: $\boxed{\text{D}}$ From Eq. (2.114), the frequency response function is $\mathbf{H}(\omega) = \frac{j\omega-1}{j\omega+1}$. Here $\omega = 1$ (from $x(t)$), so

$$\mathbf{H}(1) = \frac{j-1}{j+1} = \frac{\sqrt{2}e^{j135^\circ}}{\sqrt{2}e^{j45^\circ}} = e^{j90^\circ}.$$

$\cos(t + 90^\circ) = -\sin(t)$.

Problem 2.13 $\cos(2t) \longrightarrow \boxed{\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4y(t) = 6x(t)} \longrightarrow ?$ (Initial conditions are zero.)

(a) $\sin(2t)$
(b) $-\sin(2t)$
(c) $\cos(2t + 45^\circ)$
(d) $\cos(2t - 45^\circ)$

Solution: $\boxed{\text{A}}$ From Eq. (2.114), the frequency response function is

$$\mathbf{H}(\omega) = \frac{6}{(j\omega)^2 + 3(j\omega) + 4}.$$

Here $\omega = 2$ (from $x(t)$), so

$$\mathbf{H}(2) = \frac{6}{-4 + j6 + 4} = \frac{1}{j} = e^{-j90^\circ}.$$

$\cos(2t - 90^\circ) = \sin(2t)$.

Problem 2.14 For what value of ω is $\cos(\omega t) \longrightarrow \boxed{\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 9y(t) = 2\frac{dx}{dt}} \longrightarrow y(t) = x(t)$?

(a) 1
(b) 2
(c) 3
(d) 4

Solution: $\boxed{\text{D}}$ From Eq. (2.114), the frequency response function is

$$\mathbf{H}(\omega) = \frac{2(j\omega)}{(j\omega)^2 + 2(j\omega) + 9}.$$

Since the numerator is pure imaginary, the denominator must also be pure imaginary to make $\mathbf{H}(\omega)$ real-valued. This requires that $\omega = 3$, and $\mathbf{H}(3) = 1$, so $y(t) = x(t)$.

Problem 2.15 The ride of a car driving over a pothole is smoothest when its suspension is:

(a) Overdamped
(b) Critically damped
(c) Underdamped
(d) Undamped

Solution: $\boxed{\text{C}}$ See Fig. 2-29.

Problem 2.16 $\cos(2t) \longrightarrow \boxed{\dfrac{d^2y}{dt^2} + 5y = \dfrac{dx}{dt} + 2x} \longrightarrow y(t) =$: (Initial conditions are zero.)

(a) $\sqrt{2}\cos(2t + 45^\circ)$
(b) $\sqrt{2}\cos(2t – 45^\circ)$
(c) $2\sqrt{2}\cos(2t + 45^\circ)$
(d) $2\sqrt{2}\cos(2t – 45^\circ)$

Solution: $\boxed{\text{C}}$

$$\begin{aligned}
\mathbf{H}(\omega) &= \frac{2 + j\omega}{5 - \omega^2}, \\
\mathbf{H}(2) &= \frac{2 + j2}{5 - 2^2} = 2 + j2 = 2\sqrt{2}e^{j45^\circ}, \\
y(t) &= 2\sqrt{2}\cos(2t + 45^\circ).
\end{aligned}$$

Problem 2.17 $[e^{-3t}\ u(t)] * [3\sqrt{2}\cos(3t)] =?$ Note that this is NOT $3\sqrt{2}\cos(3t)\ u(t)$.

(a) $\cos(3t)$
(b) $\sin(3t)$
(c) $\cos(3t + 45^\circ)$
(d) $\cos(3t - 45^\circ)$

Solution: $\boxed{\text{D}}$

$$\begin{aligned}
\mathbf{H}(\omega) &= \frac{1}{j\omega + 3}, \\
\mathbf{H}(3) &= \frac{1}{3 + j3} = \frac{1}{3\sqrt{2}}e^{-j45^\circ},
\end{aligned}$$

so

$$\frac{3\sqrt{2}}{3\sqrt{2}}\cos(3t - 45^\circ) = \cos(3t - 45^\circ).$$

Problem 2.18 $x(t) = 1 \rightarrow \boxed{h(t) = e^{-t}\, u(t)} \rightarrow$? Note that the input is NOT $u(t)$.

(a) 0

(b) 1

(c) $\sqrt{2}$

(d) 2

Solution: $\boxed{\text{B}}$

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{1}{j\omega+1}, \\ \mathbf{H}(0) &= \frac{1}{0+1} = 1.\end{aligned}$$

A constant is a 0-Hz sinusoid.

Problem 2.19

$$\frac{d^2y}{dt^2} + c\frac{dy}{dt} + 4y(t) = x(t)$$

is critically damped for what value of c?

(a) 1

(b) 2

(c) 3

(d) 4

Solution: $\boxed{\text{D}}$ From Eq. (2.123), the characteristic equation is $\mathbf{s}^2 + c\mathbf{s} + 4 = 0$. The roots

$$\frac{-c \pm \sqrt{c^2 - 4(1)(4)}}{2}$$

are equal if $c = \pm 4$. But $c = -4$ makes the system unstable. Hence the only viable solution is $c = 4$.

Problem 2.20 The impulse response of a system described by an LCCDE consists of:

(a) The sum of two decaying exponential functions

(b) An exponentially decaying sinusoidal function

(c) A decaying exponential multiplied by a linear function

(d) Any one of the above

Solution: $\boxed{\text{D}}$ If the LCCDE is overdamped, the response has form A. If underdamped, it has form B. If critically damped, it has form C. So any one of these is possible.

Chapter 3

Problem 3.1 The inverse Laplace transform of

$$\frac{e^{-3\mathbf{s}}}{\mathbf{s}+2}$$

is:

(a) $e^{-2t}\, u(t)$
(b) $e^{-3t}\, u(t)$
(c) $e^{-2(t-3)} u(t-3)$
(d) $e^{-3(t-2)} u(t-2)$

Solution: $\boxed{\text{C}}$ See entry #3a of Table #3-2.

Problem 3.2 The inverse Laplace transform of

$$\frac{(\mathbf{s}+3)e^{-5\mathbf{s}}}{(\mathbf{s}+3)^2+4}$$

is:

(a) $e^{-3(t-5)} \cos(2(t-5))\, u(t-5)$
(b) $e^{-5(t-3)} \cos(2(t-3))\, u(t-3)$
(c) $e^{-3(t-5)} \sin(4(t-5))\, u(t-5)$
(d) $e^{-5(t-3)} \sin(4(t-3))\, u(t-3)$

Solution: $\boxed{\text{A}}$ See entry #14 of Table #3-2 and entry #4 of Table #3-1.

Problem 3.3 The poles and zeros of $\frac{\mathbf{s}+2}{\mathbf{s}+3}$ are, respectively,

(a) 2 and 3
(b) -2 and -3
(c) 3 and 2
(d) -3 and -2

Solution: $\boxed{\text{D}}$ See Section 3-2.

Problem 3.4 The inverse Laplace transform of

$$\frac{\mathbf{s}^2+2\mathbf{s}+2}{\mathbf{s}^2+2\mathbf{s}+1}$$

is:

(a) $te^{-t}\, u(t)$
(b) $\delta(t) + te^{-t}\, u(t)$
(c) $\delta(t) - te^{-t}\, u(t)$
(d) Undefined since the function is not strictly proper.

Solution: $\boxed{\text{B}}$

$$\frac{\mathbf{s}^2+2\mathbf{s}+2}{\mathbf{s}^2+2\mathbf{s}+1} = 1 + \frac{1}{(\mathbf{s}+1)^2}.$$

Now use entries #1 and #6 of Table 3-2.

Problem 3.5 The inverse Laplace transform of

$$\frac{1+j}{(\mathbf{s}+3+4j)} + \frac{1-j}{(\mathbf{s}+3-4j)}$$

is:

(a) $\sqrt{2}e^{-3t}\cos(4t-45°)\,u(t)$
(b) $\sqrt{2}e^{-3t}\cos(4t+45°)\,u(t)$
(c) $2\sqrt{2}e^{-3t}\cos(4t-45°)\,u(t)$
(d) $2\sqrt{2}e^{-3t}\cos(4t+45°)\,u(t)$

Solution: $\boxed{\text{C}}$ $1+j=\sqrt{2}e^{j45°}$ in entry #3 of Table 3-3 gives $2\sqrt{2}e^{-3t}\cos(4t-45°)\,u(t)$.

Problem 3.6 The inverse Laplace transform of $\frac{3}{(\mathbf{s}+1)(\mathbf{s}+4)}$ is:

(a) $e^{-t}\,u(t)+e^{-4t}\,u(t)$
(b) $e^{-t}\,u(t)-e^{-4t}\,u(t)$
(c) $e^{-4t}\,u(t)-e^{-t}\,u(t)$
(d) $3e^{-t}\,u(t)-3e^{-4t}\,u(t)$

Solution: $\boxed{\text{B}}$

$$\frac{3}{(\mathbf{s}+1)(\mathbf{s}+4)} = \frac{A_1}{\mathbf{s}+1} + \frac{A_2}{\mathbf{s}+4}.$$

Computation of residues A_1 and A_2:

$$A_1 = (\mathbf{s}+1)\frac{3}{(\mathbf{s}+1)(\mathbf{s}+4)}\bigg|_{\mathbf{s}=-1} = 1$$

and

$$A_2 = (\mathbf{s}+4)\frac{3}{(\mathbf{s}+1)(\mathbf{s}+4)}\bigg|_{\mathbf{s}=-4} = -1.$$

So

$$\frac{3}{(\mathbf{s}+1)(\mathbf{s}+4)} = \frac{1}{s+1} - \frac{1}{s+4}$$

and the inverse Laplace transform is $e^{-t}\,u(t)-e^{-4t}\,u(t)$.

Problem 3.7 Solve

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y(t) = 0$$

with initial conditions $y(0)=0$ and $\frac{dy}{dt}(0)=1$.

(a) $te^{-t}\,u(t)$
(b) $e^{-t}\,u(t)$
(c) $e^{-t}\,u(t)+te^{-t}\,u(t)$
(d) $e^{-t}\,u(t)-te^{-t}\,u(t)$

Solution: $\boxed{\text{A}}$ Taking the Laplace transform and using entries #6 and #7 of Table 3-1 gives

$$\left[\mathbf{s}^2\,\mathbf{Y}(\mathbf{s}) - \mathbf{s}y(0) - \frac{dy}{dt}(0)\right] + 2[\mathbf{s}\,\mathbf{Y}(\mathbf{s}) - y(0)] + \mathbf{Y}(\mathbf{s}) = 0.$$

Solving for $\mathbf{Y}(\mathbf{s})$ gives:

$$\mathbf{Y}(\mathbf{s}) = \frac{1}{\mathbf{s}^2 + 2\mathbf{s} + 1} = \frac{1}{(\mathbf{s}+1)^2}.$$

Using entry #6 of Table 3-2 gives $y(t) = te^{-t}\,u(t)$.

Problem 3.8 Compute the impulse response $g(t)$ of the inverse system given by

$$h(t) = \delta(t) + e^{-2t}\,u(t).$$

(a) $\delta(t) + e^{-3t}\,u(t)$

(b) $\delta(t) - e^{-3t}\,u(t)$

(c) $\delta(t) - e^{-2t}\,u(t)$

(d) $e^{-3t}\,u(t)$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}
\mathbf{H}(\mathbf{s}) &= \mathcal{L}\{h(t)\} = 1 + \frac{1}{\mathbf{s}+2} = \frac{\mathbf{s}+3}{\mathbf{s}+2}, \\
\mathbf{G}(\mathbf{s}) &= \frac{1}{\mathbf{H}(\mathbf{s})} = \frac{\mathbf{s}+2}{\mathbf{s}+3} = 1 - \frac{1}{\mathbf{s}+3}, \\
g(t) &= \mathcal{L}^{-1}\{\mathbf{G}(\mathbf{s})\} = \delta(t) - e^{-3t}\,u(t).
\end{aligned}$$

Problem 3.9 An LTI system has transfer function $\mathbf{H}(\mathbf{s}) = \frac{3(\mathbf{s}+4)}{\mathbf{s}(\mathbf{s}+3)}$. The LCCDE of the system is given by

(a) $3\dfrac{dy}{dt} + 4y = \dfrac{d^2x}{dt^2} + 3x$

(b) $3\dfrac{dy}{dt} + 12y = \dfrac{d^2x}{dt^2} + 3\dfrac{dx}{dt}$

(c) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} = 3\dfrac{dx}{dt} + 12x$

(d) $\dfrac{d^2y}{dt^2} + 3y = 3\dfrac{dx}{dt} + 4x$

Solution: $\boxed{\text{C}}$

$$\mathbf{H}(\mathbf{s}) = \frac{3\mathbf{s}+12}{\mathbf{s}^2+3\mathbf{s}} = \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})}.$$

Cross-multiplying gives $\mathbf{Y}(\mathbf{s})(\mathbf{s}^2+3\mathbf{s}) = \mathbf{X}(\mathbf{s})(3\mathbf{s}+12)$. Then taking the inverse Laplace transform gives

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} = 3\frac{dx}{dt} + 12x.$$

Problem 3.10 A minimum-phase LTI system has:

(a) All of its zeros in the left half-plane
(b) All of its poles and zeros in the left half-plane
(c) All of its zeros in the right half-plane
(d) All of its poles and zeros in the right half-plane

Solution: $\boxed{\text{B}}$ See the box above Example 3-13.

Problem 3.11 An LTI system has poles $\{-1, -2\}$ and zeros $\{-3, -4\}$, and $\mathbf{H}(0) = 18$. The LCCDE description of the system is:

(a) $3\frac{d^2y}{dt^2} + 21\frac{dy}{dt} + 36y = \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x.$
(b) $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x.$
(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 3\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 36x.$
(d) $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x.$

Solution: $\boxed{\text{C}}$

$$\mathbf{H}(\mathbf{s}) = C\frac{(\mathbf{s}+3)(\mathbf{s}+4)}{(\mathbf{s}+1)(\mathbf{s}+2)} = C\frac{\mathbf{s}^2 + 7\mathbf{s} + 12}{\mathbf{s}^+ 3\mathbf{s} + 2}$$

for some C. $18 = \mathbf{H}(0) = C\frac{12}{2}$, so $C = 3$.
Cross-multiplying

$$\mathbf{H}(\mathbf{s}) = 3\frac{\mathbf{s}^2 + 7\mathbf{s} + 12}{\mathbf{s}^2 + 3\mathbf{s} + 2} = \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})}$$

gives

$$\mathbf{Y}(\mathbf{s})(\mathbf{s}^2 + 3\mathbf{s} + 2) = \mathbf{X}(\mathbf{s})3(\mathbf{s}^2 + 7\mathbf{s} + 12).$$

Taking the inverse Laplace transform gives

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 3\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 36x.$$

Problem 3.12 The input to an RC circuit with RC = 1 s and the capacitor initially charged to 1 V is $u(t)$. The capacitor voltage is:

(a) $u(t)$
(b) $e^{-t}\, u(t)$
(c) $2e^{-t}\, u(t)$
(d) $te^{-t}\, u(t)$

Solution: $\boxed{\text{A}}$ The zero-state response is the step response $(1 - e^{-t})\, u(t)$. The zero-input response is the capacitor discharging, namely $e^{-t}\, u(t)$. The total response is the sum given by $u(t)$.

Problem 3.13 The zero-state response of an RC circuit with RC = 1 s is a function $y_{\text{ZSR}}(t)$. The capacitor is now initially charged to 3 V. The total response of the RC circuit is:

(a) $y_{\text{ZSR}}(t)$

(b) $y_{\text{ZSR}}(t) + 3e^{-t}\, u(t)$
(c) $y_{\text{ZSR}}(t) - 3e^{-t}\, u(t)$
(d) Impossible to determine without reworking the problem.

Solution: $\boxed{\text{B}}$ The total response can be partitioned into the sum of the zero-state response and the zero-input response. The zero-input response is simply the initial capacitor voltage decaying, which is $3e^{-t}\, u(t)$.

Problem 3.14 $\sqrt{2}\cos(2t+15°) \longrightarrow \boxed{\mathbf{H}(\mathbf{s}) = \frac{\mathbf{s}}{\mathbf{s}+2}} \longrightarrow$? (initial conditions are zero)

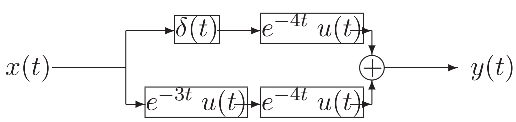
(a) $\sqrt{2}\cos(2t - 30°)$
(b) $\sqrt{2}\cos(2t + 60°)$
(c) $\cos(2t + 60°)$
(d) $\cos(2t - 30°)$

Solution: $\boxed{\text{C}}$ From Eq. (3.122), the frequency response function is $\mathbf{H}(\mathbf{s})|_{\mathbf{s}=j\omega}$. Here,

$$\mathbf{H}(j2) = \frac{j2}{2+2j} = \frac{2e^{j90°}}{2\sqrt{2}e^{j45°}} = \frac{e^{j45°}}{\sqrt{2}}.$$

The response to $\sqrt{2}\cos(2t+15°)$ is then $\frac{\sqrt{2}}{\sqrt{2}}\cos(2t+15°+45°) = \cos(2t+60°)$.

Problem 3.15 The interconnection of LTI systems shown below is equivalent to a single LTI system with impulse response $h(t) = ?$



(a) $h(t) = \delta(t)$
(b) $h(t) = e^{-3t}\, u(t)$
(c) $h(t) = e^{-4t}\, u(t)$
(d) $h(t) = e^{-3t}\, u(t) + e^{-4t}\, u(t)$

Solution: $\boxed{\text{B}}$ Using properties of cascade and parallel systems interconnections:

$$\begin{aligned} h(t) &= (e^{-3t}\, u(t)) * (e^{-4t}\, u(t)) + \delta(t) * (e^{-4t}\, u(t)) \\ &= e^{-3t}\, u(t) - e^{-4t}\, u(t) + e^{-4t}\, u(t) = e^{-3t}\, u(t). \end{aligned}$$

Much easier:

$$\mathbf{H}(\mathbf{s}) = \mathcal{L}\{h(t)\} = \frac{1}{\mathbf{s}+3}\frac{1}{\mathbf{s}+4} + (1)\frac{1}{\mathbf{s}+4} = \frac{1}{\mathbf{s}+4}\left[1 + \frac{1}{\mathbf{s}+3}\right] = \frac{1}{\mathbf{s}+4}\frac{\mathbf{s}+4}{\mathbf{s}+3} = \frac{1}{\mathbf{s}+3}.$$

Problem 3.16 The inverse Laplace transform of

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{s}+4}{(\mathbf{s}+3)^2}$$

is:

(a) $h(t) = \delta(t) + e^{-3t}\, u(t)$
(b) $h(t) = \delta(t) + te^{-3t}\, u(t)$
(c) $h(t) = e^{-3t}\, u(t) + te^{-3t}\, u(t)$
(d) $h(t) = e^{-3t}\, u(t) - te^{-3t}\, u(t)$.

Solution: $\boxed{\text{C}}$

$$\frac{\mathbf{s}+4}{(\mathbf{s}+3)^2} = \frac{1}{\mathbf{s}+3} + \frac{1}{(\mathbf{s}+3)^2}.$$

So $h(t) = e^{-3t}\, u(t) + te^{-3t}\, u(t)$.

Problem 3.17 $x(t) \longrightarrow \boxed{\mathbf{H}(\mathbf{s}) = \dfrac{e^{-3\mathbf{s}}}{\mathbf{s}}} \longrightarrow y(t) = ?$

(a) $y(t) = \displaystyle\int_{0^-}^{t} x(\tau)\, d\tau$
(b) $y(t) = \displaystyle\int_{0^-}^{t-3} x(\tau)\, d\tau$
(c) $y(t) = \displaystyle\int_{0^-}^{t+3} x(\tau)\, d\tau$
(d) $y(t) = \dfrac{dx}{dt}(t-3)$

Solution: $\boxed{\text{B}}$ Integrate, then delay by 3. See entries #4 and #8 in Table 3-1.

Problem 3.18 $e^{-3t}\, u(t) \longrightarrow \boxed{h(t)} \longrightarrow e^{-4t}\, u(t)$. Initial conditions are zero. Impulse response = ?

(a) $h(t) = \delta(t) + e^{-4t}\, u(t)$
(b) $h(t) = \delta(t) - e^{-4t}\, u(t)$
(c) $h(t) = \delta(t) + e^{-3t}\, u(t)$
(d) $h(t) = \delta(t) - e^{-3t}\, u(t)$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}
\mathbf{X}(\mathbf{s}) &= \mathcal{L}\{e^{-3t}\, u(t)\} = \frac{1}{\mathbf{s}+3}, \\
\mathbf{Y}(\mathbf{s}) &= \mathcal{L}\{e^{-4t}\, u(t)\} = \frac{1}{\mathbf{s}+4}, \\
\mathbf{H}(\mathbf{s}) &= \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})} = \frac{\mathbf{s}+3}{\mathbf{s}+4} = 1 - \frac{1}{\mathbf{s}+4}, \\
h(t) &= \mathcal{L}^{-1}\left\{1 - \frac{1}{\mathbf{s}+4}\right\} = \delta(t) - e^{-4t}\, u(t).
\end{aligned}$$

Problem 3.19 Given that $e^{-3t}\, u(t) \longrightarrow \boxed{h(t)} \longrightarrow e^{-4t}\, u(t)$ and initial conditions are zero, what is the response to input $e^{-4t}\, u(t)$?

(a) $y(t) = e^{-5t}\, u(t)$
(b) $y(t) = te^{-4t}\, u(t)$
(c) $y(t) = e^{-4t}\, u(t) - te^{-4t}\, u(t)$
(d) $y(t) = e^{-4t}\, u(t) + te^{-4t}\, u(t)$

Solution: $\boxed{\text{C}}$

$$\begin{aligned}
\mathbf{X(s)} &= \mathcal{L}\{e^{-3t}\,u(t)\} = \frac{1}{\mathbf{s}+3}, \\
\mathbf{Y(s)} &= \mathcal{L}\{e^{-4t}\,u(t)\} = \frac{1}{\mathbf{s}+4}, \\
\mathbf{H(s)} &= \frac{\mathbf{Y(s)}}{\mathbf{X(s)}} = \frac{\mathbf{s}+3}{\mathbf{s}+4} = 1 - \frac{1}{\mathbf{s}+4}, \\
h(t) &= \mathcal{L}^{-1}\{1 - \frac{1}{\mathbf{s}+4}\} = \delta(t) - e^{-4t}\,u(t).
\end{aligned}$$

The response to input $e^{-4t}\,u(t)$ is

$$h(t) * e^{-4t}\,u(t) = \delta(t) * e^{-4t}\,u(t) - e^{-4t}\,u(t) * e^{-4t}\,u(t) = e^{-4t}\,u(t) - te^{-4t}\,u(t).$$

Problem 3.20 Given that

$$\mathbf{X(s)} = \frac{\mathbf{s}^2 + 2\mathbf{s} + 3}{4\mathbf{s}^3 + 5\mathbf{s}^2 + 6\mathbf{s}}$$

and $x(t) = \mathcal{L}^{-1}\{\mathbf{X(s)}\}$, what is $x(0^+)$?

(a) $x(0^+) = \frac{1}{4}$
(b) $x(0^+) = \frac{3}{6}$
(c) $x(0^+) = \frac{3}{4}$
(d) $x(0^+) = \frac{1}{6}$

Solution: $\boxed{\text{A}}$ By the initial value theorem Eq. (3.31),

$$x(0+) = \lim_{\mathbf{s}\to\infty} \mathbf{s}\,\mathbf{X(s)},$$

which leads to $\frac{1}{4}$.

Chapter 4

Problem 4.1 The input to an RC circuit with RC = 1 s and the capacitor initially charged to 1 V is $u(t)$. Use **s**-domain circuit analysis to compute the capacitor voltage.

(a) $v(t) = u(t)$
(b) $v(t) = e^{-t}\,u(t)$
(c) $v(t) = 2e^{-t}\,u(t)$
(d) $v(t) = te^{-t}\,u(t)$

Solution: $\boxed{\text{A}}$ The **s**-domain circuit equivalent of the capacitor is impedance $\frac{1}{\mathbf{s}C}$ in series with voltage source

$$\frac{v(0^-)}{\mathbf{s}} = \frac{1}{\mathbf{s}}.$$

The step input becomes voltage source $\mathcal{L}\{u(t)\} = \frac{1}{\mathbf{s}}$.

Let $\mathcal{L}\{i(t)\} = \mathbf{I}(\mathbf{s})$. Applying KVL gives

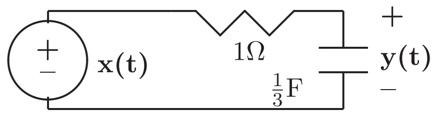
$$\frac{1}{\mathbf{s}} - \mathbf{I}(\mathbf{s})\, R - \mathbf{I}(\mathbf{s})\, \frac{1}{\mathbf{s}C} - \frac{1}{\mathbf{s}} = 0,$$

which leads to $\mathbf{I}(\mathbf{s}) = 0$. The capacitor voltage is then

$$\mathbf{I}(\mathbf{s})\, \frac{1}{\mathbf{s}C} + \frac{1}{\mathbf{s}} = \frac{1}{\mathbf{s}},$$

whose inverse Laplace transform is $u(t)$.

Problem 4.2 In the circuit shown below, $x(t) = e^{-3t}\cos(3t)\, u(t)$ and $y(0) = 0$.



Use $\mathbf{s}$-domain circuits to compute $y(t)$:

(a) $y(t) = e^{-3t}\cos(3t)\, u(t)$

(b) $y(t) = e^{-3t}\sin(3t)\, u(t)$

(c) $y(t) = e^{-3t}\cos(9t)\, u(t)$

(d) $y(t) = e^{-3t}\sin(9t)\, u(t)$

Solution: $\boxed{\text{B}}$ By voltage division,

$$\mathbf{Y}(\mathbf{s}) = \frac{1/(\mathbf{s}/3)}{1 + 1/(\mathbf{s}/3)}\, \mathbf{X}(\mathbf{s}) = \frac{3}{\mathbf{s}+3}\, \mathbf{X}(\mathbf{s}) = \frac{3}{\mathbf{s}+3}\frac{\mathbf{s}+3}{(\mathbf{s}+3)^2+3^2} = \frac{3}{(\mathbf{s}+3)^2+3^2},$$

using entry #14 of Table 3-2.

Then $y(t) = \mathcal{L}^{-1}\{\mathbf{Y}(\mathbf{s})\} = e^{-3t}\sin(3t)\, u(t)$ using entry #13 of Table 3-2.

Problem 4.3 A mass m is attached to a spring with spring constant k. The other end of the spring is connected to a surface that suddenly moves 1 m (its velocity is then $\frac{du}{dt} = \delta(t)$). Use electromechanical analogues and $\mathbf{s}$-domain circuits to compute the mass velocity.

(a) $v_c(t) = \cos(\sqrt{k/m}t)\, u(t)$

(b) $v_c(t) = \sqrt{k/m}\cos(\sqrt{k/m}t)\, u(t)$

(c) $v_c(t) = \sin(\sqrt{k/m}t)\, u(t)$

(d) $v_c(t) = \sqrt{k/m}\sin(\sqrt{k/m}t)\, u(t)$

Solution: $\boxed{\text{D}}$ Electromechanical analogues: spring→inductor with inductance $\frac{1}{k}$; mass→capacitor with capacitance m.

$\mathbf{s}$-domain circuits: inductor→impedance $\frac{\mathbf{s}}{k}$; capacitor→impedance $\frac{1}{\mathbf{s}m}$; → $\mathcal{L}\{\delta(t)\} = 1$.

Capacitor voltage

$$\mathcal{L}\{v_c(t)\} = \mathbf{V}(\mathbf{s}) = 1\frac{1/(\mathbf{s}m)}{1/(\mathbf{s}m) + \mathbf{s}/k} = \frac{k/m}{\mathbf{s}^2 + k/m}.$$

Then

$$v_c(t) = \sqrt{k/m}\sin(\sqrt{k/m}t)\, u(t)$$

is the mass velocity using entry #9 in Table 3-2.

Problem 4.4 An op-amp circuit has a capacitor in its feedback loop and a resistor in its input. The circuit can best be described as:

(a) Differentiator
(b) Integrator
(c) Inverting amplifier
(d) Inverting summer

Solution: $\boxed{\text{B}}$ See Table 4-3.

Problem 4.5 An op-amp circuit has a capacitor C in its feedback loop and a resistor R in its input. The transfer function of the op-amp circuit is:

(a) $\frac{RC}{\mathbf{s}}$
(b) $-\frac{RC}{\mathbf{s}}$
(c) $\frac{1}{RC\mathbf{s}}$
(d) $-\frac{1}{RC\mathbf{s}}$

Solution: $\boxed{\text{D}}$ See Table 4-3. Noting units immediately eliminates (a) and (b).

Problem 4.6 The advantage of Direct Form II system realization over Direct Form I is:

(a) Direct Form II requires about half as many integrators as Direct Form I.
(b) Direct Form II requires about half as many multipliers as Direct Form I.
(c) Direct Form II requires about half as many additions as Direct Form I.
(d) None: Direct Form I is better than Direct Form II.

Solution: $\boxed{\text{A}}$ Compare Fig. 4-21(b) to Fig. 4-20.

Problem 4.7 The reason for using integrators instead of differentiators in system realizations:

(a) Differentiators amplify noise; integrators in general do not.
(b) Integrators require fewer components than differentiators.
(c) Fewer integrators than differentiators are required for a system of given order.
(d) The configurations using integrators are simpler than those using differentiators.

Solution: $\boxed{\text{A}}$ This is the only true advantage of integrators.

Problem 4.8 Why do we use feedback in systems?

(a) To stabilize an unstable system.
(b) To speed up the step response of a system.
(c) To trade off high gain for bandwidth in a system.
(d) All of the above.

Solution: $\boxed{\text{D}}$ All of these are presented in Sections 4-8 to 4-10.

Problem 4.9 Stabilization of the unstable system with impulse response $h(t) = be^{at}\, u(t)$ where $a, b > 0$ requires feedback gain $K >$?

(a) $K > a$
(b) $K > \frac{b}{a}$
(c) $K > \frac{a}{b}$
(d) $K > -\frac{a}{b}$

Solution: $\boxed{\text{C}}$ $\mathbf{H}(\mathbf{s}) = \frac{b}{\mathbf{s}-a}$. Hence Eq. (4.101) is

$$\mathbf{Q}(\mathbf{s}) = \frac{\mathbf{H}(\mathbf{s})}{1 + K\mathbf{H}(\mathbf{s})} = \frac{b/(\mathbf{s}-a)}{1 + Kb/(\mathbf{s}-a)} = \frac{b}{(\mathbf{s}-a) + Kb},$$

whose pole is $a - Kb$, which is in the left half-plane if $a - Kb < 0$. See Section 4-8.3.

Problem 4.10 Stabilization of the unstable system with transfer function

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{s}+3}{\mathbf{s}^2+4}$$

requires feedback gain $K >$:

(a) $K > 0$
(b) $K > -\frac{4}{3}$
(c) $K > \frac{4}{3}$
(d) $K > -3$

Solution: $\boxed{\text{A}}$

$$\begin{aligned}\mathbf{Q}(\mathbf{s}) &= \frac{\mathbf{H}(\mathbf{s})}{1 + K\mathbf{H}(\mathbf{s})} \\ &= \frac{(\mathbf{s}+3)/(\mathbf{s}^2+4)}{1 + K(\mathbf{s}+3)/(\mathbf{s}^2+4)} = \frac{\mathbf{s}+3}{\mathbf{s}^2+4+K\mathbf{s}+3K} = \frac{\mathbf{s}+3}{\mathbf{s}^2+K\mathbf{s}+(3K+4)}.\end{aligned}$$

The roots of the denominator $\mathbf{s}^2 + K\mathbf{s} + (3K+4)$ lie in the left half-plane if and only if $K > 0$ and $(3K+4) > 0$, which combine to just $K > 0$. See Section 4-8.3.

Problem 4.11 A system is described by the LCCDE $\frac{dy}{dt} + ay = bx$, where $a, b > 0$. To speed up the impulse response by a factor of 2 we can use feedback with feedback gain $K = ?$

(a) $K = a$
(b) $K = \frac{b}{a}$
(c) $K = \frac{a}{b}$
(d) $K = -\frac{a}{b}$

Solution: $\boxed{\text{C}}$ Taking the Laplace transform of the LCCDE gives $\mathbf{Y}(\mathbf{s})\ (\mathbf{s}+a) = b\,\mathbf{X}(\mathbf{s})$. The transfer function is

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})} = \frac{b}{\mathbf{s}+a}$$

and the impulse response is $h(t) = be^{-at}\,u(t)$.

$$\mathbf{Q}(\mathbf{s}) = \frac{\mathbf{H}(\mathbf{s})}{1 + K\mathbf{H}(\mathbf{s})} = \frac{b/(\mathbf{s}+a)}{1 + Kb/(\mathbf{s}+a)} = \frac{b}{(\mathbf{s}+a) + Kb}.$$

$Kb = a$ speeds up $h(t)$ by a factor of 2.

Problem 4.12 The steady-state step response $\lim_{t\to\infty} y_{\text{step}}(t)$ of stable

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{s}^2 + a\mathbf{s} + b}{\mathbf{s}^2 + c\mathbf{s} + d}$$

is:

(a) $y_{\text{step}} = 1$
(b) $y_{\text{step}} = \frac{b}{d}$
(c) $y_{\text{step}} = 0$
(d) $y_{\text{step}} =$ indeterminate

Solution: $\boxed{\text{B}}$ Using the final-value theorem Eq. (3.32),

$$\begin{aligned}\lim_{t\to\infty} y_{\text{step}}(t) &= \lim_{\mathbf{s}\to 0} \mathbf{s}\, \mathbf{Y}_{\text{step}}(\mathbf{s}), \\ \mathbf{Y}_{\text{step}}(\mathbf{s}) &= \mathbf{H}(\mathbf{s})\ \mathbf{U}(\mathbf{s}) = \frac{\mathbf{s}^2 + a\mathbf{s} + b}{\mathbf{s}^2 + c\mathbf{s} + d}\frac{1}{\mathbf{s}}.\end{aligned}$$

Cancelling $\frac{\mathbf{s}}{\mathbf{s}}$ leads to

$$\lim_{t\to\infty} y_{\text{step}}(t) = \mathbf{H}(0) = \left.\frac{\mathbf{s}^2 + a\mathbf{s} + b}{\mathbf{s}^2 + c\mathbf{s} + d}\right|_{\mathbf{s}=0} = \frac{b}{d}.$$

See Example 3-4. This is a very useful result in control theory.

Problem 4.13 An unstable system has $h(t) = \cos(6t)\ u(t)$. The feedback gain that makes the closed loop poles $\{-4, -9\}$ is:

(a) $K = 1$
(b) $K = 3$
(c) $K = 6$
(d) $K = 13$

Solution: $\boxed{\text{D}}$

$$\mathbf{H}(\mathbf{s}) = \mathcal{L}\{\cos(6t)\ u(t)\} = \frac{\mathbf{s}}{\mathbf{s}^2 + 36}.$$

Eq. (4.101) is

$$\begin{aligned}\mathbf{Q}(\mathbf{s}) &= \frac{\mathbf{H}(\mathbf{s})}{1 + K\mathbf{H}(\mathbf{s})} \\ &= \frac{\mathbf{s}/(\mathbf{s}^2 + 36)}{1 + K\mathbf{s}/(\mathbf{s}^2 + 36)} \\ &= \frac{\mathbf{s}}{(\mathbf{s}^2 + 36) + K\mathbf{s}} = \frac{\mathbf{s}}{\mathbf{s}^2 + K\mathbf{s} + 36} = \frac{\mathbf{s}}{(\mathbf{s} + 4)(\mathbf{s} + 9)}\end{aligned}$$

if $K = 13$.

Problem 4.14 An unstable system has $h(t) = \cos(6t)\ u(t)$. The feedback gain that makes the closed loop system critically damped is:

(a) $K = 1$

(b) $K = 3$

(c) $K = 12$

(d) $K = 36$

Solution: $\boxed{\text{C}}$

$$\mathbf{H(s)} = \mathcal{L}\{\cos(6t)\ u(t)\} = \frac{\mathbf{s}}{\mathbf{s}^2 + 36}.$$

Eq. (4.101) is

$$\mathbf{Q(s)} = \frac{\mathbf{H(s)}}{1 + K\mathbf{H(s)}} = \frac{\mathbf{s}/(\mathbf{s}^2+36)}{1 + K\mathbf{s}/(\mathbf{s}^2+36)} = \frac{\mathbf{s}}{(\mathbf{s}^2+36) + K\mathbf{s}} = \frac{\mathbf{s}}{\mathbf{s}^2 + K\mathbf{s} + 36}$$

is critically damped if $K = 2\sqrt{36} = 12$. Then $\mathbf{s}^2 + K\mathbf{s} + 36 = \mathbf{s}^2 + 12\mathbf{s} + 36 = (\mathbf{s}+6)^2$.

Problem 4.15 An unstable system has $h(t) = \cos(6t)\ u(t)$. Using PI feedback

$$G(s) = K_1 + \frac{K_2}{\mathbf{s}}\ ,$$

the closed-loop system poles are $\{-8, -9\}$ for what values of (K_1, K_2)?

(a) $(K_1, K_2) = (1, 2)$

(b) $(K_1, K_2) = (17, 36)$

(c) $(K_1, K_2) = (36, 17)$

(d) $(K_1, K_2) = (17, 36)$

Solution: $\boxed{\text{D}}$

$$\mathbf{H(s)} = \mathcal{L}\{\cos(6t)\ u(t)\} = \frac{\mathbf{s}}{\mathbf{s}^2 + 36}.$$

Eq. (4.101) is

$$\begin{aligned}
\mathbf{Q(s)} &= \frac{\mathbf{H(s)}}{1 + \mathbf{G(s)}\ \mathbf{H(s)}} \\
&= \frac{\mathbf{s}/(\mathbf{s}^2+36)}{1 + (K_1 + K_2/\mathbf{s})\mathbf{s}/(\mathbf{s}^2+36)} \\
&= \frac{\mathbf{s}}{(\mathbf{s}^2+36) + (K_1 + K_2/\mathbf{s})\mathbf{s}} = \frac{\mathbf{s}}{\mathbf{s}^2 + K_1\mathbf{s} + (36 + K_2)}.
\end{aligned}$$

For $\mathbf{Q(s)}$ with poles at $\mathbf{s} = 8$ and $\mathbf{s} = 9$,

$$\frac{\mathbf{s}}{(\mathbf{s}+8)(\mathbf{s}+9)} = \frac{\mathbf{s}}{\mathbf{s}^2 + 17\mathbf{s} + 72},$$

which leads to $K_1 = 17$ and $K_2 = 36$.

Problem 4.16 An (unstable) pure integrator has $\mathbf{H(s)} = \frac{1}{\mathbf{s}}$. The feedback gain K that makes the closed-loop impulse response $q(t) = e^{-2t}\ u(t)$ is:

(a) $K = -2$

(b) $K = -1$

(c) $K = 1$

(d) $K = 2$

Solution: $\boxed{\text{D}}$ Eq. (4.101) is

$$\begin{aligned} \mathbf{Q}(\mathbf{s}) &= \frac{\mathbf{H}(\mathbf{s})}{1+K\mathbf{H}(\mathbf{s})} = \frac{1/\mathbf{s}}{1+K/\mathbf{s}} = \frac{1}{\mathbf{s}+K}, \\ q(t) &= \mathcal{L}^{-1}\left\{\frac{1}{\mathbf{s}+K}\right\} = e^{-Kt}\, u(t). \end{aligned}$$

So $K = 2$.

Problem 4.17 An unstable system has $h(t) = \sin(6t)\, u(t)$. The closed-loop system is stable for feedback gain with what value of K?

(a) $K > 6$

(b) $K < 6$

(c) $K < -6$

(d) No value of K

Solution: $\boxed{\text{D}}$

$$\mathbf{H}(\mathbf{s}) = \mathcal{L}\{\sin(6t)\, u(t)\} = \frac{6}{\mathbf{s}^2+36}.$$

Eq. (4.101) is

$$\mathbf{Q}(\mathbf{s}) = \frac{\mathbf{H}(\mathbf{s})}{1+K\mathbf{H}(\mathbf{s})} = \frac{6/(\mathbf{s}^2+36)}{1+K6/(\mathbf{s}^2+36)} = \frac{6}{(\mathbf{s}^2+36)+6K}$$

has poles $\pm j\sqrt{36+6K}$ if $36+6K > 0$ and $\pm\sqrt{-36-6K}$ if $36+6K < 0$. Either way, the closed-loop system is unstable.

Problem 4.18 An unstable system has $h(t) = \sin(6t)\, u(t)$. Using PD feedback $\mathbf{G}(\mathbf{s}) = K_2\mathbf{s}$ the closed-loop system has a double pole at -6 for what value of K_2?

(a) $K_2 = 2$

(b) $K_2 = 6$

(c) $K_2 = 12$

(d) $K_2 = 36$

Solution: $\boxed{\text{A}}$

$$\mathbf{H}(\mathbf{s}) = \mathcal{L}\{\sin(6t)\, u(t)\} = \frac{6}{\mathbf{s}^2+36}.$$

Eq. (4.101) is

$$\begin{aligned} \mathbf{Q}(\mathbf{s}) &= \frac{\mathbf{H}(\mathbf{s})}{1+\mathbf{G}(\mathbf{s})\,\mathbf{H}(\mathbf{s})} \\ &= \frac{6/(\mathbf{s}^2+36)}{1+(K_2\mathbf{s})6)/(\mathbf{s}^2+36)} \\ &= \frac{6}{(\mathbf{s}^2+36)+6K_2\mathbf{s}} = \frac{6}{\mathbf{s}^2+6K_2\mathbf{s}+36} = \frac{6}{(\mathbf{s}+6)^2} \end{aligned}$$

if $K_2 = 2$.

Problem 4.19 An unstable system has $h(t) = \sin(6t)\, u(t)$. Using PD feedback $\mathbf{G}(\mathbf{s}) = K_2\mathbf{s}$ the closed loop poles are $\{-4, -9\}$ if K_2 is:

(a) $K_2 = 1/6$

(b) $K_2 = 1$

(c) $K_2 = 13/6$

(d) $K_2 = 13$

Solution: $\boxed{\text{C}}$

$$\mathbf{H}(\mathbf{s}) = \mathcal{L}\{\sin(6t)\, u(t)\} = \frac{6}{\mathbf{s}^2 + 36}.$$

Eq. (4.101) is

$$\begin{aligned}
\mathbf{Q}(\mathbf{s}) &= \frac{\mathbf{H}(\mathbf{s})}{1 + \mathbf{G}(\mathbf{s})\, \mathbf{H}(\mathbf{s})} \\
&= \frac{6/(\mathbf{s}^2 + 36)}{1 + (K_2\mathbf{s})6/(\mathbf{s}^2 + 36)} \\
&= \frac{6}{(\mathbf{s}^2 + 36) + 6K_2\mathbf{s}} = \frac{6}{\mathbf{s}^2 + 6K_2\mathbf{s} + 36} = \frac{6}{(\mathbf{s} + 4)(\mathbf{s} + 9)}
\end{aligned}$$

if $K_2 = \frac{13}{6}$.

Problem 4.20 An (unstable) pure integrator has $\mathbf{H}(\mathbf{s}) = \frac{1}{\mathbf{s}}$. The closed-loop system is stable for what values of feedback gain K?

(a) $K > 0$

(b) $K < 0$

(c) $K > 1$

(d) $K < 1$

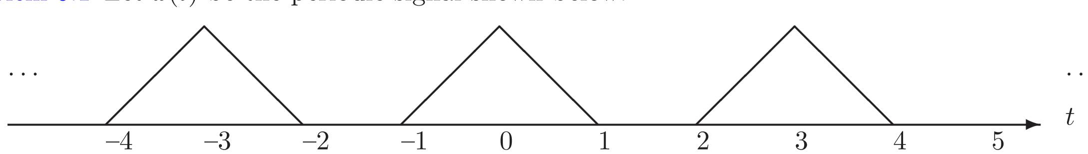
Solution: $\boxed{\text{A}}$ Eq. (4.101) is

$$\mathbf{Q}(\mathbf{s}) = \frac{\mathbf{H}(\mathbf{s})}{1 + K\mathbf{H}(\mathbf{s})} = \frac{1/\mathbf{s}}{1 + K/\mathbf{s}} = \frac{1}{\mathbf{s} + K}.$$

The pole is in the left half-plane for $K > 0$.

Chapter 5

Problem 5.1 Let $x(t)$ be the periodic signal shown below:



The form of the Fourier series expansion of $x(t)$ is (all coefficients are nonzero):

(a) $a_0 + a_1 \cos(3t) + a_2 \cos(6t) + a_3 \cos(9t) + \cdots$

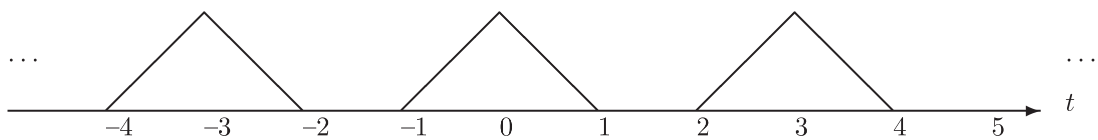
(b) $a_0 + a_1\cos((\pi/3)t) + a_2\cos((2\pi/3)t) + a_3\cos((3\pi/3)t) + \cdots$

(c) $a_0 + a_1\cos((2\pi/3)t) + a_2\cos((4\pi/3)t) + a_3\cos((6\pi/3)t) + \cdots$

(d) $b_1\sin((2\pi/3)t) + b_2\sin((4\pi/3)t) + b_3\cos((6\pi/3)t) + \cdots$

Solution: $\boxed{\text{C}}$ $x(t)$ is periodic with period $= 3$, so $\omega_0 = \frac{2\pi}{3}$.
$x(t)$ is even, so its Fourier series expansion has cosines only.

Problem 5.2 Let $x(t)$ be the periodic signal shown below:



$x(t) \rightarrow$ $\boxed{\text{Brick-wall lowpass filter with cutoff frequency 1 radians/s}}$ $\rightarrow$?

(a) 0

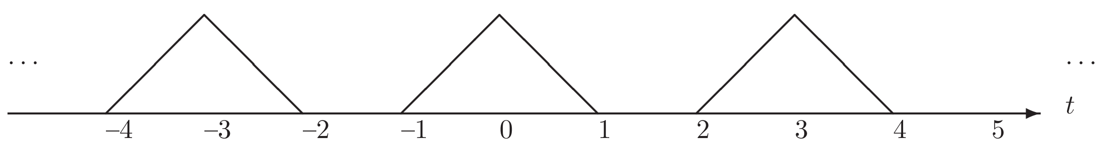
(b) a_0

(c) $a_0 + a_1\cos((2\pi/3)t)$

(d) $a_0 + a_1\cos((\pi/3)t)$

Solution: $\boxed{\text{B}}$ Both $\pi/3$ and $2\pi/3 > 1$. But there is clearly a nonzero average value a_0.

Problem 5.3 Let $x(t)$ be the periodic signal shown below:



$x(t) \rightarrow \boxed{h(t) = \frac{\sin(0.4t)}{\pi t}\cos(0.5t)} \rightarrow ?$

(a) 0

(b) a_0

(c) $a_0 + a_1\cos((2\pi/3)t)$

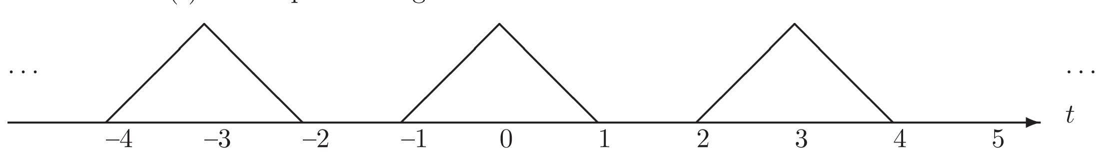
(d) $a_0 + a_1\cos((\pi/3)t)$

Solution: $\boxed{\text{A}}$ $h(t)$ is a bandpass filter that passes only

$$-0.4 + 0.5 = 0.1 < |\omega| < 0.4 + 0.5 = 0.9 < 1.$$

This excludes $\omega = 0$ and both $\pi/3$ and $2\pi/3 > 1$.

Problem 5.4 Let $x(t)$ be the periodic signal shown below:



$x(t) \rightarrow \boxed{\mathbf{H}(\omega) = 1 - e^{-j3\omega}} \rightarrow ?$

(a) 0

(b) a_0

(c) $a_0 + a_1 \cos((2\pi/3)t)$

(d) $a_0 + a_1 \cos((\pi/3)t)$

Solution: $\boxed{\text{A}}$ All you need to know is that the period of $x(t)$ is 3. Then

$$\mathbf{Y}(\omega) = \mathbf{H}(\omega)\ \mathbf{X}(\omega) = \mathbf{X}(\omega) - e^{-j3\omega}\ \mathbf{X}(\omega).$$

Then $\mathcal{F}^{-1} \rightarrow y(t) = x(t) - x(t-3) = 0!$

Problem 5.5 If $x(t)$ is a periodic signal with a period 0.5 s, what is $y(t)$ of the operation

$x(t) \rightarrow \boxed{h(t) = \frac{\sin(12t)}{\pi t}} \rightarrow y(t)$?

(a) $y(t) = 0$

(b) $y(t) = c_0$

(c) $y(t) = c_0 + c_1 \cos(2\pi t + \theta_1)$

(d) $y(t) = c_0 + c_1 \cos(2\pi t + \theta_1) + c_2 \cos(4\pi t + \theta_2)$

Solution: $\boxed{\text{B}}$ Period $= 0.5$ s$\rightarrow$ harmonics $2, 4, 6, \ldots$ Hz $= 4\pi, 8\pi, 12\pi, \ldots$ radians/s. $\mathbf{H}(\omega) = 1$ for $|\omega| < 12 < 4\pi$; 0 otherwise. Only constant ($\omega = 0$) term c_0 passes.

Problem 5.6 If $x(t) = \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots$, what is $y(t)$ of the operation

$x(t) \rightarrow \boxed{\mathbf{H}(\omega) = \begin{cases} j\omega & \text{for } 2 < |\omega| < 4 \\ 0 & \text{otherwise} \end{cases}} \rightarrow y(t)$?

(a) $y(t) = 0$

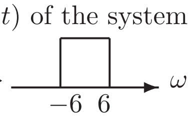
(b) $y(t) = \cos(t)$

(c) $y(t) = \sin(t) + \frac{1}{3}\sin(3t)$

(d) $y(t) = 3\sin(t) + \frac{1}{5}\sin(3t)$

Solution: $\boxed{\text{B}}$ $\mathbf{H}(\omega)$ eliminates all but the second term, and differentiates it. Output: $\cos(t)$.

Problem 5.7 What is the output $y(t)$ of the system shown below?

$3 + 2\cos(2t) + \cos(4t) \rightarrow \otimes \rightarrow$  $\rightarrow y(t)$

$\uparrow$

$2\cos(9t)$

(a) $y(t) = 0$

(b) $y(t) = 2\cos(3t)$

(c) $y(t) = \cos(5t)$

(d) $y(t) = \cos(6t)$

Solution: $\boxed{\text{C}}$ The modulated signal has components at $\pm 9 \pm 0, 2, 4 = \pm\{5, 7, 9, 11, 13\}$. Only the components at ± 5 get through the filter.

Problem 5.8 What is the output $y(t)$ of the operation

$$\cos\left(\frac{\pi}{4}\,t\right) \rightarrow \boxed{h(t)=1,\ \ -2<t<2 \text{ and } 0 \text{ otherwise}} \rightarrow y(t)\ ?$$

(a) $y(t)=0$
(b) $y(t)=\frac{2}{\pi}\cos(\frac{\pi}{4}t)$
(c) $y(t)=\frac{4}{\pi}\cos(\frac{\pi}{4}t)$
(d) $y(t)=\frac{8}{\pi}\cos(\frac{\pi}{4}t)$

Solution: $\boxed{\text{D}}$

$$\begin{aligned}\mathbf{H}(\omega) &= \mathcal{F}\{h(t)\} = 4\frac{\sin(2\omega)}{2\omega}, \\ \mathbf{H}(\pi/4) &= 4\frac{\sin(\pi/2)}{\pi/2} = \frac{8}{\pi}\ .\end{aligned}$$

Output is $\frac{8}{\pi}\cos(\frac{\pi}{4}t)$.

Problem 5.9 What is the output $y(t)$ of the operation

$$x(t)=\cos\left(\frac{\pi}{3}\,t\right) \rightarrow \boxed{y(t)=\int_{t-6}^{t} x(\tau)\,d\tau} \rightarrow y(t)\ ?$$

(a) $y(t)=0$
(b) $y(t)=\frac{3}{\pi}\cos(\frac{\pi}{3}t)$
(c) $y(t)=\frac{6}{\pi}\cos(\frac{\pi}{3}t)$
(d) $y(t)=\frac{9}{\pi}\cos(\frac{\pi}{3}t)$

Solution: $\boxed{\text{A}}$ The system has $h(t)=1$ for $0<t<6$ and 0 otherwise. To find $\mathbf{H}(\omega)$:
Let $\tilde{h}(t)=1$ with $|t|<3$ or 0 otherwise. Then from Eq. (5.88), $\tilde{\mathbf{H}}(\omega)=6\frac{\sin(3\omega)}{3\omega}$
Then $h(t)=\tilde{h}(t-3)$ and $\mathbf{H}(\omega)=6\frac{\sin(3\omega)}{3\omega}e^{-j3\omega}$ and $\mathbf{H}(\pi/3)=0$!
The reason: the system integrates the cosine over a period, and this integral is zero.

Problem 5.10

$$\int_{-\infty}^{\infty}\left(\frac{\sin(\pi t)}{\pi t}\right)^2 dt =\ ?$$

(a) 0
(b) 1
(c) $\frac{1}{2\pi}$
(d) $\frac{1}{\pi}$

Solution: $\boxed{\text{B}}$ Let $x(t)=\frac{\sin(\pi t)}{\pi t}$. Then $\mathbf{X}(\omega)=1$ for $|\omega|<\pi$ and 0 otherwise. By Parseval's theorem,

$$\int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\mathbf{X}(\omega)|^2\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}|1|^2\,d\omega = 1!$$

Problem 5.11 If $x(t) = \frac{\sin(2t)}{\pi t}$, then the energy of d^2x/dt^2 is:

(a) 0
(b) $\frac{4}{\pi}$
(c) $\frac{8}{3\pi}$
(d) $\frac{32}{5\pi}$

Solution: $\boxed{\text{D}}$

$$\begin{aligned} \mathcal{F}\{x(t)\} &= 1 \text{ for } |\omega| < 2, \\ \mathcal{F}\left\{\frac{d^2x}{dt^2}\right\} &= -\omega^2 \text{ for } |\omega| < 2. \end{aligned}$$

By Parseval's theorem,

$$\text{energy} = \frac{1}{2\pi} \int_{-2}^{2} \left|-\omega^2\right|^2 \, d\omega = \frac{32}{5\pi} .$$

Problem 5.12 The inverse Fourier transform of $8j\omega\frac{\sin(4\omega)}{4\omega}$ is:

(a) $\delta(t+4) - \delta(t-4)$
(b) $\delta(t+4) + \delta(t-4)$
(c) $\delta(t-4) - \delta(t+4)$
(d) $4\delta(t-4) - 4\delta(t+4)$

Solution: $\boxed{\text{A}}$

$$\begin{aligned} 8j\omega\frac{\sin(4\omega)}{4\omega} &= 2j\sin(4\omega) = e^{j4\omega} - e^{-j4\omega}, \\ \mathcal{F}^{-1}\left\{e^{j4\omega} - e^{-j4\omega}\right\} &= \delta(t+4) - \delta(t-4). \end{aligned}$$

Method #2:

$$\begin{aligned} \mathcal{F}^{-1}\left\{8\frac{\sin(4\omega)}{4\omega}\right\} &= \text{rect}\left(\frac{t}{8}\right) , \\ \mathcal{F}^{-1}\left\{8j\omega\frac{\sin(4\omega)}{4\omega}\right\} &= \frac{d}{dt}\,\text{rect}\left(\frac{t}{8}\right) = \delta(t+4) - \delta(t-4). \end{aligned}$$

Problem 5.13 The Fourier transform of $e^{-a|t|}$ for $a > 0$ is:

(a) $\dfrac{1}{\omega^2 + a^2}$
(b) $\dfrac{a}{\omega^2 + a^2}$
(c) $\dfrac{2a}{\omega^2 + a^2}$
(d) $\dfrac{\omega}{\omega^2 + a^2}$

Solution: $\boxed{\text{C}}$ $e^{-a|t|} = e^{-at}\, u(t) + e^{at}u(-t)$. Entries #7a and #7b in Table 5-6 are: $\mathcal{F}\{e^{-at}\, u(t)\} = \frac{1}{a+j\omega}$ and $\mathcal{F}\{e^{at}u(-t)\} = \frac{1}{a-j\omega}$. Adding,

$$\frac{1}{a+j\omega}\frac{a-j\omega}{a-j\omega} + \frac{1}{a-j\omega}\frac{a+j\omega}{a+j\omega} = \frac{2a}{\omega^2+a^2}.$$

Problem 5.14 What is $y(t)$ of the operation $x(t) \rightarrow \boxed{\mathbf{H}(j\omega) = j\omega e^{j3\omega}} \rightarrow y(t)$?

(a) $y(t) = \frac{dx}{dt}(t+3)$
(b) $y(t) = \frac{dx}{dt}(t-3)$
(c) $y(t) = \frac{dx}{dt}e^{j3t}$
(d) $y(t) = \frac{dx}{dt}2\cos(3t)$

Solution: $\boxed{\text{A}}$ $j\omega\, \mathbf{X}(\omega) \Leftrightarrow \frac{dx}{dt}$ and $\mathbf{X}(\omega)\, e^{j3\omega} \Leftrightarrow x(t{+}3)$. Combining gives $\frac{dx}{dt}(t+3)$.

Problem 5.15 What is output $y(t)$ of the operation $8\cos(2t) \rightarrow \boxed{h(t) = te^{-2t}\, u(t)} \rightarrow y(t)$?

(a) $y(t) = 4\sqrt{2}\cos(2t+45^\circ)$
(b) $y(t) = 4\sqrt{2}\cos(2t-45^\circ)$
(c) $y(t) = \sin(2t)$
(d) $y(t) = -\sin(2t)$

Solution: $\boxed{\text{C}}$ From entry #11 in Table 5-6,

$$\mathcal{F}\{te^{-2t}\, u(t)\} = \frac{1}{(j\omega+2)^2}.$$

Inserting $\omega = 2$ gives

$$\frac{1}{(j2+2)^2} = \frac{1}{(2\sqrt{2}e^{j45^\circ})^2} = \frac{1}{8}e^{-j90^\circ}.$$

Then $y(t) = \left(\frac{1}{8}\right) 8\cos(2t - 90^\circ) = \sin(2t)$.

Problem 5.16 An LTI system has gain $|\mathbf{H}(\omega)| = 1$ and phase $\angle[\mathbf{H}(\omega)] = -2\omega$. The system is given by the relationship:

(a) $y(t) = 2x(t)$
(b) $y(t) = -2x(t)$
(c) $y(t) = x(t-2)$
(d) $y(t) = x(t+2)$

Solution: $\boxed{\text{C}}$ $\mathbf{H}(\omega) = e^{-j2\omega}$ which is the frequency response of a delay by 2.

Problem 5.17 An LTI system has frequency response $\mathbf{H}(\omega) = \frac{j\omega-2}{j\omega+2}$. Which statement is true?

(a) The gain equals one for all ω.
(b) The phase equals zero for all ω.
(c) The system must be a pure delay by 2.

(d) (a) and (c).

Solution: $\boxed{\text{A}}$

$$\text{Gain} = |\mathbf{H}(\omega)| = \frac{|j\omega - 2|}{|j\omega + 2|} = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = 1.$$

This is called an all-pass system.

Problem 5.18 Which of the following cannot be the phase response of an LTI system?

(a) 3ω
(b) $3\omega^2$
(c) $-\arctan(\omega)$
(d) $\pi \cdot \text{sign}(\omega)$

Solution: $\boxed{\text{B}}$ Consider each of the four candidate phase responses:

(a) LTI system $y(t) = x(t+3)$ has transfer function $\mathbf{H}(\omega) = |\mathbf{H}(\omega)|e^{j\phi(\omega)} = e^{j3\omega}$. Hence, the phase response is

$$\phi(\omega) = 3\omega.$$

Since phase response belongs to a viable LTI system, it is not the response to the posed question.

(b) The reversal property given by Eq. (5.130) requires that the transfer function satisfy

$$\mathbf{H}(-\omega) = \mathbf{H}^*(\omega),$$

which implies that $\phi(\omega) = -\phi(-\omega)$.
Clearly, $\phi(\omega) = 3\omega^2$ does not satisfy the reversal property. Hence (b) is the answer to the posed question.

(c) LTI system $y(t) = x(t) \, * \, e^{-t} \, u(t)$ has transfer function

$$\mathbf{H}(\omega) = |\mathbf{H}(\omega)|e^{j\phi(\omega)} = \frac{1}{j\omega + 1} \; .$$

Hence, the phase response is

$$\phi(\omega) = 0 - \arctan\left(\frac{\omega}{1}\right) = -\arctan(\omega).$$

Since the phase response belongs to a viable LTI system, it is not the response to the posed question

(d) LTI system $y(t) = \frac{dx}{dt}$ has transfer function

$$\mathbf{H}(\omega) = |\mathbf{H}(\omega)|e^{j\phi(\omega)} = j\omega.$$

Hence, the phase response is

$$\phi(\omega) = \pi \; \text{sign}(\omega).$$

Since the phase response belongs to a viable LTI system, it is not the response to the posed question.

Problem 5.19 A square wave

$$x(t) = \begin{cases} \pi/4 & \text{for } k\pi < t < (k+1)\pi \\ -\pi/4 & \text{for } (k-1)\pi < t < k\pi \end{cases}$$

for integers k has Fourier series expansion

$$x(t) = \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots$$

from Example 5-4. Using this, sum the series

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$$

(a) $\pi/4$
(b) $\pi^2/4$
(c) $\pi^2/8$
(d) $\pi^2/16$

Solution: $\boxed{\text{C}}$ By Parseval's theorem,

$$\frac{1}{2}\left(1^2 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

is the average power of $x(t)$, which can be computed in the time domain as $\pi^2/16$ since

$$|x(t)|^2 = \frac{\pi^2}{16}.$$

Then

$$2\,\frac{\pi^2}{16} = \frac{\pi^2}{8}\ .$$

Problem 5.20 A square wave

$$x(t) = \begin{cases} \pi/4 & \text{for } k\pi < t < (k+1)\pi \\ -\pi/4 & \text{for } (k-1)\pi < t < k\pi \end{cases}$$

for integers k has Fourier series expansion

$$x(t) = \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots$$

from Example 5-4. Using this, compute the integral

$$\frac{1}{\pi}\int_0^{\pi} \frac{\pi}{4}\sin(3t)\,dt - \frac{1}{\pi}\int_{\pi}^{2\pi} \frac{\pi}{4}\sin(3t)\,dt$$

without computing an integral.

(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{6}$
(d) $\frac{1}{12}$

Solution: $\boxed{\text{B}}$ Eq. (5.28c) for computing coefficients $\{b_n\}$ of sine terms in a Fourier series is

$$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \sin(n\omega_0 t) \, dt,$$

where here $T_0 = 2\pi$ is the period and

$$\omega_0 = \frac{2\pi}{T_0} = 1.$$

So the desired integral is $b_3 = \frac{1}{3}$, which agrees with direct computation.

Chapter 6, Part I: Filters

Problem 6.1 A lowpass filter has a dc ($\omega = 0$) gain of 1, a corner frequency $\omega_c = 1000$ rad/s, and an order $n = 3$. What is the gain at $\omega = 10^6$ rad/s?

(a) $M(10^6) = 1$
(b) $M(10^6) = 10^{-3}$
(c) $M(10^6) = 10^{-6}$
(d) $M(10^6) = 10^{-9}$

Solution: $\boxed{\text{D}}$ The gain function for $\omega \gg \omega_c$ is

$$M(\omega) = |\mathbf{H}(\omega)| \approx \frac{\omega_c^n}{\omega^n}$$

(see Eq. (6.42) and Eq. (6.44)). Hence,

$$M(10^6) \approx \frac{(10^3)^3}{(10^6)^3} = 10^{-9}.$$

Problem 6.2 A lowpass filter has dc ($\omega = 0$) gain of 1 and a corner frequency $\omega_c = 100$ rad/s. What is the minimum order n so that the gain at $\omega = 10,000$ rad/s is 10^{-6}.

(a) $n = 1$
(b) $n = 2$
(c) $n = 3$
(d) $n = 4$

Solution: $\boxed{\text{C}}$ The gain function for $\omega \gg \omega_c$ is

$$M(\omega) = |\mathbf{H}(\omega)| \approx \frac{\omega_c^n}{\omega^n}$$

(see Eq. (6.42) and Eq. (6.44)). Here

$$10^{-6} \approx \frac{100^n}{10,000^n} = 10^{-2n},$$

which requires $n = 3$.

Problem 6.3 An octave is a factor of two in frequency, whereas a decade is a factor of ten in frequency. The gain rolloff of 20 dB/decade is equivalent to how many dB/octave?

(a) 2
(b) 3
(c) 6
(d) 10

Solution: $\boxed{\text{C}}$ $\log_{10}(2) \approx 0.3$, so $10^{0.3} \approx 2$. So an octave is 0.3 decades, and (20 dB/decade)(0.3 decade/octave) = 6 dB/octave. Both measure gain rolloff.

Problem 6.4 The system with impulse response

$$h(t) = \frac{\sin(\omega_0 t)}{\pi t}$$

is a:

(a) Lowpass filter with cutoff frequency ω_0
(b) Highpass filter with cutoff frequency ω_0
(c) Bandpass filter with cutoff frequencies ω_0 and $3\omega_0$
(d) Bandreject filter with cutoff frequencies ω_0 and $3\omega_0$

Solution: $\boxed{\text{A}}$ See Eq. (6.60).

Problem 6.5 The system with impulse response

$$h(t) = \delta(t) - \frac{\sin(\omega_0 t)}{\pi t}$$

is a:

(a) Lowpass filter with cutoff frequency ω_0
(b) Highpass filter with cutoff frequency ω_0
(c) Bandpass filter with cutoff frequencies ω_0 and $3\omega_0$
(d) Bandreject filter with cutoff frequencies ω_0 and $3\omega_0$

Solution: $\boxed{\text{B}}$ See Section 6-5.3.

Problem 6.6 The system with impulse response

$$h(t) = \frac{\sin(\omega_0 t)}{\pi t} \, 2\cos(2\omega_0 t)$$

is a:

(a) Lowpass filter with cutoff frequency ω_0
(b) Highpass filter with cutoff frequency ω_0
(c) Bandpass filter with cutoff frequencies ω_0 and $3\omega_0$
(d) Bandreject filter with cutoff frequencies ω_0 and $3\omega_0$

Solution: $\boxed{\text{C}}$ See Eq. (6.63).

Problem 6.7 The system with impulse response

$$h(t) = \delta(t) - \frac{\sin(\omega_0 t)}{\pi t}\, 2\cos(2\omega_0 t)$$

is a:

(a) Lowpass filter with cutoff frequency ω_0

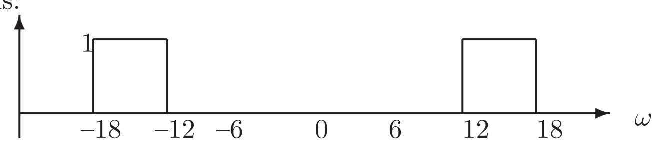
(b) Highpass filter with cutoff frequency ω_0

(c) Bandpass filter with cutoff frequencies ω_0 and $3\omega_0$

(d) Bandreject filter with cutoff frequencies ω_0 and $3\omega_0$

Solution: $\boxed{\text{D}}$ See Section 6-5.3.

Problem 6.8 The impulse response of the system with the frequency response shown below is:



(a) $h(t) = \frac{\sin(3t)}{\pi t}\, 2\cos(12t)$

(b) $h(t) = \frac{\sin(3t)}{\pi t}\, 2\cos(15t)$

(c) $h(t) = \frac{\sin(3t)}{\pi t}\, 2\cos(18t)$

(d) $h(t) = \frac{\sin(6t)}{\pi t}\, 2\cos(15t)$

Solution: $\boxed{\text{B}}$ $\omega_c = \frac{1}{2}(18+12) = 15$ and $\omega_d = \frac{1}{2}(18-12) = 3$.

$$h(t) = \frac{\sin(\omega_d t)}{\pi t}\, 2\cos(\omega_c t) = \frac{\sin(3t)}{\pi t}\, 2\cos(15t).$$

Problem 6.9 Which differential equation describes a second-order Butterworth lowpass filter with cutoff frequency $\omega_0 = 1$ rad/s?

(a) $\dfrac{d^2y}{dt^2} + \sqrt{2}\,\dfrac{dy}{dt} + y = \dfrac{d^2x}{dt^2}$

(b) $\dfrac{d^2y}{dt^2} - \sqrt{2}\,\dfrac{dy}{dt} + y = \dfrac{d^2x}{dt^2}$

(c) $\dfrac{d^2y}{dt^2} + \sqrt{2}\,\dfrac{dy}{dt} + y = x$

(d) $\dfrac{d^2y}{dt^2} - \sqrt{2}\,\dfrac{dy}{dt} + y = x$

Solution: $\boxed{\text{C}}$ The filter has poles at $\{e^{\pm j135^\circ}\}$ (see Section 6-9.3).

The transfer function is

$$\mathbf{H}(\mathbf{s}) = \frac{1}{(\mathbf{s} - e^{j135^\circ})(\mathbf{s} - e^{-j135^\circ})} = \frac{1}{\mathbf{s}^2 - \mathbf{s}2\cos(135^\circ) + 1} = \frac{1}{\mathbf{s}^2 + \sqrt{2}\mathbf{s} + 1}.$$

Equate $\mathbf{H}(\mathbf{s}) = \frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})}$, cross-multiply, and take $\mathcal{L}^{-1}$ to get the differential equation

$$\mathbf{Y}(\mathbf{s})\;(\mathbf{s}^2 + \sqrt{2}\,\mathbf{s} + 1) = \mathbf{X}(\mathbf{s}) \rightarrow \frac{d^2y}{dt^2} + \sqrt{2}\,\frac{dy}{dt} + y = x.$$

Problem 6.10 Which differential equation describes a notch filter that rejects $7\cos(100t+20°)$?

(a) $\frac{d^2y}{dt^2}+0.02\,\frac{dy}{dt}+100y=x$

(b) $\frac{d^2y}{dt^2}+0.02\,\frac{dy}{dt}+100y=\frac{d^2x}{dt^2}+100x$

(c) $\frac{d^2y}{dt^2}+0.02\,\frac{dy}{dt}+100^2y=x$

(d) $\frac{d^2y}{dt^2}+0.02\,\frac{dy}{dt}+100^2y=\frac{d^2x}{dt^2}+100^2x$

Solution: $\boxed{\text{D}}$ The filter has zeros $\{\pm j100\}$ and poles $\{-0.01\pm j100\}$ (see Section 6-7.1). Transfer function

$$\mathbf{H}(\mathbf{s})=\frac{(\mathbf{s}-j100)(\mathbf{s}+j100)}{(\mathbf{s}+0.01-j100)(\mathbf{s}+0.01+j100)}=\frac{\mathbf{s}^2+100^2}{\mathbf{s}^2+0.02\mathbf{s}+100^2+0.01^2}\approx\frac{\mathbf{s}^2+100^2}{\mathbf{s}^2+0.02\mathbf{s}+100^2}.$$

Equate $\mathbf{H}(\mathbf{s})=\frac{\mathbf{Y}(\mathbf{s})}{\mathbf{X}(\mathbf{s})}$, cross-multiply, and take $\mathcal{L}^{-1}$ to get the differential equation

$$\mathbf{Y}(\mathbf{s})\;(\mathbf{s}^2+0.02\mathbf{s}+100^2+0.01^2)=\mathbf{X}(\mathbf{s})\;(\mathbf{s}^2+100^2)$$
$$\rightarrow\frac{d^2y}{dt^2}+0.02\frac{dy}{dt}+100^2y=\frac{d^2x}{dt^2}+100^2x.$$

(-0.01 could be replaced with any negative number close to zero.)

Problem 6.11 To remove high-frequency noise from a signal, we should use a

(a) Butterworth lowpass filter
(b) Notch filter
(c) Comb filter
(d) Resonator filter

Solution: $\boxed{\text{A}}$ A Butterworth lowpass filter removes high-frequency noise.

Problem 6.12 To remove broad-spectrum noise from a periodic signal, we should use a:

(a) Butterworth filter
(b) Notch filter
(c) Comb filter
(d) Resonator filter

Solution: $\boxed{\text{D}}$ A resonator filter enhances the harmonics of a periodic signal.

Problem 6.13 To remove an interfering non-sinusoidal periodic signal, we should use a:

(a) Butterworth filter
(b) Notch filter
(c) Comb filter
(d) Resonator filter

Solution: $\boxed{\text{C}}$ A comb filter removes the harmonics of an interfering periodic signal.

Problem 6.14 To remove an interfering sinusoidal signal, we should use a:

(a) Butterworth filter
(b) Notch filter
(c) Comb filter
(d) Resonator filter

Solution: $\boxed{\text{B}}$ A notch filter removes an interfering sinusoidal signal.

Problem 6.15 What kind of filter can be regarded as a cascade connection of notch filters?

(a) Butterworth filter
(b) Bandpass filter
(c) Comb filter
(d) Resonator filter

Solution: $\boxed{\text{C}}$ A comb filter is a cascade connection of notch filters, each of which removes a harmonic of an interfering periodic signal. See Section 6-7.2.

Problem 6.16 You listen to AM radio station WBZ in Boston, which broadcasts using a carrier frequency of 670 kHz, on an AM radio whose IF filter has a center frequency of 455 kHz. The local oscillator frequency for tuning in WBZ is:

(a) 215 kHz
(b) 455 kHz
(c) 670 kHz
(d) 885 kHz

Solution: $\boxed{\text{A}}$ The local oscillator frequency is 670–455=215 kHz, which frequency converts 670 kHz down to 455 kHz, in the passband of the IF filter.

Problem 6.17 We wish to transmit four signals, each bandlimited to 10 kHz, on one wire, using DSB. The maximum frequency of the signal transmitted on the wire is:

(a) 60 kHz
(b) 70 kHz
(c) 80 kHz
(d) 100 kHz

Solution: $\boxed{\text{B}}$ The spectrum of each of the four signals is 20 kHz wide, but one of them can be at baseband:

Signal #1	Signal #2	Signal #3	Signal #4
$0 < \|f\| < 10$	$10 < \|f\| < 30$	$30 < \|f\| < 50$	$50 < \|f\| < 70$

Problem 6.18 A signal $x(t)$ is DSB-modulated using a cosine at 1 kHz, giving $y(t) = x(t)\cos(2\pi 1000t)$. Then $y(t)$ is DSB-demodulated using a sine at 1 kHz, giving $z(t) = y(t)\sin(2\pi 1000t)$. What is $z(t)$?

(a) $z(t) = 0$
(b) $z(t) = x(t)/2$
(c) $z(t) = 2x(t)$
(d) $z(t) = -x(t)$

Solution: $\boxed{\text{A}}$ The 90° phase difference between modulation and demodulation creates *fading*. See Example 6-16.

Problem 6.19 Which of the following are reasons to use a carrier, i.e., AM instead of DSB?

(a) To allow use of a simple receiver
(b) To avoid fading
(c) To reduce noise
(d) (a) and (b)

Solution: $\boxed{\text{D}}$ A is a reason, since a crystal radio, consisting of just an envelope detector and a tuner (coil) can be used to receive an AM signal. B is a reason, since fading does not occur for AM (unless the transmitter and receiver are exactly 90 degrees out of phase). C is not a reason, since both AM and DSB are vulnerable to additive noise, e.g., thunderstorms. So the answer is D (A and B).

Problem 6.20 Which of the following are reasons to use SSB instead of AM or DSB?

(a) The transmitted signal uses half the bandwidth.
(b) The transmitted signal uses half the power of DSB.
(c) Twice as many signals can be transmitted in the same amount of bandwidth.
(d) All of the above.

Solution: $\boxed{\text{D}}$ See Section 6-12.9.

Problem 6.21 A signal is bandlimited to B Hz and sampled at a rate of f_s samples/s. In order to be able to reconstruct the signal from its samples, we require:

(a) $f_s \geq B$
(b) $f_s \geq 2B$
(c) $f_s =$ Nyquist rate
(d) $f_s > 2B$

Solution: $\boxed{\text{D}}$ $f_s = 2B =$ Nyquist rate does not suffice; see Fig. 6-71.

Problem 6.22 A 300 Hz sinusoid is sampled at 500 samples/s. The frequency of the sinusoid reconstructed from these samples is:

(a) 0 Hz
(b) 100 Hz
(c) 200 Hz
(d) 300 Hz, obviously

Solution: $\boxed{\text{C}}$ The sampled signal has components at $\{\pm 300 \pm 500k\} = \pm\{200, 800, 1200 \ldots\}$ Hz, for integers k. The lowpass filter has a cutoff frequency at $\frac{500}{2} = 250$ Hz, leaving only 200 Hz.

Problem 6.23 Signal $x(t) = 3\cos(2\pi 40t) + 4\cos(2\pi 60t)$ is ideally sampled at 100 samples/s, then filtered by a brick-wall lowpass filter with a cutoff frequency of 50 Hz and a gain of 0.01:

$$3\cos(2\pi 40t) + 4\cos(2\pi 60t) \rightarrow \boxed{100 \text{ samples/s}} \rightarrow \boxed{\mathbf{H}(\omega) = \begin{cases} .01, & |\omega| < 2\pi 50 \\ 0, & |\omega| > 2\pi 50 \end{cases}} \rightarrow y(t).$$

What is the expression for $y(t)$?

(a) $y(t) = -\cos(2\pi 40t)$

(b) $y(t) = 7\cos(2\pi 40t)$

(c) $y(t) = 5\cos(2\pi 40t + 53°)$

(d) $y(t) = 5\cos(2\pi 40t - 53°)$

Solution: $\boxed{\text{B}}$ $4\cos(2\pi 60t)$ aliases to $4\cos(2\pi 40t)$ and adds to $3\cos(2\pi 40t)$, giving $7\cos(2\pi 40t)$.

Problem 6.24 The spectrum of a trumpet playing note A has a fundamental of 440 Hz and all harmonics above the tenth are negligible. The Nyquist frequency is:

(a) 2.2 kHz

(b) 4.4 kHz

(c) 8.8 kHz

(d) 17.6 kHz

Solution: $\boxed{\text{C}}$

Nyquist frequency = 2(maximum frequency) = 2(10)(440 Hz) = 8800 Hz = 8.8 kHz.

Problem 6.25 A 7-Hz sinusoid is sampled at 10 samples/s and reconstructed from its samples using a brickwall lowpass filter. The frequency of the reconstructed sinusoid is:

(a) 3 Hz

(b) 5 Hz

(c) 7 Hz

(d) 10 Hz

Solution: $\boxed{\text{A}}$ The sampled signal has components at

$$\{\pm 7 \pm 20k\} = \pm\{3, 7, 10, 13, \ldots\} \text{ Hz},$$

for integers k. The lowpass filter has cutoff frequency $\frac{10}{2} = 5$ Hz, leaving ± 3 Hz.

Problem 6.26 A signal whose spectrum = 0 for $|f| > 12$ Hz is sampled at 20 samples/s. Using a brick-wall lowpass filter, the original signal spectrum can be reconstructed from its samples up to how many Hz:

(a) 0 (none of it)

(b) 2

(c) 4

(d) 8

Solution: $\boxed{\text{D}}$ Baseband: $-12 < f < 12$ Hz. First image: $8 < f < 32$ Hz, where $20 - 12 = 8$ and $20 + 12 = 32$. So $-8 < f < 8$ kHz of baseband is unaffected by the first image.

Recall that "images" are copies of the original (baseband) spectrum induced by sampling.

Problem 6.27 Signal $x(t) = \cos(2\pi 4t)$ is ideally sampled at 1 sample/s, then filtered by a brick-wall lowpass filter with cutoff frequency 3.5 Hz and gain 1/2:

$\cos(2\pi 4t) \rightarrow \boxed{\text{1 sample/s}} \rightarrow \boxed{\mathbf{H}(\omega) = \begin{cases} 1/2, & 0 < |\omega| < 7\pi \\ 0, & |\omega| > 7\pi \end{cases}} \rightarrow y(t)$. What is the expression for $y(t)$?

(a) $y(t) = 0$

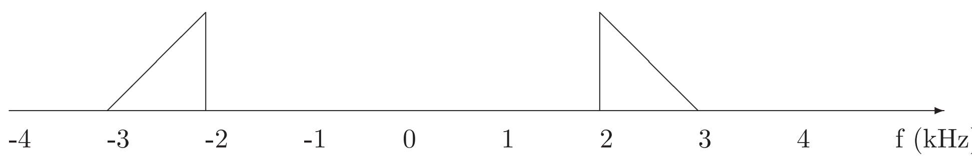
(b) $y(t) = \cos(2\pi t)$

(c) $y(t) = \cos(2\pi t) + \cos(4\pi t)$

(d) $y(t) = \cos(2\pi t) + \cos(4\pi t) + \cos(6\pi t)$

Solution: $\boxed{\text{D}}$ Sampling a 4-Hz signal at 1 sample/s creates doubled copies at $\pm\{0, 1, 2, 3, \ldots\}$ Hz. The filter eliminates 0 Hz (did you notice that?) and copies above 3.5 Hz.

Problem 6.28 A signal $x(t)$ has the Fourier transform $\mathbf{X}(2\pi f)$ plotted below.



The minimum sampling rate for which $x(t)$ can be reconstructed from its samples is:

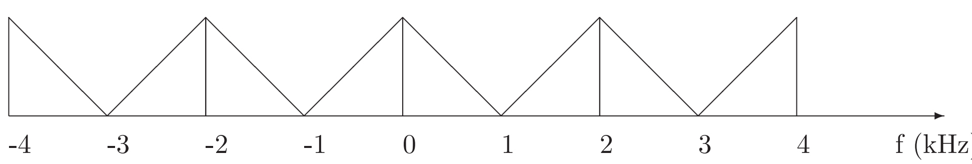
(a) 6000 samples/s

(b) 4000 samples/s

(c) 3000 samples/s

(d) 2000 samples/s

Solution: $\boxed{\text{D}}$ The spectrum of the sampled signal when sampled at 2000 samples/s:



A bandpass filter with passband $2 < |f| < 3$ kHz will recover $x(t)$ from its samples. This is an example of bandpass sampling. The spectrum is really only 1 kHz wide.

Chapter 7

Problem 7.1 For which value of Ω_0 does $\cos(\pi n/4) = \cos(\Omega_0 n)$ for all integers n?

(a) $3\pi/4$
(b) $7\pi/4$
(c) $9\pi/4$
(d) (b) and (c)

Solution: $\boxed{\text{D}}$ See Fig. 7-8 and 7-9.

Problem 7.2 The period of $x[n] = 3\cos(0.16\pi n - 1)$ is:

(a) 8
(b) 25
(c) 50
(d) Is not periodic

Solution: $\boxed{\text{B}}$

$$\begin{aligned} \frac{2\pi}{0.16\pi} &= \frac{200}{16} = \frac{25}{2}. \\ x[n+25] &= 3\cos(0.16\pi(n+25) - 1) = 3\cos(0.16\pi n + 4\pi - 1) = x[n]. \end{aligned}$$

Problem 7.3 $\{\underline{3}, 8, 4\} * \{\underline{2}, 6\} =$

(a) $\{\underline{6}, 34, 56, 24\}$
(b) $\{\underline{6}, 56, 34, 24\}$
(c) $\{\underline{6}, 54, 40, 24\}$
(d) $\{\underline{6}, 40, 54, 24\}$

Solution: $\boxed{\text{A}}$ $\{(3)(2), (3)(6) + (8)(2), (8)(6) + (4)(2), (4)(6)\} = \{\underline{6}, 34, 56, 24\}$.

Problem 7.4 $x[n] \rightarrow \boxed{w[n] = 4x[n] + x[n-1]} \rightarrow w[n] \rightarrow \boxed{y[n] = 2w[n] + 3w[n-1]} \rightarrow y[n]$.

The overall impulse response $h[n]$ of the connection of the two systems is:

(a) $h[n] = \{\underline{8}, 11, 3\}$
(b) $h[n] = \{\underline{8}, 14, 3\}$
(c) $h[n] = \{\underline{3}, 11, 8\}$
(d) $h[n] = \{\underline{3}, 14, 8\}$

Solution: $\boxed{\text{B}}$ Systems in cascade (series): Convolve impulse responses.
$\{\underline{4}, 1\} * \{\underline{2}, 3\} = \{\underline{8}, 14, 3\}$.

Problem 7.5 $\{\underline{1}, -5, 6\} * (3)^n\, u[n] = ?$

(a) $(3)^n\, u[n] + (3)^{n-2}\, u[n-2]$
(b) $(3)^n\, u[n] + 6(3)^{n-2}\, u[n-2]$
(c) $6(3)^{n-2}\, u[n]$
(d) $\{\underline{1}, -2\}$

Solution: $\boxed{\text{D}}$
$\{\underline{1},-5,6\} * (3)^n\, u[n] = 3^n\, u[n] - 5(3)^{n-1}\, u[n-1] + 6(3)^{n-2}\, u[n-2] = \{\underline{1},-2\}$.

Problem 7.6 Which statements about BIBO stability are true?

(a) All MA systems are stable.
(b) All LTI systems with impulse response $h[n] = C\mathbf{p}^n\, u[n]$ with Real$[\mathbf{p}] < 0$ are stable.
(c) All LTI systems with impulse response $h[n] = C\mathbf{p}^n\, u[n]$ with $|\mathbf{p}| < 1$ are stable.
(d) (a) and (c).

Solution: $\boxed{\text{D}}$ All MA systems are stable; see Section 7-4.5. All systems with geometric impulse responses with poles inside the unit circle are stable; see Section 7-4.6.

Problem 7.7 The $\mathbf{z}$-transform of $\{\underline{1},-2\} + (2)^n\, u[n]$ is:

(a) $(\mathbf{z}-2) + \dfrac{\mathbf{z}}{\mathbf{z}-2}$
(b) $\dfrac{2\mathbf{z}^2-4\mathbf{z}+4}{\mathbf{z}^2-2\mathbf{z}}$
(c) $\dfrac{\mathbf{z}^2-3\mathbf{z}+4}{\mathbf{z}^2-2\mathbf{z}}$
(d) $\dfrac{2\mathbf{z}^2-4\mathbf{z}+4}{\mathbf{z}^2-2\mathbf{z}}$

Solution: $\boxed{\text{B}}$

$$\mathcal{Z}\{\{\underline{1},-2\} + (2)^n\, u[n]\} = (1-2\mathbf{z}^{-1}) + \frac{\mathbf{z}}{\mathbf{z}-2} = \frac{\mathbf{z}-2}{\mathbf{z}}\frac{\mathbf{z}-2}{\mathbf{z}-2} + \frac{\mathbf{z}}{\mathbf{z}-2}\frac{\mathbf{z}}{\mathbf{z}} = \frac{2\mathbf{z}^2-4\mathbf{z}+4}{\mathbf{z}^2-2\mathbf{z}}.$$

Problem 7.8 $\{\underline{1},3,2\} \rightarrow \boxed{y[n] + 2y[n-1] = 4x[n] + 5x[n-1]} \rightarrow\ ?$

(a) $\{\underline{4},9,5\}$
(b) $(4)^n\, u[n] + u[n]$
(c) $4(4)^n\, u[n] + 9\, u[n]$
(d) $4(4)^n\, u[n] + 5u[n]$

Solution: $\boxed{\text{A}}$ Read off

$$\mathbf{H}(\mathbf{z}) = \frac{4\mathbf{z}+5}{\mathbf{z}+2}$$

and

$$\mathbf{X}(\mathbf{z}) = \frac{\mathbf{z}^2+3\mathbf{z}+2}{\mathbf{z}^2}.$$

$$\mathbf{Y}(\mathbf{z}) = \mathbf{H}(\mathbf{z})\mathbf{X}(\mathbf{z}) = \frac{4\mathbf{z}+5}{\mathbf{z}+2}\frac{(\mathbf{z}+1)(\mathbf{z}+2)}{\mathbf{z}^2} = \frac{(\mathbf{z}+1)(4\mathbf{z}+5)}{\mathbf{z}^2} = 4 + 9\mathbf{z}^{-1} + 5\mathbf{z}^{-2},$$

so $y[n] = \{\underline{4},9,5\}$.

Problem 7.9 The inverse $\mathbf{z}$-transform of

$$\frac{jz}{z-(3-j4)} - \frac{jz}{z-(3+j4)}$$

is:

(a) $2(5)^n \sin(0.927n)\, u[n]$

(b) $-2(5)^n \sin(0.927n)\, u[n]$

(c) $2(5)^n \cos(0.927n)\, u[n]$

(d) $-2(5)^n \cos(0.927n)\, u[n]$

Solution: $\boxed{\text{A}}$

$$\begin{aligned} j(3-j4)^n\, u[n] - j(3+j4)^n\, u[n] &= 2(5)^n \cos\left(-0.927n + \frac{\pi}{2}\right)\, u[n] \\ &= 2(5)^n \sin(0.927n)\, u[n]. \end{aligned}$$

Problem 7.10 The inverse $\mathbf{z}$-transform of $\frac{\mathbf{z}-1}{\mathbf{z}-2}$ is:

(a) $(2)^n\, u[n] - (2)^{n-1}\, u[n-1]$

(b) $\delta[n] + (2)^{n-1}\, u[n-1]$

(c) $\delta[n] - (2)^{n-1}\, u[n-1]$

(d) (a) and (b)

Solution: $\boxed{\text{D}}$ The same answer can be derived in different ways and in different forms.
$\frac{\mathbf{z}-1}{\mathbf{z}-2} = \frac{\mathbf{z}}{\mathbf{z}-2} - \frac{1}{\mathbf{z}-2}$. An inverse $\mathbf{z}$-transform gives $(2)^n\, u[n] - (2)^{n-1}\, u[n-1]$.
$\frac{\mathbf{z}-1}{\mathbf{z}-2} = 1 + \frac{1}{\mathbf{z}-2}$. An inverse $\mathbf{z}$-transform gives $\delta[n] + (2)^{n-1}\, u[n-1]$. Same! (Try it.)

Problem 7.11 An LTI system has transfer function $\mathbf{H}(\mathbf{z}) = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)}$.
The difference equation for the system is:

(a) $y[n] - 7y[n-1] + 12y[n-2] = x[n] - 3x[n-1] + 2x[n-2]$

(b) $12y[n] - 7y[n-1] + y[n-2] = 2x[n] - 3x[n-1] + x[n-2]$

(c) $y[n] - 3y[n-1] + 2y[n-2] = x[n] - 7x[n-1] + 12x[n-2]$

(d) $2y[n] - 3y[n-1] + y[n-2] = 12x[n] - 7x[n-1] + x[n-2]$

Solution: $\boxed{\text{C}}$

$$\frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)} = \frac{\mathbf{z}^2 - 7\mathbf{z} + 12}{\mathbf{z}^2 - 3\mathbf{z} + 2} = \frac{\mathbf{Y}(\mathbf{z})}{\mathbf{X}(\mathbf{z})}.$$

Cross-multiply: $(\mathbf{z}^2 - 3\mathbf{z} + 2)\, \mathbf{Y}(\mathbf{z}) = (\mathbf{z}^2 - 7\mathbf{z} + 12)\, \mathbf{X}(\mathbf{z})$.
An inverse $\mathbf{z}$-transform gives

$$y[n] - 3y[n-1] + 2y[n-2] = x[n] - 7x[n-1] + 12x[n-2].$$

Problem 7.12 An LTI system has transfer function $\mathbf{H}(\mathbf{z}) = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)}$.
The response of the system to input $x[n] = \{\underline{1}, -3, 2\}$ is:

(a) $(2)^n\, u[n] - u[n]$

(b) $\{\underline{1}, -7, 12\}$
(c) $(3)^n\, u[n] - (4)^n\, u[n]$
(d) $\{\underline{12}, -7, 1\}$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}\mathbf{X}(\mathbf{z}) &= \frac{\mathbf{z}^2 - 3\mathbf{z} + 2}{\mathbf{z}^2}.\\ \mathbf{Y}(\mathbf{z}) &= \mathbf{H}(\mathbf{z})\mathbf{X}(\mathbf{z})\\ &= \frac{\mathbf{z}^2 - 7\mathbf{z} + 12}{\mathbf{z}^2 - 3\mathbf{z} + 2}\frac{\mathbf{z}^2 - 3\mathbf{z} + 2}{\mathbf{z}^2} = 1 - 7\mathbf{z}^{-1} + 12\mathbf{z}^{-2}.\end{aligned}$$

An inverse $\mathbf{z}$-transform gives $y[n] = \{\underline{1}, -7, 12\}$.

Problem 7.13 An LTI system has transfer function $\mathbf{H}(\mathbf{z}) = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)}$. Its impulse response is

(a) $h(n) = 6\delta[n] + (2)^n\, u[n] - 6u[n]$
(b) $h(n) = (2)^n\, u[n] - 6u[n]$
(c) $h(n) = (2)^n\, u[n-1] - 6u[n-1]$
(d) $h(n) = (2)^n\, u[n] + u[n]$

Solution: $\boxed{\text{A}}$

$$\frac{\mathbf{H}(\mathbf{z})}{\mathbf{z}} = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{\mathbf{z}(\mathbf{z}-1)(\mathbf{z}-2)} = \frac{6}{\mathbf{z}} - \frac{6}{\mathbf{z}-1} + \frac{1}{\mathbf{z}-2}.$$

Then multiply by $\mathbf{z}$.
Computation of partial fraction residues:

$$\frac{(0-3)(0-4)}{(0-1)(0-2)} = 6; \qquad \frac{(1-3)(1-4)}{1(1-2)} = -6; \qquad \frac{(2-3)(2-4)}{2(2-1)} = 1.$$

An inverse $\mathbf{z}$-transform gives $h[n] = 6\delta[n] - 6u[n] + (2)^n\, u[n]$.

Problem 7.14 An LTI system has transfer function $\mathbf{H}(\mathbf{z}) = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)}$.
The response of the system to input $x[n] = (4)^n\, u[n] - (3)^n\, u[n]$ is:

(a) $(2)^n\, u[n] - u[n]$
(b) $(3)^n\, u[n] - (4)^n\, u[n]$
(c) $\{\underline{1}, -7, 12\}$
(d) $\{\underline{12}, -7, 1\}$

Solution: $\boxed{\text{A}}$

$$\begin{aligned}\mathbf{X}(\mathbf{z}) &= \frac{\mathbf{z}}{\mathbf{z}-4}\frac{\mathbf{z}-3}{\mathbf{z}-3} - \frac{\mathbf{z}}{\mathbf{z}-3}\frac{\mathbf{z}-4}{\mathbf{z}-4} = \frac{\mathbf{z}}{(\mathbf{z}-3)(\mathbf{z}-4)}.\\ \mathbf{Y}(\mathbf{z}) &= \mathbf{H}(\mathbf{z})\,\mathbf{X}(\mathbf{z}) = \frac{(\mathbf{z}-3)(\mathbf{z}-4)}{(\mathbf{z}-1)(\mathbf{z}-2)}\frac{\mathbf{z}}{(\mathbf{z}-3)(\mathbf{z}-4)} = \frac{\mathbf{z}}{(\mathbf{z}-1)(\mathbf{z}-2)} = \frac{\mathbf{z}}{\mathbf{z}-2} - \frac{\mathbf{z}}{\mathbf{z}-1}.\end{aligned}$$

An inverse $\mathbf{z}$-transform gives $y[n] = (2)^n\, u[n] - u[n]$.

Problem 7.15 If $x(t) = \cos(\frac{2\pi}{3}n) \rightarrow \boxed{h[n] = \{\underline{1}, 2\}} \rightarrow y(t)$, what is the expression for $y(t)$?

(a) $y(t) = \sqrt{3}\cos(\frac{2\pi}{3}n)$
(b) $y(t) = \sqrt{3}\sin(\frac{2\pi}{3}n)$
(c) $y(t) = 2\cos(\frac{2\pi}{3}n)$
(d) $y(t) = 2\sin(\frac{2\pi}{3}n)$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}\mathbf{H}(e^{j\Omega}) &= 1 + 2e^{-j\Omega}. \\ \mathbf{H}(e^{j2\pi/3}) &= 1 + 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3} = \sqrt{3}e^{-j\pi/2}.\end{aligned}$$

The output is $\sqrt{3}\cos(\frac{2\pi}{3}n - \frac{\pi}{2}) = \sqrt{3}\sin(\frac{2\pi}{3}n)$.

Problem 7.16 If $x(t) = \cos(\pi n) \rightarrow \boxed{h[n] = (\frac{1}{3})^n\, u[n]} \rightarrow y(t)$, what is the function form of $y(t)$?

(a) $y(t) = \frac{1}{3}\cos(\pi n)$
(b) $y(t) = \frac{1}{2}\cos(\pi n)$
(c) $y(t) = \frac{3}{4}\cos(\pi n)$
(d) $y(t) = \frac{3}{2}\cos(\pi n)$

Solution: $\boxed{\text{C}}$

$$\begin{aligned}\mathbf{H}(e^{j\Omega}) &= \frac{1}{1 - \frac{1}{3}e^{-j\Omega}}. \\ \mathbf{H}(e^{j\pi}) &= \frac{1}{1 - \frac{1}{3}e^{-j\pi}} = \frac{1}{1 - \frac{1}{3}(-1)} = \frac{3}{4}\,.\end{aligned}$$

Output is $\frac{3}{4}\cos(\pi n)$.

Problem 7.17 If $x(t)\cos(\frac{\pi}{3}n) \rightarrow$ $\boxed{y[n] - 2y[n-1] + y[n-2] = x[n] - 3x[n-1] + x[n-2]} \rightarrow y(t)$, what is the functional form of $y(t)$?

(a) $y(t) = 2\cos(\frac{\pi}{3}n)$
(b) $y(t) = \frac{3}{2}\cos(\frac{\pi}{3}n)$
(c) $y(t) = \frac{1}{2}\cos(\frac{\pi}{3}n)$
(d) $y(t) = \frac{1}{3}\cos(\frac{\pi}{3}n)$

Solution: $\boxed{\text{A}}$

$$\begin{aligned}\mathbf{H}(e^{j\Omega}) &= \frac{1 - 3e^{-j\Omega} + e^{-j2\Omega}}{1 - 2e^{-j\Omega} + e^{-j2\Omega}} = \frac{e^{j\Omega} - 3 + e^{-j\Omega}}{e^{j\Omega} - 2 + e^{-j\Omega}} = \frac{2\cos(\Omega) - 3}{2\cos(\Omega) - 2}. \\ \mathbf{H}(e^{j\pi/3}) &= \frac{2(\frac{1}{2}) - 3}{2(\frac{1}{2}) - 2} = 2.\end{aligned}$$

The output is $2\cos(\frac{\pi}{3}n)$.

Problem 7.18 If $x(t) = \cos(\frac{\pi}{2}n) \to$
$\boxed{y[n] - 2y[n-1] + y[n-2] = x[n] - 3x[n-1] + x[n-2]} \to y(t)$, what is the functional form of $y(t)$?

(a) $y(t) = 2\cos(\frac{\pi}{2}n)$
(b) $y(t) = \frac{3}{2}\cos(\frac{\pi}{2}n)$
(c) $y(t) = \frac{1}{2}\cos(\frac{\pi}{2}n)$
(d) $y(t) = \frac{1}{3}\cos(\frac{\pi}{2}n)$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}
\mathbf{H}(e^{j\Omega}) &= \frac{1 - 3e^{-j\Omega} + e^{-j2\Omega}}{1 - 2e^{-j\Omega} + e^{-j2\Omega}} = \frac{e^{j\Omega} - 3 + e^{-j\Omega}}{e^{j\Omega} - 2 + e^{-j\Omega}} = \frac{2\cos(\Omega) - 3}{2\cos(\Omega) - 2}\,. \\
\mathbf{H}(e^{j\pi/2}) &= \frac{2(0) - 3}{2(0) - 2} = \frac{3}{2}\,.
\end{aligned}$$

The output is $\frac{3}{2}\cos(\frac{\pi}{2}n)$.

Problem 7.19 What property of the DTFS is not true for the CTFS?

(a) It has only a finite number of terms.
(b) Conjugate symmetry among the coefficients holds for real-valued signals.
(c) It can be computed exactly by dividing the FFT of a period by its length.
(d) (a) and (c).

Solution: $\boxed{\text{D}}$ See Section 7-13.

Problem 7.20 Compute the DTFS of $x[n] = \{\ldots, 4, 2, 0, 2, \underline{4}, 2, 0, 2, 4, 2, 0, 2, \ldots\}$.

(a) $x[n] = 2 + 2\cos(\frac{\pi}{2}n)$
(b) $x[n] = 2 + 2\cos(\frac{\pi}{4}n)$
(c) $x[n] = 2 + \cos(\frac{\pi}{4}n) + \cos(\frac{\pi}{2}n)$
(d) $x[n] = 2 + 2\cos(\frac{\pi}{4}n) + \cos(\frac{\pi}{2}n)$

Solution: $\boxed{\text{A}}$ This is obvious by inspection, but it can be computed as follows: The form of the DTFS is

$$\begin{aligned}
x[n] &= \mathbf{x}_0 + \mathbf{x}_1 e^{j\pi/2n} + \mathbf{x}_2 e^{j\pi n} + \mathbf{x}_3 e^{j3\pi/2n}. \\
\mathbf{x}_0 &= \frac{1}{4}(4 + 2 + 0 + 2) = 2. \\
\mathbf{x}_1 &= \frac{1}{4}(4 - j2 + 0 + j2) = 1. \\
\mathbf{x}_2 &= \frac{1}{4}(4 - 2 + 0 - 2) = 0. \\
x[n] &= 2 + e^{j\pi/2n} + e^{j3\pi/2n} = 2 + 2\cos\left(\frac{\pi}{2}n\right),
\end{aligned}$$

since $e^{j3\pi/2n} = e^{-j\pi/2n}$ and $\mathbf{x}_3 = \mathbf{x}_1^* = 1$.

Problem 7.21 If $x[n] = \{\ldots, 4, 2, 0, 2, \underline{4}, 2, 0, 2, 4, 2, 0, 2, \ldots\} \rightarrow \boxed{h[n] = \dfrac{\sin \frac{\pi}{3}n)}{\pi n}} \rightarrow y[n]$, what is the expression for $y[n]$?

(a) $y[n] = \cos(\frac{\pi}{2}n) + 2\cos(\pi n)$

(b) $y[n] = \cos(\frac{\pi}{2}n)$

(c) $y[n] = 2\cos(\pi n)$

(d) $y[n] = \{\ldots, 2, 2, 2, \ldots\}$

Solution: $\boxed{\text{D}}$ The DTFS has the form $\mathbf{x}_0 + \mathbf{x}_1 e^{j\pi/2n} + \mathbf{x}_2 e^{j\pi n} + \mathbf{x}_3 e^{j3\pi/2n}$. $\mathbf{x}_0 = 2$.
$h[n]$ is a brick-wall lowpass filter with cutoff frequency $\Omega_0 = \frac{\pi}{3} < \frac{\pi}{2}$. Only $\mathbf{x}_0$ passes.

Problem 7.22 If $x[n] = \{\ldots, a, b, c, d, \underline{a}, b, c, d, a, b, c, d, \ldots\} \rightarrow$
$\boxed{h[n] = \dfrac{\sin(\frac{\pi}{10}n)}{\pi n} 2\cos\left(\dfrac{3\pi}{10}n\right)} \rightarrow y[n]$, what is the expression for $y[n]$?

(a) $y[n] = 0$

(b) $y[n] = \mathbf{x}_0$

(c) $y[n] = \mathbf{x}_0 + \mathbf{x}_1 e^{j\pi/2n} + \mathbf{x}_3 e^{j3\pi/2n}$

(d) $y[n] = x[n]$

Solution: $\boxed{\text{A}}$ The DTFS has the form $\mathbf{x}_0 + \mathbf{x}_1 e^{j\pi/2n} + \mathbf{x}_2 e^{j\pi n} + \mathbf{x}_3 e^{j3\pi/2n}$.
$h[n]$ is a brick-wall bandpass filter that passes $\frac{2\pi}{10} < |\Omega| < \frac{4\pi}{10} < \frac{\pi}{2}$, so nothing passes.

Problem 7.23 What property of the DTFT is not true for the CTFT?

(a) It is periodic in Ω with period 2π.

(b) It is a continuous function of Ω (or ω).

(c) It can be computed without an integral.

(d) (a) and (c).

Solution: $\boxed{\text{D}}$ Keep (a) in mind when dealing with discrete-time frequency Ω.

Problem 7.24 The DTFT $\mathbf{X}(e^{j\Omega})$ of $x[n]$ is related to:

(a) The $\mathbf{z}$-transform $\mathbf{X}(\mathbf{z})$ by $\mathbf{z} = e^{j\Omega}$.

(b) The N-point DFT $\mathbf{X}[k]$ by $\Omega = 2\pi k/N$.

(c) The continuous Fourier transform $\mathcal{F}$ by $\mathbf{X}(e^{j\omega}) = \mathcal{F}\{\sum_{n=-\infty}^{\infty} x[n]\, \delta(t - n)\}$.

(d) All of the above

Solution: $\boxed{\text{D}}$ See Section 7-14 for all of these.

Problem 7.25 The DTFT of $\{4, 2, \underline{1}, 2, 4\}$ is:

(a) $\cos(2\Omega) + \frac{1}{2}\cos(\Omega) + 1$

(b) $2\cos(2\Omega) + \cos(\Omega) + 1$

(c) $4\cos(2\Omega) + 2\cos(\Omega) + 1$

(d) $8\cos(2\Omega) + 4\cos(\Omega) + 1$

Solution: $\boxed{\text{D}}$ Plug into

$$\begin{aligned}\mathbf{X}(e^{j\Omega}) &= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} : \\ \mathbf{X}(e^{j\Omega}) &= 4e^{j2\Omega} + 2e^{j\Omega} + 1 + 2e^{-j\Omega} + 4e^{-j2\Omega} = 8\cos(2\Omega) + 4\cos(\Omega) + 1.\end{aligned}$$

Check: Setting $\Omega = 0$ gives $4 + 2 + 1 + 2 + 4 = 8 + 4 + 1$.

Problem 7.26 Compute the DTFT of $x[n] = (\frac{1}{2})^n\, u[n]$.

(a) $\dfrac{e^{j\Omega}}{e^{j\Omega} - \frac{1}{2}}$

(b) $\dfrac{e^{-j\Omega}}{e^{-j\Omega} - \frac{1}{2}}$

(c) $\dfrac{e^{j\Omega}}{e^{j\Omega} + \frac{1}{2}}$

(d) $\dfrac{e^{-j\Omega}}{e^{-j\Omega} + \frac{1}{2}}$

Solution: $\boxed{\text{A}}$

$$\mathbf{X}(\mathbf{z}) = \frac{\mathbf{z}}{\mathbf{z} - \frac{1}{2}}.$$

Setting $\mathbf{z} = e^{j\Omega}$ gives

$$\mathbf{X}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega} - \frac{1}{2}}\ .$$

Problem 7.27 If $x[n] = \{1, 2, 7, \underline{2}, 1\}$, then $\arg[X(e^{j\Omega})] =$?

(a) 0

(b) 0 or π

(c) Ω

(d) $-\Omega$

Solution: $\boxed{\text{C}}$ Let $y[n] = \{1, 2, \underline{7}, 2, 1\}$. $y[n]$ is even. Since $7 > 1+2+2+1$, $\angle\mathbf{Y}(e^{j\Omega})] = 0$. Since $x[n] = y[n+1]$, $\angle\mathbf{X}(e^{j\Omega}) = \angle\mathbf{Y}(e^{j\Omega}) + \Omega = \Omega$ using entry #2 of Table 7-7.

Problem 7.28 What is the 8-point DFT of $\{7, 8, 1, 4, 0, 0, 0, 0\}$ at index $k = 4$?

(a) $\mathbf{X}[4] = 6 + j4$

(b) $\mathbf{X}[4] = 5$

(c) $\mathbf{X}[4] = -4$

(d) $\mathbf{X}[4]$ has a $\sqrt{2}$ in it.

Solution: $\boxed{\text{C}}$

$$\mathbf{X}[4] = \sum_{n=0}^{7} x[n]\, (-1)^n = 7 - 8 + 1 - 4 + 0 - 0 + 0 - 0 = -4.$$

Problem 7.29 The 4-point DFT of $\{16, 8, 12, 4\}$ is:

(a) $\{40, 4+4j, 16, 4-4j\}$
(b) $\{10, 1+j, 4, 1-j\}$
(c) $\{40, 4-4j, 16, 4+4j\}$
(d) $\{10, 1-j, 4, 1+j\}$

Solution: $\boxed{\text{C}}$

$$\begin{aligned}
\mathbf{X}[0] &= (16+8+12+4) = 40. \\
\mathbf{X}[1] &= (16-j8-12+j4) = 4-4j. \\
\mathbf{X}[2] &= (16-8+12-4) = 16. \\
\mathbf{X}[3] &= (16+j8-12-j4) = 4+4j = \mathbf{X}^*[1].
\end{aligned}$$

Problem 7.30 The 4-point DFT of $\{12, 8, 4, 8\}$ is:

(a) $\{32, 8, 0, 8\}$
(b) $\{32, 4-j8, 16, 4+j8\}$
(c) $\{32, 4+j8, 16, 4-j8\}$
(d) $\{8, 1+j2, 4, 1-j2\}$

Solution: $\boxed{\text{A}}$

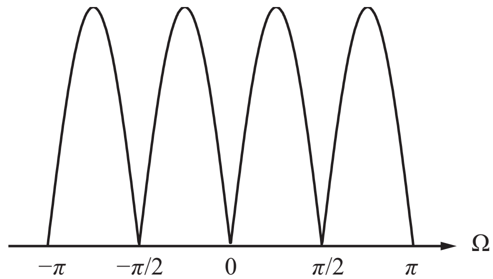
$$\begin{aligned}
\mathbf{X}[0] &= (12+8+4+8) = 32. \\
\mathbf{X}[1] &= (12-j8-4+j8) = 8. \\
\mathbf{X}[2] &= (12-8+4-8) = 0. \\
\mathbf{X}[3] &= (12+j8-4-j8) = 8.
\end{aligned}$$

Why are none of these complex numbers?
The periodic extension of $\{12, 8, 4, 8\}$ is $\{\ldots, 12, 8, 4, 8, \underline{12}, 8, 4, 8, 12, 8, 4, 8, \ldots\}$ is even.
Any Fourier transform of any kind of a real and even function is itself real and even.

Chapter 8: Discrete-Time Filtering

Problem 8.1 The impulse response $h[n]$ having the magnitude response shown below is:

(a) $h[n] = \{1, 0, 0, 0, -1\}$
(b) $h[n] = \{1, 0, 0, 0, 1\}$
(c) $h[n] = \{1, 1, -2, 1, 1\}$
(d) $h[n] = \{1, -1, 2, -1, 1\}$



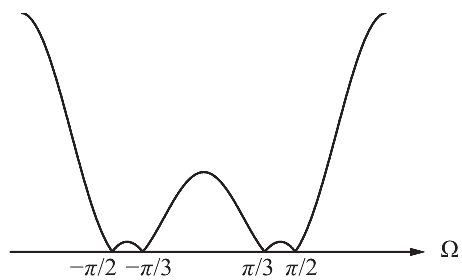
Solution: $\boxed{\text{A}}$ The plot has zeros $\{e^{j0} = 1,\ e^{\pm j\pi/2} = \pm j,\ e^{j\pi} = -1\}$.

$$\mathbf{H}(\mathbf{z}) = (\mathbf{z}-1)(\mathbf{z}+1)(\mathbf{z}-j)(\mathbf{z}+j) = \mathbf{z}^4 - 1.$$

Then $h[n] = \{1, 0, 0, 0, -1\}$.

Problem 8.2 The impulse response $h[n]$ having the magnitude response shown below is:

(a) $h[n] = \{1, 0, 0, 0, -1\}$
(b) $h[n] = \{1, 0, 0, 0, 1\}$
(c) $h[n] = \{1, 1, -2, 1, 1\}$
(d) $h[n] = \{1, -1, 2, -1, 1\}$



Solution: $\boxed{\text{D}}$ The plot has zeros $\{e^{\pm j\pi/3} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2},\ e^{\pm j\pi/2} = \pm j\}$.

$$\mathbf{H}(\mathbf{z}) = \left(\mathbf{z} - \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\left(\mathbf{z} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(\mathbf{z}-j)(\mathbf{z}+j) = (\mathbf{z}^2 - \mathbf{z} + 1)(\mathbf{z}^2 + 1).$$

Then $h[n] = \{1, -1, 1\} * \{1, 0, 1\} = \{1, -1, 2, -1, 1\}$ (2 notch filters in series).

Problem 8.3 An out-of-tune trumpet plays note A, but with a fundamental frequency of 441 Hz. The trumpet signal is sampled at 44100 samples/s. Which filter can eliminate the out-of-tune signal?

(a) $y[n] - 0.99y[n-100] = x[n] - x[n-100]$.
(b) $y[n] - y[n-100] = x[n] - 0.99x[n-100]$.
(c) $y[n] - 0.99y[n-99] = x[n] - x[n-99]$.
(d) $y[n] - 0.99y[n-101] = x[n] - x[n-101]$.

Solution: $\boxed{\text{A}}$ The signal has period $N_0 = \frac{44100}{441} = 100$. Use a comb filter (Eq. (8.31)).

Problem 8.4 Which filter rejects a 1000 Hz sinusoid sampled at 3000 samples/s?

(a) $h[n] = \{1, 0, 1\}$
(b) $h[n] = \{1, 1, 1\}$
(c) $h[n] = \{1, \sqrt{2}, 1\}$
(d) $h[n] = \{1, -\sqrt{2}, 1\}$

Solution: $\boxed{\text{B}}$ The discrete-time frequency to be rejected is $\Omega_0 = 2\pi\frac{1000}{3000} = \frac{2\pi}{3}$. The FIR notch filter that rejects Ω_0 is $h[n] = \{1, -2\cos(\Omega_0), 1\}$ (see Eq. (8.14)). $-2\cos(\Omega_0) = -2\cos(\frac{2\pi}{3}) = 1$, so $h[n] = \{1, 1, 1\}$.

Problem 8.5 Which filter rejects signal $x[n] = 5\cos(2\pi 250t) + 7\cos(2\pi 1250t)$ sampled at 3000 samples/s?

(a) $h[n] = \{1, 0, 1, 0, 1\}$
(b) $h[n] = \{1, 0, -1, 0, 1\}$
(c) $h[n] = \{1, 1, -2, 1, 1\}$
(d) $h[n] = \{1, -1, 2, -1, 1\}$

Solution: $\boxed{\text{B}}$ The frequencies to be rejected are $\Omega_1 = 2\pi\frac{250}{3000} = \frac{\pi}{6}$ and $\Omega_2 = 2\pi\frac{1250}{3000} = \frac{5\pi}{6}$. Use two FIR notch filters connected in series: $-2\cos(\frac{\pi}{6}) = -\sqrt{3}$ and $-2\cos(\frac{5\pi}{6}) = \sqrt{3}$. $h_1[n] = \{1, -\sqrt{3}, 1\}$ and $h_2[n] = \{1, \sqrt{3}, 1\}$. Then $h_1[n] * h_2[n] = \{1, 0, -1, 0, 1\}$.

Problem 8.6 The cyclic convolution $\{1, 4, 5\}$ ⓒ $\{2, 3, 7\}$ is:

(a) $\{49, 30, 41\}$
(b) $\{49, 41, 30\}$
(c) $\{45, 46, 29\}$
(d) $\{45, 29, 46\}$

Solution: $\boxed{\text{C}}$ $\{1, 4, 5\} * \{2, 3, 7\} = \{2, 11, 29, 43, 35\}$. Alias to $\{2 + 43,\ 11 + 35,\ 29\}$.

Problem 8.7 A signal is sampled at 80 samples/s and stored in MATLAB vector `X`. `length(X)` gives 16. `fft(X)` gives `[0,0,0,7,0,0,0,0,0,0,0,0,0,7,0,0]`. What is the frequency of the sinusoid in Hz?

(a) 5 Hz
(b) 6 Hz
(c) 15 Hz
(d) 16 Hz

Solution: $\boxed{\text{C}}$ Use `f=(K-1)*F/N` $= (4 - 1)80/16 = 15$ Hz. See Example 8-16.

Problem 8.8 What is the inverse DFT of $\{0, 3 + j4, 0, 0, 0, 0, 0, 3 - j4\}$?

(a) $\frac{5}{8}\cos(\frac{\pi}{8}n + 53^\circ)$
(b) $\frac{10}{8}\cos(\frac{\pi}{8}n + 53^\circ)$
(c) $\frac{5}{8}\cos(\frac{\pi}{4}n + 53^\circ)$
(d) $\frac{10}{8}\cos(\frac{\pi}{4}n + 53^\circ)$

Solution: $\boxed{\text{D}}$ See Example 8-16.

$$\frac{5}{8}\,e^{(j2\pi 1n/8+j53^\circ)}+\frac{5}{8}\,e^{(j2\pi 7n/8-j53^\circ)}=\frac{10}{8}\cos\left(\frac{\pi}{4}n+53^\circ\right).$$

Problem 8.9 A minimum phase discrete-time system:

(a) Is stable and causal.
(b) Has a stable and causal inverse system.
(c) Has all of its poles and zeros inside the unit circle.
(d) All of the above.

Solution: $\boxed{\text{D}}$ See Sections 8-4.1 and 8-4.2.

Problem 8.10 An LTI system has impulse response $h[n]=3(-3)^n\,u[n]-2(-2)^n\,u[n]$. Its inverse system has impulse response $g[n]=\,?$

(a) $g[n]=3(3)^n\,u[n]-2(2)^n\,u[n]$
(b) $g[n]=3(-\frac{1}{3})^n\,u[n]-2(-\frac{1}{2})^n\,u[n]$
(c) $g[n]=3(\frac{1}{3})^n\,u[n]-2(\frac{1}{2})^n\,u[n]$
(d) $g[n]=\{\underline{1},5,6\}$

Solution: $\boxed{\text{D}}$

$$\begin{aligned}\mathbf{H}(\mathbf{z}) &= \frac{3\mathbf{z}}{\mathbf{z}+3}\frac{\mathbf{z}+2}{\mathbf{z}+2}-\frac{2\mathbf{z}}{\mathbf{z}+2}\frac{\mathbf{z}+3}{\mathbf{z}+3}=\frac{\mathbf{z}^2}{\mathbf{z}^2+5\mathbf{z}+6}.\\ \mathbf{G}(\mathbf{z}) &= \frac{1}{\mathbf{H}(\mathbf{z})}=\frac{\mathbf{z}^2+5\mathbf{z}+6}{\mathbf{z}^2}.\end{aligned}$$

An inverse $\mathbf{z}$-transform gives $g[n]=\{\underline{1},5,6\}$

Problem 8.11 The bilateral $\mathbf{z}$-transform of $x[n]=2(\frac{1}{2})^n\,u[n]+2(3)^n\,u[-n-1]$ is:

(a) $\mathbf{X}(\mathbf{z})=\dfrac{4\mathbf{z}^2-7\mathbf{z}}{(\mathbf{z}-\frac{1}{2})(\mathbf{z}-3)}$
(b) $\mathbf{X}(\mathbf{z})=\dfrac{4\mathbf{z}-7}{(\mathbf{z}-\frac{1}{2})(\mathbf{z}-3)}$
(c) $\mathbf{X}(\mathbf{z})=\dfrac{5\mathbf{z}}{(\mathbf{z}-\frac{1}{2})(\mathbf{z}-3)}$
(d) $\mathbf{X}(\mathbf{z})=\dfrac{-5\mathbf{z}}{(\mathbf{z}-\frac{1}{2})(\mathbf{z}-3)}$

Solution: $\boxed{\text{D}}$ Recall that $\mathcal{Z}\{-\mathbf{a}^n\,u[-n-1]\}=\frac{\mathbf{z}}{\mathbf{z}-\mathbf{a}}$ (see Eqs. (8.76) and (8.81)).

$$\mathbf{X}(\mathbf{z})=\frac{2\mathbf{z}}{\mathbf{z}-\frac{1}{2}}\frac{\mathbf{z}-3}{\mathbf{z}-3}-\frac{2\mathbf{z}}{\mathbf{z}-3}\frac{\mathbf{z}-\frac{1}{2}}{\mathbf{z}-\frac{1}{2}}=\frac{-5\mathbf{z}}{(\mathbf{z}-\frac{1}{2})(\mathbf{z}-3)}.$$

Note the minus sign.

Problem 8.12 What is the ROC of the $\mathbf{z}$-transform of $2(\frac{1}{2})^n\, u[n] + 2(3)^n\, u[-n-1]$? (Note that you need not compute the function $\mathbf{H}(\mathbf{z})$ to solve this problem.)

(a) $\{|\mathbf{z}| < \frac{1}{2}\}$
(b) $\{\frac{1}{2} < |\mathbf{z}| < 3\}$
(c) $\{|\mathbf{z}| > 3\}$
(d) Does not exist.

Solution: $\boxed{\text{B}}$ ROC of $\mathcal{Z}\{-\mathbf{a}^n\, u[-n-1]\} = \frac{\mathbf{z}}{\mathbf{z}-\mathbf{a}}$ is $\{|\mathbf{z}| < |\mathbf{a}|\}$ (Eq. (8.76)). ROC of $\mathcal{Z}\{\mathbf{a}^n\, u[n]\} = \frac{\mathbf{z}}{\mathbf{z}-\mathbf{a}}$ is $\{|\mathbf{z}| > |\mathbf{a}|\}$ (Eq. (8.81)).

$$\left\{|\mathbf{z}| > \frac{1}{2}\right\} \cap \{|\mathbf{z}| < 3\} = \left\{\frac{1}{2} < |\mathbf{z}| < 3\right\}.$$

Problem 8.13 A stable LTI system can have an ROC of the form (for some constants $\mathbf{a}_1, \mathbf{a}_2$):

(a) $\{|\mathbf{z}| < |\mathbf{a}_1| > 1\}$.
(b) $\{|\mathbf{z}| > |\mathbf{a}_1| < 1\}$.
(c) $\{1 > |\mathbf{a_2}| < |\mathbf{z}| < |\mathbf{a_1}| > 1\}$.
(d) Any of the above.

Solution: $\boxed{\text{D}}$ See Section 8-7.

Problem 8.14 A system's ROC has inner radius $= 2$ and outer radius $= \infty$. The inverse $\mathbf{z}$-transform is:

(a) Stable and causal
(b) Unstable and causal
(c) Stable and 2-sided
(d) Unstable and 2-sided

Solution: $\boxed{\text{B}}$. Unit circle $\{|z| = 1\} \notin$ ROC, so the system is unstable. $\infty \in$ ROC, so the system is causal.

Problem 8.15 An ROC has inner radius $= \frac{1}{2}$ and outer radius $= 3$. The inverse $\mathbf{z}$-transform is:

(a) Stable and causal
(b) Unstable and causal
(c) Stable and 2-sided
(d) Unstable and 2-sided

Solution: $\boxed{\text{C}}$ $\{|z| = 1\} \in$ ROC, so the system is stable. ROC form: $\{|a_1| < |\mathbf{z}| < |a_2|\}$, so the system is 2-sided.

Problem 8.16 An ROC has inner radius $= 2$ and outer radius $= 3$. The inverse $\mathbf{z}$-transform is:

(a) Stable and causal
(b) Unstable and causal

(c) Stable and 2-sided

(d) Unstable and 2-sided

Solution: $\boxed{\text{D}}$ $\{|z| = 1\} \notin$ ROC, so the system is unstable. ROC form: $\{|a_1| < |\mathbf{z}| < |a_2|\}$, so the system is 2-sided.

Problem 8.17 Any rational function has an inverse bilateral **z**-transform that is:

(a) Stable

(b) Causal

(c) Anticausal

(d) Any of the above

Solution: $\boxed{\text{D}}$ See Sections 8-5 and 8-6.

Problem 8.18 Impulse response $g[n]$ of the inverse system to $h[n] = \delta[n] - (2)^{n-1}\, u[n-1]$ is:

(a) $g[n] = \delta[n] + (2)^{n-1}\, u[n-1]$

(b) $g[n] = \delta[n] - (2)^{n-1}\, u[n-1]$

(c) $g[n] = \delta[n] + (3)^{n-1}\, u[n-1]$

(d) $g[n] = \delta[n] - (3)^{n-1}\, u[n-1]$

Solution: $\boxed{\text{C}}$

$$\begin{aligned}
\mathbf{H}(\mathbf{z}) &= \mathcal{Z}\{\delta[n] - (2)^{n-1}\, u[n-1]\} = 1 - \frac{1}{\mathbf{z}-2} = \frac{\mathbf{z}-3}{\mathbf{z}-2}. \\
\mathbf{G}(\mathbf{z}) &= \frac{1}{\mathbf{H}(\mathbf{z})} = \frac{\mathbf{z}-2}{\mathbf{z}-3} = 1 + \frac{1}{\mathbf{z}-3}. \\
g[n] &= \delta[n] + (3)^{n-1}\, u[n-1].
\end{aligned}$$

Problem 8.19 Impulse response $g[n]$ of the inverse system to $h[n] = \delta[n] + (3)^{n-1}\, u[n-1]$ is:

(a) $g[n] = \delta[n] + (2)^{n-1}\, u[n-1]$

(b) $g[n] = \delta[n] - (2)^{n-1}\, u[n-1]$

(c) $g[n] = \delta[n] + (3)^{n-1}\, u[n-1]$

(d) $g[n] = \delta[n] - (3)^{n-1}\, u[n-1]$

Solution: $\boxed{\text{B}}$

$$\begin{aligned}
\mathbf{H}(\mathbf{z}) &= \mathcal{Z}\{\delta[n] + (3)^{n-1}\, u[n-1]\} = 1 + \frac{1}{\mathbf{z}-3} = \frac{\mathbf{z}-2}{\mathbf{z}-3}. \\
\mathbf{G}(\mathbf{z}) &= \frac{1}{\mathbf{H}(\mathbf{z})} = \frac{\mathbf{z}-3}{\mathbf{z}-2} = 1 - \frac{1}{\mathbf{z}-2}. \\
g[n] &= \delta[n] - (2)^{n-1}\, u[n-1].
\end{aligned}$$

Problem 8.20 Which system rejects $3\cos(2\pi 125t) + 4\cos(2\pi 250t)$ sampled at 1000 samples/s?

(a) $h[n] = \{1, -\sqrt{2}, 2, -\sqrt{2}, 1\}$

(b) $h[n] = \{1, \sqrt{2}, 2, \sqrt{2}, 1\}$

(c) $h[n] = \{1, 0, 0, 0, 0, 0, 0, -1\}$

(d) (a) or (c)

Solution: $\boxed{\text{D}}$ The frequencies to be rejected are $\Omega_1 = 2\pi\frac{125}{1000} = \frac{\pi}{4}$ and $\Omega_2 = 2\pi\frac{250}{1000} = \frac{\pi}{2}$. Use two FIR notch filters connected in series: $-2\cos(\frac{\pi}{4}) = -\sqrt{2}$ and $-2\cos(\frac{\pi}{2}) = 0$. $h_1[n] = \{1, -\sqrt{2}, 1\}$ and $h_2[n] = \{1, 0, 1\}$. Then $h_1[n] * h_2[n] = \{1, -\sqrt{2}, 2, -\sqrt{2}, 1\}$.
OR: The periods of the two sinusoids are:

$$T_1 = \frac{2\pi}{\Omega_1} = \frac{2\pi}{\pi/4} = 8$$

and

$$T_2 = \frac{2\pi}{\Omega_2} = \frac{2\pi}{\pi/2} = 4.$$

So the FIR comb filter $h[n] = \{1, 0, 0, 0, 0, 0, 0, -1\}$ also works.

Chapter 9

Problem 9.1 The result of applying a Bartlett (triangular) window to $\{7, 2, 6, 4, 5\}$ is:

(a) $\{0, 1, 6, 2, 0\}$

(b) $\{3.5, 1, 6, 4, 2.5\}$

(c) $\{0, 1, 12, 2, 0\}$

(d) $\{7, 2, 6, 4, 5\}$

Solution: $\boxed{\text{A}}$ Multiply $\{7, 2, 6, 4, 5\}$ point-by-point by $\{0, \frac{1}{2}, 1, \frac{1}{2}, 0\}$.

Problem 9.2 We observe L samples of the sum of two sinusoids. We compute the spectrum using an N-point DFT. To reduce sidelobes in the spectrum, we should:

(a) Increase N

(b) Increase L

(c) Use a data window

(d) (a) or (c)

Solution: $\boxed{\text{C}}$ Increasing N or L does not reduce sidelobes.

Problem 9.3 We observe L samples of the sum of two sinusoids. We compute the spectrum using an N-point DFT. To resolve (split the peaks) the two sinusoids, we should:

(a) Increase N

(b) Increase L

(c) Use a data window

(d) (a) or (c)

Solution: $\boxed{\text{B}}$ Increasing N does not help resolve peaks. A window makes it harder.

Problem 9.4 We observe L samples of the sum of two sinusoids. We compute the spectrum using an N-point DFT. To smooth the spectrum, we should:

(a) Increase N

(b) Increase L

(c) Use a data window

(d) (a) or (c)

Solution: $\boxed{\text{D}}$ Increasing N or using a window smoothes the spectrum.

Problem 9.5 A filter with impulse response $h[n] = \{a, b, \underline{0}, -b, -a\}$ is in general:

(a) Highpass since $\mathbf{H}(e^{j0}) = 0$.

(b) Lowpass since $\mathbf{H}(e^{j\pi}) = 0$.

(c) Bandpass since both $\mathbf{H}(e^{j0}) = 0$ and $\mathbf{H}(e^{j\pi}) = 0$.

(d) None of these in general.

Solution: $\boxed{\text{C}}$ $\mathbf{H}(e^{j0}) = a + b + 0 - b - a = 0$. $\mathbf{H}(e^{j\pi}) = a - b + 0 - (-b) + (-a) = 0$.

Problem 9.6 A filter with impulse response $h[n] = \{a, b, \underline{0}, b, a\}$ is in general:

(a) Highpass since $\mathbf{H}(e^{j0}) = 0$.

(b) Lowpass since $\mathbf{H}(e^{j\pi}) = 0$.

(c) Bandpass since both $\mathbf{H}(e^{j0}) = 0$ and $\mathbf{H}(e^{j\pi}) = 0$.

(d) None of these in general.

Solution: $\boxed{\text{D}}$ $\mathbf{H}(e^{j0}) = a + b + 0 + b + a \neq 0$. $\mathbf{H}(e^{j\pi}) = a - b - b + a \neq 0$.

Problem 9.7 A filter with impulse response $h[n] = \{a, b, -b, -a\}$ is in general:

(a) Highpass since $\mathbf{H}(e^{j0}) = 0$.

(b) Lowpass since $\mathbf{H}(e^{j\pi}) = 0$.

(c) Bandpass since both $\mathbf{H}(e^{j0}) = 0$ and $\mathbf{H}(e^{j\pi}) = 0$.

(d) None of these in general.

Solution: $\boxed{\text{A}}$ $\mathbf{H}(e^{j0}) = a + b + (-b) + (-a) = 0$. $\mathbf{H}(e^{j\pi}) = a - b + (-b) - (-a) \neq 0$.

Problem 9.8 A filter with impulse response $h[n] = \{a, b, b, a\}$ is in general:

(a) Highpass since $\mathbf{H}(e^{j0}) = 0$.

(b) Lowpass since $\mathbf{H}(e^{j\pi}) = 0$.

(c) Bandpass since both $\mathbf{H}(e^{j0}) = 0$ and $\mathbf{H}(e^{j\pi}) = 0$.

(d) None of these in general.

Solution: $\boxed{\text{B}}$ $\mathbf{H}(e^{j0}) = a + b + b + a \neq 0$. $\mathbf{H}(e^{j\pi}) = a - b + b - a = 0$.

Problem 9.9 A continuous-time lowpass filter has $\mathbf{H}(\mathbf{s}) = \frac{1}{\mathbf{s}+1}$ and $h(t) = e^{-t}\, u(t)$. Design a discrete-time lowpass filter using impulse invariance with $T = 2$.

(a) $h[n] = e^{-n}\, u[n]$
(b) $h[n] = 2e^{-2n}\, u[n]$
(c) $h[n] = \frac{1}{2}e^{-n/2}\, u[n]$
(d) $h[n] = \{\underline{\frac{1}{2}}, \frac{1}{2}\}$.

Solution: $\boxed{\text{B}}$ Using Eq. (9.74), $h[n] = Th(nT) = 2e^{-2n}\, u[n]$.

Problem 9.10 A continuous-time lowpass filter has $\mathbf{H}(\mathbf{s}) = \frac{1}{\mathbf{s}+1}$ and $h(t) = e^{-t}\, u(t)$. Design a discrete-time lowpass filter using bilinear transformation with $T = 2$.

(a) $h[n] = e^{-n}\, u[n]$
(b) $h[n] = 2e^{-2n}\, u[n]$
(c) $h[n] = \frac{1}{2}e^{-n/2}\, u[n]$
(d) $h[n] = \{\underline{\frac{1}{2}}, \frac{1}{2}\}$.

Solution: $\boxed{\text{D}}$ Using Eq. (9.84),

$$\mathbf{H}(\mathbf{z}) = \mathbf{H}\left(\mathbf{s} = \frac{2}{T}\frac{\mathbf{z}-1}{\mathbf{z}+1}\right) = \frac{1}{(\mathbf{z}-1)/(\mathbf{z}+1)+1} = \frac{\mathbf{z}+1}{(\mathbf{z}-1)+(\mathbf{z}+1)}.$$

An inverse $\mathbf{z}$-transform gives

$$h[n] = \mathcal{Z}^{-1}\left\{\frac{\mathbf{z}}{2\mathbf{z}} + \frac{1}{2\mathbf{z}}\right\} = \left\{\underline{\frac{1}{2}}, \frac{1}{2}\right\}.$$

Problem 9.11 Design a discrete-time differentiator using bilinear transformation with $T = 2$. (If you need a reminder, $\mathbf{H}(\mathbf{s}) = \mathbf{s}$.)

(a) $y[n] - y[n-1] = x[n] + x[n-1]$.
(b) $y[n] = x[n] - x[n-1]$.
(c) $y[n] = (x[n] - x[n-1])/2$.
(d) $y[n] + y[n-1] = x[n] - x[n-1]$.

Solution: $\boxed{\text{D}}$ Using Eq. (9.84),

$$\mathbf{H}(\mathbf{z}) = \mathbf{H}\left(\mathbf{s} = \frac{2}{T}\frac{\mathbf{z}-1}{\mathbf{z}+1}\right) = \frac{\mathbf{z}-1}{\mathbf{z}+1} = \frac{\mathbf{Y}(\mathbf{z})}{\mathbf{X}(\mathbf{z})}.$$

Cross-multiply: $(\mathbf{z}+1)\, \mathbf{Y}(\mathbf{z}) = (\mathbf{z}-1)\, \mathbf{X}(\mathbf{z})$. An inverse $\mathbf{z}$-transform gives $y[n] + y[n-1] = x[n] - x[n-1]$.

Problem 9.12 To design a discrete-time lowpass filter with cutoff $\Omega_0 = \frac{\pi}{2}$ from a continuous-time lowpass filter with cutoff ω_0 using bilinear transformation with $T = 0.001$, ω_0 should be:

(a) 250 rad/s
(b) 500 rad/s
(c) 1000 rad/s
(d) 2000 rad/s

Solution: $\boxed{\text{D}}$ The prewarping formula Eq. (9.98) is

$$\omega_0 = \frac{2}{T}\tan\frac{\Omega}{2} = \frac{2}{0.001}\tan\frac{\pi/2}{2} = 2000.$$

Problem 9.13 FIR filter $H(e^{j\Omega})$ minimizing

$$\int_{-\pi}^{\pi}\left|H(e^{j\Omega}) - H_{\text{ideal}}(e^{j\Omega})\right|^2 \, d\Omega$$

is designed by:

(a) MATLAB's `firpm` command

(b) Frequency sampling

(c) A Hamming window

(d) A rectangular window

Solution: $\boxed{\text{D}}$ By Parseval's theorem, this is

$$2\pi \sum_{n=-\infty}^{\infty} |h[n] - h_{\text{ideal}}[n]|^2 .$$

So set $h[n] = h_{\text{ideal}}[n]$ where possible, which amounts to using a rectangular window.

Problem 9.14 Design a discrete-time bandpass filter with cutoffs $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ using using a 5-point rectangular window:

(a) $\{\frac{1}{4}, \frac{1}{4}, 0, -\frac{1}{4}, -\frac{1}{4}\}$

(b) $\{\frac{1}{\pi}, \frac{1}{\pi}, 0, -\frac{1}{\pi}, -\frac{1}{\pi}\}$

(c) $\{0, -\frac{1}{\pi}, \frac{1}{2}, -\frac{1}{\pi}, 0\}$

(d) $\{-\frac{1}{\pi}, 0, \frac{1}{2}, 0, -\frac{1}{\pi}\}$

Solution: $\boxed{\text{D}}$

$$h_{\text{IDEAL}}[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n} 2\cos\left(\frac{\pi}{2}n\right) = \{\cdots, -\frac{1}{\pi}, 0, \frac{1}{2}, 0, -\frac{1}{\pi}, \cdots\}.$$

Truncate this.

Problem 9.15 Design a discrete-time bandpass filter with cutoffs $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ using frequency sampling with $\mathbf{H}(e^{j0}) = \mathbf{H}(e^{j\pi}) = 0$ and $|\mathbf{H}(e^{j\pi/2})| = 1$:

(a) $\{\frac{1}{2}, 1, \frac{1}{2}\}$

(b) $\{\frac{1}{2}, 1, -\frac{1}{2}\}$

(c) $\{\frac{1}{2}, 0, \frac{1}{2}\}$

(d) $\{\frac{1}{2}, 0, -\frac{1}{2}\}$

Solution: $\boxed{\text{D}}$ Use $h[n] = \{a, \underline{0}, -a\}$ for some constant a. Then $\mathbf{H}(e^{j0}) = \mathbf{H}(e^{j\pi}) = 0$ and $\mathbf{H}(e^{j\Omega}) = ae^{j\Omega} - ae^{-j\Omega} = j2a\sin(\Omega)$. Then $|\mathbf{H}(e^{j\pi/2})| = 2a\sin(\pi/2) = 1$ makes $a = \frac{1}{2}$.

Problem 9.16 The filter type designed by the MATLAB command `firpm(10,[0,0.2,0.3,0.7,0.8,1],[0,0,1,1,0,0])` is:

(a) Lowpass

(b) Bandpass

(c) Highpass

(d) Band-reject

Solution: $\boxed{\text{B}}$ Rejects $0 < \Omega < 0.2\pi$ and $0.8\pi < \Omega < \pi$; passes $0.3\pi < \Omega < 0.7\pi$.

Problem 9.17 Which of the following is an application of multirate signal processing?

(a) Changing the pitch of a musical signal.

(b) Changing the instrument playing a musical signal.

(c) Oversampling to reduce the complexity of the reconstruction lowpass filter.

(d) (a) and (c)

Solution: $\boxed{\text{D}}$ See Sections 9-10 and 9-11.

Problem 9.18 The easiest way to raise the pitch of a musical signal by one octave (a factor of 2 in frequency) is to:

(a) Downsample the signal by a factor of 2.

(b) Upsample and interpolate the signal by a factor of 2.

(c) Repeatedly upsample by a factor of 2 and downsample by a factor of 3.

(d) Repeatedly downsample by a factor of 3 and upsample by a factor of 2.

Solution: $\boxed{\text{A}}$ Deleting every other sample of a sampled sinusoid doubles its frequency.

Problem 9.19 We reconstruct from its samples a signal Nyquist-sampled at 2000 samples/s. Our reconstruction filter is poor: it passes 1 kHz and rejects only above 1 MHz. We can use this poor filter by upsampling and interpolating the signal by what factor?

(a) 10

(b) 100

(c) 500

(d) 1000

Solution: $\boxed{\text{D}}$ Interpolation by a factor of 1000 removes all copies of spectrum of the sampled signal between 1 kHz and 1 MHz. This suffices: bandwidth 1 kHz $\ll$ 1 MHz.

Problem 9.20 We reconstruct from its samples a signal Nyquist-sampled at 20000 samples/s. Our reconstruction filter is poor: it passes 10 kHz and rejects only above 90 kHz. We can use this poor filter by upsampling and interpolating the signal by what factor?

(a) 3.5

(b) 4

(c) 4.5

(d) 5

Solution: $\boxed{\text{D}}$ We need to separate the centers of the copies of the spectrum induced by sampling by 100 kHz, since baseband occupies $-10 < f < 10$ kHz and the first image occupies $S-10 < f < S+10$ kHz, where S is the sampling rate after upsampling. To reject all of the spectrum of the first image, we need $S-10 = 90 \rightarrow S = 100$ kHz, by upsampling by a factor of $\frac{100}{20} = 5$.

Problem 9.21 A sinusoid is sampled at 1000 samples/s, input into the DSP system shown, and then reconstructed.

300 Hz$\rightarrow\boxed{\uparrow \mathbf{3}}\rightarrow\boxed{\downarrow \mathbf{3}}\rightarrow y(t)$

What is the frequency in Hz of the output sinusoid?

(a) 100 Hz

(b) 300 Hz

(c) 50, 200, 300 Hz

(d) 50, 200, 300, 450 Hz

Solution: $\boxed{\text{B}}$ Upsampling inserts zeros, downsampling removes them. So output = input!

Problem 9.22 A sinusoid is sampled at 1000 samples/s, input into the DSP system shown, and then reconstructed.

300 Hz$\rightarrow\boxed{\downarrow \mathbf{3}}\rightarrow y(t)$

What is the frequency in Hz of the output sinusoid?

(a) 100 Hz

(b) 300 Hz

(c) 50, 200, 300 Hz

(d) 50, 200, 300, 450 Hz

Solution: $\boxed{\text{A}}$ 300 Hz becomes $3(300) = 900$ Hz, which aliases to $(1000-900) = 100$ Hz.

Problem 9.23 A sinusoid is sampled at 1000 samples/s, input into the DSP system shown, and then reconstructed.

200 Hz$\rightarrow\boxed{\uparrow \mathbf{4}}\rightarrow y(t)$

What is the frequency in Hz of the output sinusoid?

(a) 100 Hz

(b) 300 Hz

(c) 50, 200, 300 Hz

(d) 50, 200, 300, 450 Hz

Solution: $\boxed{\text{D}}$ 200 Hz becomes $\frac{1}{4}(200) = 50$ Hz, with copies at $\frac{1}{4}(1000-200) = 200$ Hz, $\frac{1}{4}(1000+200) = 300$ Hz, $\frac{1}{4}(2000-200) = 450$ Hz, and $\frac{1}{4}(2000-200) = 550$ Hz, which along with higher frequencies is filtered out by the reconstruction lowpass filter.

Problem 9.24 A sinusoid is sampled at 1000 samples/s and input into the DSP system shown.

500 Hz$\rightarrow\boxed{\downarrow \mathbf{2}}\rightarrow\boxed{\uparrow \mathbf{2}}\rightarrow y[n]$

What is the output?

(a) $\{\ldots, 1, -1, \underline{1}, -1, 1, \ldots\}$
(b) $\{\ldots, 1, 1, \underline{1}, 1, 1, \ldots\}$
(c) $\{\ldots, 1, 0, \underline{1}, 0, 1, \ldots\}$
(d) $\{\ldots, 0, -1, \underline{0}, -1, 0, \ldots\}$

Solution: $\boxed{\text{C}}$

$$x[n] = \cos{(2)}\,\pi \frac{500}{1000} n = \cos(\pi n) = (-1)^n.$$

Downsampling by 2 removes all of the -1's; upsampling by 2 replaces them with 0's.

Problem 9.25 The main application of autocorrelation is to:

(a) Compute the period of a noisy signal.
(b) Compute the amplitude of a noisy signal.
(c) Compute the time delay of a noisy signal.
(d) Classify a noisy signal as one of several known signals.

Solution: $\boxed{\text{A}}$ See Section 9-12.2.

Problem 9.26 The main application of cross-correlation is to:

(a) Compute the period of a noisy signal.
(b) Compute the amplitude of a noisy signal.
(c) Compute the time delay of a noisy signal.
(d) Classify a noisy signal as one of several known signals.

Solution: $\boxed{\text{C}}$ See Section 9-12.4.

Problem 9.27 The main application of correlation is to:

(a) Compute the period of a noisy signal.
(b) Compute the amplitude of a noisy signal.
(c) Compute the time delay of a noisy signal.
(d) Classify a noisy signal as one of several known signals.

Solution: $\boxed{\text{D}}$ See Section 9-12.6.

Problem 9.28 The autocorrelation of $\{\underline{1}, 2, 3\}$ is:

(a) $\{3, 8, \underline{14}, 8, 3\}$
(b) $\{1, 4, \underline{10}, 12, 9\}$
(c) $\{6, 17, \underline{32}, 23, 12\}$
(d) $\{4, 13, \underline{28}, 27, 18\}$

Solution: $\boxed{\text{A}}$ From Eq. (9.169) and Example 9-20,

$$\{\underline{1}, 2, 3\} * \{3, 2, \underline{1}\} = \{3, 8, \underline{14}, 8, 3\}.$$

Problem 9.29 The cross-correlation of $\{\underline{1}, 2, 3\}$ and $\{\underline{4}, 5, 6\}$ is:

(a) $\{3, 8, \underline{14}, 8, 3\}$
(b) $\{1, 4, \underline{10}, 12, 9\}$
(c) $\{6, 17, \underline{32}, 23, 12\}$
(d) $\{4, 13, \underline{28}, 27, 18\}$

Solution: $\boxed{\text{C}}$ From Eq. (9.177) and Example 9-21,

$$\{\underline{1}, 2, 3\} * \{6, 5, \underline{4}\} = \{6, 17, \underline{32}, 23, 12\}.$$

Chapter 10 Part I: Image Processing

Problem 10.1 Which of the following 1-D concepts generalize simply and usefully to 2-D?

(a) Continuous-time Fourier transforms and the sampling theorem
(b) Discrete-time Fourier transform and the DFT
(c) LTI systems, convolution, impulse response
(d) All of the above

Solution: $\boxed{\text{D}}$ See Section 10-1.1. DTFT becomes DSFT and impulse response becomes point-spread function.

Problem 10.2 Which of the following 1-D concepts do not generalize usefully to 2-D?

(a) Laplace transforms and differential equations
(b) Transfer functions and poles and zeros
(c) Partial fraction expansions
(d) All of the above

Solution: $\boxed{\text{D}}$ See Section 10-1.1.

Problem 10.3 An LSI system has PSF

$$h[m,n] = \begin{bmatrix} 1 & 2 & 1 \\ 2 & \underline{4} & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

What is its wavenumber response? (HINT: $h[m,n]$ is separable.)

(a) $(1 + \cos(\Omega_1))(1 + \cos(\Omega_2))$
(b) $4(1 + \cos(\Omega_1))(1 + \cos(\Omega_2))$
(c) $(1 + \sin(\Omega_1))(1 + \sin(\Omega_2))$
(d) $4(1 + \sin(\Omega_1))(1 + \sin(\Omega_2))$

Solution: $\boxed{\text{B}}$ By inspection, $h[m,n] = h[m]\, h[n]$ where $h[n] = \{1, \underline{2}, 1\}$. $h[n]$ has DTFT $\mathbf{H}(e^{j\Omega}) = 2 + 2\cos(\Omega)$.

$$\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2}) = (2 + 2\cos(\Omega_1))(2 + 2\cos(\Omega_2)).$$

Problem 10.4 The 2-D discrete Laplacian has PSF

$$h[m,n] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \underline{-4} & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Its wavenumber response is:

(a) $(1+\cos(\Omega_1))(1+\cos(\Omega_2))$

(b) $4(1+\cos(\Omega_1))(1+\cos(\Omega_2))$

(c) $\cos(\Omega_1)+\cos(\Omega_2)-1$

(d) $2\cos(\Omega_1)+2\cos(\Omega_2)-4$

Solution: $\boxed{\text{D}}$

$$\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2}) = e^{j\Omega_1} + e^{-j\Omega_1} + e^{j\Omega_2} + e^{-j\Omega_2} - 4 = 2\cos(\Omega_1) + 2\cos(\Omega_2) - 4.$$

Problem 10.5 An LSI system has PSF

$$h[m,n] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

It could be used for:

(a) Lowpass filtering

(b) Noise reduction

(c) Smoothing an image

(d) All of the above

Solution: $\boxed{\text{D}}$

$$\begin{aligned}
\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2}) &= 1 + e^{-j\Omega_1} + e^{-j\Omega_2} + e^{-j\Omega_1}e^{-j\Omega_2} \\
&= e^{-j\Omega_1/2} 2\cos\left(\frac{\Omega_1}{2}\right) + e^{-j\Omega_2}e^{-j\Omega_1/2} 2\cos\left(\frac{\Omega_1}{2}\right) \\
&= 2\cos\left(\frac{\Omega_1}{2}\right) 2\cos\left(\frac{\Omega_2}{2}\right) e^{-j\Omega_1/2}e^{-j\Omega_2/2}. \\
|\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2})| &= 4\left|\cos\left(\frac{\Omega_1}{2}\right)\cos\left(\frac{\Omega_2}{2}\right)\right|.
\end{aligned}$$

This passes low wavenumbers and rejects high ones.

Problem 10.6 An LSI system has PSF

$$h[m,n] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

It could be used for:

(a) Highpass filtering

(b) Sharpening an image

(c) Edge detection

(d) All of the above

Solution: $\boxed{\text{D}}$

$$\begin{aligned}
\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2}) &= 1 - e^{-j\Omega_1} - e^{-j\Omega_2} + e^{-j\Omega_1}e^{-j\Omega_2} \\
&= e^{-j\Omega_1/2} 2j \sin\left(\frac{\Omega_1}{2}\right) - e^{-j\Omega_2}e^{-j\Omega_1/2} 2j \sin\left(\frac{\Omega_1}{2}\right) \\
&= 2j \sin\left(\frac{\Omega_1}{2}\right) 2j \sin\left(\frac{\Omega_2}{2}\right) e^{-j\Omega_1/2}e^{-j\Omega_2/2}. \\
|\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2})| &= \left| 4 \sin\left(\frac{\Omega_1}{2}\right) \sin\left(\frac{\Omega_2}{2}\right)\right|.
\end{aligned}$$

This passes high wavenumbers and rejects low ones.

Problem 10.7 An LSI system has PSF

$$h[m,n] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \underline{0} & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

It could be used for:

(a) Bandpass filtering

(b) Bandreject filtering

(c) Smoothing an image

(d) Noise reduction

Solution: $\boxed{\text{B}}$

$$\begin{aligned}
\mathbf{H}(e^{j\Omega_1}, e^{j\Omega_2}) &= e^{j\Omega_1}e^{j\Omega_2} + e^{j\Omega_1}e^{-j\Omega_2} + e^{-j\Omega_1}e^{j\Omega_2} + e^{-j\Omega_1}e^{-j\Omega_2} \\
&= (e^{j\Omega_1} + e^{-j\Omega_1})(e^{j\Omega_2} + e^{-j\Omega_2}) \\
&= 2\cos(\Omega_1)\ 2\cos(\Omega_2).
\end{aligned}$$

This is a bandreject filter, since it rejects midrange frequencies ($\Omega_i \approx \frac{\pi}{2}$).

Problem 10.8 Which of the following PSFs are used in edge detection?

(a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

(d) (a) and (b)

Solution: $\boxed{\text{D}}$ (a) is Eq. (10.29) and (b) is Eq. (10.31).

Problem 10.9 The 2-D convolution $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} ** \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = ?$

(a) $\begin{bmatrix} 1 & 3 & 2 \\ 4 & 10 & 6 \\ 3 & 7 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 7 & 4 \\ 6 & 10 & 4 \\ 1 & 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 3 & 10 & 7 \\ 2 & 6 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 7 & 3 \\ 6 & 10 & 4 \\ 2 & 3 & 1 \end{bmatrix}$

Solution: $\boxed{\text{A}}$ From Eq. (10.10),

$$h[m,n] ** x[m,n] = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x[i,j]\, h[m-i,\ n-j].$$

$$(1)\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (1)\begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix} + (1)\begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix} + (1)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 10 & 6 \\ 3 & 7 & 4 \end{bmatrix}.$$

Problem 10.10 The (2×2) 2-D DFT of $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$ is:

(a) $\begin{bmatrix} 13 & -5 \\ 1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 13 & 1 \\ -5 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -5 \\ 1 & 13 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 1 \\ -5 & 13 \end{bmatrix}$

Solution: $\boxed{\text{B}}$ From Eq. (10.20a),

$$\mathbf{X}[k_1,k_2] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[m,n]\; e^{-j2\pi(k_1 m/M + k_2 n/N)}.$$

The (2×2) 2-D DFT of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\begin{aligned} \mathbf{X}[k_1,k_2] &= a + be^{-j\pi k_2} + ce^{-j\pi k_1} + de^{-j\pi(k_1+k_2)} \\ &= \begin{bmatrix} a+b+c+d & a-b+c-d \\ a+b-c-d & a-b+c-d \end{bmatrix}. \end{aligned}$$

The 2-D DFT of $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$ is $\begin{bmatrix} 13 & 1 \\ -5 & 3 \end{bmatrix}$.

Problem 10.11

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \rightarrow \boxed{\downarrow \mathbf{(2,2)}} \rightarrow \ ?$$

(a) $\begin{bmatrix} 2 & 4 \\ 10 & 12 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 9 & 11 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 8 \\ 14 & 16 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 7 \\ 13 & 15 \end{bmatrix}$

Solution: $\boxed{\text{B}}$ Keep columns #1 and #3 and rows #1 and #3. See Section 10-4.

Problem 10.12 Median filtering is used for:

(a) Removing 2-D white noise.
(b) Removing motion blurring.
(c) Removing salt-and-pepper (shot) noise.
(d) Removing an additive constant.

Solution: $\boxed{\text{C}}$ See Section 10-7.5.

Problem 10.13 Use the (2×2) 2-D DFT to solve the cyclic deconvolution problem

$$y[m,n] = \begin{bmatrix} 42 & 42 \\ 28 & 31 \end{bmatrix} = h[m,n] \circledcirc x[m,n] = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix} \circledcirc x[m,n].$$

Determine $x[m,n]$.

(a) $x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

(c) $x[m,n] = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(d) $x[m,n] = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Solution: $\boxed{\text{B}}$ From Eq. (10.20a),

$$\mathbf{X}[k_1,k_2] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[m,n]\, e^{-j2\pi(k_1 m/M + k_2 n/N)}.$$

The (2×2) 2-D DFT of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\mathbf{X}[k_1,k_2] = a + be^{-j\pi k_2} + ce^{-j\pi k_1} + de^{-j\pi(k_1+k_2)} = \begin{bmatrix} a+b+c+d & a-b+c-d \\ a+b-c-d & a-b+c-d \end{bmatrix}.$$

The 2-D DFT of $\begin{bmatrix} 42 & 42 \\ 28 & 31 \end{bmatrix}$ is $\begin{bmatrix} 143 & -3 \\ 25 & 3 \end{bmatrix}$, the 2-D DFT of $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$ is $\begin{bmatrix} 13 & 1 \\ -5 & 3 \end{bmatrix}$. Then

$$\mathbf{X}[k_1,k_2] = \frac{\mathbf{Y}[k_1,k_2]}{\mathbf{H}[k_1,k_2]} = \begin{bmatrix} 11 & -3 \\ -5 & 1 \end{bmatrix}$$

and, from Eq. (10.20b), $x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$.

Problem 10.14 Use the (2×2) 2-D DFT to solve the cyclic deconvolution problem

$$y[m,n] = \begin{bmatrix} 47 & 45 \\ 30 & 32 \end{bmatrix} = h[m,n] \copyright\, x[m,n] = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix} \copyright\, x[m,n].$$

Determine $x[m,n]$.

(a) $x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

(c) $x[m,n] = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(d) $x[m,n]$ cannot be determined uniquely

Solution: $\boxed{\text{D}}$ From Eq. (10.20a),

$$\mathbf{X}[k_1,k_2] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[m,n]\; e^{-j2\pi(k_1 m/M + k_2 n/N)}.$$

The (2×2) 2-D DFT of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\mathbf{X}[k_1,k_2] = a + be^{-j\pi k_2} + ce^{-j\pi k_1} + de^{-j\pi(k_1+k_2)} = \begin{bmatrix} a+b+c+d & a-b+c-d \\ a+b-c-d & a-b+c-d \end{bmatrix}.$$

The 2-D DFT of $\begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix}$ is $\begin{bmatrix} 14 & 0 \\ -6 & 4 \end{bmatrix}$. $\mathbf{H}[1,2] = 0$, so $\mathbf{X}[1,2]$ cannot be computed from

$$\mathbf{X}[k_1,k_2] = \frac{\mathbf{Y}[k_1,k_2]}{\mathbf{H}[k_1,k_2]}.$$

Problem 10.15 Tikhonov regularization:

(a) Is used in deconvolution problems.

(b) Leads to the Wiener filter for deconvolution.

(c) Trades off satisfying $y[m,n] = h[m,n] * x[m,n]$ with keeping $x[m,n]$ small.

(d) All of the above.

Solution: $\boxed{\text{D}}$ See Section 10-7.

Problem 10.16 Haar transform average component $X_1[n]$ of $x[n] = \{3,1,1,3\}$ is:

(a) $X_1[n] = \frac{1}{\sqrt{2}}\{2,2\}$

(b) $X_1[n] = \frac{1}{\sqrt{2}}\{6,2\}$
(c) $X_1[n] = \frac{1}{\sqrt{2}}\{4,4\}$
(d) $X_1[n] = \frac{1}{\sqrt{2}}\{2,6\}$

Solution: $\boxed{\text{B}}$

$$\{3,1,1,3\} \copyright \frac{1}{\sqrt{2}}\{1,1,0,0\} = \frac{1}{\sqrt{2}}\{6,4,2,4\}.$$

Downsample to $\frac{1}{\sqrt{2}}\{6,2\}$.

Problem 10.17 Haar transform detail component $x_1[n]$ of $x[n] = \{3,1,1,3\}$ is:

(a) $x_1[n] = \frac{1}{\sqrt{2}}\{2,2\}$
(b) $x_1[n] = \frac{1}{\sqrt{2}}\{2,-2\}$
(c) $x_1[n] = \frac{1}{\sqrt{2}}\{-2,2\}$
(d) $x_1[n] = \frac{1}{\sqrt{2}}\{0,0\}$

Solution: $\boxed{\text{D}}$

$$\{3,1,1,3\} \copyright \frac{1}{\sqrt{2}}\{1,-1,0,0\} = \frac{1}{\sqrt{2}}\{0,-2,0,2\}.$$

Downsample to $\{0,0\}$.
CHECK: Upsample $\frac{1}{\sqrt{2}}\{6,2\}$ to $\frac{1}{\sqrt{2}}\{6,0,2,0\}$.

$$\frac{1}{\sqrt{2}}\{6,0,2,0\} \copyright \frac{1}{\sqrt{2}}\{1,0,0,1\} = \{3,1,1,3\}$$

Upsampling $\{0,0\}$ to $\{0,0,0,0\}$ and

$$\{0,0,0,0\} \copyright \frac{1}{\sqrt{2}}\{1,0,0,-1\} = \{0,0,0,0\}.$$

Problem 10.18 Which impulse response is a quadrature mirror filter pair with $\{1,2,3,4\}$?

(a) $\{4,-3,2,-1\}$
(b) $\{-4,-3,2,1\}$
(c) $\{1,-2,3,-4\}$
(d) $\{1,2,-3,-4\}$

Solution: $\boxed{\text{A}}$ $\{a,b,c,d\}$ is a QMF pair with $\pm\{d,-c,b,-a\}$. See Ex. 7-2 and Eq. (7.52a).

Problem 10.19 Which scaling impulse response $g[n]$ satisfies the Smith-Barnwell condition?

(a) $\frac{1}{2}\{1,1,1,1\}$
(b) $\frac{1}{5\sqrt{2}}\{6,2,-1,3\}$
(c) $\frac{1}{\sqrt{30}}\{4,-3,2,-1\}$
(d) $\frac{1}{5\sqrt{2}}\{3,0,-4,5\}$

Solution: $\boxed{\text{B}}$ Smith-Barnwell condition: Autocorrelation of $g[n]$ is 0 for even $n \neq 0$. For $g[n] = \{a, b, c, d\}$: $ac + bd = 0$ and $a^2 + b^2 + c^2 + d^2 = 1$. Only **B** satisfes the former. See Section 10-11.4.

Problem 10.20 The Smith-Barnwell condition is:

(a) For perfect reconstruction in a wavelet filter bank

(b) $\mathbf{G}(-\mathbf{z})\,\mathbf{G}(-1/\mathbf{z}) + \mathbf{G}(\mathbf{z})\,\mathbf{G}(1/\mathbf{z}) = 2$

(c) $|\mathbf{G}(e^{j\Omega})|^2 + |\mathbf{G}(e^{j(\Omega\pm\pi)})|^2 = 2$

(d) All of the above.

Solution: $\boxed{\text{D}}$ (a) is Eq. (10.108) and (b) is Eq. (10.113). See Section 10-11.

Problem 10.21 Which of these signal types do Daubechies wavelet functions sparsify?

(a) Sinusoidal

(b) Polynomial

(c) Exponential

(d) (b) and (c)

Solution: $\boxed{\text{B}}$ By construction, Daubechies wavelet functions sparsify polynomials.

Problem 10.22 If $\underline{x}$ is a sparse solution to the underdetermined linear system of equations $\underline{y} = A\underline{x}$, find the solution $\underline{x}$ that minimizes

(a) $\sum_{n=1}^{N} x_n$

(b) $\sum_{n=1}^{N} x_n^2$

(c) $\sum_{n=1}^{N} |x_n|$

(d) $\min[x_n]$

Solution: $\boxed{\text{C}}$ This is the ℓ_1 norm of $\underline{x}$.

Problem 10.23 Which of the following is/are an essential ingredient of compressed sensing?

(a) Landweber iteration

(b) Wavelets

(c) ℓ_1 norm

(d) (b) and (c)

Solution: $\boxed{\text{D}}$ Wavelets result in a sparse representation, by minimizing the ℓ_1 norm. Landweber is just to solve systems of equations. Basis pursuit and IRLS can be used.

Problem 10.24 At each iteration of the ISTA, which of these operations are performed:

(a) Thresholding

(b) Shrinkage

(c) Imposing non-negativity

(d) (a) and (b)

Solution: $\boxed{\text{D}}$ See Section 10-17.3.

Problem 10.25 To denoise a signal or image, what should we do to its wavelet transform?

(a) Threshold it
(b) Shrink it
(c) Impose the Smith-Barnwell condition on it
(d) (a) and (b)

Solution: $\boxed{\text{D}}$ See Section 10-14.

Problem 10.26 The LASSO functional does which of the following?

(a) Trade off sparsity with fidelity to the data.
(b) Thresholding
(c) Shrinkage
(d) All of the above.

Solution: $\boxed{\text{D}}$ See Section 10-14.

Problem 10.27 Which of the following algorithms can be used to find a sparse solution to a linear system of equations?

(a) Iterative reweighted least squares (IRLS)
(b) Basis pursuit
(c) Iterative shrinkage and thresholding (ISTA)
(d) All of the above

Solution: $\boxed{\text{D}}$ See Sections 10-16 and 10-17.

Problem 10.28 Apply thresholding and shrinkage with $\lambda = 1$ to $\{4, 2, 0.7, -0.8, -3, -5\}$:

(a) $\{3, 1, 0, 0, -2, -4\}$
(b) $\{3, 1, 0, 0, -4, -6\}$
(c) $\{4, 2, 0, 0, -3, -5\}$
(d) $\{3, 1, -0.3, -1.8, -4, -6\}$

Solution: $\boxed{\text{A}}$ See Eq. (10.166), derived in Section 10-14.

Problem 10.29 The 2-D wavelet transform of a 256×256 image is computed and displayed as in Fig. 10-28. What is the size of Fig. 10-28?

(a) 64×64
(b) 128×128
(c) 256×256
(d) 512×512

Solution: $\boxed{\text{C}}$ Fig. 10-28 shows how to fit the wavelet transform components of various sizes together like a jigsaw puzzle.