概率论 作业十

151220129 计科 吴政亿

习题七第3题

矩估计:

$$\begin{split} E[X] &= \int_0^\infty x (2\pi\sigma^2)^{-\frac{1}{2}} x^{-1} exp(-\frac{1}{2\sigma^2} (lnx - \mu)^2) \\ &= exp(\mu + \frac{\sigma^2}{2}) \\ E[X^2] &= \int_0^\infty x^2 (2\pi\sigma^2)^{-\frac{1}{2}} x^{-1} exp(-\frac{1}{2\sigma^2} (lnx - \mu)^2) \\ &= exp(2\mu + 2\sigma^2) \\ &= exp(2\mu + 2\sigma^2) \\ &\sigma^2 = ln \frac{E[X^2]}{E[X]^2} \\ &\mu = ln \frac{E[X]^2}{\sqrt{E[X^2]}} \\ &\hat{\sigma}^2 = ln \frac{\sum X_i^2}{(\sum X_i)^2} \\ &\hat{\mu} = ln \frac{(\sum X_i)^2}{\sqrt{\sum X_i^2}} \end{split}$$

极大似然估计:

$$L(\mu,\sigma) = \prod_{i=1}^n p(X_i;\mu,\sigma)$$

$$=rac{1}{n}{\sum}(lnX_i-rac{1}{n}{\sum}lnX_i)^2$$

习题七第4题

矩估计:

$$E[X] = \int_{\mu}^{\infty} x e^{-(x-\mu)/\theta} dx = \theta + \mu$$
 $E[X^2] = 2\theta^2 + \mu^2 + 2\theta\mu$ $\hat{\theta} = \sqrt{(\sum X_i^2) - (\sum X_i)^2}$ $\hat{\mu} = (\sum X_i) - \sqrt{(\sum X_i^2) - (\sum X_i)^2}$

极大似然估计:

$$L(\mu, heta) = \prod p(X_i; \mu, heta) \ lnL(\mu, heta) = \sum (-ln heta - (X_i - \mu)/ heta) \ = -nln heta + n\mu/ heta - \sum X_i/ heta \ rac{\partial lnL}{\partial \mu} = rac{n}{ heta} \ rac{\partial lnL}{\partial heta} = -rac{n}{ heta} + rac{\sum X_i}{ heta^2} = 0 \ \hat{ heta} = rac{1}{n} \sum X_i$$

依照定义, 且 $p(x; \mu, \theta)$ 随 μ 单调递增,

$$\hat{\mu} = \min_i X_i$$

习题七第11题

$$egin{aligned} &= c \sum_{i=1}^{n-1} (D[X_{i+1}] + D[X_i] - 2cov(X_{i+1}, X_i)) \ &= c \sum_{i=1}^{n-1} (2\sigma^2) \ &= 2c(n-1)\sigma^2 \ c &= rac{1}{2(n-1)} \end{aligned}$$

习题七第12题

$$p(x; \sigma^{2}) = \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x-\mu)^{2}}{2\sigma^{2}}]$$

$$L(\sigma^{2}) = \prod p(X_{i}; \sigma^{2})$$

$$lnL = \sum (-ln(\sqrt{2\pi}) - ln\sigma - \frac{(x-\mu)^{2}}{2\sigma^{2}})$$

$$\frac{\partial lnL}{\partial \sigma} = 0$$

$$\hat{\sigma} = \frac{1}{n} \sum (X_{i} - \mu)^{2}$$

$$S^{2} = \frac{1}{n-1} \sum (X_{i} - \bar{X})^{2}$$

$$D[\hat{\sigma}] = \frac{1}{n^{2}} \sum D[(X_{i} - \mu)^{2}]$$

$$= \frac{1}{n^{2}} \sum [E[(X_{i} - \mu)^{4}] - E[(X_{i} - \mu)^{2}]^{2}]$$

$$= \frac{1}{n^{2}} \sum [3\sigma^{4} - \sigma^{4}]$$

$$= \frac{2}{n^{4}}$$

$$(n-1)S^{2} \sim \chi^{2}(n-1)$$

$$\sigma^{2} \sigma^{4} \qquad D[2] = \frac{1}{n^{2}} \sum [n-1]^{2}$$

$$L_0 = 2rac{\sigma}{\sqrt{n}}urac{lpha}{2} \leq L \ n \geq (rac{2\sigma urac{lpha}{2}}{L})^2$$

习题七第14题

$$\frac{\bar{X}-\mu}{S/\sqrt{n-1}}\sim t(n-1)$$

$$P(\bar{X}-\frac{S}{\sqrt{n-1}}t_{\frac{\alpha}{2}}(n-1)<\mu<\bar{X}+\frac{S}{\sqrt{n-1}}t_{\frac{\alpha}{2}}(n-1))=1-\alpha$$

$$\bar{X}=6,n=9,S=0.542,\alpha=0.05,t_{\frac{\alpha}{2}}(n-1)=t_{0.025}(8)=2.3060$$
 置信区间:[5.583,6.417]

习题七第18题

$$X \sim P(\lambda)$$
 $E[X] = D[X] = \lambda$
 $E[ar{X}] = E[rac{1}{n}\sum X] = \lambda$
 $D[ar{X}] = D[rac{1}{n}\sum X] = \lambda/n$
 $ar{X} \sim N(\lambda, \lambda/n)$
 $u = rac{ar{X} - \mu}{\sqrt{D[ar{X}]}} = rac{ar{X} - \lambda}{\sqrt{\lambda/n}} \sim N(0, 1)$
 $\lambda = E[rac{1}{n}\sum X_i] = ar{X}, n o \infty$
 $P(-u_{lpha/2} < rac{ar{X} - \lambda}{\sqrt{ar{X}/n}} < u_{lpha/2}) = 1 - lpha$
 $(ar{X} - u_{lpha/2} \sqrt{rac{ar{X}}{x}}, ar{X} + u_{lpha/2} \sqrt{rac{ar{X}}{x}})$