概率论 作业十

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习题七 第3题

矩估计:

$$E[X] = \int_0^\infty x(2\pi\sigma^2)^{-\frac{1}{2}}x^{-1}exp(-\frac{1}{2\sigma^2}(lnx-\mu)^2)$$
$$= exp(\mu + \frac{\sigma^2}{2})$$
$$E[X^2] = \int_0^\infty x^2(2\pi\sigma^2)^{-\frac{1}{2}}x^{-1}exp(-\frac{1}{2\sigma^2}(lnx-\mu)^2)$$
$$= exp(2\mu + 2\sigma^2)$$
$$\sigma^2 = ln\frac{E[X^2]}{E[X]^2}$$
$$\mu = ln\frac{E[X]^2}{\sqrt{E[X^2]}}$$
$$\hat{\sigma}^2 = ln\frac{\sum X_i^2}{(\sum X_i)^2}$$
$$\hat{\mu} = ln\frac{(\sum X_i)^2}{\sqrt{\sum X_i^2}}$$

极大似然估计:

$$L(\mu, \sigma) = \prod_{i=1}^{n} p(X_i; \mu, \sigma)$$

$$= \frac{1}{n}\sum(lnX_i - \frac{1}{n}\sum lnX_i)^2$$

习题七 第4题

矩估计:

$$E[X] = \int_{\mu}^{\infty} xe^{-(x-\mu)/\theta}dx = \theta + \mu$$
$$E[X^2] = 2\theta^2 + \mu^2 + 2\theta\mu$$
$$\hat{\theta} = \sqrt{(\sum X_i^2) - (\sum X_i)^2}$$
$$\hat{\mu} = (\sum X_i) - \sqrt{(\sum X_i^2) - (\sum X_i)^2}$$

极大似然估计:

$$L(\mu,\theta) = \prod p(X_i;\mu,\theta)$$
$$lnL(\mu,\theta) = \sum(-ln\theta - (X_i - \mu)/\theta)$$
$$= -nln\theta + n\mu/\theta - \sum X_i/\theta$$
$$\frac{\partial lnL}{\partial \mu} = \frac{n}{\theta}$$
$$\frac{\partial lnL}{\partial \theta} = -\frac{n}{\theta} + \frac{\sum X_i}{\theta^2} = 0$$
$$\hat{\theta} = \frac{1}{n}\sum X_i$$

依照定义, 且$p(x;\mu,\theta)$随μ单调递增,

$$\hat{\mu} = \min_i X_i$$

习题七 第11题

$$= c\sum_{i=1}^{n-1}(D[X_{i+1}] + D[X_i] - 2cov(X_{i+1}, X_i))$$

$$= c\sum_{i=1}^{n-1}(2\sigma^2)$$

$$= 2c(n-1)\sigma^2$$

$$c = \frac{1}{2(n-1)}$$

习题七 第12题

$$p(x;\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}exp[-\frac{(x-\mu)^2}{2\sigma^2}]$$

$$L(\sigma^2) = \prod p(X_i;\sigma^2)$$

$$lnL = \sum(-ln(\sqrt{2\pi}) - ln\sigma - \frac{(x-\mu)^2}{2\sigma^2})$$

$$\frac{\partial lnL}{\partial\sigma} = 0$$

$$\hat{\sigma} = \frac{1}{n}\sum(X_i-\mu)^2$$

$$S^2 = \frac{1}{n-1}\sum(X_i-\bar{X})^2$$

$$D[\hat{\sigma}] = \frac{1}{n^2}\sum D[(X_i-\mu)^2]$$

$$= \frac{1}{n^2}\sum[E[(X_i-\mu)^4] - E[(X_i-\mu)^2]^2]$$

$$= \frac{1}{n^2}\sum[3\sigma^4 - \sigma^4]$$

$$= \frac{2}{n}\sigma^4$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$D[S^2] \quad \sigma^4 \quad D[\ ^2(\ \ -1)]$$

$$L_0 = 2\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}} \le L$$
$$n \ge (\frac{2\sigma u_{\frac{\alpha}{2}}}{L})^2$$

习题七 第14题

$$\frac{\bar{X}-\mu}{S/\sqrt{n-1}} \sim t(n-1)$$
$$P(\bar{X}-\frac{S}{\sqrt{n-1}}t_{\frac{\alpha}{2}}(n-1) < \mu < \bar{X}+\frac{S}{\sqrt{n-1}}t_{\frac{\alpha}{2}}(n-1)) = 1-\alpha$$
$$\bar{X}=6, n=9, S=0.542, \alpha=0.05, t_{\frac{\alpha}{2}}(n-1)=t_{0.025}(8)=2.3060$$
置信区间:$[5.583, 6.417]$

习题七 第18题

$$X \sim P(\lambda)$$
$$E[X]=D[X]=\lambda$$
$$E[\bar{X}]=E[\frac{1}{n}\sum X]=\lambda$$
$$D[\bar{X}]=D[\frac{1}{n}\sum X]=\lambda/n$$
$$\bar{X} \sim N(\lambda, \lambda/n)$$
$$u=\frac{\bar{X}-\mu}{\sqrt{D[\bar{X}]}}=\frac{\bar{X}-\lambda}{\sqrt{\lambda/n}} \sim N(0,1)$$
$$\lambda = E[\frac{1}{n}\sum X_i]=\bar{X}, n \to \infty$$
$$P(-u_{\alpha/2} < \frac{\bar{X}-\lambda}{\sqrt{\bar{X}/n}} < u_{\alpha/2}) = 1-\alpha$$
$$(\bar{X}-u_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}, \bar{X}+u_{\alpha/2}\sqrt{\frac{\bar{X}}{n}})$$