作业九

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习题六 第3题

1. $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$
   $= \frac{1}{n-1}\sum_{i=1}^{n}[X_i^2 + X^2 - 2X_iX]$
   $= \frac{1}{n-1}(\sum_{i=1}^{n} X_i^2 + n\overline{X}^2) - 2\overline{X}\frac{1}{n-1}\sum_{i=1}^{n} X_i$
   $= \frac{1}{n-1}(\sum_{i=1}^{n} X_i^2 + nX^2) - \frac{2n}{n-1}X^2 = \frac{1}{n-1}[\sum_{i=1}^{n} X_i^2 - nX^2]$
2. $\because E(\frac{(n-1)S^2}{\sigma^2}) = n - 1 = \frac{(n-1)}{\sigma^2}E(S^2) \therefore E(S^2) = \sigma^2$

习题六 第8题

$E(Y) = \sum_{i=1}^{n} D(X_i + X_{n+i} - 2\overline{X}) + \sum_{i=1}^{n} E(X_i + X_{n+i} - 2\overline{X})^2$
$= \sum_{i=1}^{2n} D(X_i) - 4\sum_{i=1}^{n} cov(X_i, \overline{X}) - 4\sum_{i=1}^{n} cov(X_{n+i}, \overline{X}) + 4\sum_{i=1}^{n} D(\overline{X}) + 0$
$= 2n\sigma^2 - 4n[cov(X_i, \frac{X_i}{2n}) + cov(X_{n+i}, \frac{X_{n+i}}{2n})] + 2\sigma^2 = 2(n-1)\sigma^2$

习题六 第9题

$\because (n-1)S^2/\sigma^2 \sim \chi^2(n-1), \overline{X} \sim N(\mu, \frac{\sigma^2}{n}), X_{n+1} \sim N(\mu, \sigma^2)$
$\therefore X_{n+1} - \overline{X} \sim N(0, \sigma^2\frac{n+1}{n})$
$\therefore \frac{X_{n+1} - \overline{X}}{S}\sqrt{\frac{n}{n+1}} = \sqrt{\frac{n}{n+1}}\frac{X_{n+1} - \overline{X}}{\sqrt{(n-1)S^2/(n-1)}} \sim \frac{N(0, \frac{n+1}{n})}{\sqrt{\chi^2(n-1)/(n-1)}}\sqrt{\frac{n}{n+1}}$
$\sim \frac{N(0,1)}{\sqrt{\chi^2(n-1)/(n-1)}} \sim t(n-1)$

习题六 第10题

1. $\overline{X} \sim N(12, \frac{4}{5}), P(\overline{X} > 13) = P(\frac{\overline{X}-12}{\sqrt{0.8}} > \sqrt{1.25}) = 1 - \Phi(1.1180) = 0.131$

2. $P(min_{1 \le i \le 5}X_i < 10) = 1 - P(X_i \ge 10)^5 = 1 - \Phi(1)^5 = 0.5785$

3. $P(max_{1 \le i \le 5}X_i > 15) = 1 - P(X_i \le 15)^5 = 1 - \Phi(1.5)^5 = 0.2923$

习题六 第11题

设联合样本均值为Z,方差为S^2则有$\overline{Z} = \frac{n1X + n2Y}{n1 + n2}$
$\because S_1^2 = \frac{1}{n1-1}\sum_{i=1}^{n1} X_i^2 - \frac{n1}{n1-1}X^2, \quad \therefore \sum_{i=1}^{n1} X_i^2 = (n_1 - 1)S_1^2 + n_1X^2$
$S^2 = \frac{1}{n1+n2-1}(\sum_{i=1}^{n1} X_i^2 + \sum_{i=1}^{n2} Y_i^2) - \frac{n1+n2}{n1+n2-1}Z^2$
$= \frac{(n1-1)S_1^2 + n1X^2 + (n2-1)S_2^2 + n2Y^2}{n1+n2-1} - \frac{n1+n2}{n1+n2-1}(\frac{n1\overline{X} + n2\overline{Y}}{n1+n2})^2$

第二题

$E((X_1+X_2)(X_1-X_2))=E(X_1^2-X_2^2)=EX_1^2-EX_2^2$
$=(EX_1-EX_2)(EX_1+EX_2)=E(X_1+X_2)E(X_1-X_2)$
故$(X_1+X_2)^2,(X_1-X_2)^2$独立。
$\because X_1+X_2 \sim N(0,2\sigma^2),\therefore \frac{(X_1+X_2)^2}{2\sigma^2} \sim \chi(1)$,同理$\frac{(X_1-X_2)^2}{2\sigma^2} \sim \chi(1)$
故$\frac{(X1+X2)^2}{(X1-X2)^2} \sim \frac{\chi(1)}{\chi(1)} \sim F(1,1)$