作业八

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习题五第2题

设随机变量X为终端在使用的数量，则X~B(120,0.05)，
$EX = 120 * 0.05 = 6, DX = 120 * 0.05 * 0.95 = 5.7$
近似为正态分布有$\frac{X-6}{\sqrt{5.7}} \sim N(0,1)$,则$P(X \geq 10) = 1 - \Phi(\frac{10-6}{\sqrt{5.7}}) \approx 0.0465$

习题五第4题

设随机变量X_i为每个数舍入的误差，X为总的舍入的误差，

1. $X_i \sim U[-0.5, 0.5], E(X_i) = 0, D(X_i) = \frac{1}{12}$
   $E(X) = \sum_{i=1}^{n} X_i P(X_i) = 0, D(X) = D(\sum_{i=1}^{n} X_i) = \frac{n}{12} = 125$
   根据大数定律，$\frac{X}{\sqrt{125}} \sim N(0,1)$
   $P(|X| > 15) = P(|\frac{X}{\sqrt{125}}| > \frac{15}{\sqrt{125}}) \approx 2\Phi(-1.34) \approx 0.18$

2. 设最多有n个数，则$E(X) = 0, D(X) = \frac{n}{12}, \frac{X}{\sqrt{\frac{n}{12}}} \sim N(0,1)$
   $P(|X| < 10) = P(\frac{|X|}{\sqrt{\frac{n}{12}}} < \frac{10}{\sqrt{\frac{n}{12}}}) = 2\Phi(\frac{10}{\sqrt{\frac{n}{12}}}) - 1 \geq 0.96$
   $\Phi(\frac{10}{\sqrt{\frac{n}{12}}}) \geq 0.98$，查表得 $\frac{10}{\sqrt{\frac{n}{12}}} \approx 2.06, n \approx 282$

习题五第5题

$E(X) = \int_0^1 xp(x)dx = \frac{1}{2}, E(X^2) = \int_0^1 x^2 p(x)dx = \frac{3}{10}$
$D(X) = E(X^2) - E(X)^2 = \frac{3}{10} - \frac{1}{4} = \frac{1}{20}$，
$\because \frac{1}{n^2} D(\sum_{k=1}^{n} X_k) \to 0 (n \to \infty)$,故服从大数定律，
即

$$\lim_{n\to+\infty}\frac{1}{n}\sum_{k=1}^{n}X_k=E(X)=\frac{1}{2}$$

习题五第7题

$E(\ln X_i)=\int_0^1\ln x*1dx=x\ln x|_0^1-\int_0^1 xd\ln x=-1$
因为$E(\ln X_i)$存在，所以$\{\ln X_i\}$服从辛钦大数定律,
$\ln Z_n=\frac{1}{n}\sum_{i=1}^{n}\ln X_i\to^P -1$,
因为$f(x)=e^x$连续，$Z_n\to^P \frac{1}{e}$.

二

已知$X_n\to^P a, Y_n\to^P b$, 所以对于$\forall\delta_1,\delta_2$,

$$\lim_{n\to\infty}P\{|X_n-a|\le\delta_1\}=1$$
$$\lim_{n\to\infty}P\{|Y_n-b|\le\delta_2\}=1$$

因为 g 在 (a,b) 连续, $\forall\epsilon>0$,$\exists\delta_1,\delta_2$,

$$P\{|g(X_n,Y_n)-g(a,b)|\le\epsilon\}=P(|X_n-a|\le\delta_1)P(|Y_n-b|\le\delta_2)$$

所以有

$$\lim_{n\to\infty}P\{|g(X_n,Y_n)-g(a,b)|\le\epsilon\}$$
$$=\lim_{n\to\infty}P(|X_n-a|\le\delta_1)P(|Y_n-b|\le\delta_2)=1$$
$$g(X_n,Y_n)\to^P g(a,b)$$