

作业九

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习题六 第3题

$$\begin{aligned} 1. S^2 &= \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \\ &= \frac{1}{n-1} \sum_{i=1}^n [X_i^2 + \bar{X}^2 - 2X_i\bar{X}] \\ &= \frac{1}{n-1} (\sum_{i=1}^n X_i^2 + n\bar{X}^2) - 2\bar{X} \frac{1}{n-1} \sum_{i=1}^n X_i \\ &= \frac{1}{n-1} (\sum_{i=1}^n X_i^2 + n\bar{X}^2) - \frac{2n}{n-1} \bar{X}^2 = \frac{1}{n-1} [\sum_{i=1}^n X_i^2 - n\bar{X}^2] \\ 2. \because E\left(\frac{(n-1)S^2}{\sigma^2}\right) &= n-1 = \frac{(n-1)}{\sigma^2} E(S^2) \therefore E(S^2) = \sigma^2 \end{aligned}$$

习题六 第8题

$$\begin{aligned} E(Y) &= \sum_{i=1}^n D(X_i + X_{n+i} - 2\bar{X}) + \sum_{i=1}^n E(X_i + X_{n+i} - 2\bar{X})^2 \\ &= \sum_{i=1}^{2n} D(X_i) - 4 \sum_{i=1}^n \text{cov}(X_i, \bar{X}) - 4 \sum_{i=1}^n \text{cov}(X_{n+i}, \bar{X}) + 4 \sum_{i=1}^n D(\bar{X}) + 0 \\ &= 2n\sigma^2 - 4n[\text{cov}(X_i, \frac{X_i}{2n}) + \text{cov}(X_{n+i}, \frac{X_{n+i}}{2n})] + 2\sigma^2 = 2(n-1)\sigma^2 \end{aligned}$$

习题六 第9题

$$\begin{aligned} \because (n-1)S^2/\sigma^2 &\sim \chi^2(n-1), \bar{X} \sim N(\mu, \frac{\sigma^2}{n}), X_{n+1} \sim N(\mu, \sigma^2) \\ \therefore X_{n+1} - \bar{X} &\sim N(0, \sigma^2 \frac{n+1}{n}) \\ \therefore \frac{X_{n+1} - \bar{X}}{S} \sqrt{\frac{n}{n+1}} &= \sqrt{\frac{n}{n+1}} \frac{X_{n+1} - \bar{X}}{\sqrt{(n-1)S^2/(n-1)}} \sim \frac{N(0, \frac{n+1}{n})}{\sqrt{\chi^2(n-1)/(n-1)}} \sqrt{\frac{n}{n+1}} \\ &\sim \frac{N(0,1)}{\sqrt{\chi^2(n-1)/(n-1)}} \sim t(n-1) \end{aligned}$$

习题六 第10题

$$\begin{aligned} 1. \bar{X} &\sim N(12, \frac{4}{5}), P(\bar{X} > 13) = P(\frac{\bar{X}-12}{\sqrt{0.8}} > \sqrt{1.25}) = 1 - \Phi(1.1180) = 0.131 \\ 2. P(\min_{1 \leq i \leq 5} X_i < 10) &= 1 - P(X_i \geq 10)^5 = 1 - \Phi(1)^5 = 0.5785 \\ 3. P(\max_{1 \leq i \leq 5} X_i > 15) &= 1 - P(X_i \leq 15)^5 = 1 - \Phi(1.5)^5 = 0.2923 \end{aligned}$$

习题六 第11题

$$\begin{aligned} \text{设联合样本均值为 } Z, \text{ 方差为 } S^2 \text{ 则有 } \bar{Z} &= \frac{n_1 X + n_2 Y}{n_1 + n_2} \\ \therefore S_1^2 &= \frac{1}{n_1-1} \sum_{i=1}^{n_1} X_i^2 - \frac{n_1}{n_1-1} \bar{X}^2, \quad \therefore \sum_{i=1}^{n_1} X_i^2 = (n_1-1)S_1^2 + n_1\bar{X}^2 \\ S^2 &= \frac{1}{n_1+n_2-1} (\sum_{i=1}^{n_1} X_i^2 + \sum_{i=1}^{n_2} Y_i^2) - \frac{n_1+n_2}{n_1+n_2-1} \bar{Z}^2 \\ &= \frac{(n_1-1)S_1^2 + n_1\bar{X}^2 + (n_2-1)S_2^2 + n_2\bar{Y}^2}{n_1+n_2-1} - \frac{n_1+n_2}{n_1+n_2-1} (\frac{n_1\bar{X} + n_2\bar{Y}}{n_1+n_2})^2 \end{aligned}$$