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## 1 Basic

### 1.1 Default code

```
#pragma GCC (2)
#include<bits/stdc++.h>
using namespace std;
#define f first
#define s second
#define SZ(i) ((int)i.size())
#define All(i) i.begin(),i.end()
#define pb push_back
#define endl '\n'
int main(){
    ios::sync_with_stdio(false),cin.tie(0),cout.tie(0);
}
```

### 1.2 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=2 sts=2 sw=2 ls=2 mouse=a
syn on
hi cursorline cterm=none ctermbg=89
inoremap {<ENTER> {<CR>}<Esc>ko<TAB>
```

### 1.3 read

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
inline int readint(){
    int f=1,x=0;char c;
    do{c=getchar();if(c=='-')f=-1;}while(c<'0'||c>'9');
    do{x=x*10+c-'0';c=getchar();}while(c>='0'&&c<='9');
    return f*x;
}
```

### 1.4 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
struct myhash {
    const int RANDOM = (ll)(make_unique<char>().get()) ^
        chrono::high_resolution_clock::now().
        time_since_epoch().count();
    static unsigned ll hash_f(unsigned ll x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    static unsigned hash_combine(unsigned a, unsigned b){
        return a * 31 + b; }
    ll operator()(ll x) const{ return hash_f(x)^RANDOM; }
};
```

```

typedef gp_hash_table<ll, int, myhash> hashtable;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
//__int128_t, __float128_t

```

## 1.5 Texas hold'em

```

char suit[4]={'C','D','H','Y'}, ranks[13]={'2','3','4','5',
    '6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
    for(int i=0;i<13;++i)
        rk[ranks[i]]=i;
    for(int i=0;i<4;++i)
        rk[suit[i]]=i;
*/
struct cards{
    vector<pii> v;
    int suit_count[4], hands;
    void reset(){v.clear(), FILL(suit_count, 0), hands=-1;}
    void insert(char a, char b){//suit, rank
        ++suit_count[rk[a]];
        int flag=0;
        for(auto &i:v)
            if(i.Y==rk[b])
                ++i.X, flag=1;
        if(!flag) v.pb(pii(1, rk[b]));
    }
    void insert(string s){insert(s[0], s[1]);}
    void ready(){
        int Straight=0, Flush>(*max_element(suit_count,
            suit_count+4))==5;
        sort(ALL(v), [](ii a, ii b){return a>b;});
        if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
            Straight=1;
        else if(SZ(v)==5&&v[0].Y==12&&v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
            Straight=1;
        if(Straight&&Flush) hands=1;
        else if(v[0].X==4) hands=2;
        else if(v[0].X==3&&v[1].X==2) hands=3;
        else if(Flush) hands=4;
        else if(Straight) hands=5;
        else if(v[0].X==3) hands=6;
        else if(v[0].X==2&&v[1].X==2) hands=7;
        else if(v[0].X==2) hands=8;
        else hands=9;
    }
    bool operator>(const cards &a) const{
        if(hands==a.hands) return v>a.v;
        return hands<a.hands;
    }
};

```

## 2 Graph

### 2.1 BCC Vertex\*

```

vector<int> G[N]; // 1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].push_back(t);
                } while (t != v);
                bcc_id[u] = bcc_cnt;
                bcc[bcc_cnt].pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != pa)
            low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) {
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // circle-square tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}

```

### 2.2 Bridge\*

```

int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
    edge.pb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)

```

```

    if (!dfn[i]) dfs(i, -1);
}

```

## 2.3 2SAT (SCC)\*

```

struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].pb(b); }
    int rv(int a) {
        if (a > n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[i], low[u]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
            ++nScc;
        }
    }
    bool solve() {
        Time = nScc = 0;
        for (int i = 0; i < n + n; ++i)
            SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
        for (int i = 0; i < n + n; ++i)
            if (!dfn[i]) dfs(i);
        for (int i = 0; i < n + n; ++i) SCC[bln[i]].pb(i);
        for (int i = 0; i < n; ++i) {
            if (bln[i] == bln[i + n]) return false;
            istrue[i] = bln[i] < bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

## 2.4 MinimumMeanCycle\*

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {

```

```

            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

## 2.5 Virtual Tree\*

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.6 Maximum Clique Dyn\*

```

const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0,
            m = r.size();
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++)
            for (int p = cs[k]._Find_first(); p < N;
                p = cs[k]._Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;

```

```

    cur[q++] = p;
    vector<int> nr, nc;
    bitset<N> nmask = mask & a[p];
    for (int i : r)
        if (a[p][i]) nr.push_back(i);
    if (!nr.empty()) {
        if (1 < 4) {
            for (int i : nr)
                d[i] = (a[i] & nmask).count();
            sort(nr.begin(), nr.end(),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        csort(nr, nc), dfs(nr, nc, 1 + 1, nmask);
    } else if (q > ans) ans = q, copy_n(cur, q, sol);
    c.pop_back(), q--;
}
}
int solve(bitset<N> mask = bitset<N>(
    string(N, '1'))) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

## 2.7 Minimum Steiner Tree\*

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertices
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    dst[i][j] =
                        min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter);
        for (int i = 0; i < (1 << t); ++i)
            for (int j = 0; j < n; ++j) dp[i][j] = INF;
        for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
        for (int msk = 1; msk < (1 << t); ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] =
                        vcost[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int submsk = (msk - 1) & msk; submsk;
                    submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i],
                        dp[submsk][i] + dp[msk ^ submsk][i] -
                        vcost[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    tdst[i] =

```

```

                        min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for (int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
};

```

## 2.8 Dominator Tree\*

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for (int i = 2; i <= Time; ++i) {
            if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence\*

```

struct zhu_liu { // O(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // 0-base
    int pe[N], id[N], vis[N];
    ll in[N];

```

```

void init() { E.clear(); }
void add_edge(int u, int v, ll w) {
    if (u != v) E.pb(edge{u, v, w});
}
ll build(int root, int n) {
    ll ans = 0;
    for (;) {
        fill_n(in, n, INF);
        for (int i = 0; i < SZ(E); ++i)
            if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                pe[E[i].v] = i, in[E[i].v] = E[i].w;
        for (int u = 0; u < n; ++u) // no solution
            if (u != root && in[u] == INF) return -INF;
        int cntnode = 0;
        fill_n(id, n, -1), fill_n(vis, n, -1);
        for (int u = 0; u < n; ++u) {
            if (u != root) ans += in[u];
            int v = u;
            while (vis[v] != u && !~id[v] && v != root)
                vis[v] = u, v = E[pe[v]].u;
            if (v != root && !~id[v]) {
                for (int x = E[pe[v]].u; x != v;
                     x = E[pe[x]].u)
                    id[x] = cntnode;
                id[v] = cntnode++;
            }
        }
        if (!cntnode) break; // no cycle
        for (int u = 0; u < n; ++u)
            if (!~id[u]) id[u] = cntnode++;
        for (int i = 0; i < SZ(E); ++i) {
            int v = E[i].v;
            E[i].u = id[E[i].u], E[i].v = id[E[i].v];
            if (E[i].u != E[i].v) E[i].w -= in[v];
        }
        n = cntnode, root = id[root];
    }
    return ans;
}
};

```

## 2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
// matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; i++) {
        for (int j = 0; j <= N; j++) C[i][j] = G[i][j] = 0;
    }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++)
            ;
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        int u = E[t].first, v0 = E[t].second, v = v0,
            c0 = X[u], c = c0, d;
        vector<pair<int, int>> L;
        int vst[kN] = {};

```

```

        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c])
                for (a = (int)L.size() - 1; a >= 0; a--)
                    c = color(u, L[a].first, c);
            else if (!C[u][d])
                for (a = (int)L.size() - 1; a >= 0; a--)
                    color(u, L[a].first, L[a].second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d))
                ;
            if (C[u][c0]) {
                for (a = (int)L.size() - 2;
                     a >= 0 && L[a].second != c; a--)
                    ;
                for (; a >= 0; a--)
                    color(u, L[a].first, L[a].second);
            } else t--;
        }
    }
} // namespace vizing

```

## 2.11 Minimum Clique Cover\*

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0;
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

## 2.12 NumberofMaximalClique\*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];

```

```

for (int i = 0; i < sn; ++i) {
    int v = some[d][i];
    if (g[u][v]) continue;
    int tsu = 0, tnn = 0;
    copy_n(all[d], an, all[d + 1]);
    all[d + 1][an] = v;
    for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
            some[d + 1][tsu++] = some[d][j];
    for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
            none[d + 1][tnn++] = none[d][j];
    dfs(d + 1, an + 1, tsu, tnn);
    some[d][i] = 0, none[d][nn++] = v;
}
}
int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
}
};

```

## 2.13 Dijkstra

```

vector<pii> edge[mxn];
int dis[mxn], vis[mxn];
void dijkstra(int s)
{
    memset(dis, 0x7f, sizeof(dis));
    dis[s] = 0;
    priority_queue<pii, vector<pii>, greater<pii>> pq;
    pq.emplace(0, s);
    while (pq.size()) {
        int now = pq.top().Y;
        pq.pop();
        if (vis[now]) continue;
        vis[now] = 1;
        for (pii e : edge[now]) {
            if (!vis[e.X] && dis[e.X] > dis[now] + e.Y) {
                dis[e.X] = dis[now] + e.Y;
                pq.emplace(dis[e.X], e.X);
            }
        }
    }
}

```

## 2.14 Theory

$|\text{Maximum independent edge set}| = |V| - |\text{Minimum edge cover}|$   
 $|\text{Maximum independent set}| = |V| - |\text{Minimum vertex cover}|$   
 A sequence of non-negative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and
 
$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } k \text{ in } 1 \leq k \leq n.$$

# 3 Data Structure

## 3.1 Leftist Tree

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
}

```

```

a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
a->sum = sum(a->l) + sum(a->r) + a->data;
return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

## 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[10005], deep[10005], mxson[10005],
        w[10005], pa[10005];
    int t, pl[10005], data[10005], dt[10005], bln[10005],
        edge[10005], et;
    vector<pii> G[10005];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, et)), G[b].pb(pii(a, et)),
            edge[et++] = w;
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (auto &i : G[u])
            if (i.X != f) {
                dfs(i.X, u, d), w[u] += w[i.X];
                if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
                else bln[i.Y] = u, dt[u] = edge[i.Y];
            }
    }
    void cut(int u, int link) {
        data[pl[u] = t++] = dt[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (auto i : G[u])
            if (i.X != pa[u] && i.X != mxson[u])
                cut(i.X, i.X);
    }
    void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
    int query(int a, int b) {
        int ta = ulink[a], tb = ulink[b], re = 0;
        while (ta != tb)
            if (deep[ta] < deep[tb])
                /*query*/, tb = ulink[b = pa[tb]];
            else /*query*/, ta = ulink[a = pa[ta]];
        if (a == b) return re;
        if (pl[a] > pl[b]) swap(a, b);
        /*query*/
        return re;
    }
};

```

## 3.3 Centroid Decomposition\*

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])

```



```

    if (!done[e.X] && e.X != f) {
        get_cent(e.X, u, mx, c, num);
        sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
    }
    if (mx > max(mxsz, num - sz[u]))
        mx = max(mxsz, num - sz[u]), c = u;
}
void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
}

```

```

    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k),
        o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}
}

```

### 3.5 RangeOperations

```

struct node {
    ll sum;
    ll mx, mxcnt, smx;
    ll mi, micnt, smi;
    ll lazymax, lazymin, lazyadd;
    node(ll k = 0)
        : sum(k), mx(k), mxcnt(1), smx(-INF), mi(k),
          micnt(1), smi(INF), lazymax(-INF), lazymin(INF),
          lazyadd(0) {}
    node operator+(const node &a) const {
        node rt;
        rt.sum = sum + a.sum;
        rt.mx = max(mx, a.mx);
        rt.mi = min(mi, a.mi);
        if (mx == a.mx) {
            rt.mxcnt = mxcnt + a.mxcnt;
            rt.smx = max(smx, a.smx);
        } else if (mx > a.mx) {
            rt.mxcnt = mxcnt;
            rt.smx = max(smx, a.mx);
        } else {
            rt.mxcnt = a.mxcnt;
            rt.smx = max(mx, a.smx);
        }
        if (mi == a.mi) {
            rt.micnt = micnt + a.micnt;
            rt.smi = min(smi, a.smi);
        } else if (mi < a.mi) {
            rt.micnt = micnt;
            rt.smi = min(smi, a.mi);
        } else {
            rt.micnt = a.micnt;
            rt.smi = min(mi, a.smi);
        }
        rt.lazymax = -INF;
        rt.lazymin = INF;
        rt.lazyadd = 0;
        return rt;
    }
} seg[MAXC << 2];

```

```

ll a[MAXC];
void give_tag_min(int rt, ll t) {
    if (t >= seg[rt].mx) return;
    seg[rt].lazymin = t;
    seg[rt].lazymax = min(seg[rt].lazymax, t);
    seg[rt].sum -= seg[rt].mxcnt * (seg[rt].mx - t);
    if (seg[rt].mx == seg[rt].smi) seg[rt].smi = t;
    if (seg[rt].mx == seg[rt].mi) seg[rt].mi = t;
    seg[rt].mx = t;
}
void give_tag_max(int rt, ll t) {
    if (t <= seg[rt].mi) return;
    seg[rt].lazymax = t;
    seg[rt].sum += seg[rt].micnt * (t - seg[rt].mi);
    if (seg[rt].mi == seg[rt].smx) seg[rt].smx = t;
    if (seg[rt].mi == seg[rt].mx) seg[rt].mx = t;
    seg[rt].mi = t;
}
void give_tag_add(int l, int r, int rt, ll t) {
    seg[rt].lazyadd += t;
    if (seg[rt].lazymax != -INF) seg[rt].lazymax += t;
    if (seg[rt].lazymin != INF) seg[rt].lazymin += t;
    seg[rt].mx += t;
    if (seg[rt].smx != -INF) seg[rt].smx += t;
    seg[rt].mi += t;
    if (seg[rt].smi != INF) seg[rt].smi += t;
    seg[rt].sum += (ll)(r - l + 1) * t;
}
void tag_down(int l, int r, int rt) {
    if (seg[rt].lazyadd != 0) {
        int mid = (l + r) >> 1;
        give_tag_add(l, mid, rt << 1, seg[rt].lazyadd);
        give_tag_add(
            mid + 1, r, rt << 1 | 1, seg[rt].lazyadd);
        seg[rt].lazyadd = 0;
    }
    if (seg[rt].lazymin != INF) {
        give_tag_min(rt << 1, seg[rt].lazymin);
        give_tag_min(rt << 1 | 1, seg[rt].lazymin);
        seg[rt].lazymin = INF;
    }
    if (seg[rt].lazymax != -INF) {
        give_tag_max(rt << 1, seg[rt].lazymax);
        give_tag_max(rt << 1 | 1, seg[rt].lazymax);
        seg[rt].lazymax = -INF;
    }
}
void build(int l, int r, int rt) {
    if (l == r) return seg[rt] = node(a[l]), void();
    int mid = (l + r) >> 1;
    build(l, mid, rt << 1);
    build(mid + 1, r, rt << 1 | 1);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}
void modifymax(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r && t < seg[rt].smi)
        return give_tag_max(rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (L <= mid) modifymax(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifymax(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}
void modifymin(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r && t > seg[rt].smx)
        return give_tag_min(rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (L <= mid) modifymin(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifymin(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}
void modifyadd(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r)
        return give_tag_add(l, r, rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;

```

```

    if (L <= mid) modifyadd(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifyadd(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}
ll query(int L, int R, int l, int r, int rt) {
    if (L <= l && R >= r) return seg[rt].sum;
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (R <= mid) return query(L, R, l, mid, rt << 1);
    if (L > mid)
        return query(L, R, mid + 1, r, rt << 1 | 1);
    return query(L, R, l, mid, rt << 1) +
        query(L, R, mid + 1, r, rt << 1 | 1);
}
int main() {
    int n, m;
    cin >> n >> m;
    for (int i = 1; i <= n; ++i) cin >> a[i];
    build(1, n, 1);
    while (m--) {
        int k, x, y;
        ll t;
        cin >> k >> x >> y, ++x;
        if (k == 0) cin >> t, modifymin(x, y, 1, n, 1, t);
        else if (k == 1)
            cin >> t, modifymax(x, y, 1, n, 1, t);
        else if (k == 2)
            cin >> t, modifyadd(x, y, 1, n, 1, t);
        else cout << query(x, y, 1, n, 1) << "\n";
    }
}

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres

```

struct KM { // 0-base
    int w[MAXN][MAXN], h1[MAXN], hr[MAXN], slk[MAXN], n;
    int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
    bool vl[MAXN], vr[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] == 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] == 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void Bfs(int s) {
        fill(slk, slk + n, INF);
        fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        while (1) {
            int d;
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] &&
                        slk[x] >= (d = h1[x] + hr[y] - w[x][y]))
                        if (pre[x] == y, d) slk[x] = d;
                        else if (!Check(x)) return;
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) h1[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !Check(x)) return;
        }
    }
}

```



```

}
int Solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1),
    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) Bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

## 4.2 MincostMaxflow

```

struct MCMF { // 0-base
    struct edge {
        ll from, to, cap, flow, cost, rev;
    } * past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN], up[MAXN], s, t, mx, n;
    bool BellmanFord(ll &flow, ll &cost) {
        fill(dis, dis + n, INF);
        queue<ll> q;
        q.push(s), inq.reset(), inq[s] = 1;
        up[s] = mx - flow, past[s] = 0, dis[s] = 0;
        while (!q.empty()) {
            ll u = q.front();
            q.pop(), inq[u] = 0;
            if (!up[u]) continue;
            for (auto &e : G[u])
                if (e.flow != e.cap &&
                    dis[e.to] > dis[u] + e.cost) {
                    dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
                    up[e.to] = min(up[u], e.cap - e.flow);
                    if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
                }
        }
        if (dis[t] == INF) return 0;
        flow += up[t], cost += up[t] * dis[t];
        for (ll i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
        s = _s, t = _t, cost = 0;
        ll flow = 0;
        while (BellmanFord(flow, cost))
            ;
        return flow;
    }
    void init(ll _n, ll _mx) {
        n = _n, mx = _mx;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
        G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
    }
};

```

## 4.3 Maximum Simple Graph Matching\*

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V = _V;
        for (int i = 0; i <= V; ++i) {
            for (int j = 0; j <= V; ++j) el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
};

```

```

void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
}
int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while (1)
        if (u = djs[u], inp[u] = true, u == st) break;
        else u = bk[pr[u]];
    while (1)
        if (v = djs[v], inp[v]) return v;
        else v = bk[pr[v]];
    return v;
}
void upd(int u) {
    for (int v; djs[u] != nb;) {
        v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if (djs[u] != nb) bk[u] = v;
    }
}
void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if (djs[u] != nb) bk[u] = v;
    if (djs[v] != nb) bk[v] = u;
    for (int tu = 1; tu <= V; ++tu)
        if (inb[djs[tu]])
            if (djs[tu] = nb, !inq[tu])
                qe.push(tu), inq[tu] = 1;
}
void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while (!qe.empty()) {
        int u = qe.front();
        qe.pop();
        for (int v = 1; v <= V; ++v)
            if (el[u][v] && djs[u] != djs[v] &&
                pr[u] != v) {
                if ((v == st) ||
                    (pr[v] > 0 && bk[pr[v]] > 0))
                    blo(u, v, qe);
                else if (!bk[v]) {
                    if (bk[v] = u, pr[v] > 0) {
                        if (!inq[pr[v]]) qe.push(pr[v]);
                    } else return ed = v, void();
                }
            }
    }
}
void aug() {
    for (int u = ed, v, w; u > 0;)
        v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
        u = w;
}
int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for (int u = 1; u <= V; ++u)
        if (!pr[u])
            if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
}
};

```

## 4.4 Minimum Weight Matching (Clique version)\*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] = edge[v][u] = w;
    }
};

```

```

bool SPFA(int u) {
    stk[tp++] = u, onstk[u] = 1;
    for (int v = 0; v < n; ++v)
        if (!onstk[v] && match[u] != v) {
            int m = match[v];
            if (dis[m] >
                dis[u] - edge[v][m] + edge[u][v]) {
                dis[m] = dis[u] - edge[v][m] + edge[u][v];
                onstk[v] = 1, stk[tp++] = v;
                if (onstk[m] || SPFA(m)) return 1;
                --tp, onstk[v] = 0;
            }
        }
    onstk[u] = 0, --tp;
    return 0;
}

ll solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
        int found = 0;
        fill_n(dis, n, 0);
        fill_n(onstk, n, 0);
        for (int i = 0; i < n; ++i)
            if (tp = 0, !onstk[i] && SPFA(i))
                for (found = 1; tp >= 2;) {
                    int u = stk[--tp];
                    int v = stk[--tp];
                    match[u] = v, match[v] = u;
                }
        if (!found) break;
    }
    ll ret = 0;
    for (int i = 0; i < n; ++i)
        ret += edge[i][match[i]];
    return ret >> 1;
}
};

```

## 4.5 SW-mincut

```

// global min cut
struct SW { // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n) {
        n = _n, MEM(edge, 0), MEM(del, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        MEM(vst, 0), MEM(wei, 0), s = t = -1;
        while (1) {
            int mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vst[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve() {
        int res = INF;
        for (int i = 0, x, y; i < n - 1; ++i) {
            search(x, y), res = min(res, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
};

```

## 4.6 BoundedFlow(Dinic\*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                G[n + 1].pop_back(), G[i].pop_back();
            else if (cnt[i] < 0)
                G[i].pop_back(), G[n + 2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
        if (!solve()) return -1; // invalid flow
        int x = G[_t].back().flow;
        return G[_t].pop_back(), G[_s].pop_back(), x;
    }
};

```

## 4.7 Gomory Hu tree

```
struct Gomory_Hu_tree { // 0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void solve(vector<int> &v) {
        if (v.size() <= 1) return;
        int s = rand() % SZ(v);
        swap(v.back(), v[s]), s = v.back();
        int t = v[rand() % (SZ(v) - 1)];
        vector<int> L, R;
        int x = (Dinic.reset(), Dinic.maxflow(s, t));
        G[s].pb(pii(t, x)), G[t].pb(pii(s, x));
        for (int i : v)
            if (~Dinic.dis[i]) L.pb(i);
            else R.pb(i);
        solve(L), solve(R);
    }
    void build() {
        vector<int> v(n);
        for (int i = 0; i < n; ++i) v[i] = i;
        solve(v);
    }
} ght; // test by BZOJ 4519
MaxFlow &Dinic = ght.Dinic;
```

## 5 String

### 5.1 KMP

```
int F[MAXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}
```

### 5.2 Z-value

```
const int MAXn = 1e5 + 5;
int z[MAXn];
void make_z(string s) {
    int l = 0, r = 0;
    for (int i = 1; i < s.size(); i++) {
        for (z[i] = max(0, min(r - i + 1, z[i - 1]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]];
            z[i]++);
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}
```

### 5.3 Manacher\*

```
int z[MAXN];
int Manacher(string tmp) {
    string s = "&";
    int l = 0, r = 0, x, ans;
    for (char c : tmp) s.pb(c), s.pb('%');
    ans = 0, x = 0;
    for (int i = 1; i < SZ(s); ++i) {
```

```
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] == '%') x = max(x, z[i]);
    ans = x / 2 * 2, x = 0;
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] != '%') x = max(x, z[i]);
    return max(ans, (x - 1) / 2 * 2 + 1);
}
```

## 5.4 Suffix Array

```
struct suffix_array {
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n) {
        return ra[a] != ra[b] || a + k >= n ||
            b + k >= n || ra[a + k] != ra[b + k];
    }
    void radix(int *key, int *it, int *ot, int n) {
        fill_n(box, m, 0);
        for (int i = 0; i < n; ++i) ++box[key[i]];
        partial_sum(box, box + m, box);
        for (int i = n - 1; i >= 0; --i)
            ot[--box[key[it[i]]]] = it[i];
    }
    void make_sa(string s, int n) {
        int k = 1;
        for (int i = 0; i < n; ++i) ra[i] = s[i];
        do {
            iota(tp, tp + k, n - k), iota(sa + k, sa + n, 0);
            radix(ra + k, sa + k, tp + k, n - k);
            radix(ra, tp, sa, n);
            tp[sa[0]] = 0, m = 1;
            for (int i = 1; i < n; ++i) {
                m += not_equ(sa[i], sa[i - 1], k, n);
                tp[sa[i]] = m - 1;
            }
            copy_n(tp, n, ra);
            k *= 2;
        } while (k < n && m != n);
    }
    void make_he(string s, int n) {
        for (int j = 0, k = 0; j < n; ++j) {
            if (ra[j])
                for (; s[j + k] == s[sa[ra[j] - 1] + k]; ++k);
            he[ra[j]] = k, k = max(0, k - 1);
        }
    }
    void make_fa(int n) {
        for (int i = 1; i <= n; ++i) fa[0][i] = he[i - 1];
        for (int j = 1; j < 20; ++j)
            for (int i = 1; i + (1 << j) - 1 <= n; ++i)
                fa[j][i] = min(fa[j - 1][i], fa[j - 1][i + (1 << (j - 1))]);
    }
    inline int ask(int x, int y) {
        int k = Log[y - x + 1];
        return min(fa[k][x], fa[k][y - (1 << k) + 1]);
    }
    inline int lcp(int x, int y) {
        if (x > y) swap(x, y);
        return ask(x + 2, y + 1);
    }
    int sa[MAXN], ra[MAXN], he[MAXN], fa[20][MAXN], Log[
        MAXN];
    void build(const string &s) {
        int n = sz(s);
        fill_n(sa, n, 0), fill_n(ra, n, 0), fill_n(he, n,
            0);
        fill_n(box, n, 0), fill_n(tp, n, 0), m = 256;
        Log[0] = Log[1] = 0;
        for (int i = 2; i <= n; ++i) Log[i] = Log[i >> 1] + 1;
        make_sa(s, sz(s));
        make_he(s, sz(s));
        make_fa(sz(s));
    }
};
```

## 5.5 SAIS\*

```

class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|
    void build(int *s, int n, int m = 128) {
        copy_n(s, n, _s);
        _h[0] = _s[n++] = 0;
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
        SA = _sa + 1;
        H = _h + 1;
    }

private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        r[N], _sa[N * 2], _h[N];
    void mkhei(int n) {
        for (int i = 0; i < n; i++) r[_sa[i]] = i;
        for (int i = 0; i < n; i++)
            if (r[i]) {
                int ans = i > 0 ? max(_h[r[i] - 1] - 1, 0) : 0;
                while (_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                    ans++;
                _h[r[i]] = ans;
            }
    }

    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z) {
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;

#define MAGIC(XD) \
        fill_n(sa, n, 0); \
        copy_n(c, z, x); \
        XD; \
        copy_n(c, z - 1, x + 1); \
        for (int i = 0; i < n; i++) \
            if (sa[i] && !t[sa[i] - 1]) \
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1; \
        copy_n(c, z, x); \
        for (int i = n - 1; i >= 0; i--) \
            if (sa[i] && t[sa[i] - 1]) \
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1; \

        fill_n(c, z, 0);
        for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; i--)
            t[i] = (s[i] == s[i + 1] ? t[i + 1]
                : s[i] < s[i + 1]);
        MAGIC(for (int i = 1; i <= n - 1; i++)
            if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i);
        for (int i = 0; i < n; i++)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                neq = (lst < 0) ||
                    !equal(s + lst,
                        s + lst + p[q[sa[i]] + 1] - sa[i],
                        s + sa[i]);
                ns[q[lst = sa[i]]] = nmzx += neq;
            }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
            nmzx + 1);
        MAGIC(for (int i = nn - 1; i >= 0; i--)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
    }
} sa;

```

## 5.6 Aho-Corasick Automatan

```

const int len = 400000, sigma = 26;
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], pri[len], top;
    int newnode() {
        fill(nx[top], nx[top] + sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(
        string &s) { // return the end_node of string
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
            X = nx[X][c - 'a'];
        }
        return X;
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); t) {
            int R = q.front();
            q.pop(), pri[t++] = R;
            for (int i = 0; i < sigma; ++i)
                if (~nx[R][i]) {
                    int X = nx[R][i], Z = fl[R];
                    for (; Z && !nx[Z][i];) Z = fl[Z];
                    fl[X] = Z ? nx[Z][i] : 1, q.push(X);
                }
        }
    }
    void get_v(string &s) {
        int X = 1;
        fill(cnt, cnt + top, 0);
        for (char c : s) {
            while (X && !nx[X][c - 'a']) X = fl[X];
            X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
        }
        for (int i = top - 2; i > 0; --i)
            cnt[fl[pri[i]]] += cnt[pri[i]];
    }
};

```

## 5.7 Smallest Rotation

```

string mcp(string s) {
    int n = SZ(s), i = 0, j = 1;
    s += s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) ++k;
        if (s[i + k] <= s[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 5.8 De Bruijn sequence\*

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
    }
};

```

```

    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
}
} dbs;

```

## 5.9 SAM

```

const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
} sam;

```

## 5.10 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL * 2];
char pred[MAXL * 2][MAXL * 2];
inline int lcs_length(int r) {
    int i = r + al, j = bl, l = 0;
    while (i > r) {
        char dir = pred[i][j];
        if (dir == LU) l++;
        i += mov[dir][0];
        j += mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i = r, j = 1;
    while (j <= bl && pred[i][j] != LU) j++;
    if (j > bl) return;
}

```

```

    pred[i][j] = L;
    while (i < 2 * al && j <= bl) {
        if (pred[i + 1][j] == U) {
            i++;
            pred[i][j] = L;
        } else if (j < bl && pred[i + 1][j + 1] == LU) {
            i++;
            j++;
            pred[i][j] = L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if (al > bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for (int i = 0; i <= 2 * al; i++) {
        dp[i][0] = 0;
        pred[i][0] = U;
    }
    for (int j = 0; j <= bl; j++) {
        dp[0][j] = 0;
        pred[0][j] = L;
    }
    for (int i = 1; i <= 2 * al; i++) {
        for (int j = 1; j <= bl; j++) {
            if (a[i - 1] == b[j - 1])
                dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            if (dp[i][j - 1] == dp[i][j]) pred[i][j] = L;
            else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
            else pred[i][j] = U;
        }
    }
    // do cyclic lcs
    int clcs = 0;
    for (int i = 0; i < al; i++) {
        clcs = max(clcs, lcs_length(i));
        reroot(i + 1);
    }
    // recover a
    a[al] = '\0';
    return clcs;
}

```

## 6 Math

### 6.1 ax+by=gcd\*

```

pll exgcd(pll a, pll b) {
    if (b == 0) return pll(1, 0);
    else {
        pll p = a / b;
        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

### 6.2 floor and ceil

```

int floor(int a, int b) {
    return a / b - (a % b && a < 0 ^ b < 0);
}
int ceil(int a, int b) {

```

```

    return a/b+(a%b&& a<0^b>0);
}

```

### 6.3 floor sum\*

```

11 floor_sum(11 n, 11 m, 11 a, 11 b) {
    11 ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    11 y_max = (a * n + b) / m, x_max = (y_max * m - b)
        ;
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum^{n-1}_0 floor((a * i + b) / m) in Log(n + m + a
    + b)

```

### 6.4 Miller Rabin\*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(11 a, 11 n) {
    if((a = a % n) == 0) return 1;
    if((n & 1) ^ 1) return n == 2;
    11 tmp = (n - 1) / ((n - 1) & (1 - n));
    11 t = __lg(((n - 1) & (1 - n))), x = 1;
    for(; tmp; tmp >>= 1, a = mul(a, a, n))
        if(tmp & 1) x = mul(x, a, n);
    if(x == 1 || x == n - 1) return 1;
    while(--t)
        if((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

### 6.5 Big number

```

template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x,ss.str();
}
struct bigN:vector<11>{
    const static int base=1000000000,width=log10(base);
    bool negative;
    bigN(const_iterator a,const_iterator b):vector<11>(a,
        b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1,s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1;i>=0;i-=width){
            11 t=0;
            for(int j=max(0,i-width+1);j<=i;++j)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&!back())pop_back();
        if(empty())negative=0;
    }
    void carry(int _base=base){
        for(size_t i=0;i<size();++i){
            if(at(i)>=0&&at(i)<_base)continue;
            if(i+1u==size())push_back(0);
            int r=at(i)%_base;

```

```

            if(r<0)r+=_base;
            at(i+1)+=(at(i)-r)/_base,at(i)=r;
        }
    }
    int abscmp(const bigN &b)const{
        if(size()>b.size())return 1;
        if(size()<b.size())return -1;
        for(int i=int(size())-1;i>=0;--i){
            if(at(i)>b[i])return 1;
            if(at(i)<b[i])return -1;
        }
        return 0;
    }
    int cmp(const bigN &b)const{
        if(negative!=b.negative)return negative?-1:1;
        return negative?-abscmp(b):abscmp(b);
    }
    bool operator<(const bigN&b)const{return cmp(b)<0;}
    bool operator>(const bigN&b)const{return cmp(b)>0;}
    bool operator<=(const bigN&b)const{return cmp(b)<=0;}
    bool operator>=(const bigN&b)const{return cmp(b)>=0;}
    bool operator==(const bigN&b)const{return !cmp(b);}
    bool operator!=(const bigN&b)const{return cmp(b)!=0;}
    bigN abs()const{
        bigN res=*this;
        return res.negative=0, res;
    }
    bigN operator-()const{
        bigN res=*this;
        return res.negative=!negative,res.trim(),res;
    }
    bigN operator+(const bigN &b)const{
        if(negative)return -((-(*this))+(-b));
        if(b.negative)return *this-(-b);
        bigN res=*this;
        if(b.size()>size())res.resize(b.size());
        for(size_t i=0;i<b.size();++i)res[i]+=b[i];
        return res.carry(),res.trim(),res;
    }
    bigN operator-(const bigN &b)const{
        if(negative)return -((-(*this))-(-b));
        if(b.negative)return *this+(-b);
        if(abscmp(b)<0)return -(b-(*this));
        bigN res=*this;
        if(b.size()>size())res.resize(b.size());
        for(size_t i=0;i<b.size();++i)res[i]-=b[i];
        return res.carry(),res.trim(),res;
    }
    bigN operator*(const bigN &b)const{
        bigN res;
        res.negative=negative!=b.negative;
        res.resize(size()+b.size());
        for(size_t i=0;i<size();++i)
            for(size_t j=0;j<b.size();++j)
                if((res[i+j]+at(i)*b[j])>=base){
                    res[i+j+1]+=res[i+j]/base;
                    res[i+j]%=base;
                } // %a k % carry . | . , %
        return res.trim(),res;
    }
    bigN operator/(const bigN &b)const{
        int norm=base/(b.back()+1);
        bigN x=abs()*norm;
        bigN y=b.abs()*norm;
        bigN q,r;
        q.resize(x.size());
        for(int i=int(x.size())-1;i>=0;--i){
            r=r*base+x[i];
            int s1=r.size()<=y.size()?0:r[y.size()];
            int s2=r.size()<y.size()?0:r[y.size()-1];
            int d=(11(base)*s1+s2)/y.back();
            r=r-y*d;
            while(r.negative)r=r+y,--d;
            q[i]=d;
        }
        q.negative=negative!=b.negative;
        return q.trim(),q;
    }
    bigN operator%(const bigN &b)const{
        return *this-(*this/b)*b;
    }
    friend istream& operator>>(istream &ss, bigN &b){

```



```

    string s;
    return ss>>s, b=s, ss;
}
friend ostream& operator<<(ostream &ss, const bigN &b)
{
    if(b.negative)ss<<'-';
    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2;i>=0;--i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
};

```

## 6.6 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0, const ll &d=1):n(n),d(d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-(const fraction &b) const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator+(const fraction &b) const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b) const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b) const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

## 6.7 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

## 6.8 Pollard Rho

```

// does not work when n is prime
ll f(ll x, ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

## 6.9 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1){
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
}

```

```

    }
}
return ans;
}

```

## 6.10 Schreier-Sims Algorithm\*

```

namespace schreier {
int n;
vector<vector<vector<int>>> bkets, binv;
vector<vector<int>> lk;
vector<int> operator*(const vector<int> &a, const
vector<int> &b) {
vector<int> res(SZ(a));
for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
return res;
}
vector<int> inv(const vector<int> &a) {
vector<int> res(SZ(a));
for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
return res;
}
int filter(const vector<int> &g, bool add = true) {
n = SZ(bkets);
vector<int> p = g;
for (int i = 0; i < n; ++i) {
assert(p[i] >= 0 && p[i] < SZ(lk[i]));
if (lk[i][p[i]] == -1) {
if (add) {
bkets[i].pb(p);
binv[i].pb(inv(p));
lk[i][p[i]] = SZ(bkets[i]) - 1;
}
return i;
}
}
p = p * binv[i][lk[i][p[i]]];
}
return -1;
}
bool inside(const vector<int> &g) { return filter(g,
false) == -1; }
void solve(const vector<vector<int>> &gen, int _n) {
n = _n;
bkets.clear(), bkets.resize(n);
binv.clear(), binv.resize(n);
lk.clear(), lk.resize(n);
vector<int> iden(n);
iota(iden.begin(), iden.end(), 0);
for (int i = 0; i < n; ++i) {
lk[i].resize(n, -1);
bkets[i].pb(iden);
binv[i].pb(iden);
lk[i][i] = 0;
}
for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
queue<pair<pii, pii>> upd;
for (int i = 0; i < n; ++i)
for (int j = i; j < n; ++j)
for (int k = 0; k < SZ(bkets[i]); ++k)
for (int l = 0; l < SZ(bkets[j]); ++l)
upd.emplace(pii(i, k), pii(j, l));
while (!upd.empty()) {
auto a = upd.front().X;
auto b = upd.front().Y;
upd.pop();
int res = filter(bkets[a.X][a.Y] * bkets[b.X][b.Y]);
if (res == -1) continue;
pii pr = pii(res, SZ(bkets[res]) - 1);
for (int i = 0; i < n; ++i)
for (int j = 0; j < SZ(bkets[i]); ++j) {
if (i <= res) upd.emplace(pii(i, j), pr);
if (res <= i) upd.emplace(pr, pii(i, j));
}
}
}
long long size() {
long long res = 1;

```

```

for (int i = 0; i < n; ++i) res = res * SZ(bkets[i]);
return res;
}
}

```

## 6.11 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
LL g = __gcd(m1, m2);
if ((x2 - x1) % g) return -1; // no sol
m1 /= g; m2 /= g;
pair<LL, LL> p = gcd(m1, m2);
LL lcm = m1 * m2 * g;
LL res = p.first * (x2 - x1) * m1 + x1;
return (res % lcm + lcm) % lcm;
}

```

## 6.12 QuadraticResidue

```

int Jacobi(int a, int m) {
int s = 1;
for (; m > 1; ) {
a %= m;
if (a == 0) return 0;
const int r = __builtin_ctz(a);
if ((r & 1) && ((m + 2) & 4)) s = -s;
a >>= r;
if (a & m & 2) s = -s;
swap(a, m);
}
return s;
}
int QuadraticResidue(int a, int p) {
if (p == 2) return a & 1;
const int jc = Jacobi(a, p);
if (jc == 0) return 0;
if (jc == -1) return -1;
int b, d;
for (; ; ) {
b = rand() % p;
d = (1LL * b * b + p - a) % p;
if (Jacobi(d, p) == -1) break;
}
int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
for (int e = (1LL + p) >> 1; e; e >>= 1) {
if (e & 1) {
tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
p)) % p;
g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
g0 = tmp;
}
tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 %
p)) % p;
f1 = (2LL * f0 * f1) % p;
f0 = tmp;
}
return g0;
}

```

## 6.13 PiCount

```

int64_t PrimeCount(int64_t n) {
if (n <= 1) return 0;
const int v = sqrt(n);
vector<int> smalls(v + 1);
for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
int s = (v + 1) / 2;
vector<int> roughs(s);
for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
vector<int64_t> larges(s);
for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i +
1) + 1) / 2;
vector<bool> skip(v + 1);
int pc = 0;
for (int p = 3; p <= v; ++p) {

```

```

if (smalls[p] > smalls[p - 1]) {
    int q = p * p;
    pc++;
    if (1LL * q * q > n) break;
    skip[p] = true;
    for (int i = q; i <= v; i += 2 * p) skip[i] = true;
    int ns = 0;
    for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        int64_t d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
    }
    s = ns;
    for (int j = v / p; j >= p; --j) {
        int c = smalls[j] - pc;
        for (int i = j * p, e = min(i + p, v + 1); i < e; ++i) smalls[i] -= c;
    }
}
for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
}
return larges[0];
}

```

## 6.14 Primes

```

/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771 1001010013
1000512343 987654361 999991231 999888733 98789101
987777733 999991921 1010101333 1010102101
100000000039 100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/
bool sieve[mxn];
vector<int> prime;
void linear_sieve(){
    for(int i=2;i<mxn;++i){
        if(!sieve[i]) prime.pb(i);
        for(int p:prime){
            if(i*p>mxn) break;
            sieve[i*p]=1;
            if(i%p==0) break;
        }
    }
}

```

## 6.15 PrimeFactorization

```

vector<pll> f(ll x){
    vector<pll> factors;
    for(ll i = 2; i * i <= x; i++){
        if(x % i == 0){
            ll a = 0;
            for(; x % i == 0; x /= i, a++);
            factors.pb(make_pair(i, a));
        }
        if(x > 1) factors.push_back(make_pair(x, 1));
    }
    return factors;
}

```

## 6.16 Theorem

### 6.16.1 Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### 6.16.2 Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### 6.16.3 Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

## 7 Polynomial

### 7.1 Fast Fourier Transform

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans(val_t *a, int n, bool inv = false); // see NTT;
    // remember to replace LL with val_t
};

```

### 7.2 Number Theory Transform

```

//(2^16)+1, 65537, 3
//7*(2^26)+1, 469762049, 3
//7*17*(2^23)+1, 998244353, 3
//619*202879*(2^3)+1, 1004656809, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { //0
        <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {

```

```

    for (int j = i, x = 0; j < i + d1; ++j, x += dx)
    {
        LL tmp = a[j + d1] * w[x] % P;
        if ((a[j + d1] = a[j] - tmp) < 0) a[j + d1]
            += P;
        if ((a[j] += tmp) >= P) a[j] -= P;
    }
}
if (inv) {
    reverse(a + 1, a + n);
    LL invn = minv(n);
    for (int i = 0; i < n; ++i) a[i] = a[i] * invn %
        P;
}
};

```

### 7.3 Fast Walsh Transform\*

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= L * 2)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}

```

### 7.4 Polynomial Operation

```

#define fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly : vector<LL> { // coefficients in [0, P)
    using vector<LL>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int _n) : vector<LL>(_n) {
        copy_n(p.data(), min(p.n(), _n), data());
    }
    Poly& irev() { return reverse(data(), data() + n()),
        *this; }
    Poly& isz(int _n) { return resize(_n), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if (((*this)[i] += rhs[i]) >= P) (*this)
            [i] -= P;
        return *this;
    }
    Poly& imul(LL k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {

```

```

    int _n = 1;
    while (_n < n() + rhs.n() - 1) _n <= 1;
    Poly X(*this, _n), Y(rhs, _n);
    ntt(X.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) X[i] = X[i] * Y[i] % P;
    ntt(X.data(), _n, true);
    return X.isz(n() + rhs.n() - 1);
}
Poly Inv() const { // (*this)[0] != 0
    if (n() == 1) return {ntt.minv((*this)[0])};
    int _n = 1;
    while (_n < n() * 2) _n <= 1;
    Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(_n);
    Poly Y(*this, _n);
    ntt(Xi.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) {
        Xi[i] *= (2 - Xi[i] * Y[i]) % P;
        if ((Xi[i] %= P) < 0) Xi[i] += P;
    }
    ntt(Xi.data(), _n, true);
    return Xi.isz(n());
}
Poly Sqrt() const { // Jacobi((*this)[0], P) = 1
    if (n() == 1) return {QuadraticResidue((*this)[0],
        P)};
    Poly X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
    return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 +
        1);
}
pair<Poly, Poly> DivMod(const Poly &rhs) const { // (
    rhs.back() != 0
    if (n() < rhs.n()) return {{0}, *this};
    const int _n = n() - rhs.n() + 1;
    Poly X(rhs); X.irev().isz(_n);
    Poly Y(*this); Y.irev().isz(_n);
    Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = (i + 1) * (*this)[i + 1] %
        P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * (*this)[i]
        % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<LL> _eval(const vector<LL> &x, const vector<
    Poly> &up) const {
    const int _n = (int)x.size();
    if (!_n) return {};
    vector<Poly> down(_n * 2);
    down[1] = DivMod(up[1]).second;
    fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i]).
        second;
    /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev
        ()._tmul(_n, *this);
    fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
        1, down[i / 2]); */
    vector<LL> y(_n);
    fi(0, _n) y[i] = down[_n + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<LL> &x) {
    const int _n = (int)x.size();
    vector<Poly> up(_n * 2);
    fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = _n - 1; i > 0; --i) up[i] = up[i * 2].
        Mul(up[i * 2 + 1]);
    return up;
}
vector<LL> Eval(const vector<LL> &x) const {

```

```

    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector<LL> &x, const
vector<LL> &y) {
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    vector<LL> z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, _n) down[_n + i] = {z[i]};
    for (int i = _n - 1; i > 0; --i) down[i] = down[i *
2].Mul(up[i * 2 + 1]).iadd(down[i * 2 + 1].Mul
(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i]
+= P;
    return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    LL nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % P;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= n()) return Poly(n());
        nk2 %= P - 1;
    }
    if (!nk && !nk2) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * nk2);
    LL x0 = X[0];
    return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
.imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
}
static LL LinearRecursion(const vector<LL> &a, const
vector<LL> &coef, LL n) { // a_n = \sum c_j a_(n-
j)
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    LL ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 8 Geometry

### 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{pdd O; double R;};
const double eps=1e-8;
pdd operator+(const pdd &a, const pdd &b)
{ return pdd(a.X + b.X, a.Y + b.Y);}
pdd operator-(const pdd &a, const pdd &b)
{ return pdd(a.X - b.X, a.Y - b.Y);}
pdd operator*(const pdd &a, const double &b)
{ return pdd(a.X * b, a.Y * b);}
pdd operator/(const pdd &a, const double &b)
{ return pdd(a.X / b, a.Y / b);}

```

```

double dot(const pdd &a, const pdd &b)
{ return a.X * b.X + a.Y * b.Y;}
double cross(const pdd &a, const pdd &b)
{ return a.X * b.Y - a.Y * b.X;}
double abs2(const pdd &a)
{ return dot(a, a);}
double abs(const pdd &a)
{ return sqrt(dot(a, a));}
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;}
int ori(const pdd &a, const pdd &b, const pdd &c)
{ return sign(cross(b - a, c - a));}
bool collinearity(const pdd &p1, const pdd &p2, const
pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps;}
bool btw(const pdd &p1, const pdd &p2, const pdd &p3) {
    if(!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1, const pdd &p2, const
pdd &p3, const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if(a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2, const pdd &
p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X);}
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1));}

```

### 8.2 Convex hull\*

```

void hull(vector<p11> &dots) {
    sort(dots.begin(), dots.end());
    vector<p11> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(
dots[i++]))
            while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.
back(), dots[i]) <= 0)
                ans.pop_back();
            ans.pop_back(), ans.swap(dots);
}

```

### 8.3 External bisector

```

pdd external_bisector(pdd p1, pdd p2, pdd p3){//213
    pdd L1=p2-p1, L2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}

```

### 8.4 Heart

```

pdd excenter(pdd p0, pdd p1, pdd p2, double &radius){
    p1=p1-p0, p2=p2-p0;
    double x1=p1.X, y1=p1.Y, x2=p2.X, y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center), center+p0;
}
pdd incenter(pdd p1, pdd p2, pdd p3, double &radius){
    double a=abs(p2-p1), b=abs(p3-p1), c=abs(p3-p2);
}

```

```

double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
pdd l1=external_bisector(p1,p2,p3),l2=
    external_bisector(p2,p1,p3);
return radius=area/s,intersect(p1,p1+l1,p2,p2+l2),
}

pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+l1,p2,p2+l2);
}

pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd l1=p3-p2,l2=p3-p1;
    swap(l1.X,l1.Y),l1.X*=-1;
    swap(l2.X,l2.Y),l2.X*=-1;
    return intersect(p1,p1+l1,p2,p2+l2);
}

```

## 8.5 Minimum Circle Cover\*

```

pdd Minimum_Circle_Cover(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter(dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

## 8.6 Polar Angle Sort\*

```

pdd center;//sort base
int Quadrant(pdd a) {
    if(a.X > 0 && a.Y >= 0) return 1;
    if(a.X <= 0 && a.Y > 0) return 2;
    if(a.X < 0 && a.Y <= 0) return 3;
    if(a.X >= 0 && a.Y < 0) return 4;
}

bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}

bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if(fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}

```

## 8.7 Intersection of two circles\*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
}

```

```

pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
    * r1) / (2 * d2));
double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
    r2 - d) * (-r1 + r2 + d));
pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
    ;
p1 = u + v, p2 = u - v;
return 1;
}

```

## 8.8 Intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
            (r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}

double area_poly_circle(const vector<pdd> poly,const
    pdd &o,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-o,poly[(i+1)%SZ(poly)]-o,r)*ori(o,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

## 8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}

```

## 8.10 point in circle

```

// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const p11& p1, const p11& p2, const p11& p3,
    const p11& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
        p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
        p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
        p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
        sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
        sqr(p4.Y);
    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
        sqr(p4.Y);
}

```



```

__int128 det = (__int128)-u13 * u22 * u31 + (
    __int128)u12 * u23 * u31 + (__int128)u13 * u21
    * u32 - (__int128)u11 * u23 * u32 - (__int128)
    u12 * u21 * u33 + (__int128)u11 * u22 * u33;
return det > eps;
}

```

## 8.11 Half plane intersection

```

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
    return cross(l0.Y - l0.X, p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i, int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[j].X)<0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while(SZ(dq)>=2&&!isin(fin[i], dq[SZ(dq)-2], dq.back()))
            dq.pop_back();
        while(SZ(dq)>=2&&!isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while(SZ(dq)>=3&&!isin(dq[0], dq[SZ(dq)-2], dq.back()))
        dq.pop_back();
    while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}

```

## 8.12 CircleCover\*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add
            (_c){}
        bool operator<(const Teve &a)const
        {return ang < a.ang;}
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x)
    {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)
    {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
    bool contain(int i, int j) {

```

```

/* c[j] is non-strictly in c[i]. */
return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
    c[j].R) == 0 && i < j)) && contain(c[i], c[j],
    -1);
}
void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjunct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
        int E = 0, cnt = 1;
        for(int j = 0; j < C; ++j)
            if(j != i && overlap[j][i])
                ++cnt;
        for(int j = 0; j < C; ++j)
            if(i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i]
                    ].O.X);
                double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i]
                    ].O.X);
                eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa
                    , A, -1);
                if(B > A) ++cnt;
            }
        if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else{
            sort(eve, eve + E);
            eve[E] = eve[0];
            for(int j = 0; j < E; ++j){
                cnt += eve[j].add;
                Area[cnt] += cross(eve[j].p, eve[j + 1].p) *
                    .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta - sin(theta)) * c[i].R *
                    c[i].R * .5;
            }
        }
    }
}
};

```

## 8.13 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(const Point &a)
{ return sqrt(dot(a, a)); }
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }

```

## 8.14 Convexhull3D\*

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];

```

```

double dblcmp(Point &p, face &f)
{return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a
]);}
int g[N][N], num, n;
Point P[N];
void deal(int p, int a, int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
        if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
    } else
        add.a = b, add.b = a, add.c = p, add.ok = 1, g[
            p][b] = g[a][p] = g[b][a] = num, F[num++] =
            add;
}
void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
        now].b), deal(p, F[now].a, F[now].c);
}
bool same(int s, int t) {
    Point &a = P[F[s].a];
    Point &b = P[F[s].b];
    Point &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps &&
        fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
            volume(a, b, c, P[F[t].c])) < eps;
}
void init(int _n) {n = _n, num = 0;}
void solve() {
    face add;
    num = 0;
    if (n < 4) return;
    if ([&]() {
        for (int i = 1; i < n; ++i)
            if (abs(P[0] - P[i]) > eps)
                return swap(P[1], P[i]), 0;
        return 1;
    }()) || [&]() {
        for (int i = 2; i < n; ++i)
            if (abs(cross3(P[i], P[0], P[1])) > eps)
                return swap(P[2], P[i]), 0;
        return 1;
    }()) || [&]() {
        for (int i = 3; i < n; ++i)
            if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P
                [0] - P[i])) > eps)
                return swap(P[3], P[i]), 0;
        return 1;
    }()) return;
    for (int i = 0; i < 4; ++i) {
        add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
            (i + 3) % 4, add.ok = true;
        if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
        g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
            a] = num;
        F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
        for (int j = 0; j < num; ++j)
            if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                dfs(i, j);
                break;
            }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
        if (F[i].ok) F[num++] = F[i];
}
double get_area() {
    double res = 0.0;
    if (n == 3)
        return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
        res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
}
double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
        res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b
            ], P[F[i].c]);
    return fabs(res / 6.0);
}

```

```

}
int triangle() {return num;}
int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag
        , flag = 1)
        for (int j = 0; j < i && flag; ++j)
            flag &= !same(i, j);
    return res;
}
Point getcent() {
    Point ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                i].c];
            t2 = volume(temp, p1, p2, p3) / 6.0;
            if (t2 > 0)
                ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                ans.y += (p1.y + p2.y + p3.y + temp.y) *
                    t2, ans.z += (p1.z + p2.z + p3.z + temp.z
                        ) * t2, v += t2;
        }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
        );
    return ans;
}
double pointmindis(Point p) {
    double rt = 99999999;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                i].c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
                z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
                x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
                y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z
                );
            double temp = fabs(a * p.x + b * p.y + c * p.z
                + d) / sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
};

```

## 8.15 DelaunayTriangulation\*

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side)
        {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {

```

```

    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
}
bool has_chd() const { return chd[0] != 0; }
int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
}
bool contains(p11 const& q) const {
    for (int i = 0; i < 3; ++i)
        if (ori(p[i], p[(i + 1) % 3], q) < 0)
            return 0;
    return 1;
}
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri -> edge[a.side] = b;
    if(b.tri) b.tri -> edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
                    new(tris++) Tri(p11(-inf, -inf), p11(inf +
                    inf, -inf), p11(-inf, inf + inf));
    }
    Tri* find(p11 p) { return find(the_root, p); }
    void add_point(const p11 &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const p11 &p) {
        while (1) {
            if (!root -> has_chd())
                return root;
            for (int i = 0; i < 3 && root -> chd[i]; ++i)
                if (root -> chd[i] -> contains(p)) {
                    root = root -> chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, p11 const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root -> p[i], root
            -> p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1)
            );
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root -> edge[(i + 2) %
            3]);
        for (int i = 0; i < 3; ++i)
            root -> chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri -> edge[pi].tri;
        int pj = tri -> edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri -> p[0], tri -> p[1], tri -> p
        [2], trj -> p[pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri(tri -> p[(pi + 1) %
        3], trj -> p[pj], tri -> p[pi]);
        Tri* trl = new(tris++) Tri(trj -> p[(pj + 1) %
        3], tri -> p[pi], trj -> p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri -> chd[0] = trk; tri -> chd[1] = trl; tri
        -> chd[2] = 0;
        trj -> chd[0] = trk; trj -> chd[1] = trl; trj
        -> chd[2] = 0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
}

```

```

};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now -> has_chd())
        return triang.push_back(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now -> chd[i]);
}
void build(int n, p11* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

## 8.16 Triangulation Voronoi\*

```

vector<Line> ls[N];
p11 arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1)
        % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<p11, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = p11(oarr[i].X, oarr[i].Y), mp[arr[i]]
        = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t -> p[i]) != mp.end())
                p.pb(mp[t -> p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}

```

## 8.17 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
}

```

```

double h = sqrt( max( 0.0 , 1.0 - c * c ) );
for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.0 + n * c1.R;
    Pt p2 = c2.0 + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
        p2 = p1 + perp( c2.0 - c1.0 );
    ret.push_back( { p1 , p2 } );
}
return ret;
}

```

## 8.18 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots){
    vector<pll> hull;
    const double INF=1e18,qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0,Min=INF,deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0,u=1,r=1,l;i<n;++i){
        pll nw=hull[i+1]-hull[i];
        while(cross(nw,hull[u+1]-hull[i])>cross(nw,hull[u]-hull[i]))
            u=(u+1)%n;
        while(dot(nw,hull[r+1]-hull[i])>dot(nw,hull[r]-hull[i]))
            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw,hull[l+1]-hull[i])<dot(nw,hull[l]-hull[i]))
            l=(l+1)%n;
        Min=min(Min,(double)(dot(nw,hull[r]-hull[i])-dot(nw,hull[l]-hull[i]))*cross(nw,hull[u]-hull[i])/abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l],hull[u]-hull[i])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max,(double)abs(hull[r]-hull[l])*abs(hull[u]-hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min,Max);
}

```

## 8.19 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.20 Minkowski Sum\*

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

## 8.21 RotatingSweepLine

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps);
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1) / 2);
    int m = 0;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            line[m++] = pii(i, j);
    sort(ALL(line), [&](const pii &a, const pii &b)->
        bool {
            if (ps[a.X].X == ps[a.Y].X)
                return 0;
            if (ps[b.X].X == ps[b.Y].X)
                return 1;
            return (double)(ps[a.X].Y - ps[a.Y].Y) / (ps[a.X].X - ps[a.Y].X) < (double)(ps[b.X].Y - ps[b.Y].Y) / (ps[b.X].X - ps[b.Y].X);
        });
    iota(id, id + n, 0);
    sort(ALL(id), [&](const int &a, const int &b){ return ps[a] < ps[b]; });
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // meow
        tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}

```

## 8.22 Simpson

```

const double eps = 1e-9;
double F(double x){
}
double Simpson(double l, double r){
    return (r-l)*(F(r)+F(l)+4.0*F((r+l)*0.5))/6.0;
}
double simpson(double l, double r, double cur){
    double mid = (l+r)*0.5;
    double L = Simpson(l, mid), R = Simpson(mid, r);
    if(fabs(cur-L-R)<=eps)
        return L+R;
    else
        return simpson(l, mid, L)+simpson(mid, r, R);
}
double simpson_ada(double l, double r, double cur, double e){
    double mid = (l+r)*0.5;
    double L = Simpson(l, mid), R = Simpson(mid, r);
    if(fabs(cur-L-R)<=e)
        return L+R;
    else
        return simpson(l, mid, L, e*0.5)+simpson(mid, r, R, e*0.5);
}

```

## 9 Else

### 9.1 Mo's Alogrithm(With modification)

```
struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L++]);
        ans[q.id]=cur_ans;
    }
}
```

### 9.2 Mo's Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[__lg(MAXN*2)+1][MAXN*2],bln[MAXN],spt;
bitset<MAXN> inset;
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
    deep[u]=d,sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,out[u]=dft++;
}
int lca(int u,int v){
    if(bln[u]>bln[v]) swap(u,v);
    int t=__lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
    for(int i=1,x=2;x<2*n;++i,x<<=1)
        for(int j=0;j+x<2*n;++j)
```

```
        if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
            sp[i][j]=sp[i-1][j];
        else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L,q.R);
        if(c==q.L||c==q.R)
            q.L=out[c==q.L?q.R:q.L],q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c,q.L=out[q.L],q.R=in[q.R];
        else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0,R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
```

### 9.3 DynamicConvexTrick\*

```
// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 &&
        a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x -> p = kInf; return 0; }
        if (x -> a == y -> a) x -> p = x -> b > y -> b
            ? kInf : -kInf;
        else x -> p = Div(y -> b - x -> b, x -> a - y
            -> a);
        return x -> p >= y -> p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x) -> p >= y ->
            p) isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

### 9.4 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.