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#### Travail de maturité

### Les algorithmes quantiques Ou une théorie d'optimisation





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Première partie

Préambule

# Introduction

# Notions théoriques

- 2.1 Informatique
- 2.2 Physique

# Deuxième partie Les notions de base

# Un ordinateur classique

- 3.1 Logique
- 3.2 Hardware

# Un qubit

- 4.1 Superposition d'états
- 4.2 Opérations sur un qubit
- 4.3 Mesure

# Système à plusieurs qubits

- 5.1 Description du système
- 5.2 Les portes
- 5.3 Superposition
- 5.3.1 Superdense Coding
- 5.3.2 Quantum Teleportation

# Un ordinateur quantique

- 6.1 Hardware
- 6.2 Universalité

# Troisième partie Exemples d'algorithmes

# Algorithme de Deutsch-Jozsa

# Quelques protocoles

- 8.1 Phase kickback
- 8.2 Quantum Fourier Transform
- 8.3 Quantum Phase Estimation

# Algorithme de Shor

- 9.1 Principe
- 9.2 Implémentation simple
- 9.2.1 Simulation
- 9.2.2 Hardware réel
- 9.3 Application à un problème concret

Cryptographie : distribution de clés

## Algorithme de Grover

- 11.1 Principe
- 11.2 Comparaison avec une implémentation classique
- 11.2.1 2 entrées
- 11.2.2 3 entrées
- 11.2.3 4 entrées
- 11.2.4 Différence de complexité
- 11.3 Résolution d'un sudoku

Iterative Phase Estimatimation et optimiser l'algorithme de Grover

Modélisation d'un système physique

Quatrième partie

Et après...

Technologies de hardware

Sur des machines à court terme

Sur le long terme

# Conclusion

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