Analyse des données

January 13, 2024

```
[ ]: import numpy as np
     import pandas as pd
     import scienceplots
     import matplotlib.pyplot as plt

     import sympy as sp
     import scipy as sc

     from numba import jit

     plt.style.use(['science', 'notebook', 'grid'])
```

1 Études de filtres "passe-bas" et "passe-haut"

1.1 Circuit RL et filtre "passe-bas"

Le but est de faire 10 mesures entre 10 et 10'000 [Hz] (autour de 10, 20, 50, 100, ..., 5'000, 10'000). Ces mesures concernent la tension dans le circuit et l'interval de temps permettant de caractériser le déphasage. Pour le comparer avec un modèle, la tension d'entrée n'ayant pas été prise, nous allons faire une courbe de tendance et voir si cela est cohérent.

1.1.1 Mesure avec R = 5 [Ω]

Pour calculer la phase, on sait que $f = T^{-1}$ et que $\frac{\Delta t}{T} = \frac{\theta}{2\pi} \Rightarrow \theta = \Delta t \cdot 2\pi \cdot f$.

Pour l'inductance : $L = \tan\theta \cdot \frac{R}{2\pi \cdot f}$.

```
[ ]: res = 5 # Ohm
     freq = np.array([10.03,20.82,50.43,110.3,213.3,509.2,1072,2160,5035,10150]) # Hz
     tension = np.array([5.6,5.64,5.56,5.4,4.96,3.48,2,1.04,0.464,0.226]) # V
     diff_t = np.array([0,0.4,0.3,0.36,0.4,0.28,0.19,0.11,0.046,0.024])/1e3 # s

     phase = diff_t*2*np.pi*freq # rad
     inductance = res/(2*np.pi*freq) * np.tan(phase) # H

     display(pd.DataFrame({'freq [Hz]':freq,'tension [V]':tension,'del_t [ms]':
      ↪diff_t*1e3,'phase':np.round(phase, 3),'inductance [mH]':np.
      ↪round(inductance*1e3, 3)}))
```

```
print('L = {:.3f} mH'.format(np.mean(inductance[1:])*1e3))
print('Écart-type = {:.3f} mH'.format(np.std(inductance[1:])*1e3))
```

```
   freq [Hz]  tension [V]  del_t [ms]  phase  inductance [mH]
0      10.03        5.600       0.000  0.000            0.000
1      20.82        5.640       0.400  0.052            2.002
2      50.43        5.560       0.300  0.095            1.505
3     110.30        5.400       0.360  0.249            1.838
4     213.30        4.960       0.400  0.536            2.217
5     509.20        3.480       0.280  0.896            1.953
6    1072.00        2.000       0.190  1.280            2.478
7    2160.00        1.040       0.110  1.493            4.719
8    5035.00        0.464       0.046  1.455            1.362
9   10150.00        0.226       0.024  1.531            1.949
```

```
L = 2.225 mH
Écart-type = 0.937 mH
```

On note des mesures assez peu précise car l'inductance déduite est de 2.225 [mH] alors que le fabriquant en spécifie 3.

Graphes Premier graphe, l'intensité en fonction de la fréquence.

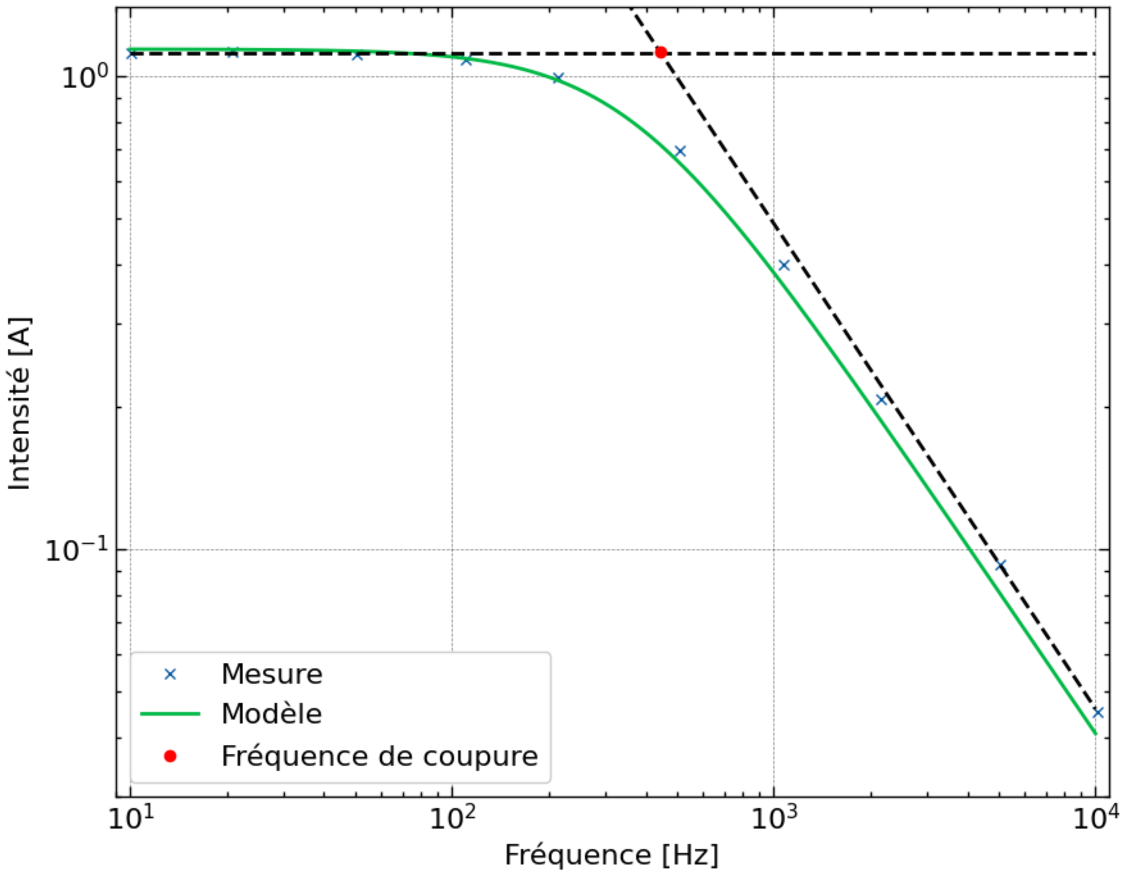
De la tension mesurée, on a l'intensité par $I = \frac{U}{R}$.

Pour celle modélisée, on va faire une courbe de tendance par la formule $I = \frac{U_{in}}{\sqrt{R^2+(2\pi \cdot f \cdot L)^2}}$.

[ ]:
```
inductance_th = np.mean(inductance[1:]) # H

plt.figure(figsize=(10,8))

plt.plot(freq, tension/res,'x',label='Mesure')

intensite_th = lambda f, U: U/np.sqrt(res**2+(2*np.pi*f * inductance_th)**2)

popt, _ = sc.optimize.curve_fit(intensite_th, freq, tension/res)

print("U_in = {:.3f} V".format(popt[0]))
plt.plot(np.linspace(10,10000,10000), intensite_th(np.linspace(10,10000,10000),
 ↪*popt), label='Modèle')

# asymptote horizontale
plt.hlines(tension[0]/res, 10, 10000, colors='k', linestyles='dashed')

# asymptote oblique
m = (np.log10(tension[-2]/res) - np.log10(tension[-1]/res)) / (np.
 ↪log10(freq[-2])- np.log10(freq[-1]))
h = np.log10(tension[-1]/res) - m * np.log10(freq[-1])
```

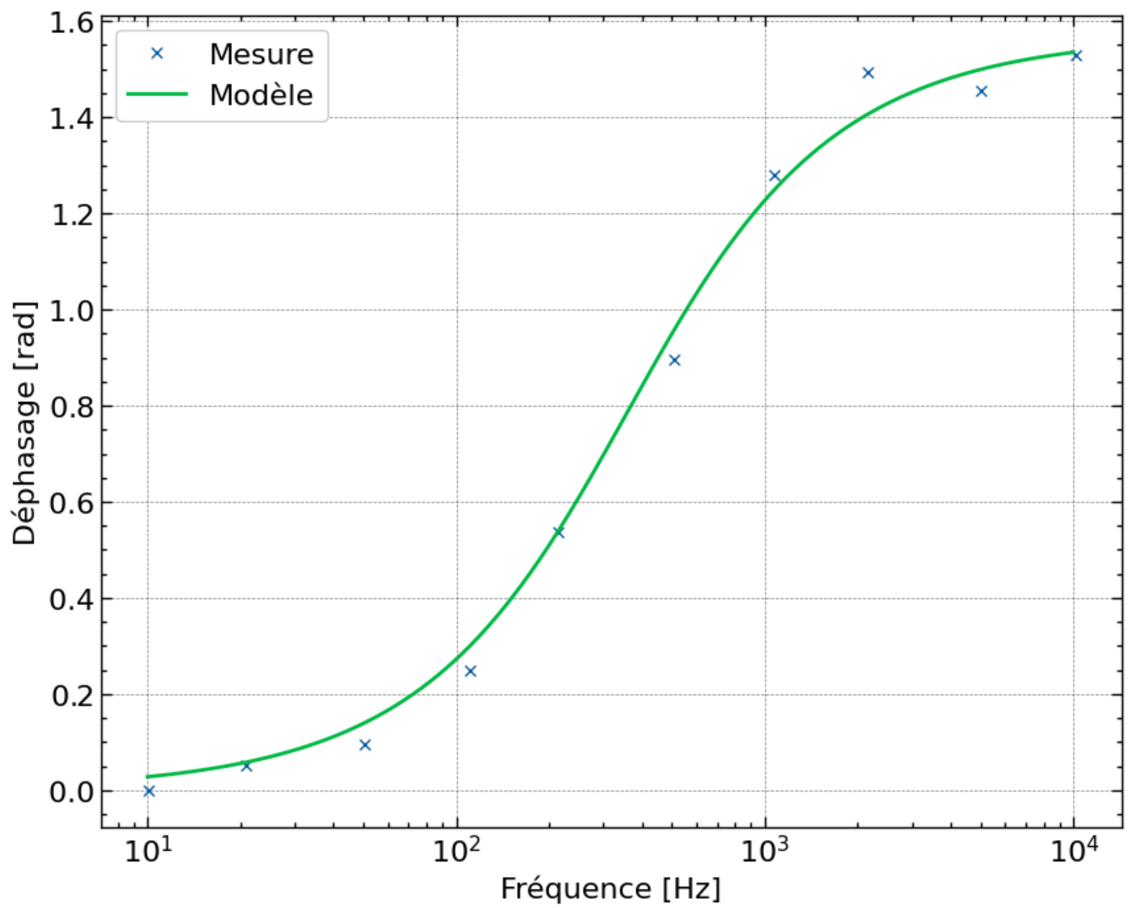
```python
line_x = 10**np.linspace(0, 4, 1000)
line_y = 10**(m*np.linspace(0, 4, 1000) + h)
plt.plot(line_x, line_y, 'k--')

# intersection
for i in range(len(line_x)-1):
    if line_y[i] < tension[0]/res < line_y[i+1] or line_y[i] > tension[0]/res >
 ↪line_y[i+1]:
        intersection = [line_x[i], line_y[i]]

print("f_c = {:.3f} Hz".format(intersection[0]))
plt.plot(intersection[0], intersection[1], 'ro', label='Fréquence de coupure')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Intensité [A]')
plt.legend(loc='lower left')

plt.xscale('log')
plt.yscale('log')

plt.xlim(9,11000)
plt.ylim(0.03,1.4)

plt.show()
```

```
U_in = 5.696 V
f_c = 443.248 Hz
```



Pour le déphasage, on graphe ce qui a été calculé par la mesure et ce qui va être calculé par la modélitation $\tan(\theta) = \frac{2\pi \cdot f \cdot L}{R}$.

[ ]:
```
plt.figure(figsize=(10,8))

plt.plot(freq, phase,'x',label='Mesure')

phase_th = lambda f: np.arctan(2*np.pi*f*inductance_th/res)

plt.plot(np.linspace(10,10000,10000), phase_th(np.linspace(10,10000,10000)),␣
 ↪label='Modèle')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Déphasage [rad]')
plt.legend(loc='upper left')

plt.xscale('log')

plt.show()
```



1.1.2 De même avec R = 50 $[\Omega]$

[ ]:
```
res = 50 # Ohm
freq = np.array([10.32,21.03,49.58,119,217.8,505.3,1187,2170,5045,10680]) # Hz
tension = np.array([8.32,8.32,8.32,8.32,8.16,8,7.68,6.52,4,2.06]) # V
diff_t = np.array([0,0.4,0.1,0.08,0.08,0.05,0.056,0.048,0.035,0.02])/1e3 # s

phase = diff_t*2*np.pi*freq # rad
inductance = res/(2*np.pi*freq) * np.tan(phase) # H

display(pd.DataFrame({'freq [Hz]':freq,'tension [V]':tension,'del_t [ms]':
  ↪diff_t*1e3,'phase':np.round(phase, 3),'inductance [mH]':np.
  ↪round(inductance*1e3, 3)}))

print('L = {:.3f} mH'.format(np.mean(inductance[2:])*1e3))
print('Écart-type = {:.3f} mH'.format(np.std(inductance[2:])*1e3))
```

	freq [Hz]	tension [V]	del_t [ms]	phase	inductance [mH]
0	10.32	8.32	0.000	0.000	0.000

```
1      21.03          8.32        0.400  0.053            20.019
2      49.58          8.32        0.100  0.031             5.002
3     119.00          8.32        0.080  0.060             4.005
4     217.80          8.16        0.080  0.109             4.016
5     505.30          8.00        0.050  0.159             2.521
6    1187.00          7.68        0.056  0.418             2.975
7    2170.00          6.52        0.048  0.654             2.814
8    5045.00          4.00        0.035  1.109             3.173
9   10680.00          2.06        0.020  1.342             3.201
```

```
L = 3.463 mH
Écart-type = 0.764 mH
```

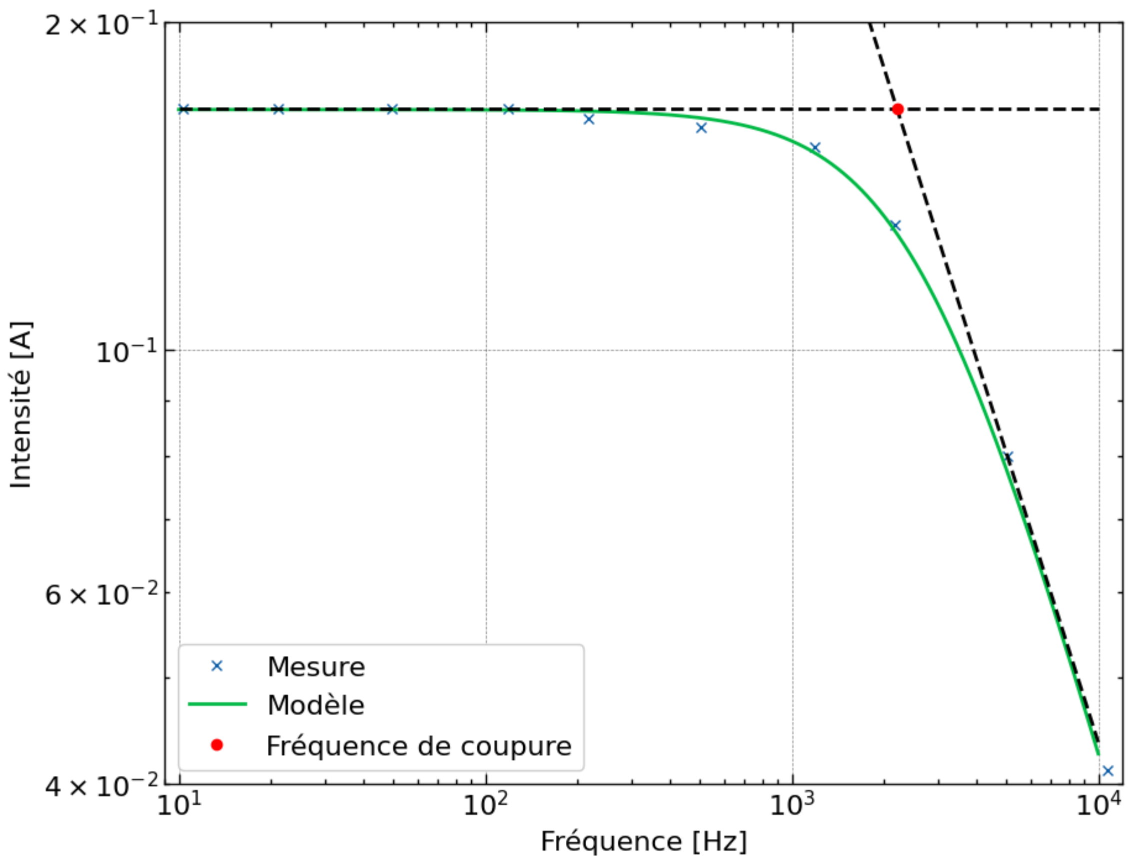
On peut noter que ce coup-ci l'inductance est un peut trop élevée, mais que sur les deux volée de mesure on a une moyenne autour de celle du fabricant.

Graphes

```
[ ]: inductance_th = 3e-3 # H

plt.figure(figsize=(10,8))

plt.plot(freq, tension/res,'x',label='Mesure')

intensite_th = lambda f, U: U/np.sqrt(res**2+(2*np.pi*f * inductance_th)**2)

popt, _ = sc.optimize.curve_fit(intensite_th, freq, tension/res)

print("U_in = {:.3f} V".format(popt[0]))
plt.plot(np.linspace(10,10000,10000), intensite_th(np.linspace(10,10000,10000),
  *popt), label='Modèle')

# asymptote horizontale
plt.hlines(tension[0]/res, 10, 10000, colors='k', linestyles='dashed')

# asymptote oblique
m = (np.log10(tension[-2]/res) - np.log10(tension[-1]/res)) / (np.
  log10(freq[-2])- np.log10(freq[-1]))
h = np.log10(tension[-1]/res) - m * np.log10(freq[-1])
line_x = 10**np.linspace(0, 4, 1000)
line_y = 10**(m*np.linspace(0, 4, 1000) + h)
plt.plot(line_x, line_y, 'k--')

# intersection
for i in range(len(line_x)-1):
    if line_y[i] < tension[0]/res < line_y[i+1] or line_y[i] > tension[0]/res >
  line_y[i+1]:
        intersection = [line_x[i], line_y[i]]
```

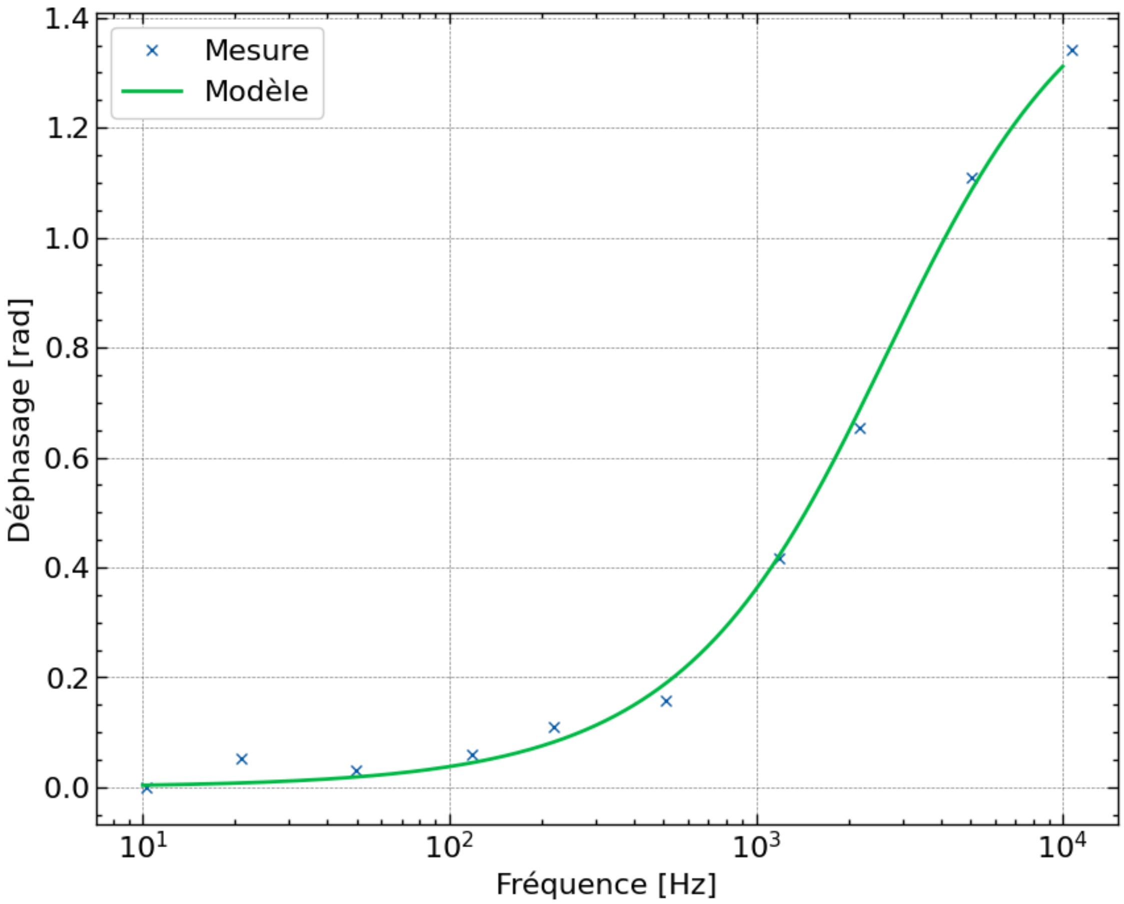
```
print("f_c = {:.3f} Hz".format(intersection[0]))
plt.plot(intersection[0], intersection[1], 'ro', label='Fréquence de coupure')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Intensité [A]')
plt.legend(loc='lower left')

plt.xscale('log')
plt.yscale('log')

plt.xlim(9,12000)
plt.ylim(0.04,0.2)

plt.show()
```

```
U_in = 8.310 V
f_c = 2204.669 Hz
```



[ ]:
```
plt.figure(figsize=(10,8))
```

```
plt.plot(freq, phase,'x',label='Mesure')

phase_th = lambda f: np.arctan(2*np.pi*f*inductance_th/res)

plt.plot(np.linspace(10,10000,10000), phase_th(np.linspace(10,10000,10000)),␣
 ↪label='Modèle')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Déphasage [rad]')
plt.legend(loc='upper left')

plt.xscale('log')

plt.show()
```



1.2 Circuit RC et filtre "passe-haut"

Pour celle-ci, nous avons toutes les données pour la tension, mais nous n'avons qu'une série de mesure pour R = 50 [Ω].

La phase est corrigée car elle a été prise selon le plus grand écart dans la mesure.

[ ]:
```
res = 50 # Ohm
freq = np.array([5.102,10.95,21.1,51.87,104.
 ↪3,208,516,1034,2185,5060,10980,20460]) # Hz
tension = np.array([0.222,0.28,0.184,0.344,0.62,0.848,1.86,3.3,5.88,8.4,9.2,9.
 ↪2]) # V
tension_in = np.array([10.2,10.1,10.2,10,9.92,9.92,9.92,9.92,10.1,9.92,9.76,9.
 ↪36]) # V
diff_t = np.array([100,52.4,29,13.6,6.24,3.28,1.44,0.76,0.38,0.18,0.088,0.
 ↪0476])/1e3 # s

phase = diff_t*2*np.pi*freq - 2*np.pi # rad
capacite = -1/((2*np.pi*freq*res) * np.tan(phase)) # F

display(pd.DataFrame({'freq [Hz]':freq,'tension [V]':tension,'tension_in [V]':
 ↪tension_in,'del_t [ms]':diff_t*1e3,'phase':np.round(phase,3), 'capacite␣
 ↪[µF]':np.round(capacite*1e6,3)}))

print('C = {:.3f} µF'.format(np.mean(capacite[7:])*1e6))
print('Écart-type = {:.3f} µF'.format(np.std(capacite[7:])*1e6))
```

	freq [Hz]	tension [V]	tension_in [V]	del_t [ms]	phase	capacite [µF]
0	5.102	0.222	10.20	100.0000	-3.078	-9721.527
1	10.950	0.280	10.10	52.4000	-2.678	-581.496
2	21.100	0.184	10.20	29.0000	-2.439	-177.986
3	51.870	0.344	10.00	13.6000	-1.851	-17.648
4	104.300	0.620	9.92	6.2400	-2.194	-21.930
5	208.000	0.848	9.92	3.2800	-1.997	-6.940
6	516.000	1.860	9.92	1.4400	-1.615	-0.270
7	1034.000	3.300	9.92	0.7600	-1.346	0.705
8	2185.000	5.880	10.10	0.3800	-1.066	0.804
9	5060.000	8.400	9.92	0.1800	-0.560	1.002
10	10980.000	9.200	9.76	0.0880	-0.212	1.346
11	20460.000	9.200	9.36	0.0476	-0.164	0.940

```
C = 0.960 µF
Écart-type = 0.219 µF
```

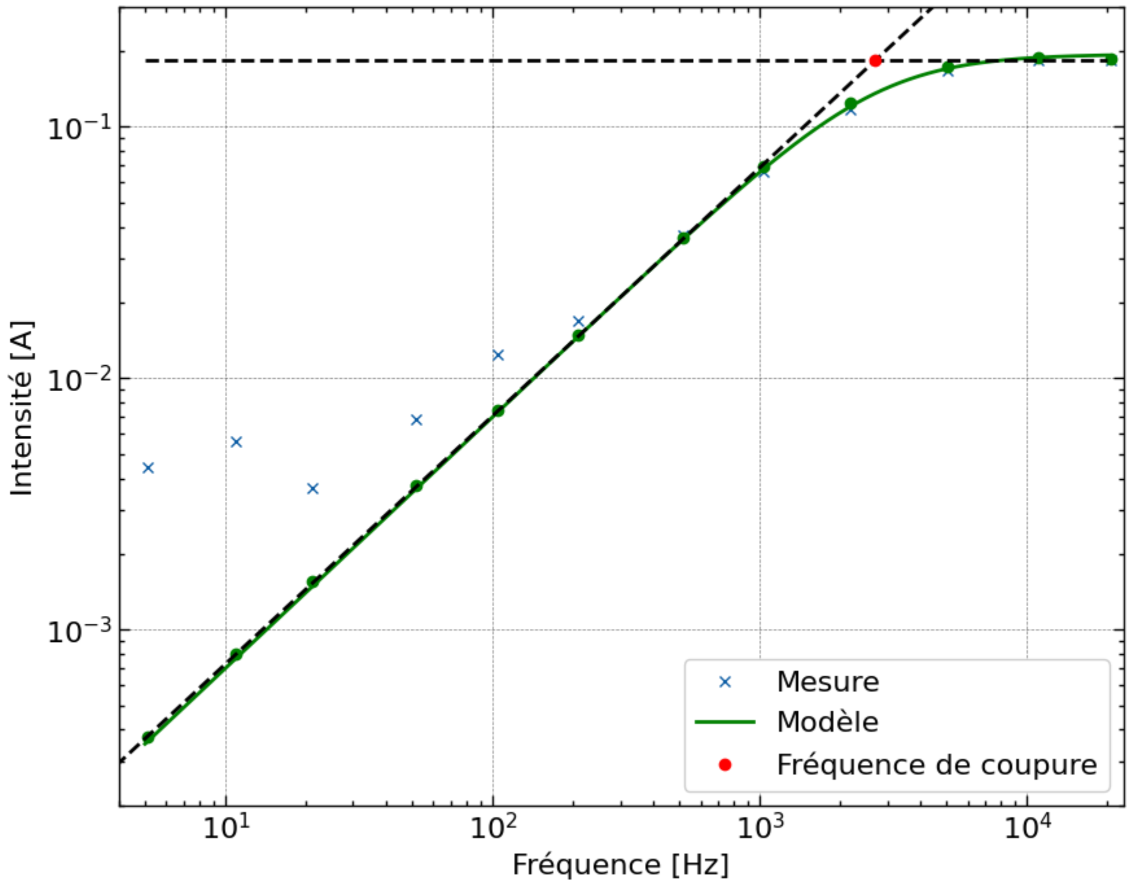
1.2.1 Graphes

Pour la tension de mesure, c'est toujours la même formule. Pour celle théorique, on utilise la tension d'entrée et la formule $I = \frac{U}{\sqrt{R^2 + \frac{1}{(2\pi \cdot f \cdot C)^2}}}$.

[ ]:
```
capacite = 1.15e-6 # F

plt.figure(figsize=(10,8))
```

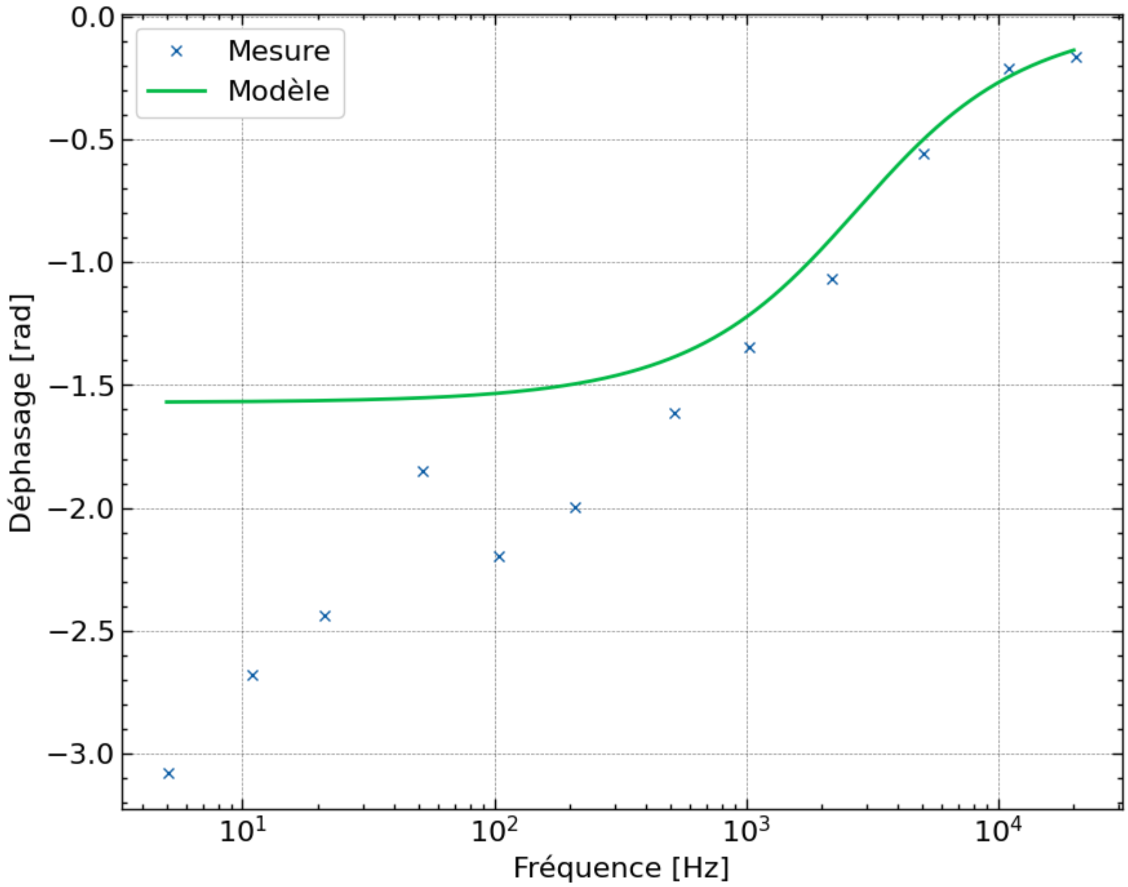
```python
plt.plot(freq, tension/res,'x',label='Mesure')

intensite_th = lambda f, U: U/np.sqrt(res**2+1/(2*np.pi*f * capacite)**2)

popt, _ = sc.optimize.curve_fit(intensite_th, freq, intensite_th(freq, tension_in))

print("U_in = {:.3f} V".format(popt[0]))
plt.plot(np.linspace(5,20000,10000), intensite_th(np.linspace(5,20000,10000), *popt), 'g-', label='Modèle')
plt.plot(freq, intensite_th(freq, tension_in), 'go')

# asymptote horizontale
plt.hlines(tension[-1]/res, 5, 20000, colors='k', linestyles='dashed')

# asymptote oblique
m = (np.log10(intensite_th(freq, tension_in)[0]) - np.log10(intensite_th(freq, tension_in)[1])) / (np.log10(freq[0])- np.log10(freq[1]))
h = np.log10(intensite_th(freq, tension_in)[1]) - m * np.log10(freq[1])
line_x = 10**np.linspace(0, 4, 1000)
line_y = 10**(m*np.linspace(0, 4, 1000) + h)
plt.plot(line_x, line_y, 'k--')

# intersection
for i in range(len(line_x)-1):
    if line_y[i] < tension[-1]/res < line_y[i+1] or line_y[i] > tension[-1]/res > line_y[i+1]:
        intersection = [line_x[i], line_y[i]]

print("f_c = {:.3f} Hz".format(intersection[0]))
plt.plot(intersection[0], intersection[1], 'ro', label='Fréquence de coupure')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Intensité [A]')
plt.legend(loc='lower right')

plt.xscale('log')
plt.yscale('log')

plt.xlim(4,23000)
plt.ylim(0.0002,0.3)

plt.show()
```

```
U_in = 9.725 V
f_c = 2700.421 Hz
```



Pour le déphasage, on a une équation similaire à précédemment : $\tan\theta = -\frac{1}{2\pi \cdot f \cdot R \cdot C}$.

[ ]:
```
plt.figure(figsize=(10,8))

plt.plot(freq, phase,'x',label='Mesure')

phase_th = lambda f: np.arctan(-1/(2*np.pi*f*res*capacite))

plt.plot(np.linspace(5,20000,10000), phase_th(np.linspace(5,20000,10000)),␣
↪label='Modèle')

plt.xlabel('Fréquence [Hz]')
plt.ylabel('Déphasage [rad]')
plt.legend(loc='upper left')

plt.xscale('log')

plt.show()
```


Mesure
Modèle
Déphasage [rad]
Fréquence [Hz]
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
10^1
10^2
10^3
10^4