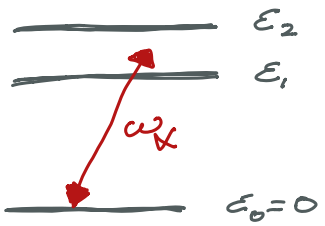

IN THE LAB FRAME:

$$H = \mathcal{E}_1 |1\rangle\langle 1| + \mathcal{E}_2 |2\rangle\langle 2| + \hat{d}\cdot\hat{E}_0 \cos\omega_\alpha t =$$

$$= \mathcal{E}_1 |1\rangle\langle 1| + \mathcal{E}_2 |2\rangle\langle 2| + \sum_{k,\ell} |k\rangle\langle k|\hat{d}\hat{E}|\ell\rangle\langle \ell| \cos\omega_\alpha t$$

$$= \mathcal{E}_1 |1\rangle\langle 1| + \mathcal{E}_2 |2\rangle\langle 2| + \Omega_{01}\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)\cos\omega_\alpha t +$$
$$+ \Omega_{02}\left(|0\rangle\langle 2| + |2\rangle\langle 0|\right)\cos\omega_\alpha t +$$
$$+ \Omega_{12}\left(|1\rangle\langle 2| + |2\rangle\langle 1|\right)\cos\omega_\alpha t$$

RWA - APPROXIMATION

IN THE INTERACTION PICTURE:

$$\tilde{H}_c = \cos\omega_\alpha t \Big\{ \Omega_{01}\left(e^{-i\mathcal{E}_1 t}|0\rangle\langle 1| + e^{i\mathcal{E}_1 t}|1\rangle\langle 0|\right) +$$
$$+ \Omega_{02}\left(e^{-i\mathcal{E}_2 t}|0\rangle\langle 2| + e^{i\mathcal{E}_2 t}|2\rangle\langle 0|\right) +$$
$$+ \Omega_{12}\left(e^{-i(\mathcal{E}_2-\mathcal{E}_1)t}|1\rangle\langle 2| + e^{i(\mathcal{E}_2-\mathcal{E}_1)t}|2\rangle\langle 1|\right)\Big\}$$

IF $\mathcal{E}_1 \approx \mathcal{E}_2 \approx \omega_\alpha$:

(A) $$H_c \approx \frac{1}{2}\Omega_{01} e^{i\omega_\alpha t}|0\rangle\langle 1| + \frac{1}{2}\Omega_{02} e^{i\omega_\alpha t}|0\rangle\langle 2| + h.c.$$

IF $\mathcal{E}_1 \approx \omega_\alpha$ & $\mathcal{E}_2 - \mathcal{E}_1 \approx \omega_\alpha$

(B) $$H_c \approx \frac{1}{2}\Omega_{01} e^{i\omega_\alpha t}|0\rangle\langle 1| + \frac{1}{2}\Omega_{12} e^{i\omega_\alpha t}|1\rangle\langle 2| + h.c.$$

ROTATING FRAME TRANSFORMATION

$$\tilde{H} = UHU^{+} + i\dot{U}U^{+}$$

(A) $U = \exp\left\{-i\omega_{\alpha} t\, |0\rangle\langle 0|\right\}$

$$H = \begin{bmatrix} 0 & \frac{1}{2}\Omega_{01}e^{i\omega_1 t} & \frac{1}{2}\Omega_{02}e^{i\omega_2 t} \\ \frac{1}{2}\Omega_{01}e^{-i\omega_1 t} & \varepsilon_1 & 0 \\ \frac{1}{2}\Omega_{02}e^{-i\omega_2 t} & 0 & \varepsilon_2 \end{bmatrix} \rightarrow \tilde{H} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_{01} & \frac{1}{2}\Omega_{02} \\ \frac{1}{2}\Omega_{01} & \varepsilon_1 - \omega_{\alpha} & 0 \\ \frac{1}{2}\Omega_{02} & 0 & \varepsilon_2 - \omega_{\alpha} \end{bmatrix}$$

(B) $U = \exp\left\{-i\omega_{\alpha} t\, (|0\rangle\langle 0| - |2\rangle\langle 2|)\right\}$

$$H = \begin{bmatrix} 0 & \frac{1}{2}\Omega_{01}e^{i\omega_1 t} & 0 \\ \frac{1}{2}\Omega_{01}e^{-i\omega_1 t} & \varepsilon_1 & \frac{1}{2}\Omega_{12}e^{i\omega_2 t} \\ 0 & \frac{1}{2}\Omega_{12}e^{-i\omega_2 t} & \varepsilon_2 \end{bmatrix} \rightarrow \tilde{H} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_{01} & 0 \\ \frac{1}{2}\Omega_{01} & \varepsilon_1 - \omega & \frac{1}{2}\Omega_{12} \\ 0 & \frac{1}{2}\Omega_{12} & \varepsilon_2 - 2\omega \end{bmatrix}$$