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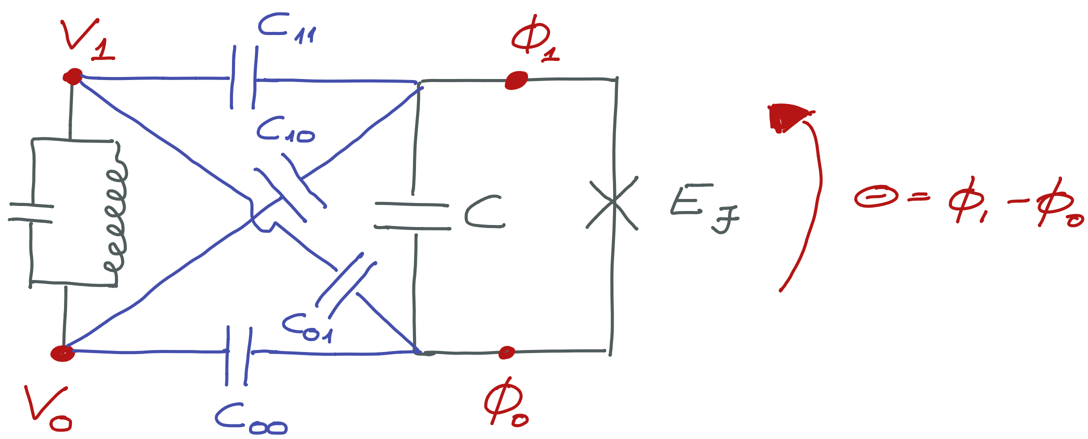
TRANSMON COUPLED TO A RESONATOR



THE LAGRANGIAN OF THE CIRCUIT

→ FLUX NODE BASIS:

$$\mathcal{L} = \frac{1}{2}\dot{\underline{\phi}}^T \underline{\underline{C}}_\phi \dot{\underline{\phi}} - \dot{\underline{\phi}}^T \underline{\underline{C}}_V^\phi \underline{V} - U(\underline{\phi})$$

$$\underline{\underline{C}}_\phi = \begin{bmatrix} C + C_{00} + C_{01} & -C \\ -C & C + C_{10} + C_{11} \end{bmatrix},$$

$$\underline{\underline{C}}_V^\phi = \begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix}$$

$$\underline{\phi} = \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix}, \quad \underline{V} = \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} \qquad U(\underline{\phi}) = -E_J \cos(\phi_0 - \phi_1)$$

QUBIT MODE BASIS:

THE TRANSFORMATION OF THE LAGRANGIAN:

$$\mathcal{L} = \frac{1}{2}\dot{\underline{\Theta}}^T \underline{\underline{C}}_\Theta \dot{\underline{\Theta}} - \dot{\underline{\Theta}}^T \underline{\underline{C}}_V^\Theta \underline{V}_\Theta - U(\underline{\Theta})$$

$$\boxed{\begin{aligned} \underline{\Theta} &= \underline{\underline{R}}\,\underline{\phi} \qquad & \underline{\underline{C}}_\Theta &= \left[\underline{\underline{R}}^{-1}\right]^T \underline{\underline{C}}_\phi \underline{\underline{R}}^{-1} \\ \underline{V}_\Theta &= \underline{\underline{R}}\,\underline{V}_\phi \qquad & \underline{\underline{C}}_V^\Theta &= \left[\underline{\underline{R}}^{-1}\right]^T \underline{\underline{C}}_V^\phi \underline{\underline{R}}^{-1} \end{aligned}}$$

WE USE THE FOLLOWING TRANSFORMATION:

$$\underline{\underline{R}} = \begin{bmatrix} -1 & 1 \\ \dfrac{C_{00}+C_{01}}{C_{00}+C_{01}+C_{10}+C_{11}} & \dfrac{C_{10}+C_{11}}{C_{00}+C_{01}+C_{10}+C_{11}} \end{bmatrix}.$$

THE HAMILTONIAN OF THE CIRCUIT:

AFTER LEGENDRE TRANSFORMATION $\left(\underline{q} = \frac{\partial \mathcal{L}}{\partial \dot{\underline{\Theta}}}\right)$:

$$\boxed{H = \frac{1}{2}\left(\underline{q} + \underline{\underline{C}}_V^\Theta\right)^T \underline{\underline{C}}_\Theta^{-1}\left(\underline{q} + \underline{\underline{C}}_V^\Theta\right) + U(\underline{\Theta}).}$$

WE CAN DIVIDE THE HAMILTONIAN AS:

$$H_0 = \frac{1}{2} \underline{q}^T \underline{\underline{C}}_{\Theta}^{-1} \underline{q} + U(\underline{\Theta})$$

$$H_c = \underline{q}^T \underline{\underline{C}}_{\Theta}^{-1} \underline{\underline{C}}_{V}^{\Theta} \cdot \underline{V}_{\Theta}$$

INTRODUCING THE NUMBER OF COOPER PAIRS $\underline{n}$

$\underline{q} \longrightarrow 2e\,\underline{n}$, AND IGNORING THE CENTER OF MASS:

$$\underline{n} = \begin{bmatrix} n \\ \cancel{n_2} \end{bmatrix}$$

$$\boxed{\begin{aligned} H_0 &= 4E_c n^2 - E_J \cos\Theta \\ H_c &= 2e\beta V_{\Theta} n \end{aligned}}$$

WHERE $E_c = \dfrac{e^2}{2C_\phi}$ $\quad \beta = \dfrac{C_c}{C_\phi}$

$$C_\phi = 1 \Big/ \left[\underline{\underline{C}}_{\Theta}^{-1}\right]_{00} = C + \frac{(C_{00} + C_{01})(C_{10} + C_{11})}{C_{00} + C_{01} + C_{10} + C_{11}}$$

$$C_c = \left[\underline{\underline{C}}_{V}^{\Theta}\right]_{00} = \frac{C_{11} C_{00} - C_{10} C_{01}}{C_{00} + C_{01} + C_{10} + C_{11}}$$

$$E_c[\mathrm{GHz}] = 19.37 / C_\phi[\mathrm{fF}]$$

$$g\,[\mathrm{GHz}] = 0.053\,\beta \cdot f_R[\mathrm{GHz}]\,(E_J/E_c)^{1/4}$$