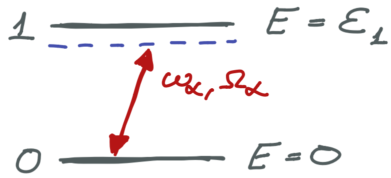

TWO-LEVEL SYSTEM COUPLED TO CLASSICAL DRIVE:

IN LAB FRAME — WITHIN DIPOLE APPROXIMATION.

$$H = H_{ATOM} + H_c = \varepsilon_1 |1\rangle\langle 1| + \bar{d}\cdot\bar{E}(t) =$$

$$= \varepsilon_1 |1\rangle\langle 1| + \underbrace{\langle 1|\bar{d}\,\bar{E}(t)|0\rangle}\, |1\rangle\langle 0| + \langle 0|\bar{d}\,\bar{E}(t)|1\rangle\, |0\rangle\langle 1| =$$

$$= \varepsilon_1 |1\rangle\langle 1| + \overset{\Omega_\alpha^*(t)}{\Omega_\alpha(t)}\, |0\rangle\langle 1| + \Omega_\alpha^*(t)\, |1\rangle\langle 0|$$

$$\boxed{H = \varepsilon_1 |1\rangle\langle 1| + \Omega_\alpha \cos(\omega_\alpha t)\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big).}$$

RWA - APPROXIMATION

- TRANSFORMING INTO INTERACTION PICTURE

$$H_0 = \varepsilon_1 |1\rangle\langle 1|$$

$$H_c = \Omega_\alpha \cos(\omega_\alpha t)\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big)$$

$$\tilde{H}_c = e^{iH_0 t} H_c e^{-iH_0 t} = H_c + [iH_0 t, H_c] + \frac{1}{2}\big[iH_0 t, [iH_0 t, H_c]\big] + \ldots$$

$$\big[|1\rangle\langle 1|, |0\rangle\langle 1|\big] = -|0\rangle\langle 1|$$

$$\big[|1\rangle\langle 1|, |1\rangle\langle 0|\big] = |1\rangle\langle 0|$$

$$H_c = \Omega_\alpha \cos(\omega_\alpha t)\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big)$$

$$[H_0, H_c] = \Omega_\alpha \cos(\omega_\alpha t)\big(-i\varepsilon_1 t|0\rangle\langle 1| + i\varepsilon_1 t|1\rangle\langle 0|\big)$$

$$\Rightarrow \tilde{H}_c = \Omega_\alpha \cos(\omega_\alpha t)\left[e^{-i\varepsilon_1 t}|0\rangle\langle 1| + e^{i\varepsilon_1 t}|1\rangle\langle 0|\right] =$$

$$= \frac{1}{2}\Omega_\alpha\left[\left(e^{i(\omega_\alpha-\varepsilon_1)t} + \underbrace{e^{-i(\omega_\alpha+\varepsilon_1)t}}_{\to 0}\right)|0\rangle\langle 1| + \right.$$
$$\left. + \left(\underbrace{e^{i(\omega_\alpha+\varepsilon_1)t}}_{\to 0} + e^{-i(\omega_\alpha-\varepsilon_1)t}\right)|1\rangle\langle 0|\right]$$

TOO FAST TERMS

- TRANSFORMING BACK TO SCHRÖDINGER PICTURE

$$\boxed{H = \varepsilon_1|1\rangle\langle 1| + \frac{1}{2}\Omega_\alpha\left(e^{i\omega_\alpha t}|0\rangle\langle 1| + e^{-i\omega_\alpha t}|1\rangle\langle 0|\right)}$$

ROTATING FRAME TRANSFORMATION

$$\tilde{H} = UHU^+ + i\dot{U}U^+$$

WE LOOK FOR U SO THAT $\tilde{H}$ IS TIME-INDEPENDENT.

$$U = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\omega_1 t}\end{bmatrix}. \qquad \omega_1 = ?$$

$$H = \begin{bmatrix} 0 & \frac{1}{2}\Omega_\alpha e^{i\omega_\alpha t} \\ \frac{1}{2}\Omega_\alpha e^{-i\omega_\alpha t} & \varepsilon_1 \end{bmatrix}.$$

$$U H U^{+} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_\alpha e^{it(\omega_\alpha - \omega_1)} \\ \frac{1}{2}\Omega_\alpha e^{-it(\omega_\alpha - \omega_1)} & \varepsilon_1 \end{bmatrix}$$

$$\Rightarrow \omega_\alpha = \omega_1$$

$$\tilde{H} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_\alpha \\ \frac{1}{2}\Omega_\alpha & \varepsilon_1 - \omega_\alpha \end{bmatrix}$$

$$\boxed{\tilde{H} = (\overbrace{\varepsilon_1 - \omega_\alpha}^{\Delta}) |1\rangle\langle 1| + \frac{1}{2}\Omega_\alpha \sigma_x}$$

IF THERE IS AN ADDITIONAL PHASE:

$$\Omega(t) = \Omega_\alpha \cos(\omega_\alpha t + \phi)$$

$$\tilde{H} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_\alpha e^{i\phi} \\ \frac{1}{2}\Omega_\alpha e^{-i\phi} & \Delta \end{bmatrix} = \Delta |1\rangle\langle 1| + \frac{1}{2}\Omega_\alpha (\cos\phi\, \sigma_x + \sin\phi\, \sigma_y).$$