School of Computer Science and Technology
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Exercise-2 for
A Guide to Formal Methods
2022 Spring

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From the last page of the **Chap 3's PPT**.

Exercise 1
Prove the validity of the following sequents:

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(1) $(p \wedge q) \wedge r,\ s \wedge t \vdash q \wedge s$

1 $(p \wedge q) \wedge r$	*premise*
2 $p \wedge q$	$\wedge e_1 1$
3 q	$\wedge e_2 2$
4 $s \wedge t$	*premise*
5 s	$\wedge e_1 4$
6 $q \wedge s$	$\wedge i\ 3, 5$

(2) $q \to r \vdash (p \to q) \to (p \to r)$

1 $q \to r$	*premise*
2 $p \to q$	*assumption*
3 p	*assumption*
4 q	$\to e\ 2, 3$
5 r	$\to e\ 1, 4$
6 $p \to r$	$\to i\ 3 - 5$
7 $(p \to q) \to (p \to r)$	$\to i\ 2 - 6$

(3) $\vdash q \to (p \to (p \to (q \to p)))$

1 q	*assumption*
2 p	*assumption*
3 $q \to p$	$\to i\ 1, 2$
4 $p \to (q \to p)$	$\to i\ 2, 3$
5 $p \to (p \to (q \to p))$	$\to i\ 2 - 4$
6 $q \to (p \to (p \to (p \to q)))$	$\to i\ 1 - 5$

(4) $p \to q \wedge r \vdash (p \to q) \wedge (p \to r)$

1 p	*assumption*
2 $p \to (q \wedge r)$	$\to$ *premise*
3 $q \wedge r$	$\to e\ 1, 2$
4 q	$\wedge e_1 3$
5 r	$\wedge e_2 3$
6 $p \to q$	$\to i\ 1 - 4$
7 $p \to r$	$\to i\ 1 - 5$
8 $(p \to q) \wedge (p \to r)$	$\wedge i\ 6, 7$

(5) $p \wedge \neg p \vdash \neg(r \to q) \wedge (r \to q)$

1 $p \wedge \neg p$	*premise*
2 p	$\wedge e_1 1$
3 $\neg p$	$\wedge e_2 1$
4 $\bot$	$\neg e\ 2, 3$
5 $\neg(r \to q) \wedge (r \to q)$	$\bot\ e\ 4$

Exercise 2
Prove the validity of the following sequents in predicate logic, where P and Q have arity 1, and S has arity 0 (a 'propositional atom'):

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(1) $\exists x(S \to Q(x)) \vdash S \to \exists x Q(x)$

1		$\exists x(S \to Q(x))$	*premise*
2	x_0	$S \to Q(x_0)$	*assumption*
3		S	*assumption*
4		$Q(x_0)$	$\to e\ 2, 3$
5		$\exists x Q(x)$	$\exists x\ i\ 4$
6		$S \to \exists x Q(x)$	$\to i\ 3 - 5$
7		$S \to \exists x Q(x)$	$\exists e\ 1, 2 - 6$

(2) $\forall x P(x) \to S \vdash \exists x(P(x) \to S)$

1 $\forall x P(x) \to S$	*premise*
2 $P(t) \to S$	$\forall x\ e\ 1,\ x\ is\ not\ free\ in\ S$
3 $\exists x(P(x) \to S)$	$\exists x\ i\ 2$

(3) $\forall x(P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$

1	$\forall x(P(x) \wedge Q(x))$	*premise*
2	$P(t) \wedge Q(t)$	$\forall x\ e\ 1$
3	$P(t)$	$\wedge e_1\ 2$
4	$Q(t)$	$\wedge e_2\ 2$
5	$\forall x P(x)$	$\forall x\ i\ 3$
6	$\forall x Q(x)$	$\forall x\ i\ 4$
7	$\forall x P(x) \wedge \forall x Q(x)$	$\wedge i\ 5, 6$

(4) $\neg\forall x \neg P(x) \vdash \exists x P(x)$

1		$\neg\forall x \neg P(x)$	*premise*
2		$\exists x \neg\neg P(x)$	*semantically understanding*
3	x_0	$\neg\neg P(x_0)$	*assumption*
4		$P(x_0)$	$\neg\neg e\ 3$
5		$\exists x P(x)$	$\exists x\ i\ 4$
6		$\exists x P(x)$	$\exists e\ 2, 3-5$

(5) $\forall x \neg P(x) \vdash \neg\exists x P(x)$

1		$\forall x \neg P(x)$	*premise*
2		$\exists x P(x)$	*assumption*
3	x_0	$P(x_0)$	*assumption*
4		$\neg P(x_0)$	$\forall x\ e\ 1$
5		$\bot$	$\neg e\ 3, 4$
6		$\bot$	$\exists x\ e\ 2, 3-5$
7		$\neg\exists x P(x)$	$\neg i\ 2-6 (MT)$