Q1: $P_{x_j|w_i}(x_j|w_i) = \frac{1}{\Gamma(p_i)} \lambda_i^{p_i} x_j^{p_i} e^{-\lambda_j x_j}$, $p_i, \lambda_j > 0$

(a) $P(w_i | X_1 = x_1, X_2 = x_2 \cdots X_k = x_k) = \frac{P(X_1 = x_1, \cdots X_k = x_k | w_i)}{P(x_1, x_2, w_i)}$

$\propto P(x_1 | w_i) P(x_2 | w_i) \cdots P(x_k | w_i) P(w_i)$

$= \prod_{j=1}^{k} \frac{1}{\Gamma(p_i)} \lambda_j^{p_i} x_j^{p_i - 1} e^{-\lambda_j x_j} P(w_i)$

$= \frac{P(w_i)}{\Gamma(p_i)} \prod_{j=1}^{k} (\lambda_j)^{p_i} (x_j)^{p_i - 1} e^{-\lambda_j x_j}$

$\frac{P(w_i)}{\Gamma(p_i)} \prod_{j=1}^{k} (\lambda_j)^{p_i} (x_j)^{p_i - 1} e^{-\lambda_j x_j} > \frac{P(w_i)}{\Gamma(p_i)} \prod_{j=1}^{k} (\lambda_i)^{p_2} (x_j)^{p_2 - 1} e^{-\lambda_j x_j}$

$\therefore \frac{P(w_i)}{\Gamma(p_i)} \prod_{j=1}^{k} (\lambda_j)^{p_i} (x_j)^{p_2 - 1} > \frac{P(w_i)}{\Gamma(p_i)} \prod_{j=1}^{k} (\lambda_i)^{p_2} (x_j)^{p_2 - 1}$

(b). linear when $|p_2 - p_1| = 1$

(c) $P_1 = 4,\ P_2 = 2,\ c = 2,\ k = 4,\ \lambda_1 = \lambda_3 = 1$

$\lambda_2 = \lambda_4 = 2$

$$\frac{P(w_1)}{\Gamma(P_1)} \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_2 - 1} \underset{}{\overset{w_1}{>}} \frac{P(w_2)}{\Gamma(w_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2 - 1}$$

$$\frac{1}{6} (4)^4 \left((0.1)(0.2)(0.3)4 \right)^3 \overset{w_1}{>} \frac{1}{1} (4)^2 \left[(0.1)(0.2)(0.3)4 \right]$$

$$\frac{16}{6} (0.024)^2 \overset{w_1}{>} 1$$

$$0.001536 \underset{w_2}{<} 1$$

$$c(x) = 2$$

(d) $P_1 = 3.2,\ P_2 = 8,\ c = 2,\ k = 1,\ \lambda_1 = 1$

$$\frac{P(w_1)}{\Gamma(P_1)} \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_1 - 1} \overset{w_1}{>} \frac{P(w_2)}{\Gamma(P_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2 - 1}$$

$$\frac{1}{\Gamma(3.2)} 1^{3.2} x^{2.2} \overset{w_1}{>} \frac{1}{\Gamma(8)} 1^{8} \cdot x^{7}$$

$$\frac{\Gamma(8)}{\Gamma(3.2)} = x^{4.8}$$

$$\therefore \text{as for } x^* = x = \sqrt[4.8]{\frac{5040}{2.432}} = 4.91$$

for Type - 1 error

$$T_1 = 1 - P(w_1|x) = 1 - \frac{\frac{P(w_1)}{\Gamma(3.2)} x^{2.2}}{\frac{P(w_1)}{\Gamma(3.2)} x^{2.2} + \frac{P(w_2)}{\Gamma(8)} x^{7}}$$

$$= 1 - \frac{1}{1 + \frac{\Gamma(3.2)}{\Gamma(8)} x^{4.8}} = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Type - 2 error =

$$T_2 = 1 - P(w_2|x)$$

$$= 1 - \frac{\frac{P(w_2)}{\Gamma(8)} x^{7}}{\frac{P(w_1)}{\Gamma(3.2)} x^{2.2} + \frac{P(w_2)}{\Gamma(8)} x^{7}} = 1 - \frac{1}{\frac{\Gamma(8)}{\Gamma(3.2)} x^{-4.8}} = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

(e). $P_1 = P_2 = 4,\ c = 2,\ k = 2,\ \lambda_1 = 8,\ \lambda_2 = 0.3,\ P(w_1) = \frac{1}{4}$

$P(w_2) = \frac{3}{4}$

$$\frac{P(w_1)}{\Gamma(P_1)} \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_1 - 1} = \frac{P(w_2)}{\Gamma(P_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2 - 1}$$

$$\left(\frac{1}{4}\right) [8(0.3)]^4 (x_1 x_2)^3 = \frac{3}{4} [8(0.3)]^4 (x_1 x_2)^3$$

$$2(x_1 x_2)^3 = 0$$

$$f(x_1, x_2) = 0 \text{ when } x_1 x_2 = 0$$

Q_2; $x_i | w_j \sim Lap(m_{ij}, \lambda_i)$

$$P(x_i|w_j) = \frac{\lambda_i}{2} e^{-\lambda_i |x_i - m_{ij}|}, \quad \lambda_i > 0 \begin{cases} i \in \{1,2,\ldots k\} \\ j \in \{1,2,\ldots c\} \end{cases}$$

$$P(w_1) = P(w_2) = \cdots P(w_c)$$

$$P(w_i|x) \overset{w_i}{>} P(w_j|x) \quad \forall j \neq i$$

$$\frac{P(x|w_i)P(w_i)}{P(x)} \overset{w_i}{>} \frac{P(x|w_i)P(w_j)}{P(x)}$$

$$P(w_i) \prod_{j=1}^{k} P(x_j|w_i) \overset{w_i}{>} P(w_y) \prod_{j=1}^{k} P(x_j|w_y) \quad \forall y \neq i$$

$$\sum_{j=1}^{k} -\lambda_j |x_j - m_{ji}| \ln(e) \overset{w_i}{>} \sum_{j=1}^{k} -\lambda_j |x_j - m_{jy}|$$

$$\sum_{j=1}^{k} [\lambda_j (|x_j - m_{ji}| - |x_j - m_{jy}|)] \overset{w_i}{>} 0 \quad \forall y \neq i$$

which is min manhattan distance classifier

when $m_{ji} = m_{jy}$

Q3 :

(a) $R(\alpha_i|x) = \sum_{j=1}^{4} \lambda(\alpha_i|w_j) P(w_j|x)$

$P(x_1) = P(x_1|w_1)P(w_1) + \cdots + P(x_1|w_4)P(w_4)$

$= \frac{1}{3}\cdot\frac{1}{10} + \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{6}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{5} = 0.2967$

$P(x_2) = \frac{1}{3}\cdot\frac{1}{10} + \frac{1}{4}\cdot\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{5} = 0.33$

$P(x_3) = \frac{1}{3}\cdot\frac{1}{10} + \frac{1}{4}\cdot\frac{1}{5} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{5}\cdot\frac{1}{5} = 0.3733$

substitute to the equation

$R(\alpha_i|x) = \frac{1}{P(x)} \cdot \sum_{j=1}^{4} [\lambda(\alpha_i|w_j) P(x|w_j) P(w_j)]$

$R(\alpha_1|x_1) = 2.5952$

$R(\alpha_2|x_1) = 2.5506$

$R(\alpha_3|x_1) = 1.5506$

$R(\alpha_4|x_1) = 1.8539$

$R(\alpha_{\cancel{1}}|x_2) = 2.7879$

$R(\alpha_2|x_2) = 2.5455$

$R(\alpha_3|x_2) = 1.0909$

$R(\alpha_4|x_2) = 1.4646$

$R(\alpha_1|X_3) = 2.7054$

$R(\alpha_2|X_3) = 1.6161$

$R(\alpha_3|X_3) = 0.7500$

$R(\alpha_4|X_3) = 1.5179$

(b) $R = \sum_{i=1}^{3} R(\alpha(x_i)|x_i)P(x_i)$

$= R(\alpha(x_1)|x_1)P(x_1) \cdots + R(\alpha(x_3)|x_3)P(x_3)$

$= 1.5506(0.2967) + 1.0807(0.33) + 0.75(0.3733)$

$R = 1.1$

Q4:

(a) $P(w_1) = \frac{3}{10}$ $P(w_2) = \frac{7}{10}$

x for income

(b) $P(x|w_i) = N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma_i}\right)^2\right]$

$$\mu_1 = \frac{88+90+85}{3} = 87.67$$

$$\mu_2 = \frac{122+77+106+210+72+117+60}{7} = 109.143$$

$$\sigma_1^2 = \frac{\Sigma(x-\mu_1)^2}{n-1} = 6.33$$

$$\sigma_1 = 2.52$$

$$\sigma_2^2 = \frac{\Sigma(x-\mu_2)^2}{n-1} = 2539.5$$

$$\sigma_2 = 50.3932$$

$$P(x|w_1) = \frac{1}{6.3082} e\left[-\frac{1}{2}\left(\frac{x-87.67}{2.5166}\right)^2\right]$$

$$P(x|w_2) = \frac{1}{126.317} e^{-\frac{1}{2}\left(\frac{x-109.143}{50.3932}\right)^2}$$

c. $X_1 \to \{$ refund, no refund $\}$

$P(X_1 = \text{refund} \mid w_1) = 0$ $P(X_1 = \text{refund} \mid w_2) = \frac{3}{7}$

$P(X_1 = \text{no refund} \mid w_1) = 1$ $P(X_1 = \text{no refund} \mid w_2) = \frac{4}{7}$

$X_2 \to \{$ single, marry, divorce $\}$

$P(X_2 = \text{single} \mid w_1) = \frac{2}{3}$ $P(X_2 = \text{single} \mid w_2) = \frac{2}{7}$

$P(X_2 = \text{marry} \mid w_1) = 0$ $P(X_2 = \text{marry} \mid w_2) = \frac{4}{7}$

$P(X_2 = \text{divorce} \mid w_1) = \frac{1}{3}$ $P(X_2 = \text{divorce} \mid w_2) = \frac{1}{7}$

No

(d) Consider the features are conditionally independence

$P(x \mid w_i) = P(x_1 \mid w_i) P(x_2 \mid w_i) \cdots P(x_k \mid w_i) P(w_i)$

If any one of the conditional probability goes 0, then $P(x \mid w_i) = 0$. Laplace correction corrects this problem

(e)

$$P(w_1|x) \overset{w_1}{>} P(w_2|x)$$

$$\frac{\left[\prod_{i=1}^{k} P(x_i|w_1)\right] P(w_1)}{P(x)} \overset{w_1}{>} \frac{\left[\prod_{i=1}^{k} P(x_i|w_2)\right] P(w_2)}{P(x)}$$

$$P(w_1)\prod_{i=1}^{k} P(x_i|w_1) - P(w_2)\prod_{i=1}^{k} P(x_i|w_2) \overset{w_1}{>} 0$$

$$P(w_1)P(x_1|w_1)P(x_2|w_1)P(x_3|w_1) > P(w_2)P(x_1|w_2)P(x_2|w_2)P(x_3|w_3)$$

x_1 : refund x_2 : Marital status x_3 : Taxable Income

$$P(x_1 = \text{refund}|w_1) = \frac{1}{5}$$

$$P(x_1 = \text{refund}|w_2) = \frac{4}{9}$$

$$P(x_1 = \text{no refund}|w_1) = \frac{4}{5}$$

$$P(x_1 = \text{no refund}|w_2) = \frac{5}{9}$$

$$P(x_2 = \text{single}|w_1) = \frac{1}{2} \qquad P(x_2 = \text{single}|w_2) = \frac{3}{10}$$

$$P(x_2 = \text{marry}|w_1) = \frac{1}{6} \qquad P(x_2 = \text{marry}|w_2) = \frac{1}{2}$$

$$P(x_2 = \text{diverce}|w_1) = \frac{1}{3} \qquad P(x_2 = \text{diverce}|w_2) = \frac{1}{5}$$

$$P(x_3|w_1) = \frac{1}{6.3082} e^{-\frac{1}{2}\left(\frac{x-87.67}{2.5166}\right)^2}$$

$$P(x_3|w_2) = \frac{1}{126.317} e^{-\frac{1}{2}\left(\frac{x-109.143}{50.3932}\right)^2}$$