ADVANCED BAYESIAN MODELING

Further Topics in Model Checking

External Validation

Ultimate test of a model: How well does it predict future data?

Checking new data against model predictions is **external validation**.

For example:

If other two-way 2016 presidential polls (for the same period) were considered, would their Clinton leads be mainly within the range predicted by the model?

Would 90% of them be within a 90% posterior prediction interval, for example?

Marginal Predictive Checks

Consider posterior predictive p-values from test statistics for single observations, such as

$$p_i \quad = \quad \Pr(y_i^{\text{rep}} \leq y_i \mid y)$$

If p_i is near 0 or 1, y_i could be an outlier.

If p_is concentrate near 0 or 1, data are overdispersed relative to model.

More info: BDA3, Sec. 6.3

Mixed Predictive Checks

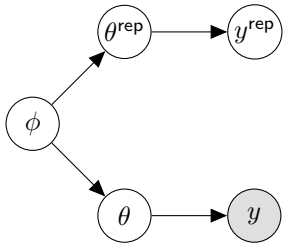
Consider a hierarchical model: The full parameter vector (ϕ, θ) includes hyperparameter ϕ.

Eg: $\phi = (\mu, \tau)$ for the 2016 polls model

Some model checking might concern replicate data y^{rep} not based on θ but on a replicate parameter θ^{rep} (with same hyperparameter ϕ).

Eg: "new" polls not in the data set, but from the same "population" of polls (exchangeable with the others)

The "new" polls have different means θ than the polls in the data.



Remark: θ^{rep} need not be same size as θ, nor y^{rep} same size as y
(just need the same distributional structure)

Can define *mixed* predictive p-values for checking both the data and parameter (θ) distributions:

$$\Pr\big(T(y^{\mathsf{rep}}, \theta^{\mathsf{rep}}, \phi) \ \geq \ T(y, \theta, \phi) \mid y\big)$$

or even just the parameter distribution:

$$\Pr\big(T(\theta^{\mathsf{rep}}, \phi) \ \geq \ T(\theta, \phi) \mid y\big)$$

Eg: check normality of poll means $\{\theta_j\}$ relative to a replicate set $\{\theta_j^{\mathsf{rep}}\}$