ADVANCED BAYESIAN MODELING

WAIC for 2016 Polls Data

Just for illustration, we use JAGS to compute WAIC for the hierarchical model for the 2016 polls data ...

Need extra deterministic nodes in JAGS model for posterior sampling of

$$p(y_j \mid \theta_j) \qquad \text{and} \qquad \log p(y_j \mid \theta_j) \qquad j = 1, \ldots, 7$$

to approximate

$$p_{\text{post}}(y_j) = \mathrm{E}_{\text{post}}\big(p(y_j \mid \theta_j)\big)$$

$$\mathrm{E}_{\text{post}}\big(\log p(y_j \mid \theta_j)\big) \qquad \qquad \mathrm{var}_{\text{post}}\big(\log p(y_j \mid \theta_j)\big)$$

In `polls2016waic.bug`:

```
model {

  for (j in 1:length(y)) {
    y[j] ~ dnorm(theta[j], 1/sigma[j]^2)
    theta[j] ~ dnorm(mu, 1/tau^2)

    p.y[j] <- dnorm(y[j], theta[j], 1/sigma[j]^2)
    log.p.y[j] <- log(p.y[j])
  }

  mu ~ dunif(-1000,1000)
  tau ~ dunif(0,1000)

}
```

```
> library(rjags)
...

> m <- jags.model("polls2016waic.bug", d)
...

> update(m, 2500)  # burn-in
  |**************************************************| 100%

> x <- coda.samples(m, c("p.y","log.p.y"), n.iter=100000)
  |**************************************************| 100%

> sx <- summary(x)
```

Approximate values of

$$\mathrm{E}_{\mathsf{post}}\big(p(y_j \mid \theta_j)\big) \qquad \mathrm{E}_{\mathsf{post}}\big(\log p(y_j \mid \theta_j)\big) \qquad \mathrm{var}_{\mathsf{post}}\big(\log p(y_j \mid \theta_j)\big)$$

and log pointwise predictive density:

```
> p.y.means <- sx$statistics[paste("p.y[",1:nrow(d),"]", sep=""),"Mean"]

> log.p.y.means <- sx$statistics[paste("log.p.y[",1:nrow(d),"]", sep=""),"Mean"]

> log.p.y.vars <- sx$statistics[paste("log.p.y[",1:nrow(d),"]", sep=""),"SD"]^2

> lppd <- sum(log(p.y.means))
```

Approximate p_{WAIC1}, p_{WAIC2}, and the corresponding versions of WAIC:

```
> ( pWAIC1 <- 2 * sum(log(p.y.means) - log.p.y.means) )
[1] 1.398974

> ( pWAIC2 <- sum(log.p.y.vars) )
[1] 1.829402

> ( WAIC1 <- -2 * lppd + 2 * pWAIC1 )
[1] 25.10937

> ( WAIC2 <- -2 * lppd + 2 * pWAIC2 )
[1] 25.97023
```

Note: Effective number of parameters for WAIC is only about half what it was for DIC.

Remark: Could approximate WAIC for no-pooling and complete-pooling models, using direct simulation from posteriors.