ADVANCED BAYESIAN MODELING

Mixed Models

In an ordinary (non-hierarchical) regression model, coefficients are sometimes called **fixed effects**.

This is to distinguish them from **random effects** (in a hierarchical model).

A **mixed model** has both types of effects ...

Matrix Formulation

A normal-theory linear mixed model:

$$\begin{aligned} y \mid \beta_f, \beta_r, \sigma_y^2, X_f, X_r &\sim \mathrm{N}(X_f\beta_f + X_r\beta_r,\ \sigma_y^2 I) \\ \beta_r \mid \Sigma_\beta &\sim \mathrm{N}(0,\ \Sigma_\beta) \end{aligned}$$

$\beta_f \;=\;$ fixed effects $\qquad \beta_r \;=\;$ random effects

X_f and X_r are known or observed.

X_r typically contains indicator variables of groups (or their interactions).

Σ_β is often diagonal, and contains variance components.

Q: Why assume β_r is multivariate normal?

A: It is (partially) conjugate. (Convenient, if not always correct.)

Q: Why assume β_r has (prior) mean zero?

A: If not, its mean could be moved into the fixed effects portion of the model.

Q: Where is the intercept?

A: In the fixed portion $X_f\beta_f$ (possibly implicitly)

Priors

To be noninformative:

$$
\begin{aligned}
p(\sigma_y^2) &\propto (\sigma_y^2)^{-1} \qquad \sigma_y^2 > 0 \\
p(\beta_f) &\propto 1
\end{aligned}
$$

Hyperprior for Σ_β depends on its structure.

For example, if $\Sigma_\beta = \tau^2 I$, might choose

$$
p(\tau) \propto 1 \qquad\qquad \tau > 0
$$

Remarks:

- General structure of $X_r \beta_r$ allows more than one grouping criterion, and even group hierarchies (groups of groups).

- Higher levels of linear model hierarchy are possible (BDA3, Sec. 15.3).

- Response variance (σ_y^2) may also vary by group.

For examples, see BDA3, Sec. 15.2–4.