$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}=f'(x_0)$$

$$f(x_0+h)-f(x_0)=f'(x_0)h+o(h)=[Df_{x_0}](h)+o(h)$$

$$\because f(x_0+h)-f(x_0)=f(x_0)+f'(x_0)+\frac{f''(x_0)}{2!}h^2+\frac{f'''(x_0)}{3!}h^3+\dots$$

$$\therefore f(\overline{x_0}+\bar{h})-f(\overline{x_0})=\sum_{j=1}^{m}\frac{\partial f}{\partial x_m}|_{x=\overline{x_0}}\,h_i=[Df_{x_0}](\overline{h})+o(||\overline{h}||)$$

$$\because f(\overline{x_0}+\bar{h})=f(\overline{x_0})+(\nabla f)_{x_o}\cdot\bar{h}+\frac{1}{2}\bar{h}^T Hf(x_0)\bar{h}+\dots$$

$$\therefore f(\overline{x_0}+\bar{h})-f(\overline{x_0})=(\nabla_{x_0}f)\cdot+o(||\overline{h}||)$$