# 1.

$$X \backsim N(\mu, \sigma^2), \sigma\ is\ give.$$

## a

$$\bar{x} \pm 2.81 \cdot \frac{\sigma}{\sqrt{n}}, 100\%(1-\alpha) =?$$
$$\hat{\mu} = \bar{x}$$
$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \backsim N(0,1)$$
$$P(Z_{1-\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}) = P(Z < 2.81) - P(Z < -2.81) = 0.9974 - 0.0026 = 0.9948$$
$$100\%(1-\alpha) = 99.48\%$$

## b

$$\bar{x} \pm 1.44 \cdot \frac{\sigma}{\sqrt{n}}, 100\%(1-\alpha) =?$$
$$P(Z_{1-\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}) = P(Z < 1.44) - P(Z < -1.44) = 0.925 - 0.075 = 0.85$$
$$100\%(1-\alpha) = 85\%$$

## c

$$100\%(1-\alpha) = 99.7\%,\ Z_{\frac{\alpha}{2}} =?$$
$$\alpha = 0.003 \implies \frac{\alpha}{2} = 0.0015$$
$$CDF : F_Z(z) = P(Z < z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}}\ e^{-\frac{t^2}{2}} dt$$
$$F_Z(z) = P(Z < z) = 1 - 0.0015 = 0.9985$$
$$Z_{\frac{\alpha}{2}} = Z_{0.9985} = F_Z^{-1}(0.9985) = 2.96$$

## d

$$100\%(1-\alpha) = 75\%,\ Z_{\frac{\alpha}{2}} =?$$
$$\alpha = 0.25 \implies \frac{\alpha}{2} = 0.125$$
$$F_Z(z) = P(Z < z) = 1 - 0.125 = 0.825$$
$$Z_{\frac{\alpha}{2}} = Z_{0.825} = F_Z^{-1}(0.825) = 1.5$$

# 2.

$$X \backsim N(\mu, 0.75^2)$$

## a

$$n = 20, \bar{x} = 4.85, 100\%(1-\alpha) = 95\%, (\theta_L, \theta_U) = ?$$
$$Z_{0.025} = 1.96 \implies \bar{x} \pm Z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 4.85 \pm 0.3287$$
$$(\theta_L, \theta_U) = (4.5213, 5.1787)$$

## b

$$n = 16, \bar{x} = 4.56, 100\%(1-\alpha) = 98\%, (\theta_L, \theta_U) = ?$$
$$Z_{0.001} = 2.326 \implies \bar{x} \pm Z_{0.001} \cdot \frac{\sigma}{\sqrt{n}} = 4.56 \pm 0.436$$
$$(\theta_L, \theta_U) = (4.124, 4.996)$$

## c

$$length = E(\theta_L, \theta_U) = 0.4, 100\%(1-\alpha) = 95\%, n = ?$$
$$E(\theta_L, \theta_U) = 2 \cdot Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.4 \implies \sqrt{n} = 7.35$$
$$n = 54$$

## d

$$error = \frac{E(\theta_L, \theta_U)}{2} < 0.2, 100\%(1-\alpha) = 99\%, n = ?$$
$$\frac{E(\theta_L, \theta_U)}{2} = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < 0.2 \implies \sqrt{n} > 12.88$$
$$n = 165.6 \approx 166$$

# 3.

$$X \backsim N(\mu, \sigma^2), \bar{x} = 654.16, S = 164.43, n = 50$$

## a

$$100\%(1-\alpha) = 95\%, (\theta_L, \theta_U) = ?$$
$$\bar{x} \backsim N(\mu, \frac{\sigma}{\sqrt{n}}) \implies \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \backsim N(0,1), \sigma\ is\ not\ give$$
$$\frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \backsim t_{n-1} \implies P(t_{1-\frac{\alpha}{2}, n-1} < T < t_{\frac{\alpha}{2}, n-1}) \implies \bar{x} \pm T_{\frac{\alpha}{2}, n-1} \cdot \frac{\sigma}{\sqrt{n}}$$
$$T_{\frac{\alpha}{2}, n-1} = 2.0227 \implies \bar{x} \pm T_{\frac{\alpha}{2}, n-1} \cdot \frac{\sigma}{\sqrt{n}} = 654.16 \pm 2.0227 \cdot \frac{164.43}{\sqrt{50}}$$
$$(\theta_L, \theta_U) = (607.1243, 701.1957)$$

## b

$$S = 175, 100\%(1-\alpha) = 95\%, E(\theta_L, \theta_U) = 50, n = ?$$
$$E(\theta_L, \theta_U) = 2 \cdot T_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \implies 50 = 2 \cdot T_{\frac{\alpha}{2}, n-1} \frac{175}{\sqrt{n}} \implies n = 49 \cdot T^2$$
$$\begin{cases} n\ is\ big \implies T_{\frac{\alpha}{2}, n-1} = 1.96 \\ n\ is\ small \implies T_{\frac{\alpha}{2}, n-1} = 2.262 \end{cases} \implies \begin{cases} \bar{T} = 2.111 \\ T_{\frac{\alpha}{2}, 50-1} = 2.01 \end{cases} \implies \begin{cases} n = 49 \cdot \bar{T} \approx 219 \\ n = 49 \cdot T_{\frac{\alpha}{2}, 50-1} \approx 198 \end{cases}$$
$$n = 198$$

# 4.

$$n=356, suc=201, 100\%(1-\alpha)=95\%, (\theta_L,\theta_U)=?$$

$$\begin{array}{c|cc} x_i & 0 & 1 \\ \hline & 1-p & p \end{array}$$

$$E(x_i)=p, D(x_i)=p(1-p)$$

$$n\cdot p>10, n(1-p)>10, n>30 \implies n\ is\ big$$

$$X \backsim N(P,\frac{p(1-p)}{n}) \implies \frac{\bar{x}-p}{\sqrt{\frac{p(1-p)}{n}}} \backsim N(0,1)$$

$$Z_{0.025,356}=1.96, \hat{p}=\frac{201}{356} \implies \frac{201}{356} \pm 1.96\sqrt{\frac{\frac{201}{356}\cdot\frac{155}{356}}{356}}=0.5726\pm 0.0515$$

$$(\theta_L,\theta_U)=(0.5211, 0.6231)$$

# 5.

$$\mu,\ T_{\frac{\alpha}{2},n-1}=?, \sigma\ is\ not\ give$$

## a

$$100\%(1-\alpha)=99\%, n=5$$

$$\mu \implies \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} \backsim t_{n-1}, T_{\frac{\alpha}{2},n-1}=P(1-\frac{\alpha}{2})$$

$$T_{0.005,4}=4.604$$

## b

$$100\%(1-\alpha)=98\%, n=38$$

$$T_{0.001,37}=2.431$$

# 6.

$$x_n=[418,434,454,421,437,463,421,439,465,422,446,425,447,427,448,431,453,]$$

## a

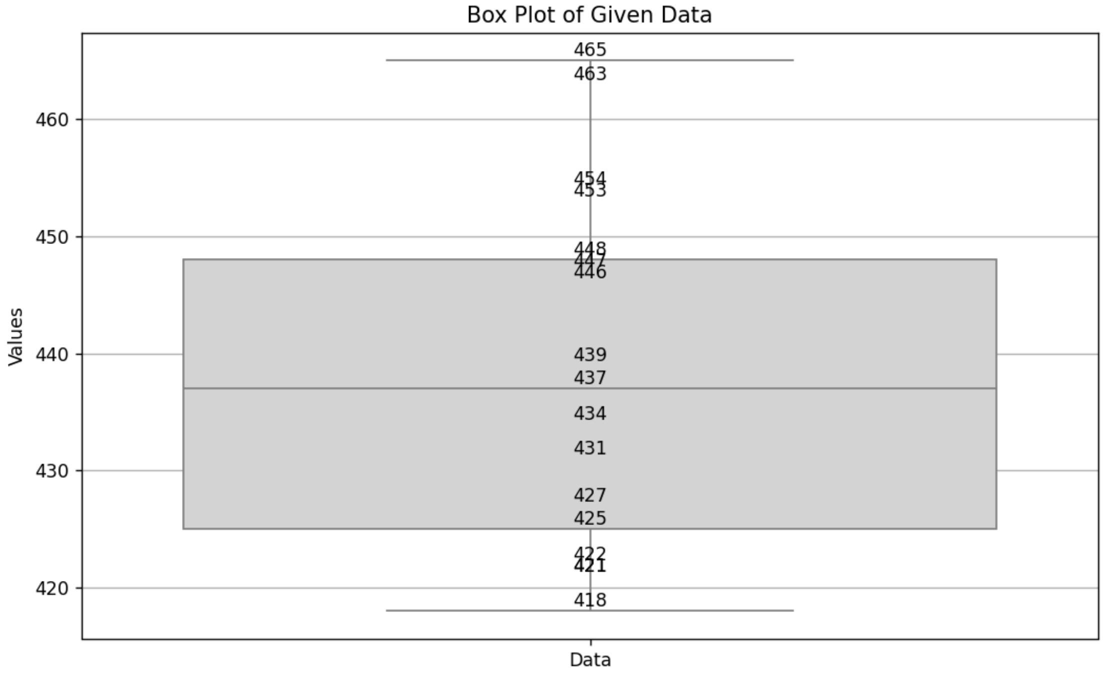
$$[418,421,421,422,425,427,431,434,437,439,446,447,448,453,454,463,465]$$

$$Q_2=437, Q_1=425, Q_3=448$$

$$IQR=Q_3-Q_1=23$$

$$Error=[Q_1-1.5IQR, Q_3+1.5IQR]=[390.5, 482.5]$$

No anomalies, moderate dispersion




## b

$$H_0 : x \backsim N(\mu, \sigma^2)$$
$$H_1 : x \not\backsim N(\mu, \sigma^2)$$
$$\alpha = 0.05$$
$$W = \frac{(\sum a_i x_i)^2}{\sum(x_i - \bar{x})},\ a_i = \frac{m_i}{\sqrt{\sum m_j^2}},\ m_i = \phi^{-1}(\frac{i - 0.375}{n + 0.25})$$
$$use\ python, W_{n=17} = 0.9541, P\_ = 0.28 > \alpha$$
$$accept\ H_0$$

## c

$$100\%(1-\alpha) = 95\%, n = 17,\ \ 440、450 \in (\theta_L, \theta_U)?$$
$$\bar{x} = 436.24, S = 12.5$$
$$\bar{x} \pm T_{\frac{\alpha}{2}, n-1}\frac{S}{\sqrt{n}} = 436.24 \pm 2.12\frac{12.5}{\sqrt{17}} = 436.24 \pm 6.427$$
$$(\theta_L, \theta_U) = (429.813, 442.637)$$
$$440 \in (429.813, 442.637), 450 \notin (429.813, 442.637)$$

# 7.

$$x_n = [211, 16, 29, 35, 42, 24, 24, 35], \sigma^2 \& \ \sigma\ \ of\ (\theta_L, \theta_U)$$
$$n = 8, S^2 = 37.54, \alpha = 0.05, \chi^2_{0.025,7} = 1.69, \chi^2_{0.975,7} = 16.013, \chi^2_{0.95,7} = 14.067$$
$$\hat{\sigma}^2 = S^2 \implies \frac{(n-1)S^2}{\sigma^2} \backsim \chi^2_{n-1} \implies \chi^2_{1-\frac{\alpha}{2}, n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\frac{\alpha}{2}, n-1}$$
$$\implies \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}} \implies (17.69, 147.21)$$
$$\hat{\sigma} \implies \sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}}} < \sigma < \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}} \implies (25.075, 72.33)$$

# 8.

$$x_n = [2.0, 15.7, 1.3, 0.7, 6.0, 4.8, 1.9, 1.9, 5.1, 12.2, 0.4, 5.3, 1.0, 0.6, 5.3]; n = 15,$$

$$X \backsim Exp, 100\%(1-\alpha) = 95\%,\ (\theta_L, \theta_U) = ?$$

$$\bar{x} = 4.21$$

$$\left(\frac{2n\bar{x}}{\chi^2_{1-\frac{\alpha}{2},n-1}}, \frac{2n\bar{x}}{\chi^2_{\frac{\alpha}{2},n-1}}\right) = \left(\frac{30 \times 4.21}{46.98}, \frac{30 \times 4.21}{16.79}\right) = (2.69, 7.52)$$

$$H_0 : x \backsim Exp$$

$$H_1 : x \not\backsim Exp$$

$$\alpha = 0.05$$

$$CV = \sqrt{-\frac{1}{2n}\ln(\alpha)} = 0.316$$

$$D_n = [EDF(x_i) - F(x_i)]_{MAX} \approx 0.0624 < CV$$

$$accept\ H_0$$