Questions

1. What is the difference between type I and type II errors?

$$\begin{array}{l|ll} & H_0(True) & H_0(False) \\ \hline \{X_n = x_n\}reject & I & True \\ \{X_n = x_n\}accpet & True & II \end{array}$$

> Type I error: Rejecting a true null hypothesis.
> Type II error: Accepting a false null hypothesis.

2. What is the difference between critical value approach and p-value approach? Explain what does the critical value mean and what does the p-value mean?

> $$\text{Critical Value approach:}$$
> $$\text{By given } \alpha \text{ to find } Z_\alpha \text{ which is reject area}$$
> $$\text{Top-Value approach:}$$
> $$\text{By } H_0 \text{ is true to find } \alpha_{min} \text{ and}$$
> $$\text{to compare given } \alpha \text{ to find statistical is significance or not}$$

> $$\text{Critical value } :$$
> $$\text{Is a threshold used to decide whether to reject } H_0$$
> $$\text{P-value } :$$
> $$\text{It's a probability value,when } H_0 \text{ is true}$$
> $$\text{the probability of extreme values}$$

3. What does the significance level in hypothesis testing mean?

$$\alpha = P\{Type\ I\ error\} = P\{Reject\ a\ true\ H_0\}$$

Problems

1)

$$H_0 : \mu = 5 | H_1 : \mu > 5$$
$$a. 1.42\ b. 0.90\ c. 1.96\ d. 2.48\ e. -0.11$$
$$find\ P\_\ in\ Z$$

$$P_{\_} = P\{\bar{X} \geq \bar{x} | \mu = 5\} = P(Z \geq z) = 1 - P(Z < z) = 1 - \phi(z)$$
$$P_a = 1 - \phi(1.42) \approx 1 - 0.9222 = 0.0778$$
$$P_b = 1 - \phi(0.9) \approx 1 - 0.8159 = 0.1841$$
$$P_c = 1 - \phi(1.96) \approx 1 - 0.975 = 0.025$$
$$P_d = 1 - \phi(2.48) \approx 1 - 0.9934 = 0.0066$$
$$P_e = 1 - \phi(-0.11) \approx 1 - 0.4562 = 0.5438$$

2)

$$n = 150, A = 82, \hat{p} = \frac{82}{150} | p = 0.4?$$

$$H_0 : p_0 = p = 0.4$$

$$H_1 : p \neq 0.4$$

$$\implies Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \backsim N(0,1)$$

$$\implies Z = \frac{0.5467 - 0.4}{\sqrt{\frac{0.4 \cdot 0.6}{150}}} = 3.6675$$

$$\alpha = 0.01$$

$$CV : Z > Z_{0.005} = 2.576$$

$$P_{-} = P\{\bar{X} \geq 0.5467 | p = 0.4\} = 1 - \phi(3.6675) = 0.0001 < 0.01$$

$$Accept\ H_0$$

$$\alpha = 0.05$$

$$CV : Z > Z_{0.025} = 1.96$$

$$P_{-} = P\{\bar{X} \geq 0.5467 | p = 0.4\} = 1 - \phi(3.6675) = 0.0001 < 0.05$$

$$Accept\ H_0$$

3)

$$X \backsim T_{n-1} \quad H_0 : \mu = 20 | H_1 : \mu > 20$$

> $$a)n = 15, t = 3.2, \alpha = 0.05$$
> $$CV : T_{0.05,15-1} \approx 1.761 < 3.2$$
> $$P_{-} = P(T_{15-1} \geq 3.2) = 0.0032 < 0.05$$
> $$reject$$

> $$b)n = 9, t = 1.8, \alpha = 0.01$$
> $$CV : T_{0.01,9-1} \approx 2.896 > 1.8$$
> $$P_{-} = P(T_{9-1} \geq 1.8) = 0.055 > 0.05$$
> $$accpet$$

> $$c)n = 24, t = 2.2$$
> $$if\ \alpha = 0.01$$
> $$CV : T_{0.01,24-1} \approx 2.5 > 2.2$$
> $$P_{-} = P(T_{24-1} \geq 2.2) = 0.019 > 0.01$$
> $$accpet$$
> $$if\ \alpha = 0.05$$
> $$CV : T_{0.05,24-1} \approx 1.714 < 2.2$$
> $$P_{-} = P(T_{24-1} \geq 2.2) = 0.019 < 0.05$$
> $$reject$$

4)

$$\{X_n\} = \{159\ 120\ 480\ 149\ 270\ 547\ 340\ 43\ 228\ 202\ 240\ 218\}$$
$$\bar{X} = 249.7, S = 145.1, \alpha = 0.05, \mu > 200?$$
$$\frac{S}{\sqrt{n}} = 41.887$$
$$H_0 : \mu_0 = 200 | H_1 : \mu > 200$$
$$T = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = 1.1865$$
$$CV : T_{0.05,12-1} \approx 1.796 > 1.1865$$
$$P_- = P(T_{12-1} \geq 1.1865) = 0.13 > 0.05$$
$$reject\ H_0$$

5)

$$\{X_n\} = \{112.3\ 97.0\ 92.7\ 99.2\ 95.8\ 103.5\ 86.0\ 89.0\ 102.0\ 86.7\}$$

$$a) X \backsim N(\mu, \sigma^2)?$$
$$\hat{\mu} = \bar{x} = 96.42\ \ \hat{\sigma} = S = 8.25$$
$$[86.0, 86.7, 89.0, 92.7, 95.8, 97.0, 99.2, 102.0, 103.5, 112.3] k = 10$$

Category	*class*	*Obs*	*Exp*	*Q*
$Z_{0.1}$	$(-\infty, 85.8)$	0	1	0.1
$Z_{0.2}$	$(85.8, 89.5)$	3	1	0.4
$Z_{0.3}$	$(89.5, 92.1)$	0	1	0.1
$Z_{0.4}$	$(92.1, 94.3)$	1	1	0
$Z_{0.5}$	$(94.3, 96.4)$	1	1	0
$Z_{0.6}$	$(96.4, 98.5)$	1	1	0
$Z_{0.7}$	$(98.5, 100.8)$	1	1	0
$Z_{0.8}$	$(100.8, 103.4)$	1	1	0
$Z_{0.9}$	$(103.4, 107)$	1	1	0
Z_1	$(107, \infty)$	1	1	0
		$n = 10$	$n = 10$	$Q_9 = 0.6$

n is not big

$$Q_9 = 0.6 \not> \chi^2_{10-1,0.05} = 16$$
$$reject$$

$$b) hypotheses\ \mu < 100$$
$$H_0 : \mu_0 = 100 | H_1 : \mu < 100$$
$$T = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = -0.1615$$
$$T_{9,0.05} = 1.833; P_- = P(Z < -0.1615) = \phi(-0.1615) = 1 - \phi(0.1615) = 0.438$$
$$|Z| < T_{9,0.05}\ and\ P_- < 0.05 \implies accpet$$
$$T_{9,0.01} = 2.821; P_- = P(Z < -0.1615) = \phi(-0.1615) = 1 - \phi(0.1615) = 0.438$$
$$|Z| < T_{9,0.05}\ but\ P_- < 0.01 \implies reject$$

6)

$$n=1361, \begin{array}{c|c|c|c} winter & spring & summer & fall \\ \hline 328 & 344 & 372 & 327 \end{array}$$

$$H_0: p_0 = 0.25 | H_1: p \neq 0.25$$

$$\alpha = 0.01$$

	$winter$	$spring$	$summer$	$fall$
p_i	0.25	0.25	0.25	0.25
X	328	344	372	327
E	340.25	340.25	340.25	340.25

$$Q_3 = \sum_{i=1}^{5} \frac{(X_i - np_i)^2}{np_i} = 0.294$$

$$\chi^2_{3,0.01} = 11.3449 > Q_3$$

$$accpect$$

7)

$$n = 541, stand = 30$$

$(0,25)$	$(25,30]$	$(30,\infty)$
262	159	120

$$\text{more than 20\% is fat?}$$

> $$a)H_0: p_0 = 0.2 | H_1: p > 0.2$$
>
> $$\alpha = 0.05$$
>
> | | $(0,25)(25,30]$ | $(30,\infty)$ |
> |---|---|---|
> | X | $262+159$ | 120 |
> | p | 0.8 | 0.2 |
> | E | 432.8 | 108.2 |
>
> $$\chi^2_{0.05,2} = 5.99$$
>
> $$Q_2 = \sum \frac{(X_i - E_i)^2}{E_i} = 1.61$$
>
> $$accept$$

> b)
>
> The first type of error : mistakenly believing that the proportion of obese individuals is greater than 20%, even though it is not.
>
> The second type of error : mistaken belief that the proportion of obese individuals is no more than 20%, even though it is

> $$\mu = 0.25, \hat{p} = X/N = 0.2$$
>
> $$Z = \frac{\hat{p} - \mu}{\sqrt{\frac{\mu(1-\mu)}{n}}} \backsim N(0,1) \implies Z = \frac{0.2 - 0.25}{\sqrt{\frac{0.25 \cdot 0.75}{n}}} = -0.115\sqrt{n}$$
>
> $$\text{if n } = 541, Z = -2.67, cdf = 0.0038$$