

## Neural Network Control of Mobile Manipulators

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### Abstract

In this paper, a novel neural-net (NN) based control methodology is developed for the motion control of mobile manipulators subject to kinematic constraints. The dynamics of the mobile manipulator is assumed to be unknown and is to be identified by the NN on-line estimators. No preliminary learning stage of NN weight matrices is required. The controller is capable of disturbance-rejection in the presence of unknown bounded disturbances. Closed-loop stability of the control system and convergence of the NN learning processes are both guaranteed. Experimental tests on a two-DOF manipulator arm illustrate that the proposed control is significantly better than conventional robust control.

### 1. Introduction

Mobile manipulators have been suggested for various applications, such as tasks involving hazardous environments, waste management, outdoor exploration, and space operations. In recent years, there has been a growing interest in the motion control of mobile manipulators (see [1] for a general survey on this subject). Most of previous approaches require a precise knowledge of the dynamics of mobile manipulators, or, they simplify the dynamics model by ignoring complex dynamics, such as vehicle dynamics, payload dynamics, dynamics interactions between the base and the arm, or unknown disturbances such as dynamics caused by terrain irregularity. Nonetheless, the precise dynamics model of a mobile manipulator is normally unobtainable. However, when the mobile manipulator moves at a relatively high speed, ignoring the vehicle dynamics and the dynamics interactions between the arm and the base may cause unbearable vibrations in the system [2]. Finally, when the robot moves on uneven terrain (e.g., for outdoor exploration), ignoring disturbances generated by terrain irregularities may cause tip-over (instability).

In recent years, neural networks, with their strong learning capability, have proven to be a suitable tool of controlling complex nonlinear dynamical systems. They have been widely adopted in the modeling and control of robotic manipulators (See [3] for a survey). The basic idea behind NN-based control is to use NN estimators to identify and compensate for the unknown dynamics. The NN-based approach can deal with the

control of nonlinear systems that may not be linearly parameterizable. Very few results reported have been proven experimentally in the domain of NN control.

In this paper, an NN-based control methodology is proposed for the position control of a mobile manipulator. Two NN controllers are developed to control the arm and the base independently. The NN controllers identify on-line and compensate for unknown dynamics caused by parameter uncertainty and dynamical coupling. No preliminary learning stage is required for the NN weight matrices. The control scheme is capable of disturbance-rejection in the presence of bounded disturbances. Also, experimental results are presented to illustrate the effectiveness of the proposed theory in real applications.

The paper is organized as follows. The necessary mathematical preliminaries and the dynamics of mobile manipulators subject to kinematic constraints are given in Section 2. The proposed theory is developed in Section 3. Section 4 presents the experimental results to verify the theory. Some remarks are made in the final section.

### 2. Mathematical Preliminaries

Let  $\delta(\varepsilon)$  denotes the real numbers,  $\mathbb{R}^n$  the real  $n$  vector. The norm of a vector  $x \in \mathbb{R}^n$  is defined as

$$\|x\| = \sqrt{x^T x}. \quad (1)$$

The norm of a matrix is defined as

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)}, \quad (2)$$

where  $\lambda_{\max}(\circ)$  is the maximum eigenvalue. The Frobenius norm is defined by

$$\|A\|_F^2 = \text{tr}(A^T A). \quad (3)$$

The trace of matrix  $A$ , i.e.,  $\text{tr}(A)$ , satisfies  $\text{tr}(A) = \text{tr}(A^T)$  for  $A \in \mathbb{R}^{n \times n}$ . For any  $B \in \mathbb{R}^{m \times n}$  and  $C \in \mathbb{R}^{n \times m}$ ,  $\text{tr}(BC) = \text{tr}(CB)$ . Suppose that  $A$  is positive definite, then for any  $B \in \mathbb{R}^{m \times n}$ ,

$$\text{tr}(BAB^T) \geq 0, \quad (4)$$

with equality iff  $B$  is the  $m \times n$  zero matrix, and

$$\frac{d}{dt} \{\text{tr}(A(t))\} = \text{tr}\left(\frac{dA(t)}{dt}\right). \quad (5)$$

Consider a nonlinear system  $\dot{x} = f(x, t)$ , where  $x \in \mathbb{R}^n$ ,  $A$  solution  $x(t)$  is said to be uniformly bounded

with respect to  $B(0,d)$ , a hyperball of radial  $d$  centered at the origin of the state space, if there exists a positive constant  $\delta(\varepsilon)$  such that  $\|x_0\| < \delta(\varepsilon)$  implies  $\|x(t)\| \leq d$  for all time. If  $\delta = \infty$ , the solution is said to be *globally uniformly bounded* [3].

The dynamics of a mechanical system subject to kinematic constraints can be obtained using a Lagrangian approach in the form [3]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) + A^T(q)\lambda + \tau_d = E(q)\tau, \quad (6)$$

with  $r$  kinematic constraints described by

$$A(q)\dot{q} = 0, \quad (7)$$

where  $q \in \mathcal{R}^n$  denotes the  $n$  generalized coordinates;  $M(q) \in \mathcal{R}^{n \times n}$  the symmetric and positive definite inertia matrix;  $C(q, \dot{q}) \in \mathcal{R}^{n \times n}$  the centripetal and Coriolis matrix;  $F(q, \dot{q}) \in \mathcal{R}^n$  the friction and gravitational vector;  $A(q) \in \mathcal{R}^{r \times n}$  the constraint matrix denoting  $r$  kinematic constraints;  $\lambda \in \mathcal{R}^r$  the Lagrange multiplier vector which denotes the vector of constraint forces;  $E(q) \in \mathcal{R}^{n \times r}$  the input transformation matrix;  $\tau_d$  denotes bounded unknown disturbances including unstructured dynamics; and  $\tau \in \mathcal{R}^r$  is the torque input vector.

The generalized coordinates  $q$  may be separated into two subsets  $q = \begin{pmatrix} q_v^T & q_r^T \end{pmatrix}^T$ , where  $q_v \in \mathcal{R}^m$  describes the generalized coordinates appearing in the constraint equations (6), and  $q_r \in \mathcal{R}^n$  the free generalized coordinates free of them. Therefore, (7) can be simplified to

$$A_v(q_v)\dot{q}_v = 0, \quad (8)$$

with  $A_v(q_v) \in \mathcal{R}^{r \times m}$ . Assume that the robot is fully actuated, and therefore, (6) can be further rewritten as

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \ddot{q}_v \\ \ddot{q}_r \end{pmatrix} + \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \dot{q}_v \\ \dot{q}_r \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} + \begin{pmatrix} A_v^T(q_v)\lambda \\ 0 \end{pmatrix} + \begin{pmatrix} \tau_{d1} \\ \tau_{d2} \end{pmatrix} = \begin{pmatrix} E_v \tau_v \\ \tau_r \end{pmatrix}, \quad (9)$$

where  $\tau_v \in \mathcal{R}^{m-r}$  represents the actuated torque vector of the constrained coordinates, those related to the constrained motion of the wheels, the joints, and the end-effector. For simplicity in the theoretical derivation, hereinafter we consider only the case where the vehicle motion is constrained. However, the proposed theory can be easily extended to include joint or end-effector constraints.  $E_v \in \mathcal{R}^{m \times (m-r)}$  represents the input transformation matrix,  $\tau_r \in \mathcal{R}^n$  the actuated torque

vector on the free coordinates,  $\tau_{d1}$  and  $\tau_{d2}$  are disturbance torques bounded by  $\|\tau_{d1}\| \leq \tau_{N1}$  and  $\|\tau_{d2}\| \leq \tau_{N2}$ , with  $\tau_{N1}$  and  $\tau_{N2}$  positive constants. The constraint equation (8) and the first row of (9) form the vehicle sub-dynamics, and the second row of (9) represents the arm sub-dynamics.

Let  $S_v(q_v) \in \mathcal{R}^{m \times (m-r)}$  denote a full rank matrix formed by  $(m-r)$  smooth vector fields that span the null space of  $A_v(q_v)$  defined in (9), i.e.,

$$S^T(q_v)A_v^T(q_v) = 0. \quad (10)$$

From (10), we can find an auxiliary vector  $v(t) \in \mathcal{R}^{m-r}$  such that for all  $t$ ,

$$\dot{q}_v = S(q_v)v(t). \quad (11)$$

The vehicle sub-dynamics and arm sub-dynamics can further be obtained as follows.

*Vehicle sub-dynamics:*

$$\bar{M}_{11}\ddot{v} + \bar{C}_{11}\dot{v} + S^T(M_{12}\ddot{q}_r + C_{12}\dot{q}_r + F_1 + \tau_{d1}) = \bar{\tau}_v, \quad (12)$$

where  $\bar{M}_{11} = S^T M_{11} S$ ,  $\bar{C}_{11} = S^T C_{11} S + S^T M_{11} \dot{S}$ , and  $\bar{\tau}_v = S^T E_v \tau_v$ .

Equation (11) and (12) represent the complete vehicle sub-dynamics;

*Arm sub-dynamics:*

$$M_{22}\ddot{q}_r + C_{22}\dot{q}_r + (M_{21}\ddot{q}_v + C_{21}\dot{q}_v + F_2 + \tau_{d2}) = \tau_r. \quad (13)$$

In this paper, we assume that reference joint-space trajectories have already been derived based on desired task-space trajectories. The main concern is to provide joint-torque input that can guarantee stable joint-space tracking in the presence of parameter uncertainty and unknown disturbances. The dynamics interaction between the base and the arm is also taken into account in the consideration of vibration inhibition.

### 3. Theory

Consider the vehicle sub-dynamics represented by (11) and (12). Tracking control of the steering system (11) has been well investigated in the literature [4][5]. Define the tracking error as  $\tilde{q}_v = q_{vd} - q_v$ . Then, there exist a *Lyapunov* function  $V_1(\tilde{q}_v, t)$ , a positive continuous function  $W_1(t) > 0$ , and a reference feedback velocity control law  $v = \alpha(\cdot)$ , such that

$$\left. \frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial \tilde{q}_v} \dot{\tilde{q}_v} \right|_{\dot{q}_v = S(q_v)\alpha} \leq -W_1(t) \text{ when } \tilde{q}_v \neq 0. \quad (14)$$

The objective hereinafter is to derive proper torque input  $\tilde{\tau}_v(t)$  in (12), such that  $v$  tracks the reference velocity  $\alpha(\cdot)$ .

Define the velocity tracking error  $z$  as

$$z = v - \alpha. \quad (15)$$

Differentiating (15) and multiplying both sides by  $\bar{M}_{11}$  yields

$$\bar{M}_{11}\dot{z} = \bar{M}_{11}\dot{v} - \bar{M}_{11}\dot{\alpha} \quad (16)$$

(16) represents the vehicle dynamics in terms of tracking errors.

Now consider the arm sub-dynamics (13). Define the tracking error and its derivative as

$$e = q_{rd} - q_r, \quad \dot{e} = \dot{q}_{rd} - \dot{q}_r. \quad (17)$$

Define the filtered tracking error as

$$r = \dot{e} + ke, \quad (18)$$

where  $k = k^T > 0$ . Similarly, the arm sub-dynamics can be expressed in terms of the tracking error  $r$  as:

$$M_{22}\dot{r} = -\tau_r + (M_{22}k - C_{22})(r - ke) + M_{21}\ddot{q}_v + C_{21}\dot{q}_v + M_{22}\ddot{q}_{rd} + C_{22}\dot{q}_{rd} + F_2 + \tau_{d2} \quad (19)$$

Choose the *Lyapunov* function as

$$V = V_1 + \frac{1}{2} \begin{pmatrix} Sz \\ -r \end{pmatrix}^T M \begin{pmatrix} Sz \\ -r \end{pmatrix}. \quad (20)$$

Differentiating (20) along system trajectories, and after lengthy derivations, one obtains

$$\dot{V} \leq -W_1(t) + z^T \{\bar{\tau}_v - \Psi_1\} + r^T \{-\tau_r + \Psi_2\}, \quad (21)$$

with the unknown nonlinear terms

$$\begin{aligned} \Psi_1 &= \bar{M}_{11}\dot{\alpha} + \bar{C}_{11}\alpha + \bar{f}_1 + S^T \{C_{12}ke + M_{12}k(r - ke)\} \\ \Psi_2 &= M_{22}kr + (C_{22} - M_{22}k)ke + \bar{f}_2 + M_{21}S\dot{\alpha} \\ &\quad + M_{21}S\alpha + C_{21}S\alpha \end{aligned} \quad (22)$$

where

$$\begin{aligned} \bar{f}_1 &= S^T \{M_{12}\ddot{q}_{rd} + C_{12}\dot{q}_{rd} + F_1 + \tau_{1d}\} \\ \bar{f}_2 &= M_{22}\ddot{q}_{rd} + C_{22}\dot{q}_{rd} + F_2 + \tau_{d2} \end{aligned} \quad (23)$$

The nonlinear terms  $\Psi_1$  and  $\Psi_2$  in (22) are identified on-line using NN estimators. In the development of NN on-line estimators, *Radial Basis Function* (RBF) networks with fixed centers and widths are employed. It is not difficult to extend the results to the case when *Multiple-Layer Perceptions* (MLP) networks are employed.

In light of strong nonlinear approximation ability of RBF networks,  $\Psi_1$  and  $\Psi_2$  can be identified using RBF nets with sufficiently many hidden-layer-neurons:

$$\begin{aligned} \Psi_1 &= W_1^T h_1(x) + \varepsilon_1(x) \\ \Psi_2 &= W_2^T h_2(y) + \varepsilon_2(y) \end{aligned} \quad (24)$$

where  $W_1 \in \mathbb{R}^{n_1 \times n}$ ,  $W_2 \in \mathbb{R}^{n_2 \times n}$  are  $n_1 \times n$  and  $n_2 \times n$  constant ideal weight matrices, respectively;  $x$  and  $y$  are input patterns of the neural networks, which are determined based on the properties of the functions.  $n_1$  and  $n_2$  are the numbers of hidden-layer neurons in each approximation network; the estimation errors  $\varepsilon_1 \in \mathbb{R}^n$ ,  $\varepsilon_2 \in \mathbb{R}^n$  are bounded by  $\|\varepsilon_1\| \leq \varepsilon_{1N}$  and

$\|\varepsilon_2\| \leq \varepsilon_{2N}$ , with  $\varepsilon_{1N}$  and  $\varepsilon_{2N}$  two positive constants:

$h_1(x) \in \mathbb{R}^{n_1}$  and  $h_2(y) \in \mathbb{R}^{n_2}$  are properly chosen basis functions for the hidden-layer neurons. One candidate for the basis functions is *Gaussian* functions defined as

$$\varphi_i(x) = \exp\left(-\frac{\|x - c_i\|^2}{\sigma_i^2}\right), \quad i=1,2,\dots,N \quad (25)$$

where  $c_i$  are centers and  $\sigma_i$  are widths that are chosen *a priori* and kept fixed. Therefore, only the weight matrices  $W_1$  and  $W_2$  are adjustable during the learning process.

The estimations of  $\Psi_1$  and  $\Psi_2$  are given by

$$\begin{aligned} \hat{\Psi}_1 &= \hat{W}_1^T h_1(x) \\ \hat{\Psi}_2 &= \hat{W}_2^T h_2(y) \end{aligned} \quad (26)$$

The torque input for (12) and (13) is chosen as

$$\begin{aligned} \bar{\tau}_v &= -k_1 z + \hat{\Psi}_1 \\ \tau_r &= k_2 r + \hat{\Psi}_2 \end{aligned} \quad (27)$$

and the weight updating laws for the two neural nets are

$$\begin{aligned} \dot{\hat{W}}_1 &= -\beta_1 h_1 z^T \\ \dot{\hat{W}}_2 &= \beta_2 h_2 r^T \end{aligned} \quad (28)$$

where  $\beta_1 > 0$ ,  $\beta_2 > 0$  are the learning rates of the two neural networks. The main contribution of this paper is located in the following theorem.

**Theorem:** The control law (27) and NN learning law (28) guarantee the global uniform boundedness of all tracking errors and weight estimation errors.

**Proof:** The proof is straightforward after choosing the overall Lyapunov function to be

$$\bar{V} = V + \frac{1}{2\beta_1} \text{tr}\{\tilde{W}_1 \tilde{W}_1^T\} + \frac{1}{2\beta_2} \text{tr}\{\tilde{W}_2 \tilde{W}_2^T\}, \quad (29)$$

where  $V$  is defined in (20), and  $\tilde{W}_1 = W_1 - \hat{W}_1$ ,  $\tilde{W}_2 = W_2 - \hat{W}_2$  are the weight estimation errors.

#### 4. Experiments

Extensive experiments have been performed with a real manipulator arm in order to verify the effectiveness of the proposed NN controller, specifically, NN-based arm controller.

The experiments were carried out on the IRIS RoboTwin (Fig. 1), which is a modular and reconfigurable robot first introduced in [6]. RoboTwin is a dual arm that comprises at least two robotic manipulators, each having several modular rotary joints. In our experiments, two joints were used for the purpose of verifying the proposed control schemes. The dynamics parameters of the arm are assumed to be completely unknown. The robot is equipped with high precision position sensors. The joint velocities are estimated using numerical position differentiation.

Two groups of experiments were implemented to

verify the proposed NN controller. To illustrate their effectiveness more convincingly, additional experiments were performed with the saturation-type control (STC) [3] employed for comparison. All controllers were assumed to have the same PID compensator. Therefore, the experimental results directly show the effects of the nonlinear control portion implemented by including the STC or NN controllers.

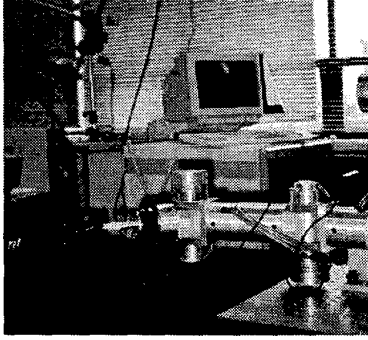


Fig.1. IRIS RoboTwin

The structure of the saturation-type controller (STC) is defined by the following [3]:

$$\tau = k_v \dot{e} + k_p e + v_r(p, e, \dot{e}, \varepsilon), \quad (30)$$

where  $e$  is the position errors,  $k_v$  the differential gains,  $k_p$  the proportional gains, and

$$v_r = \begin{cases} (e/2 + \dot{e})p(\|e/2 + \dot{e}\|)^{-1} & \text{if } \|e/2 + \dot{e}\| > \varepsilon \\ (e/2 + \dot{e})p^2 / \varepsilon & \text{if } \|e/2 + \dot{e}\| \leq \varepsilon, \end{cases} \quad (31)$$

$$p = \delta_0 + \delta_1 \|e\| + \delta_2 \|\dot{e}\|^2$$

where the parameters  $\varepsilon$ ,  $\delta_0$ ,  $\delta_1$  and  $\delta_2$  must be designed uniquely for a specific robot arm. In the experiments, the stability of the closed-loop system is found to be highly sensitive to all these parameters: a small variation in any of them may cause instability, e.g., arm-shaking. No such pathological phenomena were observed in the experiments when the proposed NN controller was used. However, due to the inaccuracy of estimated velocity signals by using numerical position differentiation, high-frequency noises were introduced into the control loop as shown in the figures.

Fig. 2 to Fig. 5 show the experimental results from tracking a sinusoidal curve using STC and NN controller, respectively. The left column in the figures shows the results of using the STC controller, and the right column shows those of NNC. A RBF network with 10 input neurons, 40 hidden neurons and 2 output neurons is constructed to estimate the unknown nonlinearity on line. The input pattern of the NN

controller is chosen as  $(q_{rd}, \dot{q}_{rd}, \ddot{q}_{rd}, e, r)$ ; and the outputs of the RBF net represent the compensation torques. The weight matrices of the RBF net are simply initialized as zeros and are updated timely at each control cycle, with the system sampling frequency set at 500HZ. and the learning rate of the NN weight matrices chosen as 0.4. It is shown that the tracking performance is improved over time using the proposed on-line NN controller, which verifies the self-learning and adaptation capability of the proposed NN learning law; while STC does not provide such advantage. The comparison illustrates that NNC works significantly better than STC.

## 5. Conclusions

In this paper, a systematic methodology has been developed for the tracking control of general mobile manipulators subject to kinematic constraints using *neural networks*. The dynamics of the mobile manipulator is assumed to be completely unknown and is identified on-line by NN estimators. The dynamics interactions between the mobile platforms and the mounted manipulator arms were taken into account. The controller has the capability of treating unknown and bounded disturbances. No preliminary learning stage of NN weight matrices is required. Global uniform boundedness of all tracking errors is guaranteed. Experimental results verify the effectiveness of the proposed NN controller. More experiments will be carried out on a real mobile manipulator to verify the proposed combined arm/vehicle NN controllers.

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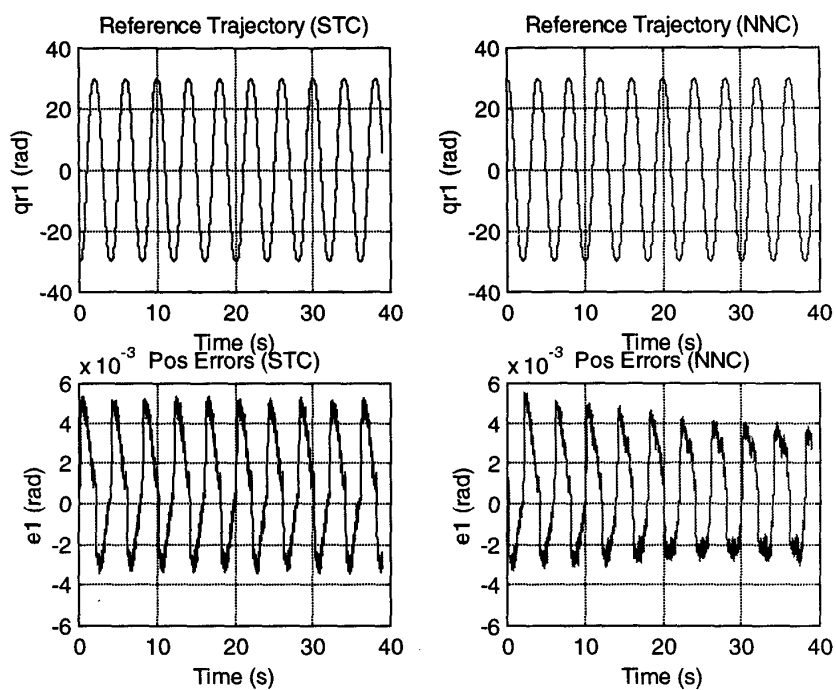


Fig.2. Desired Trajectory and Position Errors (Joint 1)

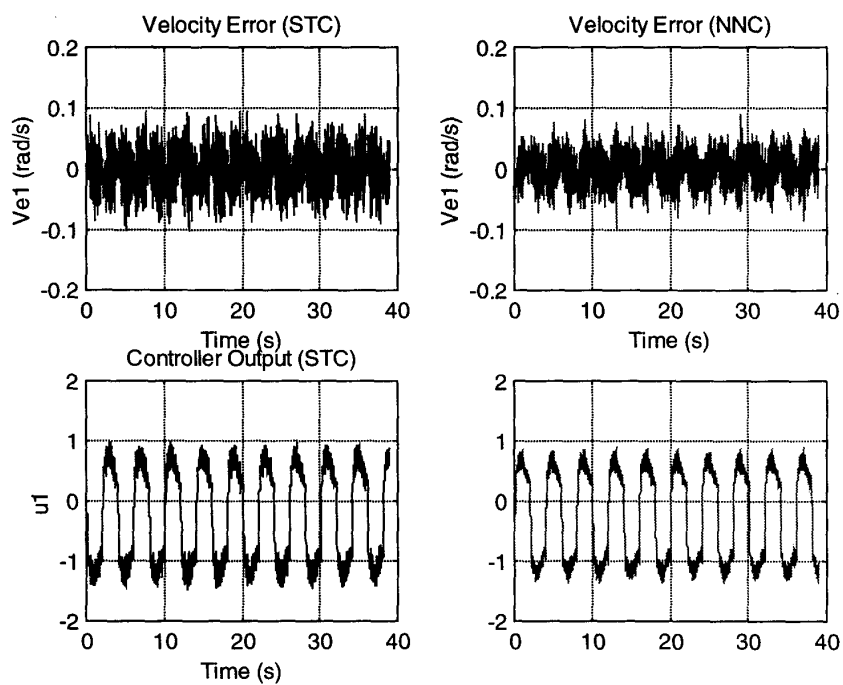


Fig.3. Velocity Errors and Controller Outputs (Joint 1)

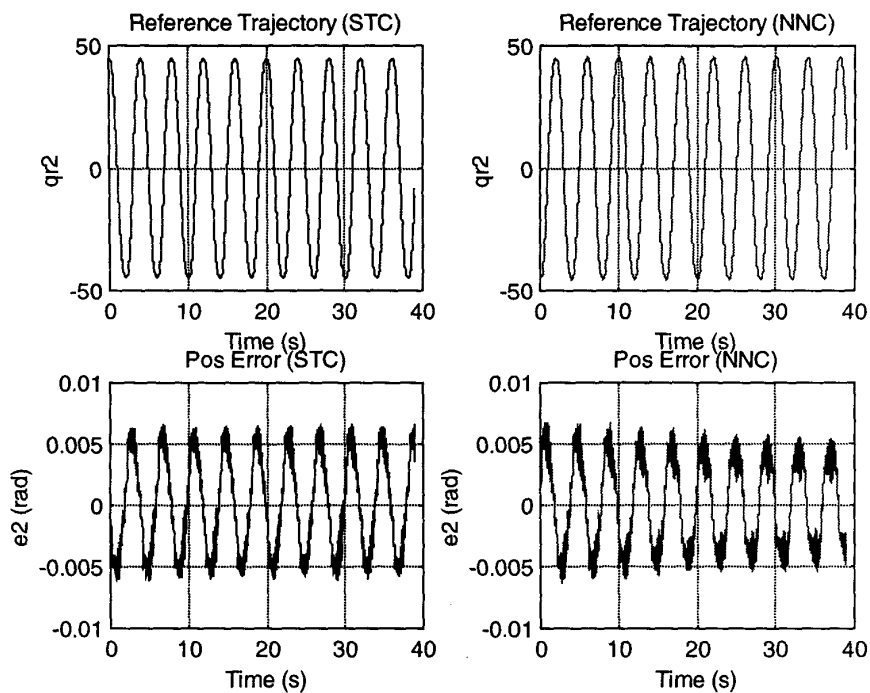


Fig.4. Desired Trajectory and Position Errors (Joint 2)

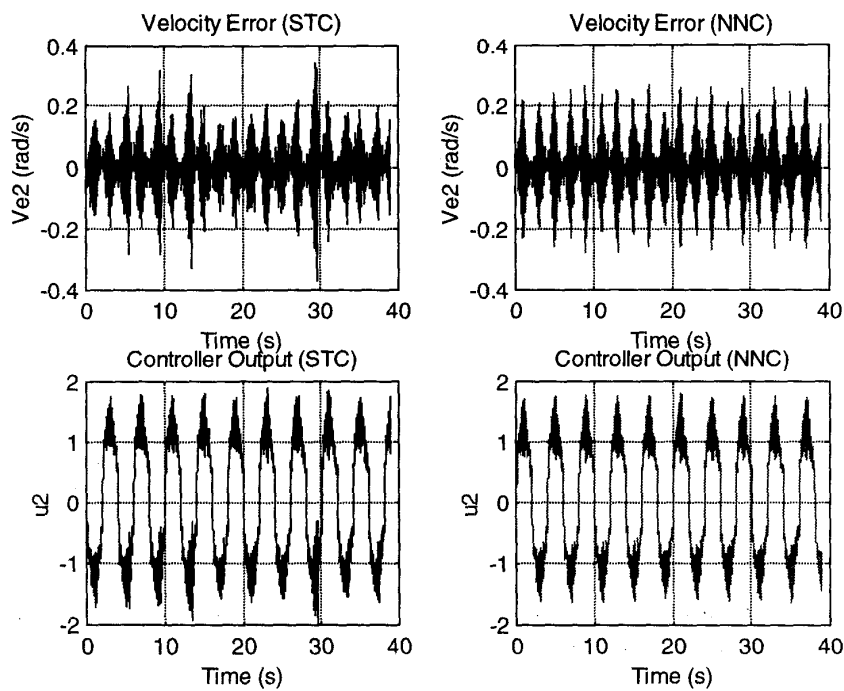


Fig.5. Velocity Errors and Controller Outputs (Joint 2)