1.

$$X \backsim N(\mu, \sigma^2), \sigma \ is \ give.$$

a

$$egin{aligned} ar{x} \pm 2.81 \cdot rac{\sigma}{\sqrt{n}}, 100\% (1-lpha) = ? \ & \hat{\mu} = ar{x} \ & Z = rac{ar{x} - \mu}{rac{\sigma}{\sqrt{n}}} \backsim N(0,1) \ & P(Z_{1-rac{lpha}{2}} < Z < Z_{rac{lpha}{2}}) = P(Z < 2.81) - P(Z < -2.81) = 0.9974 - 0.0026 = 0.9948 \ & 100\% (1-lpha) = 99.48\% \end{aligned}$$

b

$$ar{x}\pm 1.44\cdot rac{\sigma}{\sqrt{n}}, 100\%(1-lpha)=?$$
 
$$P(Z_{1-rac{lpha}{2}}< Z< Z_{rac{lpha}{2}})=P(Z<1.44)-P(Z<-1.44)=0.925-0.075=0.85$$
 
$$100\%(1-lpha)=85\%$$

C

$$egin{aligned} 100\%(1-lpha) &= 99.7\%, \ Z_{rac{lpha}{2}} = ? \ &lpha = 0.003 \implies rac{lpha}{2} = 0.0015 \ CDF: F_Z(z) &= P(Z < z) = \int_{-\infty}^z rac{1}{\sqrt{2\pi}} \ e^{-rac{t^2}{2}} dt \ F_Z(z) &= P(Z < z) = 1 - 0.0015 = 0.9985 \ Z_{rac{lpha}{2}} &= Z_{0.9985} = F_Z^{-1}(0.9985) = 2.96 \end{aligned}$$

d

$$egin{align} 100\%(1-lpha) &= 75\%, \;\; Z_{rac{lpha}{2}} = ? \ &lpha = 0.25 \implies rac{lpha}{2} = 0.125 \ &F_Z(z) = P(Z < z) = 1 - 0.125 = 0.825 \ &Z_{rac{lpha}{2}} = Z_{0.825} = F_Z^{-1}(0.825) = 1.5 \ \end{align}$$

2.

$$X \backsim N(\mu, 0.75^2)$$

a

$$egin{align} n = 20, ar{x} = 4.85, 100\% (1-lpha) = 95\%, ( heta_L, heta_U) = ? \ Z_{0.025} = 1.96 \implies ar{x} \pm Z_{0.025} \cdot rac{\sigma}{\sqrt{n}} = 4.85 \pm 0.3287 \ ( heta_L, heta_U) = (4.5213, 5.1787) \ \end{cases}$$

b

$$n=16, ar{x}=4.56, 100\%(1-lpha)=98\%, ( heta_L, heta_U)=?$$
 $Z_{0.001}=2.326 \implies ar{x}\pm Z_{0.001}\cdot rac{\sigma}{\sqrt{n}}=4.56\pm 0.436$ 
 $( heta_L, heta_U)=(4.124, 4.996)$ 

C

$$length = E(\theta_L, \theta_U) = 0.4, 100\%(1-lpha) = 95\%, n = ?$$
 $E(\theta_L, \theta_U) = 2 \cdot Z_{\frac{lpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.4 \implies \sqrt{n} = 7.35$ 
 $n = 54$ 

d

$$egin{align} error &= rac{E( heta_L, heta_U)}{2} < 0.2, 100\%(1-lpha) = 99\%, n = ? \ &rac{E( heta_L, heta_U)}{2} = Z_{rac{lpha}{2}} rac{\sigma}{\sqrt{n}} < 0.2 \implies \sqrt{n} > 12.88 \ &n = 165.6 pprox 166 \ \end{aligned}$$

3.

$$X\backsim N(\mu,\sigma^2), \bar{x}=654.16, S=164.43, n=50$$

a

$$egin{align} 100\%(1-lpha) &= 95\%, ( heta_L, heta_U) =? \ ar{x} &\backsim N(\mu, rac{\sigma}{\sqrt{n}}) \implies rac{ar{x}-\mu}{rac{\sigma}{\sqrt{n}}} \backsim N(0,1), \sigma \ is \ not \ give \ ar{rac{x}-\mu} & \backsim t_{n-1} \implies P(t_{1-rac{lpha}{2},n-1} < T < t_{rac{lpha}{2},n-1}) \implies ar{x} \pm T_{rac{lpha}{2},n-1} \cdot rac{\sigma}{\sqrt{n}} \ T_{rac{lpha}{2},n-1} &= 2.0227 \implies ar{x} \pm T_{rac{lpha}{2},n-1} \cdot rac{\sigma}{\sqrt{n}} = 654.16 \pm 2.0227 \cdot rac{164.43}{\sqrt{50}} \ ( heta_L, heta_U) &= (607.1243, 701.1957) \ \end{array}$$

b

$$S=175,100\%(1-lpha)=95\%, E( heta_L, heta_U)=50, n=? \ E( heta_L, heta_U)=2\cdot T_{rac{lpha}{2},n-1}rac{S}{\sqrt{n}}\implies 50=2\cdot T_{rac{lpha}{2},n-1}rac{175}{\sqrt{n}}\implies n=49\cdot T^2 \ \left\{ egin{align*} n~is~big &\Longrightarrow T_{rac{lpha}{2},n-1}=1.96 \ n~is~small &\Longrightarrow T_{rac{lpha}{2},n-1}=2.262 \end{array} 
ight. \Longrightarrow \left\{ ar{T}=2.111 \ T_{rac{lpha}{2},50-1}=2.01 \end{array} 
ight. \Longrightarrow \left\{ egin{align*} n=49\cdot ar{T}pprox 219 \ n=49\cdot T_{rac{lpha}{2},50-1}pprox 198 \end{array} 
ight.$$

$$n=356, suc=201, 100\%(1-lpha)=95\%, ( heta_L, heta_U)=? \ rac{x_i \mid 0}{\mid 1-p \mid p} \ E(x_i)=p, D(x_i)=p(1-p) \ n\cdot p>10, n(1-p)>10, n>30 \implies n \ is \ big \ X\backsim N(P,rac{p(1-p)}{n}) \implies rac{ar{x}-p}{\sqrt{rac{p(1-p)}{n}}}\backsim N(0,1) \ Z_{0.025,356}=1.96, \hat{p}=rac{201}{356} \implies rac{201}{356}\pm 1.96\sqrt{rac{201}{356}\cdotrac{155}{356}}=0.5726\pm 0.0515 \ ( heta_L, heta_U)=(0.5211,0.6231)$$

5.

$$\mu, T_{\frac{\alpha}{2},n-1} = ?, \sigma \ is \ not \ give$$

a

$$100\%(1-lpha) = 99\%, n = 5$$
  $\mu \implies rac{ar{x}-\mu}{rac{S}{\sqrt{n}}} \backsim t_{n-1}, T_{rac{lpha}{2},n-1} = P(1-rac{lpha}{2})$   $T_{0.005.4} = 4.604$ 

b

$$100\%(1-\alpha) = 98\%, n = 38$$

$$T_{0.001,37} = 2.431$$

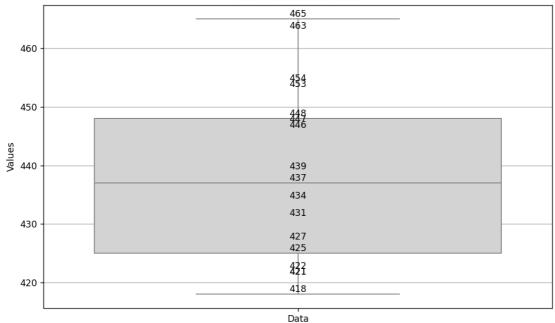
6.

$$x_n = [418, 434, 454, 421, 437, 463, 421, 439, 465, 422, 446, 425, 447, 427, 448, 431, 453,]$$

a

$$[418,421,421,422,425,427,431,434,437,439,446,447,448,453,454,463,465]$$
 
$$Q_2=437,Q_1=425,Q_3=448$$
 
$$IQR=Q_3-Q_1=23$$
 
$$Error=[Q_1-1.5IQR,Q_3+1.5IQR]=[390.5,482.5]$$
 No anomalies, moderate dispersion





b

$$H_0: x \backsim N(\mu, \sigma^2) \ H_1: x \not\backsim N(\mu, \sigma^2) \ lpha = 0.05 \ W = rac{(\sum a_i x_i)^2}{\sum (x_i - ar{x})}, \ a_i = rac{m_i}{\sqrt{\sum m_j^2}}, \ m_i = \phi^{-1}(rac{i - 0.375}{n + 0.25}) \ use \ python, W_{n=17} = 0.9541, P_- = 0.28 > lpha \ accept \ H_0$$

C

$$100\%(1-lpha)=95\%, n=17, \ 440, \ 450\in( heta_L, heta_U)?$$
  $ar{x}=436.24, S=12.5$   $ar{x}\pm T_{rac{lpha}{2},n-1}rac{S}{\sqrt{n}}=436.24\pm2.12rac{12.5}{\sqrt{17}}=436.24\pm6.427$   $( heta_L, heta_U)=(429.813,442.637)$   $440\in(429.813,442.637),450
otin (429.813,442.637)$ 

**7**.

$$x_{n} = [211, 16, 29, 35, 42, 24, 24, 35], \sigma^{2} \& \sigma \text{ of } (\theta_{L}, \theta_{U})$$

$$n = 8, S^{2} = 37.54, \alpha = 0.05, \chi_{0.025,7}^{2} = 1.69, \chi_{0.975,7}^{2} = 16.013, \chi_{0.95,7}^{2} = 14.067$$

$$\hat{\sigma}^{2} = S^{2} \implies \frac{(n-1)S^{2}}{\sigma^{2}} \backsim \chi_{n-1}^{2} \implies \chi_{1-\frac{\alpha}{2},n-1}^{2} < \frac{(n-1)S^{2}}{\sigma^{2}} < \chi_{\frac{\alpha}{2},n-1}^{2}$$

$$\implies \frac{(n-1)S^{2}}{\chi_{\frac{\alpha}{2},n-1}^{2}} < \sigma^{2} < \frac{(n-1)S^{2}}{\chi_{1-\frac{\alpha}{2},n-1}^{2}} \implies (17.69, 147.21)$$

$$\hat{\sigma} \implies \sqrt{\frac{(n-1)S^{2}}{\chi_{\frac{\alpha}{2},n-1}^{2}}} < \sigma < \sqrt{\frac{(n-1)S^{2}}{\chi_{1-\frac{\alpha}{2},n-1}^{2}}} \implies (25.075, 72.33)$$

$$\begin{split} x_n &= [2.0, 15.7, 1.3, 0.7, 6.0, 4.8, 1.9, 1.9, 5.1, 12.2, 0.4, 5.3, 1.0, 0.6, 5.3]; n = 15, \\ X &\sim Exp, 100\% (1-\alpha) = 95\%, \;\; (\theta_L, \theta_U) = ? \\ &\bar{x} = 4.21 \\ &(\frac{2n\bar{x}}{\chi_{1-\frac{\alpha}{2},n-1}^2}, \frac{2n\bar{x}}{\chi_{\frac{\alpha}{2},n-1}^2}) = (\frac{30\times4.21}{46.98}, \frac{30\times4.21}{16.79}) = (2.69, 7.52) \\ &H_0: x \backsim Exp \\ &H_1: x \not\sim Exp \\ &H_1: x \not\sim Exp \\ &\alpha = 0.05 \\ &CV = \sqrt{-\frac{1}{2n}\ln(\alpha)} = 0.316 \\ &D_n = [EDF(x_i) - F(x_i)]_{MAX} \approx 0.0624 < CV \\ &accept \; H_0 \end{split}$$