MA 677 Homework 5 MLE

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1. ① $n=43$ to have 5.

$\hat{p}_{MLE} = \frac{5}{43}$.

② $\hat{p}_{MLE} = \frac{2}{58}$.

2. $X_1 \sim X_n \sim U(0,\theta)$

$L(\theta|x) = \prod_{i=1}^{n} \frac{1}{\theta} = \theta^{-n}$, $\frac{\partial \cdot L(\theta|x)}{\partial \theta} = -\frac{n}{\theta} < 0$

$\therefore \hat{\theta} = X_n$

3. $X_1 \sim X_n \sim N(\mu, \sigma^2)$

MLE: $\mu + 1.64\sigma$.

4. $X_1 \sim X_n \sim N(\mu, \sigma^2)$, $MLE = P(X>2) = P(Z > \frac{2-\mu}{\sigma})$

$= 1 - \phi(\frac{2-\mu}{\sigma})$

5. $f(x_i|\theta) = \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2}$ $L(\theta, -x) = -n\log\pi - \Sigma \log(1+(x_i-\theta)^2)$

$\therefore \frac{\partial L(\theta|x)}{\partial \theta} = 0$. $\hat{\theta} = 0.407$

6. $\hat{\mu} = 6.75$

7. $X_1 \sim X_n \sim Poi(\lambda)$, $f(x|\lambda) = \frac{e^{-n\lambda}\lambda^{\Sigma x_i}}{x_1! \sim x_n!}$ $\rightarrow \hat{\lambda} = \frac{\Sigma x_1}{n}$.

$\hat{sd} = \sqrt{\lambda} = \sqrt{\frac{\Sigma x_1}{n}}$.

8. $X_1 \sim X_n \sim exp(\beta)$ $\therefore l(\beta|x) = -n \cdot \ln(\beta) - \frac{1}{\beta}\Sigma x_i$

$\frac{\partial l}{\partial \beta} = 0$. $\therefore \hat{\beta} = \frac{\Sigma x_i!}{n}$.

$\therefore$ median $= \ln 2 \cdot \sqrt{\frac{\Sigma x_i!}{n}}$.