Homework 2 Chaoqun Yin U55151797

1. $Y = 1, 2, 3, \cdots, 8$.

$$P_Y(k) = \begin{cases} \frac{1}{42}, & k=1 \\ \frac{2}{42} = \frac{1}{21}, & k=2 \\ \frac{6}{42} = \frac{1}{7}, & k=3 \\ \frac{4}{42} = \frac{2}{21}, & k=4 \\ \frac{6}{42} = \frac{1}{7}, & k=6 \\ \frac{7}{42} = \frac{1}{6}, & k=7 \\ \frac{16}{42} = \frac{8}{21}, & k=8 \end{cases}$$

$$EY = \sum_1^k k \cdot P_Y(k) = \frac{252}{42} = 6.$$

2. $E(XY) = \int_0^1 \int_0^1 xy f(x,y)\, dy\, dx$

$$= \int_0^1 \int_0^x xy(12y^2)\, dy\, dx.$$

$$= \int_0^1 \left(12x \cdot \frac{y^4}{4}\right]_0^x dx$$

$$= \int_0^1 3x^5\, dx = \frac{1}{2}.$$

3. $E[(X_1 - 2X_2 + X_3)^2] = \int_0^1 \int_0^1 \int_0^1 (X_1 - 2X_2 + X_3)^2 \, dx_1 dx_2 dx_3$

$$= \int_0^1 \int_0^1 \left(\frac{1}{6} - 2x_2 + x_3 + 4x_2^2 - 4x_2x_3 + x_3^2\right) dx_2 dx_3$$

$$= \int_0^1 \left(\frac{1}{3} - 1 + x_3 + \frac{4}{6} - 2x_3 + x_3^2\right) dx_3$$

$= \frac{1}{3} - 1 + \frac{1}{2} + \frac{4}{5} - 1 + \frac{1}{3} = \frac{1}{2}$

4. $E(e^{\frac{3}{4}X}) = \int_0^{+\infty} e^{\frac{3}{4}x} \cdot e^{-x} dx$

$= \int_0^{+\infty} e^{-\frac{1}{4}x} dx$

$= [-4e^{-\frac{1}{4}x}]_0^{\infty} = 4.$

5. $EY = \frac{1}{6}(3+9+19+33+51+73)$

$= \frac{94}{3}.$

6. $E(Y^2) = E[(2X+1)^2] = E(4X^2+4X+1)$ $= 4EX^2 + 4EX + 1$

$EX = \int_0^1 x f(x) dx = \int_0^1 x(2-2x) dx = \frac{1}{3}.$

$EX^2 = \int_0^1 x^2 f(x) dx = \int_0^1 x^2(2-2x) dx = \frac{1}{6}$

$E(Y^2) = 4 \times \frac{1}{6} + 4 \times \frac{1}{3} + 1 = 3.$

7. $E[(aX+b)^n] = E[\sum_{k=0}^{n} \binom{n}{k} (aX)^{n-k} \cdot b^k] = E[\sum_{i=1}^{n} \binom{n}{i} a^{n-i} X^{n-i} b^i]$

$= \sum_{i=0}^{n} \binom{n}{i} a^{n-i} \cdot b^i E(X^{n-i}).$

8. $EX = np \quad EY = n(1-p) = n - np.$

$E(X-Y) = EX - EY = 2np - n. \quad n = 20, p = 0.05.$

$E(X-Y) = 2 \times 20 \times 0.05 - 20 = -18.$