MA 677 Homework 6 Hypothesis Testing.

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1. (a) If $\beta \geq 1$, $p = \alpha(\beta) = P(X \geq 1) = 1 - (1 - e^{-\lambda}) = e^{-\frac{1}{\beta}}$.

If $\beta < 1$, $p = \beta(\beta) = 1 - P(X < 1) = e^{-\frac{1}{\beta}}$.

$\therefore$ power function $= e^{-\frac{1}{\beta}}$.

(b) $\alpha = \sup_{\beta \geq 1} e^{-\frac{1}{\beta}} = 1$.

2. If $p = 0.2$

power $= \alpha(p) = P(Y \geq 7) + P(Y \leq 1) = 1 - P(Y \geq 7) + P(Y \leq 1)$

≈ 0.156.

If $p \neq 0.2$

power $= \beta(p) = 1 - P(1 < Y < 7) = 1 - \sum_{i=1}^{6} C_{20}^{i} p^{i} (1-p)^{20-i}$

when $p = 0$, $\beta(p) = 1$; when $p = 0.1$, $\beta(p) = 0.394$; ...

... when $p = 0.9$ or 1, $\beta(p) = 1$

$\therefore \alpha = 0.156$

3. $\alpha(\mu) = P(T(X) > c) = P(|\bar{X}_n - \mu_0| > c) = 0.05$.

$\therefore p = 0.05$.

$\sqrt{n} \cdot c = 1.96$, $n = 25$ $\therefore c = 0.392$

4. (a) $P(Y \leq c_1) + P(Y \geq c_2) < 0.1$

$\rightarrow \sum_{i=0}^{c_1} C_9^i 0.4^i 0.6^{9-i} + 1 - \sum_{i=0}^{c_2-1} C_9^i 0.4^i 0.6^{9-i} < 0.1$

when $c_1 = 1$ & $c_2 = 7$, $p = 0.1$.

(b) $\alpha(0.4) = 0.1$