MA 677 Homework 4 Chaoqun Yin

1. MLE of P: $\frac{58}{70} = 0.83$.

2. $X_1 - X_n \sim Bern(\theta) = \theta^x (1-\theta)^{1-x}$

$L(\theta, x_1 - x_n) = \theta^{\Sigma x} (1-\theta)^{n-\Sigma x}$.

$l(\theta, x_1 - x_n) = \Sigma x \log(\theta) + (n - \Sigma x) \log(1-\theta)$

$\Rightarrow \hat{\theta} = \frac{\Sigma x}{n}$. equal to 0 or 1.

3. $X_1 - X_n \sim Poisson(\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$.

$l(\lambda, x_1 - x_n) = \prod_{i=1}^{n} \exp(-\lambda) \frac{1}{x_i} \lambda^{x_i}$

$\frac{d\, l(\lambda, x_1 - x_n)}{d\lambda} = 0 \Rightarrow \hat{\lambda} = \frac{\Sigma x_i}{n} = \overline{X}_n$. If every observed value is 0, then MLE of λ equals 0.

4. $X_1 - X_n \sim N(\mu, \sigma^2)$.

$L(\mu, \sigma, x_1 - x_n) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{\Sigma}(x_i - \mu))^2$

$\frac{d(\mu, \sigma, x_1 - x_n)}{d\sigma} = -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{\sigma^3} = 0$

$\Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$.