# Monte Carlo Simulation of NFIP Surplus

Chanyang Ryoo, Xinlang Yue

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# Introduction: How NFIP Works

Because the United States suffers from large hurricanes, the government created an insurance program that assists homeowners called the National Flood Insurance Program (NFIP). There are about 5 million policyholders who are covered under this insurance, and there is about 3.6 billion dollars in revenue from premium every year [1]. Just like any insurance company, NFIP has two main types of cash flow: premium income and claims liability.

The uniqueness of NFIP is that it issues what is called a catastrophe bond, or 'cat bond' for short. This is a major way of transferring risk to the capital market; in other words, this is how NFIP obtains an initial surplus to operate their insurance coverage. When a bond is sold, then a fixed amount of money is given to the bond issuer, and in return, the bond issuer gives coupon payments (interest) to the bond buyers and returns the whole amount at maturity. Another uniqueness about NFIP's operation is that it is reinsured. Reinsurance is insurance coverage for an insurance company. Due to the large amount of claims both in number and magnitude, NFIP also needs insurance. The reinsurers are private companies who receive annual premium from NFIP and in return, cover a fraction of NFIP's claims. The condition on the reinsurer's coverage is that the claim is a single event that exceeds a threshold amount (e.g. 8 billion). There are specific rates of coverage depending on the magnitude of the claim. In the past, NFIP would borrow large amounts of money from the US Treasury in cases of big losses. But starting from 2017, NFIP started to rely on reinsurers and a limit is soon going to be set on the amount of money that it can borrow from US Treasury (1 billion) [2].

Another way of possibly increasing income for NFIP is investing its premium and initial surplus in the capital markets. However, the difference between the NFIP and other private insurance companies is the sheer amount of policyholders and claims. This will affect the effectiveness of investments.

# Model

## Without reinsurance or rare large event:

First we summarize the model in words. The model estimates the monthly surplus of NFIP. Therefore, it makes sense to write the model as a difference equation. In this way, we can model easily the effect of incremental cash inflow and outflow. In the beginning month, we will receive the proceeds from the cat bond sale. We will keep $p_1$ of the proceeds as cash, invest $p_2$ in stocks and $p_3$ in bonds. Each month we receive premium $P$ from $N$ policyholders and keep $p_1$ as cash and invest as we did the initial proceeds (into the *same* stock and bond). The stock and bonds produce income in the form of capital gain and coupon each month as well. Each month there will be liabilities in the form of claims, coupon payments to NFIP's bond buyers, and operating costs. So all in all we have:

$$U(t) = U(t-1) + \text{INC}(t) - \text{LIA}(t)$$

where INC($t$) is the income and LIA($t$) is the liability, and

$$\begin{cases} \text{INC}(t) & = \text{PREMIUM} + \text{BOND} + \text{STOCKS} \\ \text{PREMIUM} & = p_1 NP \\ \text{BOND} & = \mathbb{O}(t > t_\mu) \cdot p_3 U_0 \frac{\kappa}{12} + \mathbb{O}(t > t_\mu)\mathbb{I}(t \geq 2) \cdot p_3 NP \frac{\kappa}{12} + \\ & \mathbb{I}(t = t_\mu)(p_3 U_0 + (t_\mu - 1) p_3 NP) \\ \text{STOCKS} & = R(t,\omega)(p_2 U_0) + \mathbb{I}(t \geq 2) \cdot R(t,\omega)(p_2 NP) \\ \text{LIA}(t) & = C(t,\omega) + \mathbb{O}(t > t_M) \frac{\hat{c}}{12} U_0 + \mathbb{I}(t = t_M) U_0 + Op \end{cases}$$

$$
\text{with}
\begin{cases}
U(t) & \text{is surplus at time } t, \\
U_0 & \text{is initial surplus from selling cat bond,} \\
\kappa & \text{is the coupon rate of bond investment} \\
t_\mu & \text{is the maturity date of bond investment} \\
R(t,\omega) & \text{is interest rate from SP 500,} \\
P & \text{is monthly premium from insurance holders,} \\
Op & \text{is monthly operating fees, i.e. 2019 standard 14.69 million} \\
p_1 & \text{is the percentage of surplus held as cash,} \\
p_2 & \text{is the percentage of surplus held as stock,} \\
p_3 & \text{is the percentage of surplus held as bonds,} \\
N & \text{is the number of insurance holders (customers)} \\
C(t,\omega) & \text{is estimated claim amount at month } t \\
\hat{c} & \text{is the coupon rate to investors} \\
\mathbb{I}(T(t)) & \text{is the indicator function that } T(t) \text{ is true} \\
\mathbb{O}(T(t)) & \text{is 0 when } T(t) \text{ is true}
\end{cases}
$$

The initial condition will be

$$U(0) = U_0 - (p_2 + p_3)U_0 = p_1 U_0.$$

**Investment: Stock model**

The stock pricing model follows the Black Scholes model:

$$S(t,\omega) = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right).$$

We choose $S_0$ to be the initial price of the historical data set. Here $W_t$ is random Brownian motion. The parameters $\mu, \sigma$ are chosen by looking at the historical monthly prices and taking their average and standard deviation, respectively.

To take the return percent from this, we simply take:

$$R(t,\omega) = \frac{S(t+1,\omega) - S(t,\omega)}{\Delta t}.$$

**Claims**

The monthly claims model follows the compound Poisson process:

$$C(t,\omega) = \sum_{i=0}^{N(t,\omega)} X_i(t,\omega)$$

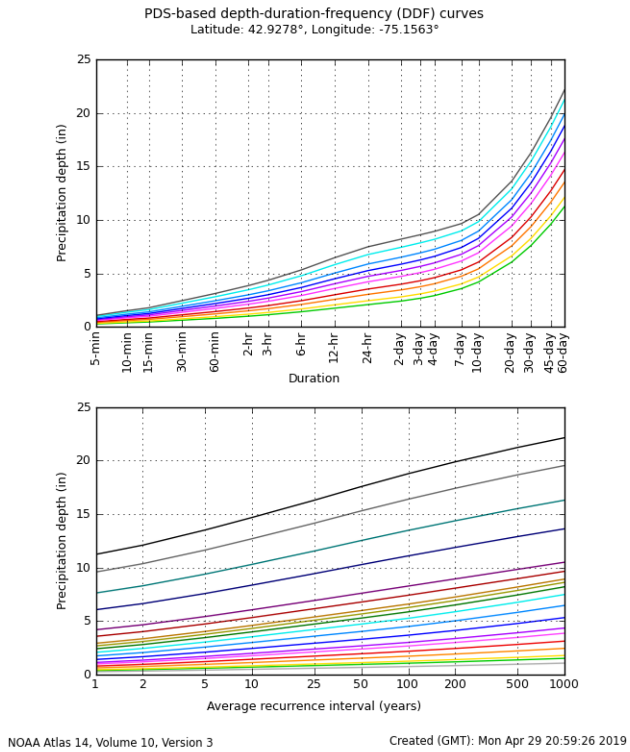
$$N(t,\omega) \sim \mathrm{Po}(\lambda_t)$$

$$X_i(t,\omega) \sim N(\mu_t,\sigma)$$

where $N(t,\omega)$ is the number of claims modeled after Poisson process with parameter $\lambda_t$, which will be sample means of historical numbers of claims for each month over 2017-18, and $X_i(t,\omega)$ will be modeled after $N(\mu_t,\sigma)$, where $\mu_t$ is the historical average claim amount for each month over same period, $\sigma$ is heuristically chosen.

## With Reinsurance and rare large event:

The only addition to the previous model here will be the annual reinsurance premium and the coverage of reinsurance. Here we make two assumptions. First we use NOAA's return curve to estimate the average occurrence of a one week storm with 7 inches of rainfall–our model rare, large event. We see that such a storm happens every 25 years, or every 300 months, or a probability of 0.0033.




We also assume that such an event will have a damage around 80 billion dollars, and we give the loss a normal distribution with mean 80 billion and standard deviation 1 billion. This is chosen heuristically and also by taking note of the losses of large hurricanes such as Harvey, Katrina, Sandy and Irma [3]. Second assumption is that the reinsurer will cover 56 % of the loss, which is a reasonable estimate [4]. This model does not take into account the 'step nature' of the reinsurance programs, where the coverage is different for different intervals of losses (e.g. for loss between \$8-10 billion, coverage is 40 %, and for loss between \$10-12 billion, coverage is 50 %).

We modify the model as follows:

$$\begin{cases} U(t) = & U(t-1) + \mathrm{INC}(t) - \mathrm{LIA}(t) - \text{Reinsurance Cost} + \text{Reinsurance Coverage} \\ U(0) = & U_0 - (p_2 + p_3)U_0 = p_1 U_0 \end{cases}$$

where everything remains same except:

$$\begin{cases} \text{Reinsurance cost} = & \mathbb{I}(t \bmod 12 = 0) P_{re} \\ \text{Reinsurance coverage} = & \mathbb{I}(U[0,1] < 0.0033)(0.54\mathcal{C} - \mathcal{C}) \end{cases}$$

and $\begin{cases} P_{re} & \text{is the premium of reinsurance} \\ \mathcal{C} = \mathcal{N}(8^{10}, 10^9) & \text{is the claim model of big event} \end{cases}$

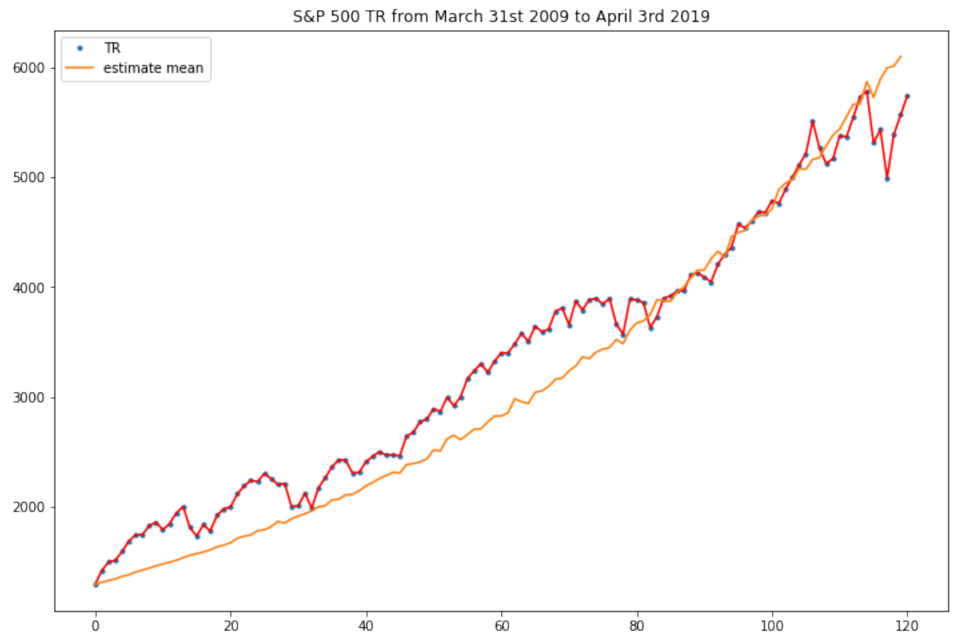
# Verification

Because we do not have access to NFIP's bank account, we cannot exactly compare the result of our model with its real time surplus. However, we can verify components of the model. Recall the model:

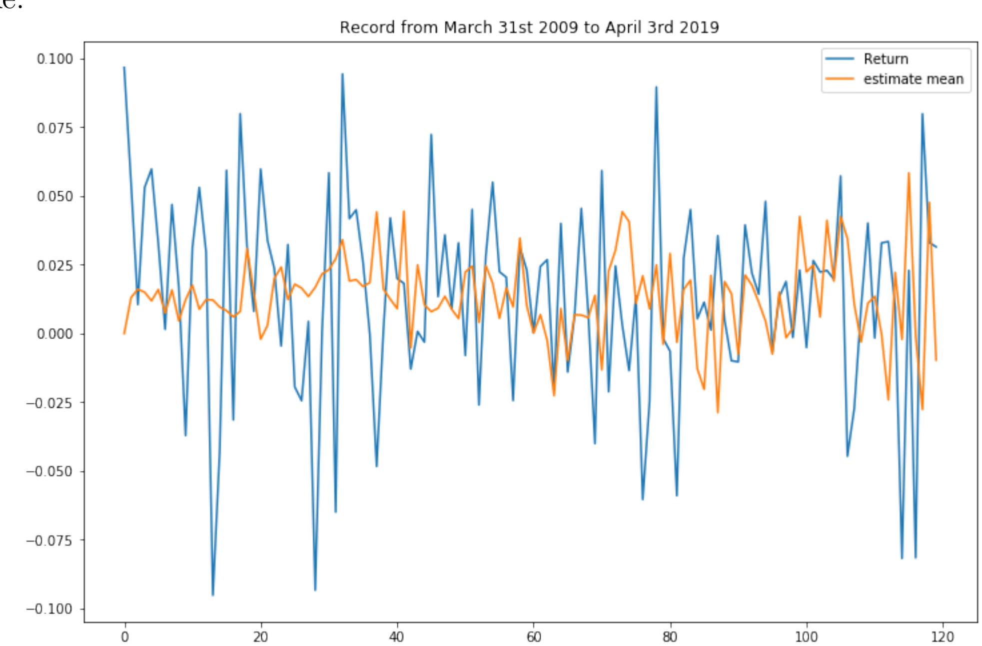
$$U(t) = U(t-1) + \text{INC}(t) - \text{LIA}(t)$$

$$\begin{cases} \text{INC}(t) & = \text{PREMIUM} + \text{BOND} + \text{STOCKS} \\ \text{PREMIUM} & = p_1 NP \\ \text{BOND} & = \mathbb{O}(t > t_\mu) \cdot p_3 U_0 \frac{\kappa}{12} + \mathbb{O}(t > t_\mu)\mathbb{I}(t \geq 2) \cdot p_3 NP \frac{\kappa}{12} + \\ & \mathbb{I}(t = t_\mu)(p_3 U_0 + (t_\mu - 1)p_3 NP) \\ \text{STOCKS} & = R(t,\omega)(p_2 U_0) + \mathbb{I}(t \geq 2) \cdot R(t,\omega)(p_2 NP) \\ \text{LIA}(t) & = C(t,\omega) + \mathbb{O}(t > t_M)\frac{\hat{c}}{12} U_0 + \mathbb{I}(t = t_M) U_0 + Op \end{cases}$$

First we can verify the model for stock investment return $R(t,\omega)$. To do so we compare our stock pricing model $S(t,\omega)$ first to historical data. This model is a stochastic process which will give different results every simulation, so we take the mean of 1000 simulations and compare the mean to the historical data, and we obtain the following plot:




Now we also check the mean of 1000 simulations of $R(t, \omega)$, which looks like:




Second we can verify the model for the total cost of claims in a year with the historical data given by FEMA. We can accumulate the total cost of the claims by adding up the monthly claim model and we obtain a figure of $8.82 \times 10^9$ dollars whereas FEMA gives $8.7 \times 10^9$ dollars for the year of 2017 [5].

# Numerical Experiment

Four types of numerical experiments were conducted, along with estimates of final surplus distribution and the value at risk. The important value is the probability of ruin, which is the probability that the wealth of NFIP will drop below 0. This can be estimated by Monte Carlo methods, namely by simulating many times and seeing how many times the simulation gave a scenario of bankruptcy.

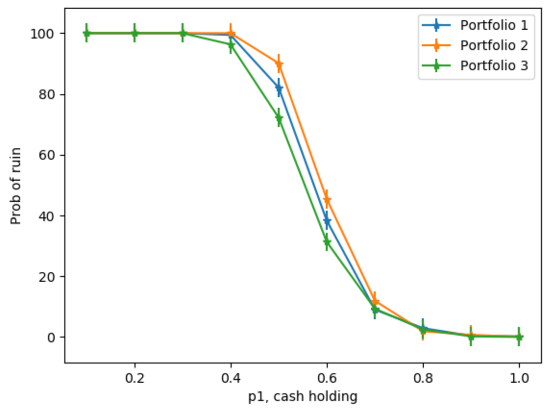
## Experiment 1: Portfolio evaluation

There are three types of investment practices for the purpose of our project: (1) equal ratio of stocks and bonds, (2) more bonds than stocks (3:1), or (3) more stocks than bonds (again 3:1). Then we can let $p_1$, the cash holding amount, vary and see how the probability of ruin changes in a one dimensional plot.

**Parameter Setup:**

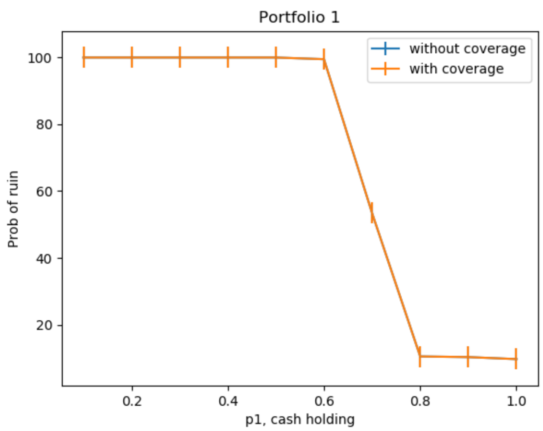
As for the parameters, we set our $U_0 = 800 \times 10^6$ as NFIP issued a cat bond of that amount, $\kappa$, which is the rate of coupon for the invesments NFIP makes, to be some number between 10-11%, and $\hat{c}$, which is the coupon rate for the investors of cat bond, to be some number between 8-10%. We lowered the coupon rate for the investors of cat bond compared to the bond investments of NFIP to lower the risk of ruin. There are about $N = 5 \times 10^6$ policyholders covered by NFIP. We set the premium to be 80 dollars and we look over a period of 28 months, with maturity time for cat bond being 24 and maturity time for NFIP bond investments being 23. We set the maturity times as such so that the bond investments of NFIP would mature earlier than the cat bond, which is more desirable since it frees up more capital for NFIP to give back to the cat bond investors.

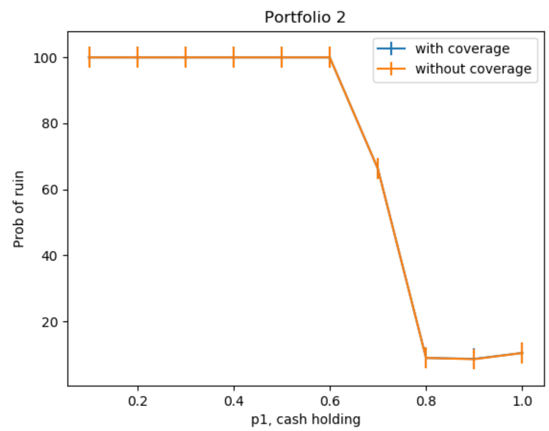
The resulting plot is

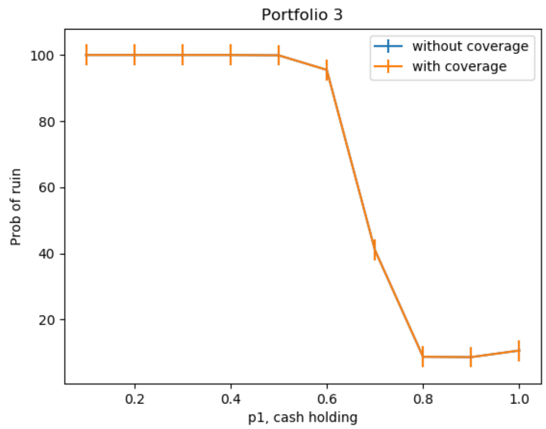



We can see that portfolio 3, the one with more stock investments than bond does the best, since it has overall the lowest probability of ruin throughout the $p_1$ interval.

For model with reinsurance: we obtain



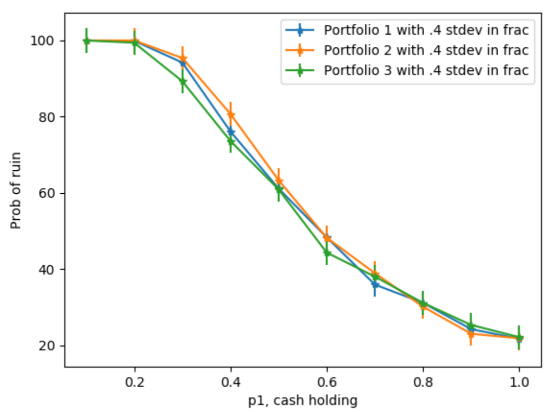






The similar result of more cash holding, lower probability of ruin is still seen with the reinsurance model. However, there is a slight preference toward having 80% cash, rather than having all cash, as was predicted in the model without reinsurance.

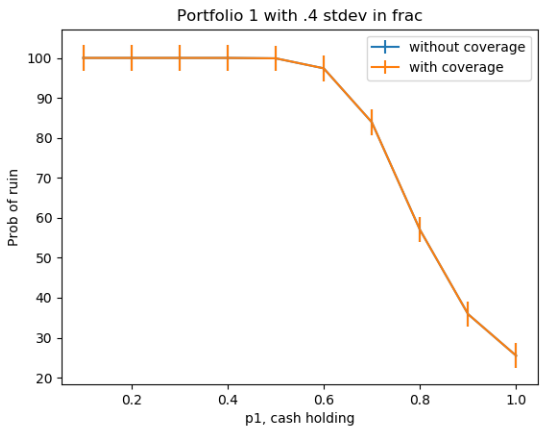
## Experiment 2: Portfolio evaluation with high uncertainty in number of premium payers

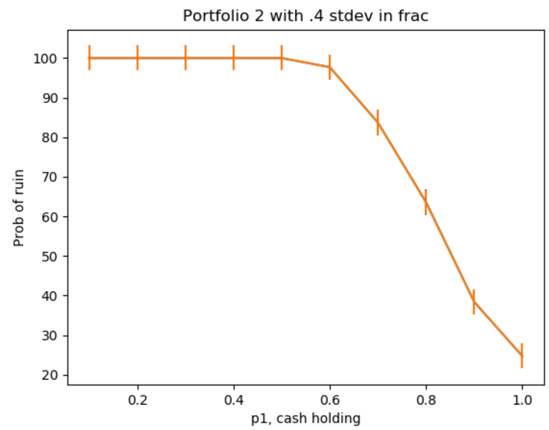
Here we see if the portfolio's performance changes when there is uncertainty in the number of payers. The number of policyholders is modeled after normal distribution with mean of 80% of $N$ and standard deviation of 40%. The resulting plot is:

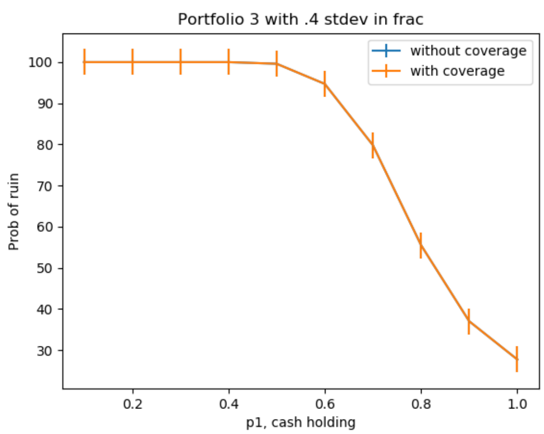



In this experiment, we see a similar performance throughout all the porfolios. Also, the lower bound of the overall curves is increased by 20%. This is because the main source of income which is premium is negatively affected by this uncertainty.

For model with reinsurance, we obtain:



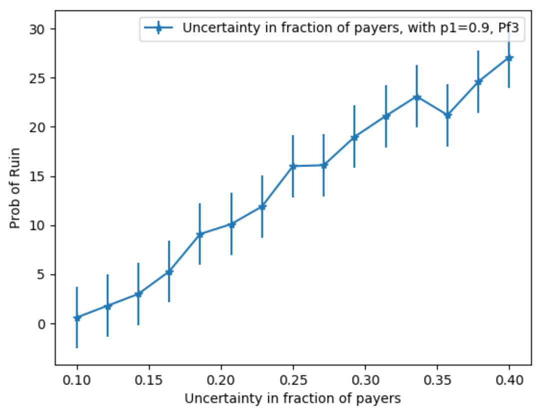




The overall results compared to those of the model without reinsurance are similar. The lowest probability of ruin increases to 20% and the performance is similar throughout all the portfolios.

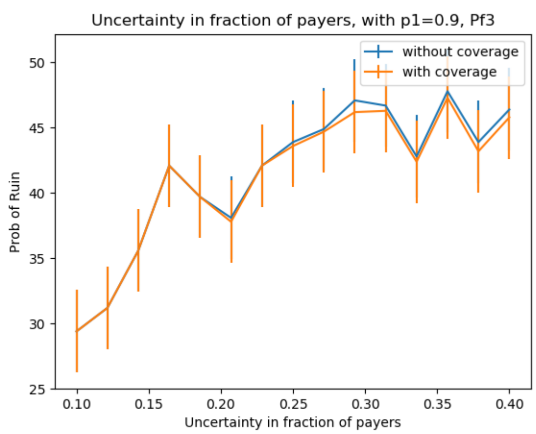
## Experiment 3: Uncertainty in fraction of payers

In this experiment, we more closely examine the effect of the increasing standard deviation in the model of the number of premium payers on the probability of ruin. We choose to observe the probability of ruin using portfolio three with cash holding $p_1$ of 0.9.




We can see that the uncertainty in the number of premium payers plays a big role in determining probability of ruin. This shows that steadiness of premium inflow plays a big role in determining probability of ruin.

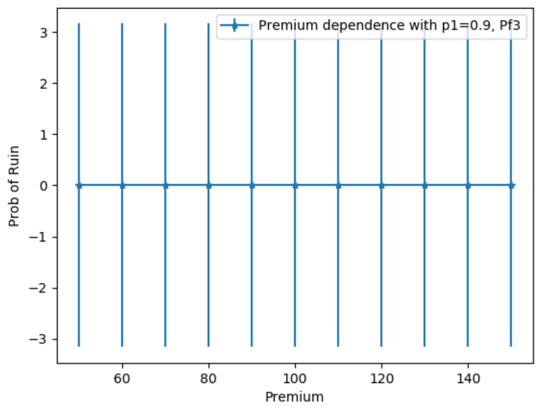
For model with reinsurance, we obtain:




The coverage provided by reinsurance lowers the probability of ruin slightly as uncertainty in fraction of premium payers increases.

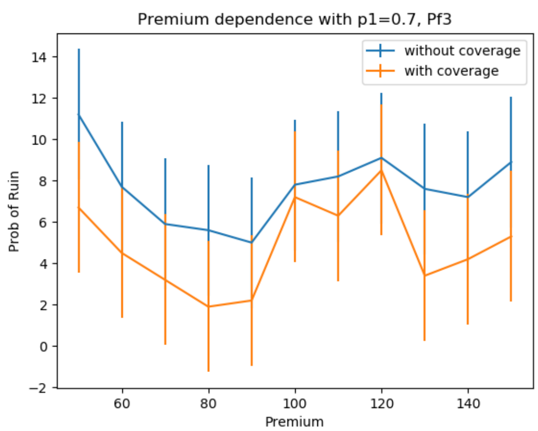
## Experiment 4: Premium

In addition to the steadiness of premium payers, we observe the effect of the amount of the premium. In this experiment, we assume that most policy-holders will pay their premium ($\sim N(0.9, 0.1)$).




Contrary to what one might expect, if there is a steady flow of premium, the probability of ruin will constantly be low regardless of the actual premium amount.

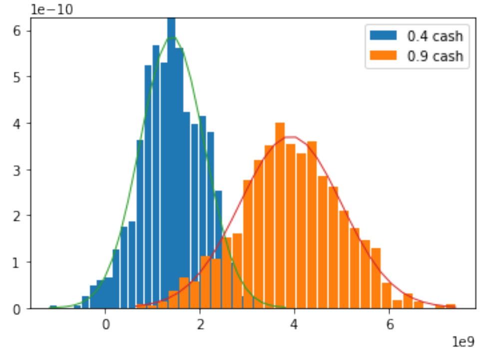
For model with reinsurance, we obtain:




We can clearly see the benefit of having reinsurance coverage here. The parameter set is using portfolio 3 with cash holding of $p_1 = 0.7$. There is about four percent in improvement in the probability of ruin.

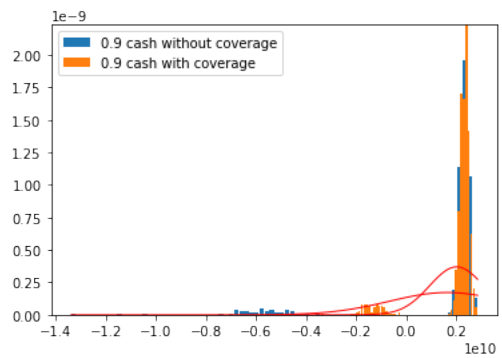
## Final surplus distribution

Our results indicate that holding more cash is safer to manage all the claims. We wanted to then observe if the surplus amount at the final time (in our case, 28) would be higher or lower depending on whether we hold more cash in our investment practice. Again we choose portfolio 3. The result is:




We see that holding more cash also results in a distribution of final surplus with a higher mean. However, the width of the distribution is wider when holding more cash. This may be because even though the initial surplus in the case of holding more cash is higher (and therefore the probability of ruin is lower), there is less income from investments that offset the claims. So in essence the surplus can be depleted much or little depending on the magnitude of the random claims. In the case of having less cash and more investments, the incomes offset the claims and the surplus stays near the vicinity of where the initial surplus was.

In the model with reinsurance, we have:




We see that although NFIP does not gain more profit at the end by having a reinsurance, it saves a lot in losses, indicated by the left tail.

## Value at Risk

The value at risk is a way of measuring risk, that gives a percentage of confidence and an idea of the magnitude of loss outside that level of confidence.

So if we have a high percentage of confidence and low value of loss, then the risk is low. In our experiment we evaluated the value at risk as follows:

$$\mathrm{VaR}_{1-p} = \max_n \{ \min_t U^{(n)}(t) : \text{where } U^{(n)} \text{ is such that } \exists t^*, U^{(n)}(t^*) < 0 \}$$

After running many simulations, we select the simulations that had a bankruptcy. We take the maximum loss (the most negative) from such simulation. Then we count how many simulations had a bankruptcy (the rate of which is given by the estimate probability of ruin, $p$). If we select the lowest loss $L$ (the least negative) of this subset of simulations, then we say that we are $1-p$ percent confident that the loss will not exceed $L$. This $L$ is then our $\mathrm{VaR}_{1-p}$.

From our simulation, we see that without reinsurance coverage, our value at risk with 90.7 percent confidence is $4.09 \times 10^9$. With reinsurance, our value at risk with the same percent of confidence is $456 \times 10^6$. So we see that our value at risk decreases ten-fold when we have reinsurance.

# Conclusion

In conclusion, we summarize our findings and the rationale behind them.

First we have found that whether or not NFIP obtains reinsurance, it is best to have more than half of surplus as cash. We believe this is due to the amount of claims that come in, i.e. the large magnitude of liabilities requires that there be lot of cash at NFIP's disposal. Interestingly, there is a preference for 80% cash when there is a risk of large calamity and NFIP is reinsured.

Second we have found that having reinsurance reduces the value at risk by ten times. The reinsurance coverage absorbs a significant portion of the large claim in the case of a rare event.

Third we have found that the probability of ruin does not depend so much on the amount of premium, as long as there is constant flow of premium from the policyholders. There are about 5 million policyholders, and their premium payments can add up to a large sum even with monthly rate of $60, for example.

Fourth we have found that in the case where most policyholders pay their premium, investing more in stocks than bonds seems to lower the probability of ruin more than having more bonds than stocks. This is probably because our model assumes that NFIP invests in the same bond every time it invests $p_3$ of its premium, which decreases in value as time goes on (because of maturity). However, the stock market has no such problems; most of the time, there is a good profit that can be made from the stock market based on the Black Schole based return rates.

Finally we have found that uncertainty in the number of premium payers adversely affects NFIP's probability of ruin. At an uncertainty level of 40%, there is almost no possibility of achieving near 0 probability of ruin, regardless of which portfolio is used. This shows that NFIP must securely insure that most of its policyholders pay the required premium to avoid bankruptcy.

# Future improvement

For future addition or improvement to the model, we can add possibility of stock market crashing since more stock seems to do better in general. Or we can allow the investment model to invest in different bonds with different maturities, so that investment is not made to a bond that decreases in value. Also we can add the complex nature of actual reinsurance model.

# Codes

All the codes can be found at the authors' Github links: `https://www.github.com/cr2940/UQ` or `https://www.github.com/xlyue92/UQ`.

# Reference


[1] "Facts and Statistics: Flood Insurance" by Insurance Information Institute, `https://www.iii.org/fact-statistic/facts-statistics-flood-insurance#National%20Flood%20Insurance%20Program`

[2] "National Flood Insurance Program: Selected Issues and Legislation in the 115th Congress", Diane P. Horn, Congressional Research Service Report, `crs.gov`

[3] "Hurricane Sandy Facts, Damage and Economic Impact," Kimberly Amadeo, https://www.thebalance.com/hurricane-sandy-damage-facts-3305501

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[5] "Policy and Claim Statistics" by FEMA, https://www.fema.gov/policy-claim-statistics-flood-insurance