

7 The Ontology of Relations

In chapter 6, we introduced the basic categories of BFO: *continuant* and BFO: *occurrent*, and their respective subtypes. In this chapter we will introduce the central ontological relations in BFO, and provide examples of how definitions for such relations are to be formulated.

BFO Relations

As has been noted in earlier chapters, providing definitions of the terms representing universals and defined classes alone is normally not sufficient to capture all of the important scientific information about a given domain. The relations that obtain between and among them need to be defined also, and we further need to provide axioms, for example, representing how specific categories are related to each other within the ontology. Definitions and axioms can then be combined together for purposes of reasoning.

Many of our principles of ontology good practice—for example, the principle of single inheritance and of Aristotelian definitions—draw on the central architectural role of the *is_a* relation in ontology construction. Some relations, such as identity and parthood, are primitive; they are so basic to our understanding of reality that it is impossible to conceive of there being anything more basic in terms of which to define them. Here axioms are indispensable if the terms in question are to play more than a trivial role in reasoning about entities in the domain. BFO also includes other relations, such as instantiation, identity, parthood (including both *part_of* and *has_part*), dependence (including both *generic* and *specific dependence*), *located_in*, and a series of further relations pertaining to space and time.¹

As we discussed in chapter 1, there are three basic kinds of relations that need to be taken into account when designing an ontology and defining the relations that it will represent. These are

- relations holding between one universal and another (the relations represented in the ontology itself);
- relations holding between a particular and a universal—for example, the relationship of *instantiation*, which comes into play where the ontology is applied to some specific portion of reality, for instance in annotating clinical data pertaining to a specific group of patients;
- relations holding between one particular and another—for example, when asserting that Mary's leg is a continuant part of Mary.

Having these three kinds of relations at our disposal allows us to use an ontology in conjunction with information about particulars in the world to reason about those particulars. A paradigm case of this in biomedicine would be a software tool that could allow domain-specific ontologies of biology and medicine to help in guiding decisions as to diagnosis and treatment of specific patients.

It is also important when defining relations to specify what categories of objects form the domain and range of the relation (or in other words what are valid expressions to figure as its left- and right-hand terms, respectively). For example, the relation *instantiates* always holds between a particular and a universal, as in Fido *instantiates* Labrador Retriever. The parthood relation, on the other hand, comes in two forms, the first of which holds between two particulars, the second between two universals. Because these relations behave differently according to whether they obtain between continuants or occurrents, BFO distinguishes further between continuant parthood and occurrent parthood, as we shall explain in more detail.

Relations: Formal Properties and Conventions

First we need to introduce some conventions that will enable us to define the relations between and among universals and particulars:

- the upper-case variables C, D, . . . will be used to represent continuant universals
- the lower-case variables c, d, . . . will be used to represent continuant particulars
- the upper-case variables P, Q, . . . will be used to represent occurrent universals
- the lower-case variables p, q, . . . will be used to represent occurrent particulars
- a relation that holds between two universals will be represented in *italics*, as in: C *is_a* D; P *is_a* Q
- a relation holding between a particular and a universal will be represented in bold-face, as in: c **instance_of** C; p **instance_of** P

Box 7.1**Three Major Families of Relations**

1. universal-universal

continuant examplescancer *is_a* diseasedisease *is_a* dispositionnosocomial infection *is_a* infection object *is_a* independent continuant*occurrent* examplesmeiosis *is_a* cell divisionactive transport *is_a* membrane transportbreathing air *is_a* respirationprocess *is_a* occurrent

2. particular-universal

continuant examplesthis cell **instance_of** cellthis red (here, of this ball) **instance_of** redthis myelomeningocele (here, in this girl) **instance_of** myelomeningocele*occurrent* examplesthis waltz (being danced here, in Palermo) **instance_of** waltzthis process of tanning **instance_of** slowing the putrefaction of skin

3. particular-particular

continuant examplesthis atom of hydrogen **continuant_part_of** this water moleculethis portion of helium gas **continuant_part_of** the sunthis ion channel **continuant_part_of** this cell membrane*occurrent* examplesthis rupturing of ovarian follicle **occurrent_part_of** this process of ovulation here in this fawnthis process of gamma-glutamylcysteine synthesis **occurrent_part_of** this process of glutathione synthesis

- a relation obtaining between two particulars will also be written in bold type, as in:
c **continuant_part_of** c at t; or: p **occurrent_part_of** q

The three families of relations and the conventions for representing them are summarized in box 7.1.

Consider the case of Fido and the parthood relation obtaining between Fido (c) and his tail (d). The information communicated by “Fido’s tail is a part of the dog Fido” could be represented formally using the conventions just described as follows:

- d **continuant_part_of** c
- d **instance_of** *tail*
- c **instance_of** *dog*

Note that Fido might at some point in time lose his tail, and we will need to address this temporal feature of parthood for continuants. Note, too, that our discussion of Fido here is an example provided only for purposes of illustration. BFO does not assume that *dog* is a universal—since BFO is a domain-neutral ontology that leaves it to biologists to construct domain-specific ontologies in its terms. As already mentioned, an influential school of thought in biology holds that species such as *dog* are more properly to be conceived as evolving dynamic populations of organisms,² and a view along these lines could be formulated using the BFO category *object aggregate*. Even then, however, the formulation of such a view would still need to utilize terms designating universals (such as *organism*, *sexual reproduction*, *population*, and so forth) to capture what is involved in being a member of a species population, and for these terms the more traditional relation of instantiation between particulars and universals would still be required.

Primitive Instance-Level Relations

We have noted that the categories of any ontology should represent universals in reality. Yet, we will not be able to define what it means for one universal to stand in some relation to another universal—for example, in some parthood relation—without consideration of the underlying relations among instantiations of those universals on the side of particulars. We will show in what follows how universal-universal relations are to be defined in terms of previously accepted primitive relations of the particular-particular and particular-universal sort. A key part of the strategy for understanding universal-universal relations will be to interpret them as being true only if certain other things are true of their instances. Thus we will understand

- phototransduction *occurrent_part_of* visual perception,
- portion of carbon *continuant_part_of* portion of glutathione, and
- phospholipid bilayer *continuant_part_of* mitochondrion

to be true *only if* each particular instance of the universals *phototransduction*, *portion of carbon*, and *phospholipid bilayer* stands in an instance-level part relation to a corresponding instance of the universals *visual perception*, *portion of glutathione*, and *mitochondrion*, respectively.

Some of our definitions of relations will involve reference to spatial or temporal regions. Above all, assertions of relations involving continuant particulars will need to include a reference to time. This is so because continuants may change their relations to other entities from one time to the next. We will use the following:

- variables r, r', \dots to represent three-dimensional spatial regions
- variables t, t', \dots to represent instants of time

We can now identify the following primitive instance-level relations and their definitions:

- **c instance_of C at t.** This is a primitive relation obtaining at a specific time between a continuant instance *c* and a continuant universal *C* when the former instantiates the latter at that time. For example: Fido **instance_of** *Labrador Retriever* **at** the present.
- **p instance_of P.** This is a primitive relation obtaining between a process instance and a process universal that it instantiates. (This relation holds independently of time.) For example: John's life **instance_of** *human life*.
- **c continuant_part_of d at t.** This is a primitive relation obtaining between two continuant instances and a time at which the one is part of the other. For example: this cell nucleus **continuant_part_of** this cell **at** the present.
- **p occurrent_part_of q.** This is a primitive relation of parthood, holding independently of time, between process instances when one is a subprocess of the other). For example: this tumor's growth **occurrent_part_of** Mary's life.
- **r continuant_part_of r'.** This is a primitive relation of parthood, holding independently of time, between spatial regions (one a subregion of the other). For example: the spatial region occupied by the surface of the Northern hemisphere **continuant_part_of** the spatial region occupied by the whole surface of the earth.
- **c inheres_in d at t.** This is a primitive relation obtaining between a specifically dependent continuant and an independent continuant at a particular time. For example: the shape of John's body **inheres_in** John **at** July 26, 2006.
- **c located_in r at t.** This is a primitive relation between a continuant instance, a spatial region that it occupies, and a time. For example: John **located_in** the region occupied by the dining room **at** dinnertime.
- **r adjacent_to r'.** This is a primitive relation of proximity between two spatial regions. For example: Northern hemisphere **adjacent_to** Southern hemisphere.
- **c derives_from d.** This is a primitive relation between two distinct material continuants when one succeeds the other across a temporal divide. For example: this blastocyst **derives_from** this ovum.
- **p has_participant c.** This is a primitive relation between a process and a continuant. For example: John's life **has_participant** John.

While these instance-level relations cannot be defined, their meanings can be elucidated informally through the provision of examples, and formally by adding axioms.

For example, having accepted the instance-level relation of continuant parthood (**c continuant_part_of d at t**), it is possible to specify its logical properties by explicitly adopting axioms such as

if **c continuant_part_of d at t**, then **c** and **d exist at t**, and

if **c instance_of continuant** then there is no **d** such that **c occurrent_part_of d at t**,
and so on.

Universal-Universal Relations in BFO

In the previous section, we considered some of the primitive instance-level relations that scientists—with or without realizing it—draw on in their work. In order now to define what it means for one universal to stand in relation to another we need to consider the particular instantiations of those universals. This should not be seen as standing in conflict with our view that ontologies are representational artifacts whose representations are intended to designate some combination of *universals* and the relations between them. For in defining the relations between universals, the reference to particulars is entirely general—we will be referring in effect to *all* particulars instantiating given universals. This corresponds to the way in which science as a whole concerns itself with *generalities* when formulating its types, laws, and relations. It tests its hypotheses concerning such generalities by examining particulars in experiments; but references to specific particulars do not play a role when representing scientific laws.

In 2005 Smith, together with a group of influential researchers in the biomedical ontology field,³ compiled a list of ten basic universal-universal relations under the categories of

- foundational relations
- spatial relations
- temporal relations
- participation relations

proposing them as a common basis for the further development of biomedical ontologies in separate disciplinary communities. These relations have to a large degree been reused in the OBO Foundry and in a number of related ontologies (see box 7.2), while at the same time the list has been expanded with further relations also recommended for common use.⁴ Work is also ongoing in the context of the formulation of the OWL version of BFO 2.0 to create a complete system of formal definitions of all the relations between BFO categories.

Box 7.2

Core Relations in BFO

Foundational Relations

1. *is_a* (is a subtype of)
 - *portion of deoxyribonucleic acid (DNA) is_a portion of nucleic acid*
 - *photosynthesis is_a physiological process*
2. *continuant_part_of*
 - *cell nucleus continuant_part_of cell*
 - *heart continuant_part_of cardiovascular system*
3. *occurent_part_of*
 - *neurotransmitter release part_of synaptic transmission*
 - *gastrulation part_of animal development*

Spatial Relations

4. *located_in*
 - *intron located_in gene*
 - *chlorophyll located_in thylakoid*
5. *adjacent_to*
 - *Golgi apparatus adjacent_to endoplasmic reticulum*
 - *periplasm adjacent_to plasma membrane*

Temporal Relations

6. *derives_from*
 - *mammal derives_from gamete*
 - *triple oxygen molecule derives_from oxygen molecule*
7. *preceded_by*
 - *translation preceded_by transcription*
 - *digestion preceded_by ingestion*

Participation Relations

8. *has_participant*
 - *death has_participant organism*
 - *breathing has_participant thorax*

In the rest of this chapter, we will examine these relations as well as provide definitions (where possible) and examples of each. We shall conclude by providing some examples of axioms illustrating how these relations are treated formally within the larger BFO framework.

Foundational Relation: *is_a*

The foundational relation *is_a* has been discussed at length already. Examples include

- *myelin is_a lipoprotein*
- *beer is_a alcoholic beverage*
- *eukaryotic cell is_a cell*
- *site is_a independent continuant*

for *continuant* entities, and

- *gonad development is_a organogenesis*
- *binge drinking is_a drinking*
- *intracellular signaling cascade is_a signal transduction*

for *occurent* entities.

We define the *is_a* relations in terms of the primitive relation **instance_of** introduced previously. For occurrents this reads as follows:

A is_a B = def. A and B are universals and for all x (if x **instance_of** A then x **instance_of** B)

The corresponding definition for continuants is created by using the temporally qualified relation **instance_of at t**, as in the definition of *continuant_part_of* on p. 139.

Thus *diploid cell is_a cell* means: for any particular continuant c, c **instance_of diploid cell at t** implies c **instance_of cell at t**. *Lung cancer development is_a cancer development* means: for any particular occurrent p, p **instance_of lung cancer development** implies p **instance_of cancer development**.

Foundational Relations: *continuant_part_of* and *occurent_part_of*

BFO distinguishes two foundational parthood relations namely *continuant_part_of* and *occurent_part_of*. Examples include

- *axon continuant_part_of neuron*
- *cell nucleus continuant_part_of cell*
- *neuronal death occurrent_part_of dementia*
- *bird song occurrent_part_of avian mating behavior*

These relations can be defined in terms of the instance-level parthood relations as follows:

$C \text{ *continuant_part_of* } D$ = def. for every particular continuant c and every time t , if c **instance_of** C **at** t , then there is some d such that d **instance_of** D **at** t and c is a **continuant_part_of** d **at** t

So, for example, to say that *cell nucleus* *continuant_part_of* *cell* is to say that for every particular cell nucleus and every time in which that cell nucleus exists, there is some instance of *cell* which this cell nucleus is an instance-level **continuant_part_of** at that time. Notice that this does not require that every instance of *cell* have some instance of nucleus as part.

For occurrent universals we have

$P \text{ *occurrent_part_of* } Q$ = def. for every particular occurrent p , if p **instance_of** P , then there is some particular occurrent q such that q **instance_of** Q and p **occurrent_part_of** q

Thus, for example, *human neurulation* *occurrent_part_of* *human fetal development* just in case every particular process of human neurulation is an instance-level **occurrent_part_of** some particular process of human fetal development.

Spatial and Temporal Relations

One important influence on the development of BFO was the Region Connection Calculus (RCC), a simple framework to support qualitative reasoning about spatial relations⁵ and currently integrated into the GeoSPARQL standard for representation and querying of geospatial linked data for the Semantic Web.⁶ The Allen Interval Algebra is an analogous and similarly influential framework for reasoning about temporal relations.⁷

In what follows we provide sample definitions of spatial and temporal relations in BFO, which are designed to serve as a basis for defining all the relations defined in RCC and in the Allen calculus—and further analogous spatiotemporal relations according to need. A particularly ambitious set of such relations for spatial adjacency and connectedness is defined in the BFO-conformant Foundational Model of Anatomy (FMA) Ontology.⁸ We provide here a description of the treatment of location and adjacency in conformance with BFO, which is formulated in terms of the relations between the spatial regions that independent continuants occupy. The strategy employed here can then be generalized to spatial relations of other sorts.

Both *located_in* and *adjacent_to* connect one spatially extended entity to another in terms of the relations between the spatial regions (r, r', \dots) that they occupy. Examples of *located_in* include

- *ribosome located_in cytoplasm*
- *Golgi body located_in cell*

Defining the universals-level relation *located_in* comes in two steps. First, we introduce the instance-level primitive relation **c located_in r at t** (obtaining between a continuant instance, a spatial region, and a time). Second, we define an instance-level relation (obtaining between two continuant particulars and a time) **c located_in d at t**, and then use this relation to define the *located_in* relation at the level of universals.

c located_in d at t = def. there are two spatial regions *r* and *r'* such that the particular continuant *c* is **located_in r at t** and the particular continuant *d* is **located_in r' at t**, and the region *r* is a **continuant_part_of** the region *r'*

For example, John's kidney is **located_in** John's torso at present because the region in which John's kidney is **located** is a **continuant_part_of** the region in which his torso is located.

Given this definition, we can now define

C located_in D = def. for every particular continuant *c* and every time *t*, if **c instance_of C at t**, then there is some *d* such that **d instance_of D at t** and **c located_in d at t**

Thus, *kidney located_in torso* means that, for every (instance of) kidney and for every time *t* at which that kidney exists, there is some instance of torso at that time such that the kidney is **located_in** the torso at that time.

As we have already seen in our discussion, for example, of bacteria, while all continuant parts of spatially extended entities are located in those entities not all continuant entities located in the interiors of spatially extended entities are parts thereof. Note that (for example, because of kidney transplants) great care must be taken when incorporating assertions such as *kidney located_in torso* into an ontology. In the FMA, the problem is solved by treating assertions of this sort as holding, not of the actual human anatomy instantiated by you or me, but rather of what is called *canonical* human anatomy, defined as the arrangement and structure of body parts that is generated by the coordinated expression of the structural genes of the human organism.⁹

Spatial Relation: *adjacent_to*

The relation *adjacent_to* is a relation of proximity between disjoint continuants. Examples include the following:

- *nuclear membrane adjacent_to cytoplasm*
- *seminal vesicle adjacent_to urinary bladder*
- *ovary adjacent_to parietal pelvic peritoneum*

This relation can now be defined in similar pattern:

$C \text{ adjacent_to } D = \text{def. for every particular continuant } c \text{ and for every time } t, \text{ if } c \text{ instance_of } C \text{ at } t, \text{ then there is some } d \text{ such that } d \text{ instance_of } D \text{ at } t \text{ and } c \text{ adjacent_to } d \text{ at } t$

Thus, *liver adjacent_to falciform ligament* means that, for every instance of *liver* and every time t , if the liver exists at t , then there exists an instance of *falciform ligament* at t , and the liver is **adjacent_to** the falciform ligament **at** t . **adjacent_to** for material entities can itself be defined in terms of the relation of adjacency between the regions they occupy as defined in RCC.

Temporal Relation: *derives_from*

The temporal relation *derives_from* is used to assert that each instance of one continuant universal is derived from some instance of a second universal. Different *derives_from* relations have been proposed by biologists working in different disciplines. The relation we consider here interprets the relation to hold in those cases where a biologically significant portion of the matter contained in the earlier instance is inherited by the later instance. Examples include the following:

- *plasma cell derives_from B lymphocyte*
- *portion of tyrosine derives_from portion of henylalanine*

The underlying instance-level **derives_from** relation in cases of this sort can be understood as meaning that c is such that in the first moment of its existence it occupies a spatial region that substantially overlaps the spatial region occupied by d in the last moment of its existence. We can then define:

$C \text{ derives_from } D = \text{def. for every particular continuant } c \text{ and every time } t, \text{ if } c \text{ instance_of } C \text{ at } t, \text{ then there is some } d \text{ and some earlier time } t' \text{ such that } d \text{ instance_of } D \text{ at } t', \text{ and } c \text{ derives_from } d$

We can think of the relation so defined as *immediate* derivation, and define from there different sorts of *mediated* derivation (so that we could infer, for example, from $C \text{ derives_from } D$ and $D \text{ derives_from } E$ that $C \text{ mediately_derives_from } E$).

Temporal Relation: *preceded_by*

The temporal relation *preceded_by* is used in assertions such as

- *translation preceded_by transcription*
- *aging preceded_by development*
- *neurulation preceded_by gastrulation*

The underlying instance-level **preceded_by** relation is to be understood in the obvious way (and in conformity with the Allen calculus) as meaning that the temporal region occupied by the process *p* is later than the temporal region occupied by the process *q*.

Preceded_by as a relation between occurrent universals can then be defined as follows:

P preceded_by Q = def. for every particular occurrent *p*, if *p* **instance_of** *P*, then there is some *q*, such that *q* **instance_of** *Q* and *p* **preceded_by** *q*

Participation Relation: *has_participant*

The relation *has_participant* holds between a process and a continuant entity when the latter participates in or is involved in the former. Examples include the following:

- *cell transport has_participant cell*
- *translation has_participant portion of amino acid*
- *cell division has_participant chromosome*

Thus, every instance of *cell transport* (occurrent) has some instance of *cell* (continuant) as participant, every instance of *translation* (occurrent) has some instance of *portion of amino acid* (continuant) as participant, and so on. We can accordingly define the following:

P has_participant C = def. for every particular occurrent *p*, if *p* **instance_of** *P*, then there is some *c* and some time *t* such that *c* **instance_of** *C* at *t* and *p* **has_participant** *c* at *t*

Some Further Top-Level Relations

We have now described the primitive-level instance relations and universal-universal relations that are recognized by BFO, providing both definitions (where possible) and examples. We now consider some additional relations designed for use in specific domains and proposed for inclusion in the Relation Ontology (RO).¹⁰

proper_continuant_part_of and *proper_occurent_part_of*

To speak of a “proper part of” something at the instance level is to speak of something that is a part of but not identical with something else. With respect to continuant universals, an example is this: human uterus *proper_continuant_part_of* human. We define as follows:

C proper_continuant_part_of D = def. for every particular continuant *c* and every time *t*, if *c* **instance_of** *C* at *t*, then there is some *d* such that *d* **instance_of** *D* at *t*, and *c* **proper_continuant_part_of** *d* at *t*

With respect to occurrents:

$P \text{ proper_occurrent_part_of } Q = \text{def. for every particular occurrent } p, \text{ if } p \text{ instance_of } P,$
 then there is some particular q such that $q \text{ instance_of } Q, p \text{ proper_occurrent_part_of } q$

Examples are

- *swallowing proper_occurrent_part_of eating*
- *anaphase proper_occurrent_part_of mitosis*

Has_continuant_part and integral_continuant_part; has_occurrent_part and integral_occurrent_part

To assert of two universals that the first has the second as part is to assert that the former has instances that are wholes, and that each such whole has an instance of the latter as part. Thus

$C \text{ has_continuant_part } D = \text{def. for all particular continuants } c, \text{ and all times } t, \text{ if } c \text{ instance_of } C \text{ at } t \text{ then there is some } D, \text{ such that } d \text{ instance_of } D \text{ at } t, \text{ and } d \text{ continuant_part_of } c \text{ at } t$

$P \text{ has_occurrent_part } Q = \text{def. for all particular occurrents } p, \text{ if } p \text{ instance_of } P \text{ then there is some } q, \text{ such that } q \text{ instance_of } Q, \text{ and } q \text{ occurrent_part_of } p$

As we shall see, the relation between *has_part* and *part_of* is logically not quite trivial. One can speak of the relation of “integral parthood” holding between two universals A and B when $A \text{ part_of } B$ and $B \text{ has_part } A$. Thus for the continuant case:

$C \text{ integral_continuant_part_of } D = \text{def. } C \text{ continuant_part_of } D \text{ and } D \text{ has_continuant_part } C$

and similarly for occurrents

Examples are

- *brain integral_continuant_part_of mammal*
- *systole integral_occurrent_part_of cardiac cycle*

Relations and Definitions of Categories

It is important to note that well-defined formal relations can be used to help more precisely define the nature of the universals that they relate. For example, a universal can be asserted to be an entity that is not an *instance_of* anything, while a particular can be asserted to be an entity that is an **instance_of** other entities, but does not itself have

any entities standing in the **instance_of** relation to it. Axioms of these sorts are provided in a section to follow. Similarly, an independent continuant can be asserted to be an entity to which other entities stand in the **inheres_in** relation, but which does not itself **inhere_in** any other entity. Such relational assertions can be used in definitions and thereby help the ontology to support formal reasoning.

The All-Some Rule

Apart from *is_a* all the relations defined as obtaining between universals adhere to what we call the all-some rule. If a universal A bears such a relation to a universal B, then **all** relevant instances of A must bear the relevant instance-level relation to **some** instance of B. This point can be captured simply by saying that relations obtaining among universals should not admit exceptions.

Consider the definition of *continuant_part_of*:

C *continuant_part_of* *D* = def. for every particular continuant *c* and every time *t*, if *c* **instance_of** *C* at *t*, then there is some *d* such that *d* **instance_of** *C* at *t* and *c* is a **continuant_part_of** *d* at *t*

What this says is that for one universal to be *continuant_part_of* another, *all* instances of the one must at the pertinent times be **continuant_part_of** *some* instance of the other. Thus to say that *human heart* *continuant_part_of* *human circulatory system* is to say that for every particular human heart, at every time at which the heart exists there is some instance of human circulatory system which that human heart is part of at that time. Note that it is not implied that it is the same instance of human circulatory system that is involved from one time to the next.

An analogous feature holds of the *is_a* relation in that here, too, we are interested in what holds universally. *Prokaryotic cell* *is_a* *cell* is from this point of view in perfect order as it stands. However, *cancer* *is_a* *terminal disease* fails to respect the test of universality, since not all instances of *cancer* are instances of *terminal disease*.

Inversion and Reciprocity

As will be clear from our discussion of “has part,” it is possible to ask in regard to any relation defined in an ontology whether there is another relation that goes, as it were, in the other direction—generally referred to as the *inverse* relation. The inverse of a relation *R* is defined as the relation that obtains between each pair of relata of *R* when taken in reverse order.

So, if *C* *is_a* *D*, the relation between *D* and *C* that goes in the opposite direction is the relation *has_subuniversal*, defined by

C has_subuniversal D = def. D is_a C,

as in

cell has_subuniversal prokaryotic cell

(again with suitable modifications when account is taken of defined classes as relata of the *is_a* relation).

However, for most of the relations that we have defined, it is not possible to define an inverse relation in this direct fashion. This makes it necessary to define what have been called *reciprocal* relations.

To see the problem consider the assertion

human testis continuant_part_of human,

which passes the all-some test. When we attempt to formulate the assertion in the opposite direction, however, the result

human has_continuant_part human testis

fails by the all-some test, because not all humans have testes. *Has_continuant_part* is not the inverse but rather what we call the reciprocal of *continuant_part_of*. Something similar holds for the *adjacent_to* relation, for while we have for adult humans

uterine artery adjacent_to urinary bladder

we do not have

urinary bladder adjacent_to uterine artery

since male humans do not have a uterine artery.

The relations *is_a* and *has_subuniversal* allow us to reason both upward and downward through the taxonomical backbone of an ontology, following the principle of inheritance. If *A has_subuniversal B* then everything that holds of every instance of *A* will hold also of every instance of *B*. Inferences of this sort are possible also by using other relations in an ontology, but as can be seen from the examples of parthood and adjacency, they must be used with care.

Some Examples of Axioms

Our purpose in this chapter has been to give only an outline of what a theory of relations for purposes of ontology construction within the BFO framework looks like. It is not to develop a full axiomatic theory. However, we here provide for purposes of illustration some examples of axioms for BFO, formulated in English and in first-order logic.¹¹

Entity

All *entities* exist at some *temporal region*.

$$\forall x (\text{Entity}(x) \rightarrow \exists t (\text{TemporalRegion}(t) \wedge \text{exists_at}(x, t)))$$

Material Entity

Every *material entity* has a *history*.

$$\forall x (\text{MaterialEntity}(x) \rightarrow \exists y \text{ has_history}(x, y))$$

Every *entity* that has a *material entity* as continuant part is a *material entity*.

$$\forall y \forall x \forall t ((\text{continuant_part_of}(x, y, t) \wedge \text{MaterialEntity}(x)) \rightarrow \text{MaterialEntity}(y))$$

Every *material entity* exists at some *temporal interval*.

$$\forall x (\text{MaterialEntity}(x) \rightarrow \exists t (1\text{DTemporalRegion}(t) \wedge \text{exists_at}(x, t)))$$

Occurrent

Every *occurrent* occupies some *spatiotemporal region*.

$$\forall x (\text{Occurrent}(x) \rightarrow \exists y (\text{SpatioTemporalRegion}(y) \wedge \text{occupies_spatiotemporal_region}(x, y)))$$

SpatioTemporalRegion

Every *spatiotemporal region* occupies *some temporal region*.

$$\forall x (\text{SpatioTemporalRegion}(x) \rightarrow \exists t (\text{TemporalRegion}(t) \wedge \text{occupies_temporal_region}(x, t)))$$

Every *spatiotemporal region* occupies itself.

$$\forall x (\text{SpatioTemporalRegion}(x) \rightarrow \text{occupies_spatiotemporal_region}(x, x))$$

TemporalRegion

All *temporal regions* are either *zero-* or *one-dimensional* (i.e., either instants or intervals).

$$\forall x (\text{TemporalRegion}(x) \leftrightarrow (1\text{DTemporalRegion}(x) \wedge 0\text{DTemporalRegion}(x)))$$

Every *temporal region* occupies itself.

$$\forall x (\text{TemporalRegion}(x) \rightarrow \text{occupies_temporal_region}(x, x))$$

Universal

Something is a universal if and only if it is instantiated by something.

$$\forall X (\text{Universal}(X) \leftrightarrow \exists y \text{ inst}(X, y))$$

Continuant_part_of

The part of relation for *continuants* is anti-symmetric.

$$\forall x \forall y \forall t ((\text{continuant_part_of}(x, y, t) \wedge \text{continuant_part_of}(y, x, t)) \rightarrow x = y)$$

The part of relation for *continuants* is transitive.

$$\forall x \forall y \forall z \forall t ((\text{continuant_part_of}(x, y, t) \wedge \text{continuant_part_of}(y, z, t)) \rightarrow \text{continuant_part_of}(x, z, t))$$

The part of relation for *continuants* is reflexive.

$$\forall x \forall t ((\text{Continuant}(x) \wedge \text{exists_at}(x, t)) \rightarrow \text{continuant_part_of}(x, x, t))$$

Weak supplementation: If *x* is a proper (continuant) part of *y*, then there is some (continuant) part of *y* that does not overlap *x*.

$$\forall x \forall y \forall t (\text{proper_continuant_part_of}(x, y, t) \rightarrow \exists z (\text{continuant_part_of}(z, y, t) \wedge \neg \text{continuant_overlap}(z, x, t)))$$

Unique product: If one *continuant* overlaps another *continuant* at some time, then there is a unique mereological product (intersection) of those *continuants* at that time.

$$\forall x \forall y \forall t (\text{continuant_overlap}(x, y, t) \rightarrow \exists z (\text{continuant_mereological_product}(z, x, y, t) \wedge \forall w (\text{continuant_mereological_product}(w, x, y, t) \rightarrow w = z)))$$

If some *continuant* is part of a *continuant* at some time, then both continuants exist at that time.

$$\forall x \forall y \forall t (\text{continuant_part_of}(x, y, t) \rightarrow (\text{exists_at}(x, t) \wedge \text{exists_at}(y, t)))$$

Occupies_spatial_region

Something can only occupy one *spatial region* at a time.

$$\forall x \forall r_1 \forall r_2 \forall t ((\text{occupies_spatial_region}(x, r_1, t) \wedge \text{occupies_spatial_region}(x, r_2, t) \rightarrow r_1 = r_2)$$

All entities that occupy a *spatial region* at a time exist at that time.

$$\forall x \forall r \forall t (\text{occupies_spatial_region}(x, r, t) \rightarrow \text{exists_at}(x, t))$$

Box 7.3**Properties of Instance-Level Relations in BFO**

<i>Relation</i>	<i>Transitive</i>	<i>Symmetric</i>	<i>Reflexive</i>	<i>Antisymmetric</i>
is_a	+	-	+	+
part_of	+	-	+	+
located_in	+	-	+	-
adjacent_to	-	-	-	-
derives_from	-	-	-	-
preceded_by	+	-	-	-
has_participant	-	-	-	-

Occupies_spatiotemporal_region

Something can occupy only one *spatiotemporal region*.

$$\forall x \forall r_1 \forall r_2 ((\text{occupies_spatiotemporal_region}(x, r_1) \wedge \text{occupies_spatiotemporal_region}(x, r_2)) \rightarrow r_1 = r_2)$$

Reflexivity, Symmetry, and Transitivity

We conclude by summarizing a number of well-understood properties of relations that should be taken into account when defining further relations within the BFO framework. (See Box 7.3.) Here A, B, . . . range over all entities, whether universals, defined classes or particulars.

- To say that a relation R is *reflexive* is to say that anything A that bears the relation R to something else, B, also bears that relation to itself. The relation “is as tall as” is reflexive, because when John is as tall as Jill, he also stands in this same relation to himself: John is as tall as John.
- To say that a relation R is *symmetric* is to say that if A stands in R to B then B also stands in R to A. The instance-level relation **adjacent_to** is symmetric, because if John is next to Mary, then Mary is also next to John. On the level of universals, however, *adjacent_to* is not symmetric.
- To say that a relation R is *transitive* is to say that if a thing A bears R to B, and if B bears R to C, then A also bears R to C. A simple example of a transitive relation is “is taller than.” If John is taller than Mary, and Mary is taller than Steve, then John is taller than Steve.
- To say that a relation R is *antisymmetric* is to say that if A bears R to B and B bears R to A, then A and B are identical.

Further Reading on Relations

Bennett, Brandon. V. Chaudhri, and N. Dinesh. "A Vocabulary of Topological and Containment Relations for a Practical Biological Ontology." In *Spatial Information Theory: Proceedings of COSIT 2013*, Lecture Notes in Computer Science, vol. 8116, ed. J. Stell, T. Tenbrink, and Z. Wood, 418–437. Scarborough, UK: Springer, 2014.

Bittner, Thomas, and Maureen Donnelly. "Logical Properties of Foundational Relations in Bio-ontologies." *Artificial Intelligence in Medicine* 39 (2007): 197–216.

Donnelly, Maureen, Thomas Bittner, and Cornelius Rosse. "A Formal Theory for Spatial Representation and Reasoning in Biomedical Ontologies." *Artificial Intelligence in Medicine* 36 (2006): 1–27.

Smith, Barry, Werner Ceusters, Bert Klagges, Jacob Köhler, Anand Kumar, Jane Lomax, Chris Mungall, Fabian Neuhaus, Alan L. Rector, and Cornelius Rosse. "Relations in Biomedical Ontologies." *Genome Biology* 6 (5) (2005). doi:10.1186/gb-2005-6-5-r46. Accessed September 25, 2014.