LU

September 28, 2022

Outer Products and LU

$$LU = \begin{bmatrix} | & & | \\ l_1 & \cdots & l_n \\ | & & | \end{bmatrix} \begin{bmatrix} \text{—} & u_1^* & \text{—} \\ & \cdots & \\ \text{—} & u_n^* & \text{—} \end{bmatrix} = l_1 u_1^* + \cdots + l_n u_n^* \qquad (1)$$

where remember the outer product of vectors gives a matrix:

$$\mathrm{ab}^* = \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix} \begin{bmatrix} b_1 & \cdots & b_m \end{bmatrix} = \begin{bmatrix} a_1 b_1 & a_1 b_2 & \cdots & a_1 b_m \\ a_2 b_1 & a_2 b_2 & \cdots & a_2 b_m \\ \cdots & \cdots & \cdots & \cdots \\ a_m b_1 & a_m b_2 & \cdots & a_m b_m \end{bmatrix} \tag{2}$$

What L_k do

Remember the L_k are the matrix operators that reduce the matrix A one row at a time. Suppose

$$A = \begin{bmatrix} 3 & 0 & -1 & 1 \\ 2 & -1 & 1 & 0 \\ -2 & 2 & 3 & 3 \\ 7 & 0 & 0 & 2 \end{bmatrix} \tag{3}$$

And then of course we choose to divide each row by $A_{1,1} = 3$.