You have a function `rand7()` that generates a random integer from 1 to 7. Use it to write a function `rand5()` that generates a random integer from 1 to 5.

`rand7()` returns each integer with equal probability. `rand5()` must also return each integer with equal probability.

Gotchas

Your first thought might be to simply take the result of `rand7()` and take a modulus:

```java
public int rand5() {
    return rand7() % 5 + 1;
}
```

However, this won't give an equal probability for each possible result. We can write out each possible result from `rand7()` (each of which is equally probable, per the problem statement) and see that some results for `rand5()` are more likely because they are caused by more results from `rand7()`:

`rand7()`	`rand5()`
1	2
2	3
3	4
4	5
5	1
6	2
7	3

So we see that there are two ways to get 2 and 3, but only one way to get 1, 4, or 5. This makes 2 and 3 twice as likely as the others.

The answer takes worst-case infinite time. However, we can get away with worst-case $O(1)$ space. **Does your answer have a non-constant space cost?** If you're using recursion (and your language doesn't have tail-call optimization), you're potentially incurring a worst-case infinite space cost in the call stack . But replacing your recursion with a loop avoids this.

Breakdown

`rand5()` must return each integer with equal probability, but we don't need to make any guarantees about its runtime...

In fact, the solution has a small possibility of *never* returning...

Solution

We simply "re-roll" whenever we get a number greater than 5.

```java
public int rand5() {
    int result = 7; // arbitrarily large
    while (result > 5) result = rand7();
    return result;
}
```

So each integer 1,2,3,4, or 5 has a probability $\frac{1}{7}$ of appearing at each roll.

Complexity

Worst-case $O(\infty)$ time (we might keep re-rolling forever) and $O(1)$ space.

Note that if we weren't worried about the potential space cost (nor the potential stack overflow) of recursion, we could use an arguably-more-readable recursive approach with $O(\infty)$ space cost:

```java
public int rand5Recursive() {
    int roll = rand7();
    return (roll <= 5) ? roll : rand5Recursive();
}
```

Bonus

This kind of math is generally outside the scope of a coding interview, but: if you know a bit of number theory you can *prove* that there exists no solution which is guaranteed to terminate. Hint: it follows from "the fundamental theory of arithmetic".

What We Learned

Making sure each possible result has *the same probability* is a big part of what makes this one tricky.

If you're ever struggling with the math to figure something like that out, don't be afraid to *just count*. As in, write out all the possible results from `rand7()`, and label each one with its final outcome for `rand5()`. Then count up how many ways there are to make each final outcome. If the counts aren't the same, the function isn't uniformly random.
