Why is the spin of an electron $\frac{1}{2}$?

$$L = \pm\sqrt{\ell(\ell+1)} \cdot \hbar$$

$$where$$
$$L\ is\ the\ quantised\ angular\ momentum$$
$$\ell\ is\ the\ orbital\ quantum\ number\ (= 0,1,2,\cdots)$$
$$\hbar = \frac{h}{2\pi}$$

$$Let\ m_s = \pm\sqrt{\ell(\ell+1)}$$

$Assume\ the\ smallest\ integer\ difference\ in\ angular\ momentum\ between\ the\ two\ states\ (demonstrated\ by\ the\ two\ points\ of\ accumulation\ in\ the\ Stern-Gerlach\ experiment)\ and\ thus\ let:$

$$m_{s+} = 1s$$
$$m_{s-} = -1s$$

$$\therefore\ \hbar \cdot ((m_{s+}) - (\ m_{s-})) = 1\hbar$$
$$\hbar \cdot \left((1s) - (-1s)\right) = 1\hbar$$
$$\hbar \cdot (2s) = 1\hbar$$
$$2s = 1$$
$$s = \frac{1}{2}$$

$$\therefore m_{s+} = \frac{1}{2}\ and\ m_{s-} = -\frac{1}{2}$$
$$\therefore L = \pm\frac{1}{2}\hbar\ \blacksquare$$