Why is the spin of an electron $\frac{1}{2}$?

$$L = \pm\sqrt{\ell(\ell+1)} \cdot \hbar$$

$where$

$$L\ is\ the\ quantised\ angular\ momentum$$
$$\ell\ is\ the\ orbital\ quantum\ number\ (=0,1,2,\cdots)$$
$$\hbar = \frac{h}{2\pi}$$

$$Let\ m_s = \pm\sqrt{\ell(\ell+1)}$$

$Assume\ the\ smallest\ integer\ difference\ in\ angular\ momentum\ between\ the\ two\ states\ (demonstrated\ by\ the\ two\ points\ of\ accumulation\ in\ the\ Stern-Gerlach\ experiment)\ and\ thus\ let:$

$$m_{s+} = 1s$$
$$m_{s-} = -1s$$

$$\therefore\ \hbar \cdot ((m_{s+}) - (\ m_{s-})) = 1\hbar$$
$$\hbar \cdot \big((1s) - (-1s)\big) = 1\hbar$$
$$\hbar \cdot (2s) = 1\hbar$$
$$2s = 1$$
$$s = \frac{1}{2}$$

$$\therefore m_{s+} = \frac{1}{2}\ and\ m_{s-} = -\frac{1}{2}$$
$$\therefore L = \pm\frac{1}{2}\hbar\ \blacksquare$$

References:

- *https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Book%3A_Quantum_States_of_Atoms_and_Molecules_(Zielinksi_et_al)/08%3A_The_Hydrogen_Atom/8.05%3A_Discovering_Electron_Spin (Contributed by David M. Hanson, Erica Harvey, Robert Sweeney, Theresa Julia Zielinski*

  Quantum States of Atoms and Molecules at Chemical Education Digital Library (ChemEd DL))

- *<http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/qangm.html>*